A.6 Simulation of a 2D Ising model by the Metropolis algorithm

Program Write a program that simulates a 2D Ising model with periodic boundary conditions by using the Metropolis acceptance matric a_{ij} and the matrix Γ_{ij} based on the local spin-flip (Glauber) proposed move.

Simulation By assuming $k_B = 1$ and $J = 1$ simulate the 2D Ising model for different values of temperatures (at least 3, one below, one above, and one close to the critical temperature $T_c = \frac{2}{\ln(1+\sqrt{2})}$ and 3 different values of L (for instance 25,50 and 100).

Equilibration time, averages and fluctuations After having determined the equilibrium time and disregarding the samples for $t < \tau_{eq}$ estimate the ensemble averages of the magnetisation per spin, the energy per spin, and the corresponding fluctuations (specific heat and magnetic susceptibility).

Integrated correlation time and critical slowing-down Estimate the autocorrelation time of the magnetisation and the energy for the MC simulations proposed above and estimate the errors accordingly.

Finite-size analysis and estimates of the critical exponents By following the procedure sketched in section 1.9 and in the lecture perform a finite size scaling analysis of the specific heat, the magnetisation, and the magnetic susceptibility for the 2D Ising model simulated above. Provide a first estimate of the γ and β exponents.