Project 3

1)

a) The properties of MA don't depend on time: E[Yt] is constant over time and Cov(Yt, Yt-k) only depends on k, not time. So, it is stationary.

b)

$E[Y_t]=0$

$Var(Y_t)=Var(\varepsilon_t) + Var(w_1 * \varepsilon_{t-1}) + Var(w_2 * \varepsilon_{t-2}) = \sigma^2(1+w^2_1+w^2_2)$

c)

$E[Y_t\ Y_t] = \sigma^2(1+w^2_1+w^2_2)$

$E[Y_t\ Y_{t-1}] = \sigma^2(w_1+ w_1w_2)$

$E[Y_t\ Y_{t-2}] = \sigma^2 * w_2$

$E[Y_t\ Y_{t-k}] =0$, k=3,4,5.....

$Corr(Y_t , Y_{t+1}) = E[Y_t\ Y_{t-1}] / E[Y_t\ Y_t] = (w_1 + w_{1*}\ w_2)/ (1+ w^2_1 + w^2_2)$

$Corr(Y_t , Y_{t+2}) = E[Y_t\ Y_{t-2}] / E[Y_t\ Y_t] = w_2 /(1+ w^2_1 + w^2_2)$

$Corr(Y_t , Y_{t+k}) = 0$, k=3,4,5...

2)

a)

AR(1) : $Y_t = \mu+ \varphi * Y_{t-1} + \varepsilon_t$

$= 10 + 0.7 * Y_{t-1} + \varepsilon_t$

$Y_{20} = 12.5$

$Y_{21|20} = E[Y_{21} \mid Y_{20}] = E[10 + 0.7 * Y_{20} + \varepsilon_{21} \mid Y_{20}] = 10 + 0.7*E[Y_{20} \mid Y_{20}] + E[\varepsilon_{21} \mid Y_{20}]$

$=10 + 0.7*12.5 = 18.75$

$Y_{22|20} = E[Y_{22} \mid Y_{20}] = E[10 + 0.7 * Y_{21} + \varepsilon_{22} \mid Y_{20}] = 10 + 0.7* E[Y_{21} \mid Y_{20}]+ E[\varepsilon_{22} \mid Y_{20}]$

$=10+0.7*18.75 = 23.125$

$Y_{23|20} = E[Y_{23} \mid Y_{20}] = E[10 + 0.7 * Y_{22} + \varepsilon_{23} \mid Y_{20}] = 10 + 0.7* E[Y_{22} \mid Y_{20}]+ E[\varepsilon_{23} \mid Y_{20}]$

$=10+0.7*23.125 = 26.1875$

b) $Y_{21} - Y_{21|20}$ = 10 + 0.7* Y_{20}+ ε_{21} - 10 - 0.7* Y_{20} = ε_{21}

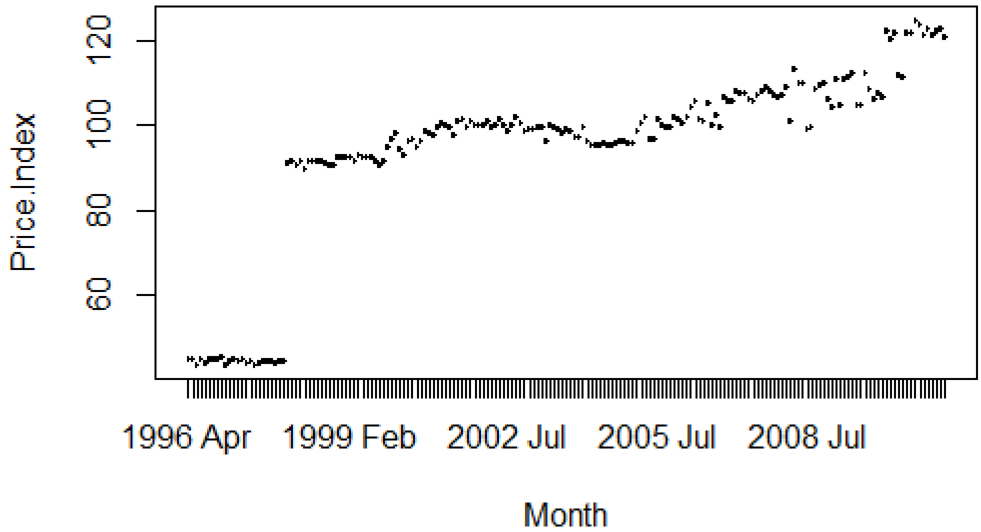
Y_{22} -$Y_{22|20}$ = 10 + 0.7* Y_{21}+ ε_{22} - 10 - 0.7* $Y_{21|20}$ = ε_{22} + 0.7* ε_{21}

Y_{23} -$Y_{23|20}$ = 10 + 0.7* Y_{22}+ ε_{23} - 10 - 0.7* $Y_{22|20}$ = ε_{23} + 0.7* ε_{22} + 0.49* ε_{22}

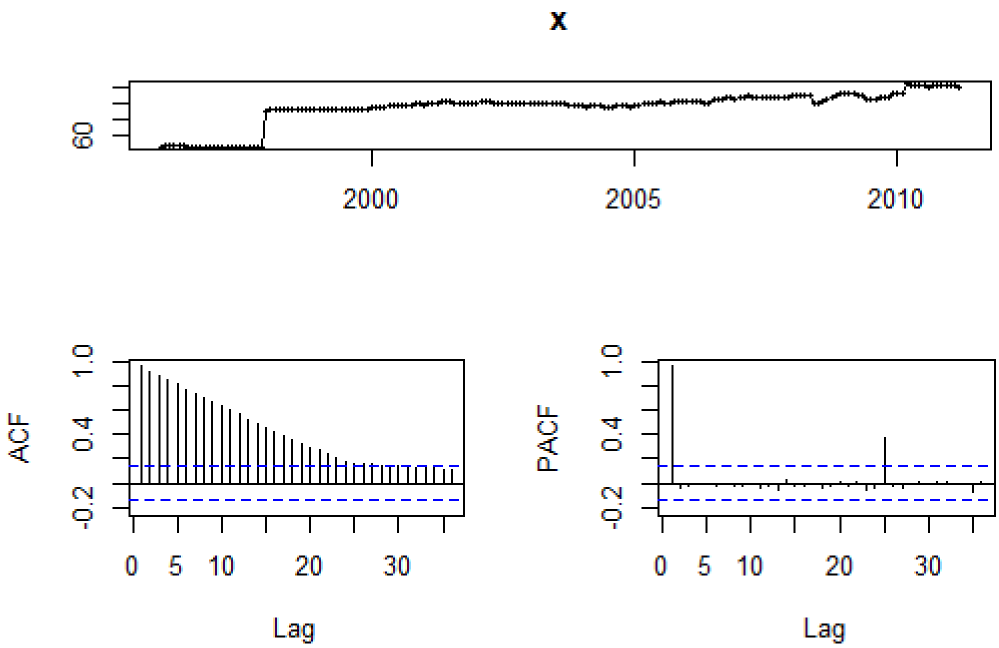
3)

a)

```
setwd("C:/Users/gülce/Desktop/3-2/IE360/assignment3")
res<-read.csv("UKPlasticPrices.csv",skip=1,header=T)
plot(res)
```



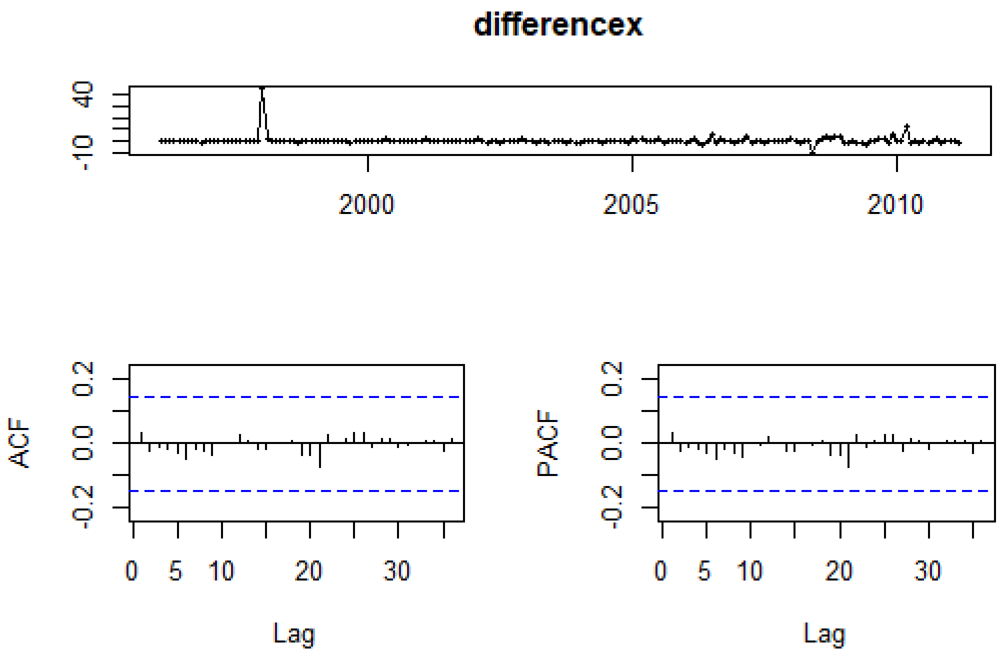

```
x<-ts(res$Price.Index,freq=12,start=1996)
library(forecast)
tsdisplay(x)
```



As it can be seen from the linear decrease in ACF, there is a trend in data which we should get rid of.

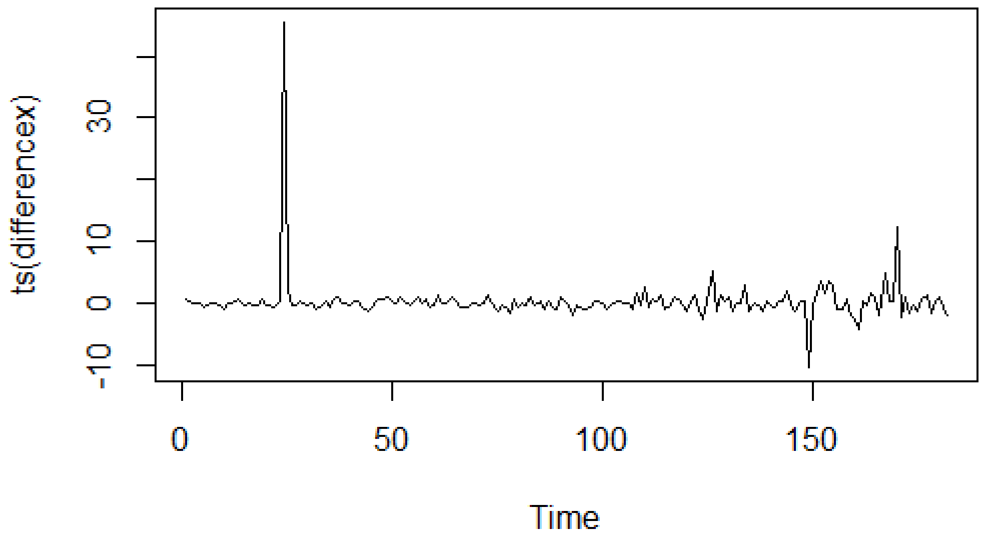
differencex<-diff(x)

tsdisplay(differencex)

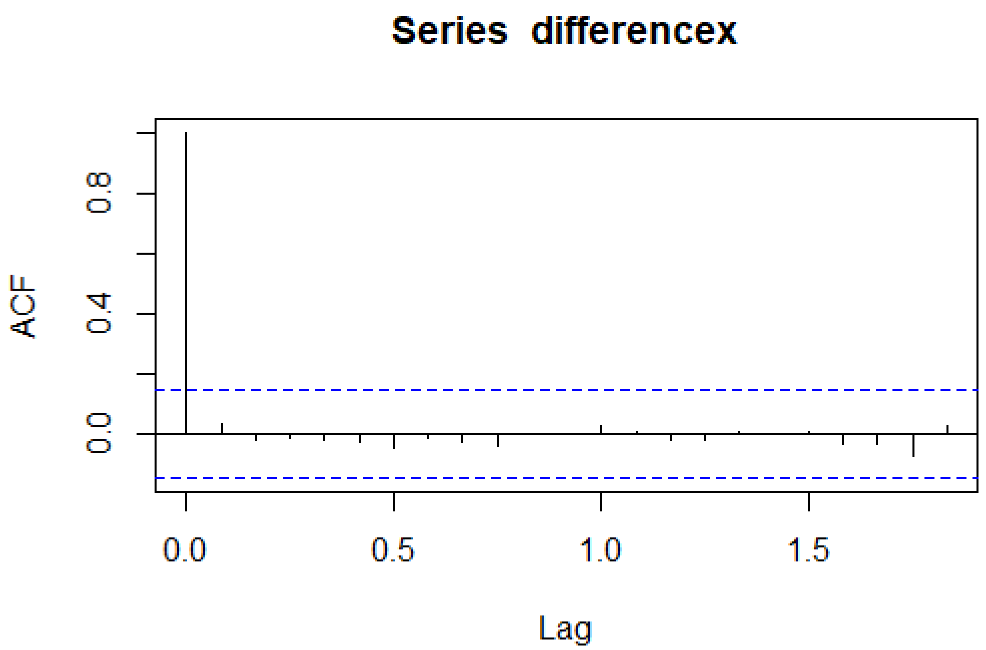


#Graphs seem ok.

plot(ts(differencex))

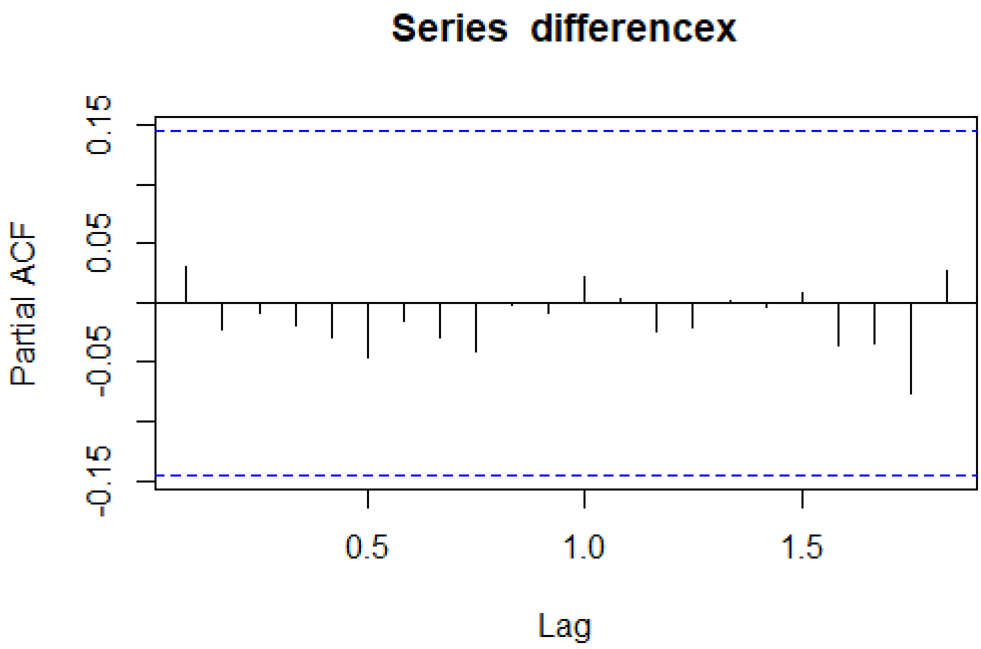


acf(differencex)



#As it can be seen from ACF, MA(1) is suitable for the data.

pacf(differencex)



#As it can be seen from PartialACF, AR seems to be zero.

fit1<-Arima(ts(x),c(0,1,1),include.drift=T)
fit1

```
 #Series: ts(x)
 #ARIMA(0,1,1) with drift
 #
 #Coefficients:
 #          ma1   drift
 #       0.0309  0.4189
 #s.e.   0.0755  0.2841
 #
 #sigma^2 estimated as 13.98:  log likelihood=-497.27
 #AIC=1000.53   AICc=1000.67   BIC=1010.14
```

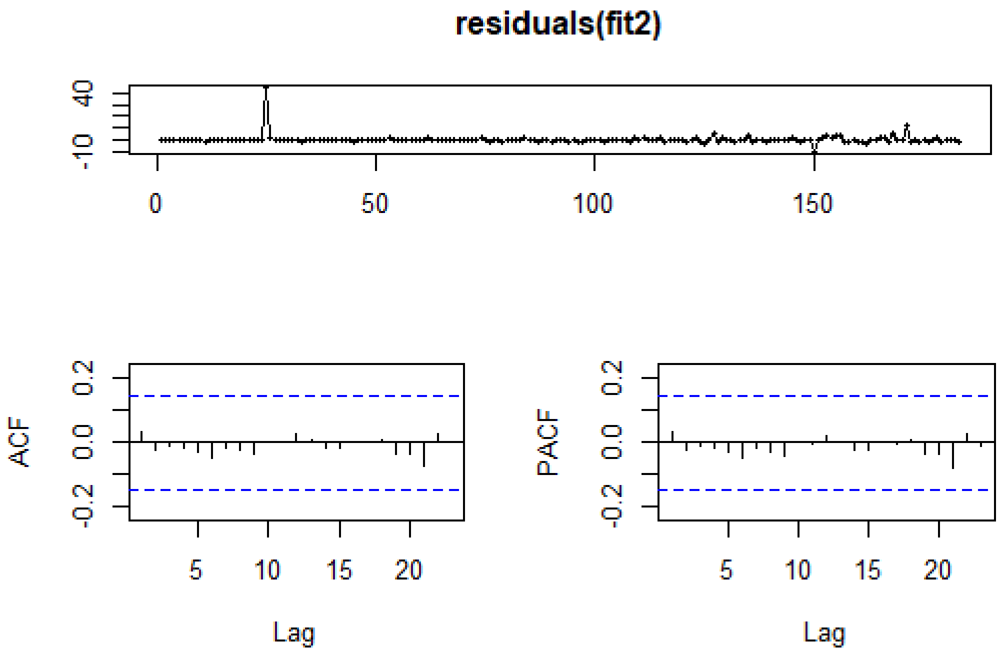
fit2<-Arima(ts(x),c(0,1,0),include.drift=T)

fit2

```
#Series: ts(x)
#ARIMA(0,1,0) with drift
#
#Coefficients:
#        drift
#       0.4192
#s.e.   0.2758
#
#sigma^2 estimated as 13.92:  log likelihood=-497.35
#AIC=998.7   AICc=998.77   BIC=1005.11
```

fit3<-Arima(ts(x),c(1,1,0),include.drift=T)

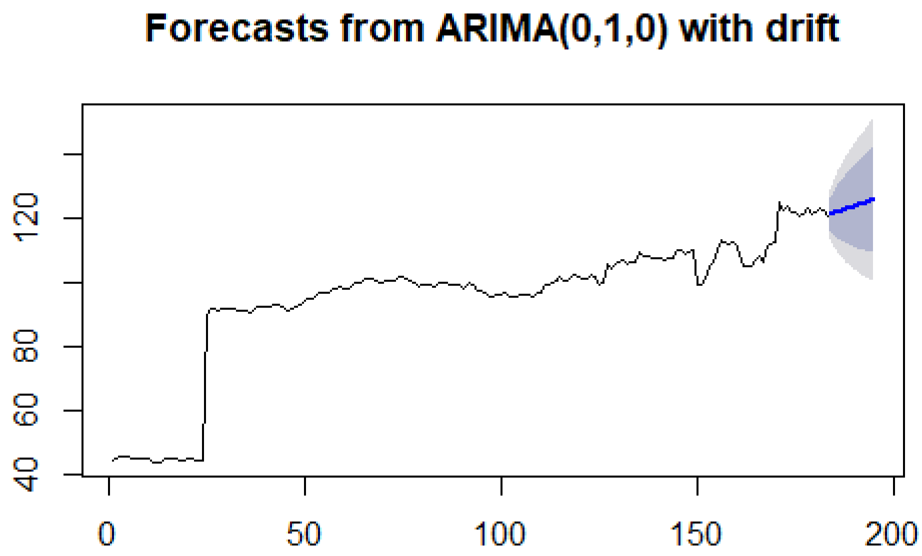
```
fit3

#Series: ts(x)
#ARIMA(1,1,0) with drift
#
#Coefficients:
#         ar1   drift
#      0.0296  0.4189
#s.e.  0.0740  0.2840
#
#sigma^2 estimated as 13.98:  log likelihood=-497.27
#AIC=1000.54   AICc=1000.68   BIC=1010.15

fit4<-Arima(ts(x),c(0,0,1),include.drift=T)
fit4

#Series: ts(x)
#ARIMA(0,0,1) with drift
#
#Coefficients:
#         ma1  intercept   drift
#      0.8207    65.1318  0.3179
#s.e.  0.0303     2.0209  0.0190
#
#sigma^2 estimated as 57.54:  log likelihood=-629.52
#AIC=1267.03   AICc=1267.26   BIC=1279.87

#It can be seen that taking the difference is important!

#Since the lowest AIC and BIC values are in fit2, fit2 is chosen.

tsdisplay(residuals(fit2))
```




```
#Graphs seem ok.


b)
forecastfit2<-(forecast(fit2, h=12))
```

```
forecastfit2
#     Point Forecast    Lo 80    Hi 80    Lo 95    Hi 95
#184        121.0192 116.2386 125.7999 113.7078 128.3306
#185        121.4385 114.6776 128.1994 111.0986 131.7784
#186        121.8577 113.5773 130.1381 109.1940 134.5214
#187        122.2769 112.7156 131.8383 107.6541 136.8997
#188        122.6962 112.0062 133.3861 106.3474 139.0450
#189        123.1154 111.4052 134.8256 105.2062 141.0246
#190        123.5346 110.8861 136.1831 104.1905 142.8788
#191        123.9538 110.4321 137.4756 103.2741 144.6336
#192        124.3731 110.0311 138.7151 102.4389 146.3073
#193        124.7923 109.6745 139.9101 101.6716 147.9130
#194        125.2115 109.3558 141.0672 100.9623 149.4607
#195        125.6308 109.0700 142.1915 100.3033 150.9582
```

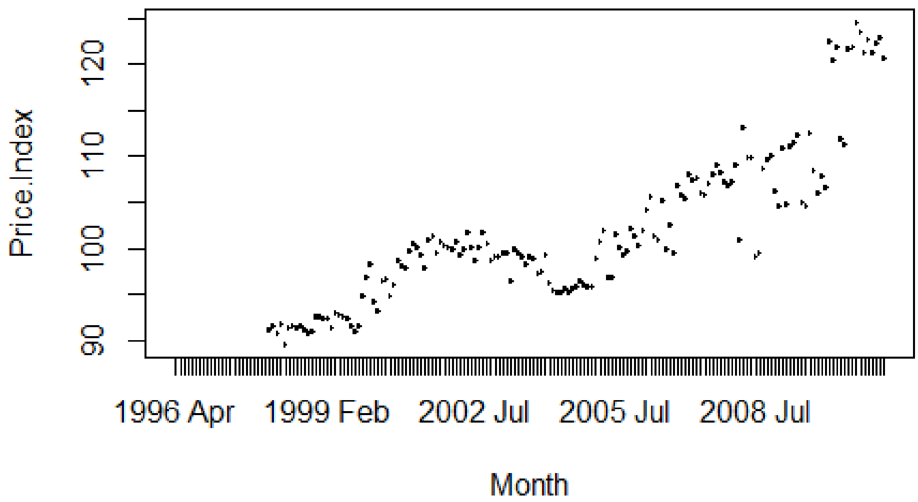
```
plot(forecastfit2)
```



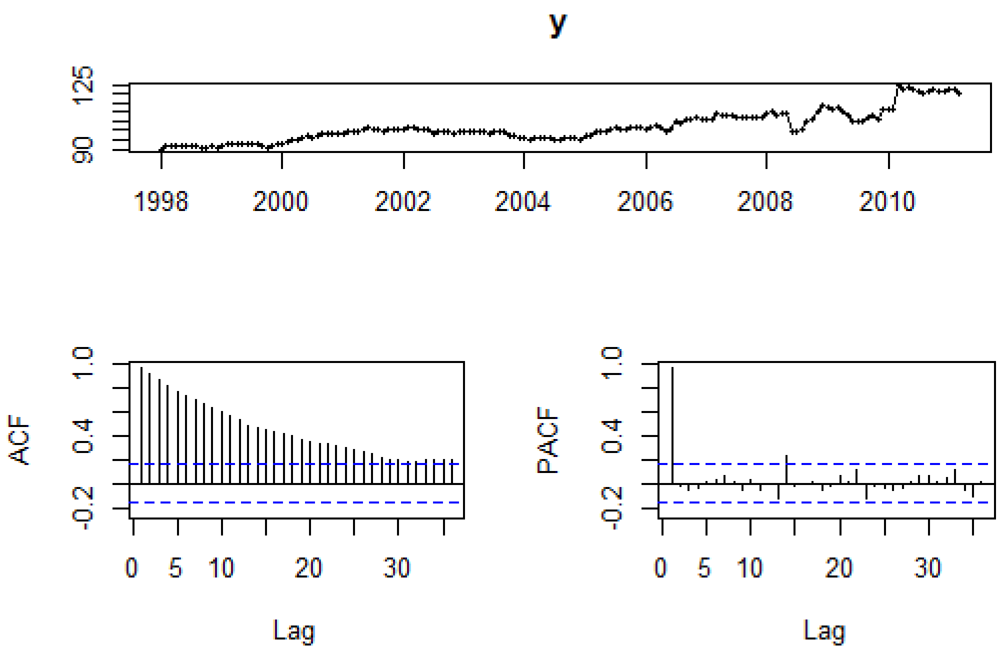
c)

```
res2<-res[c(-1:-24),]
```

plot(res2)

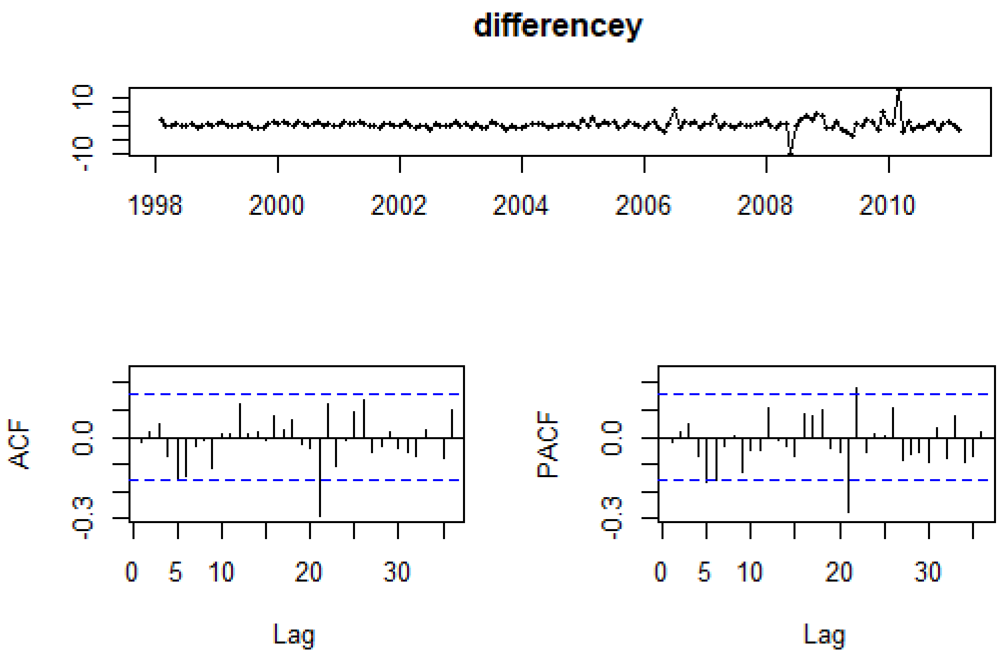


```
y<-ts(res2$Price.Index,freq=12,start=1998)

tsdisplay(y)
```

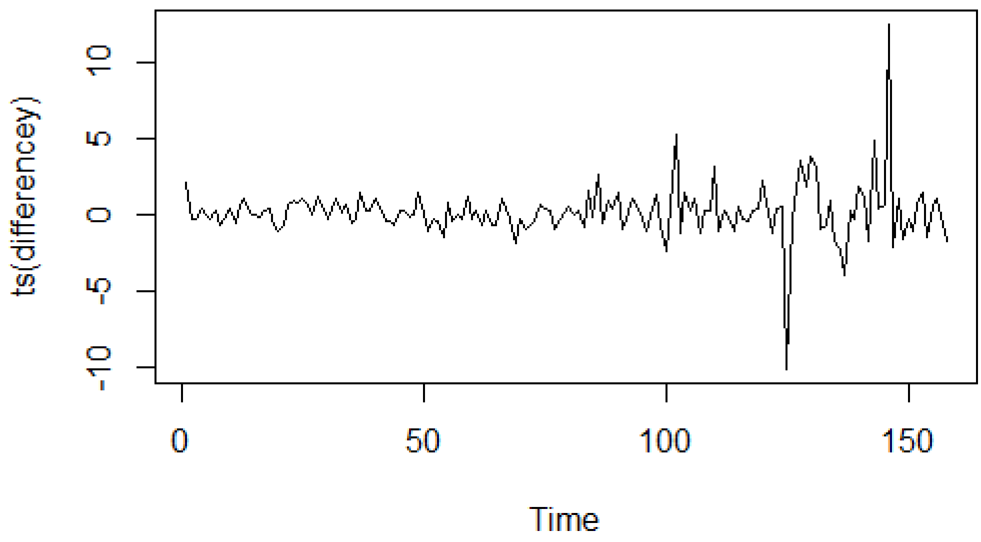


As it can be seen from the linear decrease in ACF, there is a trend in data which we should get rid of.

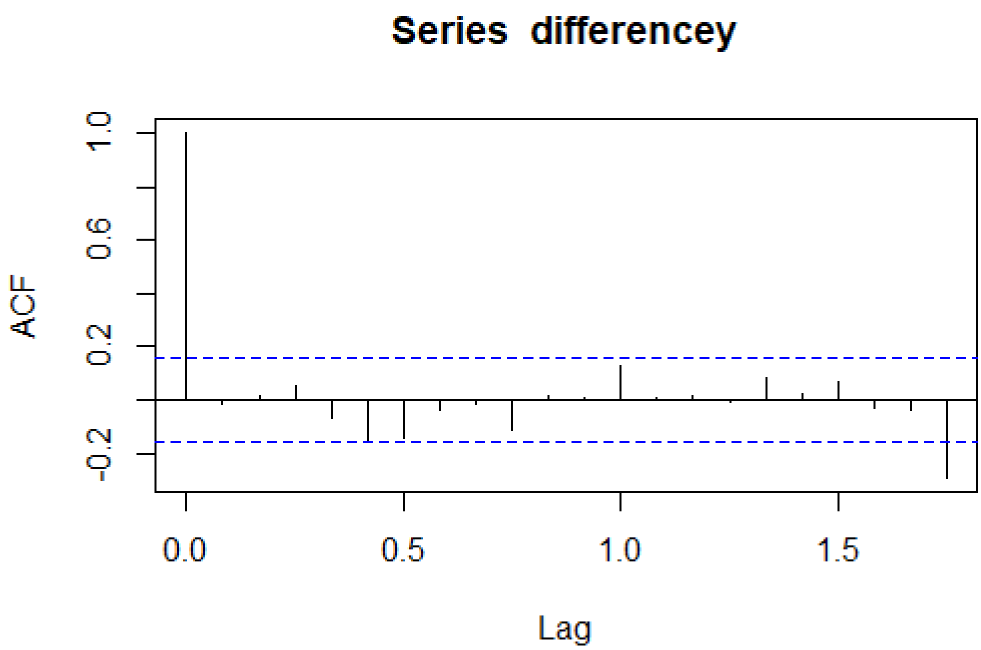
```
differencey<-diff(y)

tsdisplay(differencey)
```



plot(ts(differencey))

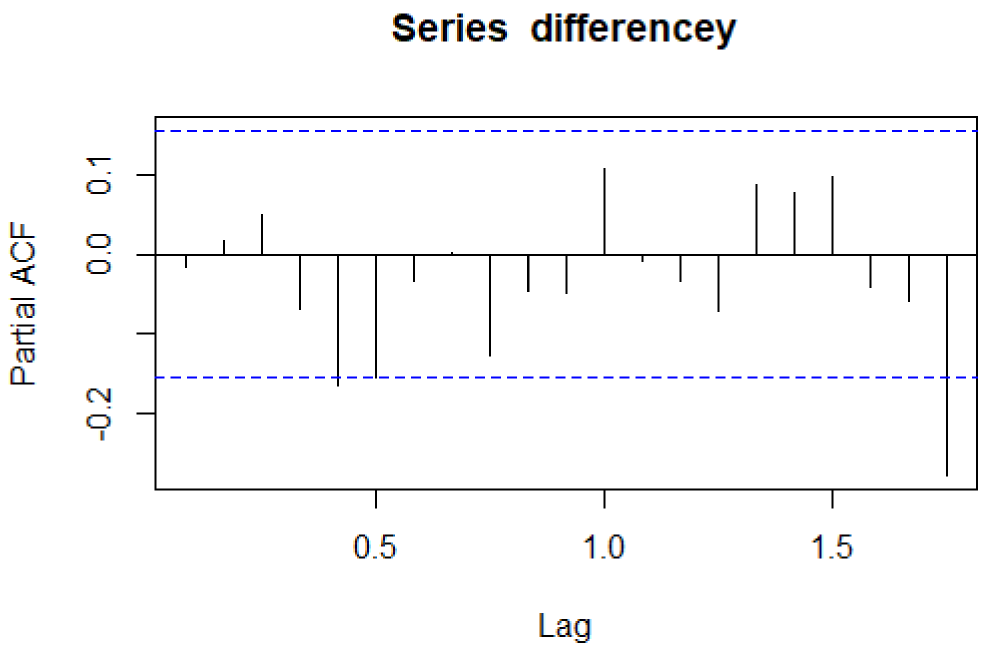


acf(differencey)



#As it can be seen from ACF, MA(1) is suitable for the data.

pacf(differencey)




```
auto.arima(ts(y), seasonal=FALSE)

#Series: ts(y)
#ARIMA(0,1,0)
#
#sigma^2 estimated as 3.134:  log likelihood=-314.44
#AIC=630.87   AICc=630.9   BIC=633.93


fity<- auto.arima(ts(y), seasonal=FALSE)

forecastfity<-(forecast(fity, h=12))
forecastfity

#  Point Forecast    Lo 80    Hi 80    Lo 95    Hi 95
#160          120.6 118.3312 122.8688 117.1302 124.0698
#161          120.6 117.3915 123.8085 115.6930 125.5070
#162          120.6 116.6704 124.5296 114.5901 126.6099
#163          120.6 116.0624 125.1376 113.6604 127.5396
#164          120.6 115.5269 125.6731 112.8413 128.3587
#165          120.6 115.0426 126.1574 112.1008 129.0992
#166          120.6 114.5974 126.6026 111.4198 129.7802
#167          120.6 114.1829 127.0171 110.7859 130.4141
#168          120.6 113.7937 127.4063 110.1906 131.0094
#169          120.6 113.4255 127.7745 109.6275 131.5725
#170          120.6 113.0753 128.1247 109.0920 132.1080
#171          120.6 112.7407 128.4593 108.5803 132.6197


plot(forecastfity)
```

Forecasts from ARIMA(0,1,0)