NMSM Homework Exercises

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{·1} Sampling random points within *d*-dimensional domains by hit and miss

I skipped the integration on the rectangle, solving only the disk case. The source code is in `A01b_disk_hit_miss.c`; I implemented the main part of the algorithm like this:

```
for (n = 0; n < n_plot; ++n) {
    hits = 0; // reset hit counter
    throws = (1 + n) * dn;
    for (i = 0; i < throws; ++i) {
        x = RngStream_RandU01(rngs);
        y = RngStream_RandU01(rngs);
        if (x * x + y * y < 1)
            ++hits;
    }
    mc_pi = 4 * (double)hits / throws;

    // write to file: throws + \t + relative error + \n
    fprintf(file, "%d\t", throws);
    fprintf(file, "%g\n", fabs(1 - mc_pi * M_1_PI));
}
```

The error as a function of the number of throws is shown in Fig. 1.1. It is comfortably under 1 % with around 25 000–30 000 iterations.

{·2} Sampling random numbers from a given distribution

The idea is to sample from the probability distribution $\rho_n(x) = cx^n$ in $[0, 1]$. First, using the normalization condition we can find out what c should be:

$$1 = \int_0^1 cx^n \, dx = \frac{c}{n+1} \implies c = n + 1. \tag{2.1}$$

Then, we find the expression of the associated cumulative density function:

$$F_n(x) = (n+1) \int_0^x y^n \, dy = x^{n+1}, \tag{2.2}$$

and invert it:

$$p = x^{n+1} \implies x = p^{1/(n+1)}. \tag{2.3}$$

So, inside the code `A02a_inversion_method.c` I sample a random `double` from a uniform distribution between 0 and 1 using `drand48()`, and I raise it to the power of $1/(n+1)$ to get x:

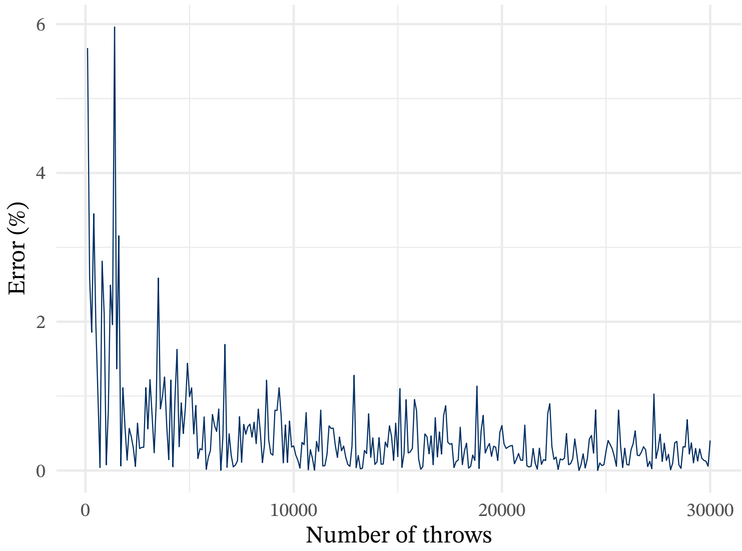


Figure 1.1: error in the Monte Carlo estimation of the area of a unit disk, as a function of the number of 'throws'.

```
double* x = malloc(n_smp * sizeof(*x));
for (int i = 0; i < n_smp; ++i)
    x[i] = pow(drand48(), 1.0 / (n + 1));
```

A histogram of 100 000 points sampled from ρ with $n = 3$ is displayed in Fig. 2.2.

Inside `A02b_inversion_method.c` I modified the code to sample from $\rho_2(x) = cx^2$ in $[0, 2]$. c is different this time, of course:

$$1 = \int_0^2 cx^2\,dx = \frac{8}{3}c \implies c = \frac{3}{8}. \tag{2.4}$$

The cumulative is then

$$F_2(x) = \frac{3}{8}\int_0^x y^2\,dy = \frac{x^3}{8} \implies x = 2p^{1/3}. \tag{2.5}$$

Once again, you can see the comparison between a 100 000-points histogram and the theoretical curve in Fig. 2.3.

{·3} Sampling via transformation of coordinates

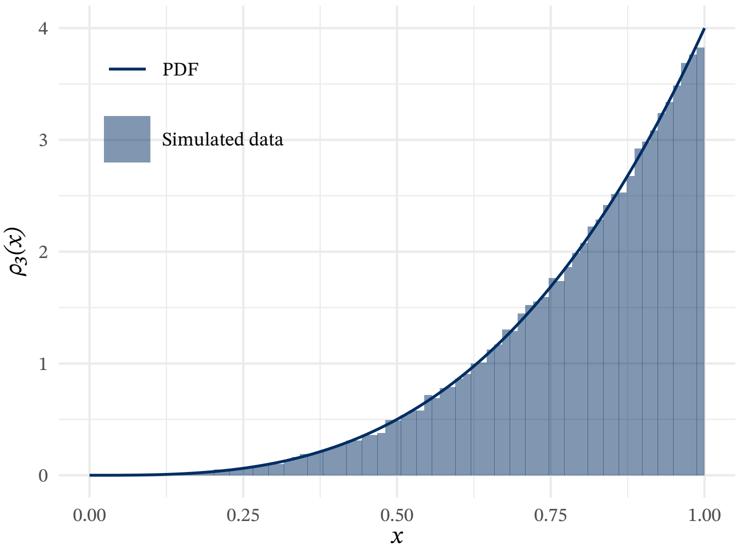


Figure 2.2: histogram of 100 000 points sampled from the probability distribution $4x^3$ in $[0, 1]$.

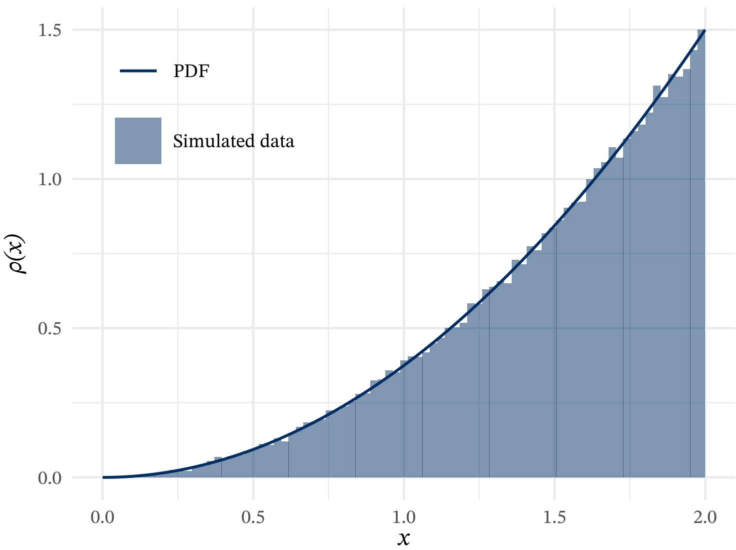
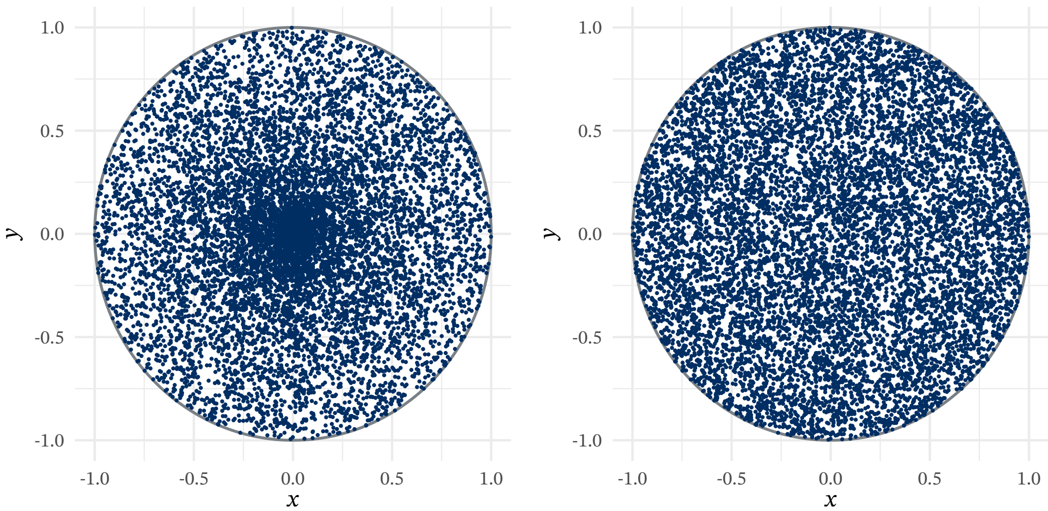


Figure 2.3: histogram of 100 000 points sampled from the probability distribution $3x^2/8$ in $[0, 2]$.



(a) (b)

Figure 3.4: 10 000 points sampled on the unit disk with the wrong coordinate transformation (a) and with the correct one (b).