Portfolio choice with heterogeneous risks

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Abstract

Wealthier individuals tend to allocate a higher percentage of their wealth to equity, a pattern that standard life cycle portfolio choice models cannot explain without decreasing relative risk aversion (DRRA) preferences. While DRRA preferences align with observed data, evidence from long-run consumption and risk premia suggests that relative risk aversion should not depend on wealth. I propose an alternative explanation: wealthier individuals benefit from higher Sharpe ratios due to reduced risk through more effective diversification and more accurate return estimations, and subject to lower per-period portfolio monitoring costs. By incorporating these factors into a quantitative life-cycle model, I show how these changes bring the model closer to explaining the portfolio allocation choices of wealthier households.

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1 Introduction

Optimal asset allocation and portfolio choice are central research topics in finance, crucial for understanding household preferences and explaining the demand for equity and asset pricing dynamics. Heterogeneity in preferences plays a key role in understanding the choice patterns of different household groups and has significant implications at the aggregate level for wealth inequality and the relationship between equity demand and asset prices. Life cycle portfolio choice models that incorporate realistic processes are widely used in the literature to estimate household preferences and document optimal portfolio allocation.[1]

What explains the higher percentage allocation of wealthier households to risky assets? Decreasing relative risk aversion (DRRA) preferences offer one explanation (Wachter and Yogo, 2010; Carroll, 1998; Meeuwis, 2020). DRRA preferences suggest that as wealth increases, relative risk aversion decreases, leading to proportionally higher risk-taking behaviour among wealthier households. This mechanism could rationalize their higher percentage allocation to risky assets.

Even though DRRA preferences can bring the model closer to explaining data observations of top wealth percentiles allocating a greater percentage of their wealth to risky assets, long-run patterns of interest rates and risk premia suggest that relative risk aversion cannot strongly depend on wealth (Cambell and Viceira (2002)). Historically, households have become wealthier over time. DRRA preferences imply that rising overall wealth levels would have significantly altered consumption, interest rates and risk premia over the long run. However, the data show that long-run patterns in interest rates and risk premia have remained relatively constant, which is inconsistent with the implications of DRRA preferences.

[1] See, for instance, Gourinchas and Parker (2002); Cocco *et al.* (2005); Gomes and Michaelides (2005); Campbell *et al.* (2001); Viceira (2001)

I propose an alternative explanation following three facts about differences in financial market outcomes across wealth distribution. First, Campbell *et al.* (2019) provide empirical evidence showing that less wealthy households face higher risks due to an inability to diversify their stock portfolios effectively. Second, Gomes *et al.* (2021) highlight that less wealthy households incur higher fees. In addition, although low-cost diversification options are available, navigating these options is more complex and confusing for them, partly because the number of funds exceeds the number of individual stocks, leading to lower per-period portfolio monitoring costs with wealth. Similarly, Lusardi and Mitchell (2014) demonstrate the relationship between financial literacy and stock market participation costs. Third, using highly granular Scandinavian administrative data, Fagereng *et al.* (2017) show that wealthier households earn higher returns on their equity investments. These higher realized returns may influence their expectations, leading them to anticipate higher expected returns.

Following the literature, I propose that as households become wealthier, equity risk decreases, per-period participation costs are reduced, and expected returns are higher. In the model, relative risk aversion is fixed, but the riskiness of equity, per-period participation costs and risk premium change with wealth. Thus, the model can match data observations of portfolio shares with CRRA preferences, which is more suitable for the long-term behaviour of interest rates and risk premia.

I use different experiments to decompose the quantitative importance of the main ingredients of the model: higher expected returns, lower risk, or reduced per period stock market participation costs. Following empirical findings of Fagereng *et al.* (2017), first, I include higher expected returns for wealthier. Then, I introduce per-period participation cost into the model and make it heterogeneous with respect to wealth. Lastly, the model is modified to incorporate lower equity risk for wealthier households. In the final specification, I combine the three heterogeneities, reduced participation costs, lower equity risks and higher expected returns from equity.

Results show that while each mechanism individually moves the model toward the higher equity shares held by wealthier households, none is sufficient on its own to match the patterns observed in the data. Heterogeneous per-period participation costs generate differences in percentage allocation between the top and bottom wealth percentiles, but this effect is limited to very young ages when accumulated wealth is low. Heterogeneous risk and returns have a similar impact on model simulations: each flattens the risky asset share across the wealth distribution by reducing the high equity allocation of low wealth households, thereby aligning the model more closely with the data. Lastly, combining the three heterogeneities creates a divergence in allocation between the top and bottom wealth percentiles across all working ages. Heterogeneous costs capture differences at younger ages when accumulated wealth is low, while the heterogeneous risk and returns become important closer to retirement, as accumulated wealth reaches sufficient levels to impact equity risk and return.

A relative risk aversion level of 6 with standard deviations of equity returns decreasing from 0.21 to 0.16, per-period costs reduced from 1.4% of yearly income to 0.1% and risk premium rising from 3% to 4.5% with the lowest wealth level to the highest, generates a higher percentage allocation to risky assets for top wealth percentiles. Relative risk aversion of 6 is within the range of literature estimates that incorporate countercyclical labor income risk and DRRA preferences (Meeuwis (2020); Catherine (2022)).[2] The reduction in per-period participation cost from 1.4% to 0.1% of yearly labor income is also within the estimates of Catherine (2022), where fixed per-period participation cost is estimated as 0.8% of yearly labor income.[3]

Lower risk, reduced per-period participation costs and higher returns for wealthier households result in them allocating a higher percentage of their wealth to risky assets and receiving higher returns on their portfolios, exacerbating wealth inequality. Standard life cycle portfolio choice models can't match with wealth inequality dynamics, even with realistic settings (Parker *et al.*, 2022; Meeuwis, 2020). Following the empirical findings of Kartashova (2014) and Fagereng *et al.* (2020), which demonstrate

[2]Fagereng *et al.* (2017) estimate a higher relative risk aversion of approximately 10 when considering only tail event risk. Choukhmane and de Silva (2024) provide a more detailed analysis of risk aversion estimations.

[3]With cyclical skewness of labor income shocks, the estimate is 0.5% of yearly labor income.

higher returns from private wealth than public equity, I incorporate type dependence into the model where high-type households have private wealth in their risky assets and obtain a higher return than others from their private wealth to match with wealth inequality dynamics. After the addition of type dependence in the baseline model, results show that heterogeneities in risk, returns and costs increase the top 1% wealth share of total wealth from 12% in the standard model estimates to 21%.

Standard life cycle portfolio choice models estimate a lower percentage allocation to risky assets for greater wealth levels (Cocco *et al.* (2005)). Merton (1971) rule suggests the same percentage allocations to risky assets for all agents with the same level of risk aversion, irrespective of the wealth level. Including risky labor income in a dynamic setting makes human capital act as an implicit bond because of the lower risk of labor income than equity; therefore, where the ratio of human capital to total wealth is high, the model estimates an optimal percentage allocation to risky assets of almost a hundred percent (Cocco *et al.* (2005); Gomes and Michaelides (2003)).

This dynamic leads to a decreasing percentage allocation to equity investments with age, a strategy central to target date funds (TDFs), as wealth accumulation reduces the relative share of human capital in total wealth until retirement. Recent literature focuses on the adaptation of TDF products in retirement wealth and shows that younger cohorts' portfolio allocation decisions align more with simple life cycle portfolio allocation rules (Parker *et al.* (2022)). In contrast, this paper shifts the focus toward total wealth rather than retirement wealth specifically.

This paper is linked to three main strands of the literature. The first strand focuses on explaining heterogeneity in risks, returns, and costs. Empirically, Bach *et al.* (2020) and Fagereng *et al.* (2020) demonstrate the wealth effect in Scandinavia using highly granular administrative data. They show that wealthier households obtain higher returns from the same investments than their less wealthy counterparts. Xavier (2021) extends this analysis to the US using the SCF. I contribute to this literature by demonstrating the wealth effect on returns from capital gains for public equity in the US.

Campbell *et al.* (2019) show that poorer households have less effective diversification in their stock portfolios.

Theoretically, Gabaix *et al.* (2016), Benhabib *et al.* (2011), Hubmer *et al.* (2021) show the effect of heterogeneous returns on explaining wealth inequality. Kacperczyk *et al.* (2019) link capital income inequality to heterogeneity in financial sophistication. Similarly, Lusardi *et al.* (2017) attribute differences in equity market participation to financial sophistication. In their model, individuals endogenously choose to invest in financial knowledge, which allows them to access sophisticated financial products with higher expected returns. This paper includes heterogeneity in risk and participation costs in the model along with higher expected returns, and links them with wealth directly. In addition, whereas Lusardi *et al.* (2017) focus on explaining wealth inequality with disparity in financial sophistication, I focus on explaining the higher equity share of the wealthy. The main contribution of this paper is to quantitatively estimate the impact of these channels by using them in a theoretical model to assess their impact on household decisions.

A second strand of literature addresses the positive correlation between wealth and equity share. While low participation rates among less wealthy households are largely attributed to participation costs (Gomes and Michaelides, 2005; Vissing-Jorgensen, 2002), this paper focuses on the intensive margin: the variation in equity shares across the wealth distribution after participation. The main contribution of this paper to this literature is to explain the higher equity share of the wealthy without relying on DRRA preferences, which is inconsistent with long-term macroeconomic data. While prior studies employ DRRA either directly (Meeuwis, 2020; Calvet and Sodini, 2014) or indirectly through non-separable utility from luxury goods and capitalist spirit motives (Wachter and Yogo, 2010; Carroll, 1998; Bakshi and Chen, 1996; Carroll, 2000), I provide an alternative mechanism that maintains consistency with empirical wealth allocation patterns.

Lastly, the paper contributes to the growing body of literature focused on explaining portfolio allocation decisions and estimating household preferences (Gourinchas and Parker (2002); Cocco *et al.* (2005); Gomes and Michaelides (2005); Cocco (2005); Yao and Zhang (2005); Benzoni *et al.* (2007); Dimmock *et al.* (2016); Fagereng *et al.* (2017); Duarte *et al.* (2021); Catherine (2022); Shen (2024)). Cocco (2005) shows that having a large portion of housing wealth in household portfolios reduces equity holdings, and Yao and Zhang (2005) extends the literature by including rental markets and showing differences in equity shares between renters and homeowners. Recent advancements in this field, notably Catherine (2022) and Shen (2024), attribute the rising equity share over the working life to countercyclical labor income risk. However, while these models capture the aggregate age profile, they do not account for the within-age heterogeneity in equity shares across the wealth distribution observed in the data. I demonstrate that wealthier households hold a higher equity share within the same age groups.

The paper is organized as follows. The next section covers data, sample selection and portfolio-allocation evidence by wealth percentile. Section 3 presents the baseline model and extensions for heterogeneous costs, risks, returns and both combined. Section 4 details model calibrations. Sections 5 and 6 report policy functions and simulation results. Section 7 shows aggregate effects of heterogeneity in risk, returns and costs in the economy. Finally, Section 8 concludes.

2 Data

I use the Survey of Consumer Finances (SCF) from 1989 to 2022. SCF is a triennial cross-sectional survey that collects detailed information about the wealth and portfolio allocations of US households.[4] I use the data for two main purposes: calculating stock returns and obtaining information on the life-cycle profiles of equity shares across top and bottom wealth percentiles. Different sample selections are used for each purpose.

[4]SCF overweights wealthy, therefore I use household weights to make the estimates representative of all US households.

2.1 Sample selection

The sample used for calculating the life cycle profile of equity shares includes households aged between 26 and 64. This paper focuses on the pre-retirement period, as standard life-cycle portfolio choice models more effectively capture wealth accumulation and investment behavior before retirement (Gomes, 2020). Household wealth is represented as net worth, which is assets that include housing wealth minus debt and is recorded as *networth* in the documentation of the SCF. Total equity holdings are given as *equity* in SCF documentation. Thus, equity share is calculated as the percentage of equity holdings in net worth, *equity*/*networth*. The sample is restricted to households that have a net worth greater than $1,000, who are not business owners, and have wage income ($wageinc$) greater than $100. Summary statistics are given in Table 1.

[Table 1 about here.]

2.2 Equity share profile

As shown by Ameriks and Zeldes (2004), age, time and cohort have perfect multicollinearity because of the linear relationship between the three: age = time - cohort. The multicollinearity makes it challenging to disentangle the relationship between the three and capture the effects separately from each other. In order to break the multicollinearity and obtain the age profile of equity share, following Deaton and Paxson (1994) and Fagereng *et al.* (2017), I assume that time trends add up to zero by modifying time effects to be orthogonal to each other and sum to zero.

Following Ameriks and Zeldes (2004), I construct three-year age and cohort groups in the SCF. Wealth percentiles are calculated using *networth* of households across all survey waves from 1989 to 2022 within each age group. Then, wealth percentile subsamples are established. Finally, OLS regressions using Deaton and Paxson (1994) methodology are run on equity share with time, age and cohort dummies for the four wealth percentile groups. Figure 1 shows average estimated equity shares with age and wealth.

[Figure 1 about here.]

Results show that the whole sample follows an increasing pattern with age. Catherine (2022) and Shen (2024) show the same pattern and explain the reason for this pattern with countercyclical labor income risk.[5] Meeuwis (2020) shows the same increasing pattern for different wealth percentile groups with a restricted sample of investors who have considerable retirement wealth in the SCF.

Figure 1 exhibits large differences between the top and bottom wealth percentiles: the top 25 wealth percentile has a larger equity share than the whole sample average and the bottom. The bottom 25 wealth percentile only allocates approximately 10% of their wealth to equity, whereas at the top wealth percentile, equity share is sharply increasing with age from 20% to 40%. This paper aims to understand the impact of decreasing equity risk and portfolio monitoring cost with wealth on the observed differences in equity share between the top and bottom wealth percentiles.

2.3 Diversification

Campbell *et al.* (2019) show empirically that wealthier Indian households diversify better than less wealthy. Because India is a developing country, findings from India may not be directly applicable to developed economies. Using the SCF that represents a large sample of the US population, Figure 2 demonstrates the number of different companies in household portfolios for different wealth percentiles. The SCF question is asked as follows: ”In how many different companies do you (or your family living

[5] Fagereng *et al.* (2017), Gomes and Smirnova (2021) and Poterba *et al.* (1997) show hump-shape age pattern of equity share.

here) own stock?”. The question further states that the SCF wants the number of companies in which respondents own stock, not the number of individual shares.

[Figure 2 about here.]

Figure 2 shows the average number of different companies in stock portfolios of households that are equity participants for different wealth percentiles. First, wealth percentiles are calculated using *networth* of households for each survey wave from 1989 to 2022. Then, averages of the number of different companies are calculated using household weights for each wealth percentile. Two distinct patterns are observed. The first one is the increase with wealth percentiles; households that are at the higher wealth percentiles have a larger number of different companies in their stock portfolios. Second is the sharp increase at the top, the number of different companies invested by households after the 85th wealth percentile. Until the 85th wealth percentile, the number of different companies rises from 1 to 6, whereas from the 85th to the top, it increases to approximately 20 different companies in the stock portfolio.

Although not perfectly linking the pattern observed in Figure 2 to diversification, wealthier households having a larger number of different companies in their stock portfolios can be linked with better diversification ability of the wealthier for a large sample of US households, and it is shown that the highest number of different companies in household stock portfolios is concentrated at the top of the wealth distribution.

[Figure 3 about here.]

Figure 3 plots the weighted average share of stock mutual funds in household stock portfolios, where stock portfolio = directly-held stocks + stock mutual funds. Wealthier households hold a higher percentage of mutual funds, which increases the diversification of their stock portfolios and reduces overall stock market risk.

3 Model

The model is an extension of Gomes and Michaelides (2005) by introducing heterogeneous riskiness and per-period participation costs that vary with wealth to income level of households. First, I demonstrate the theoretical background of the optimal portfolio allocation. Next, I present the standard life cycle portfolio choice problem, introduce heterogeneous per-period participation costs and risk, and finally integrate both forms of heterogeneity into the model.

3.1 Life-cycle portfolio choice model

The benchmark model uses the framework by Gomes and Michaelides (2005) with tail event risk following Fagereng *et al.* (2017). The model is modified to include heterogeneous per-period participation costs and the riskiness of equity investment. I solve the model for three main specifications. The first one uses higher expected returns for wealthier households. Second, I introduce heterogeneous per-period participation cost that varies with the level of wealth. The third approach is incorporating heterogeneous risks. Finally, the model combines the three specifications.

3.1.1 Households

In the model, households start life at age 20 and start receiving labor income immediately. At each age t, they have two choices: the level of consumption C_t and percentage allocation to risky assets α_t. Households live for T years and life cycle ends at the terminal period with survival probabilities calibrated from Cocco *et al.* (2005). Until the terminal period, they derive utility from consumption with Epstein-Zin preferences, risk aversion parameter γ and elasticity of substitution ψ.

In order to reduce the complexity of the model, all parameters are divided by the average permanent income, which would help to prevent labor income from becoming a state variable. All parameters are normalized to permanent income. Thus, the model has two state variables (time and wealth on hand) and two choice variables (consumption and risky portfolio share). In a recursive decision setting, the household maximization problem becomes:

$$V_t = \max_{C_t, \alpha_t} \left\{ (1-\beta) C_t^{1-1/\psi} + \beta E_t \left(\pi_{t+1} V_{t+1}^{1-\gamma} \right)^{\frac{1-1/\psi}{1-\gamma}} \right\}^{\frac{1}{1-1/\psi}} \tag{1}$$

Which is subject to budget constraints:

$$X_{t+1} = (R^f(1-\alpha_t) + \alpha_t R^s_{t+1})(X_t - C_t) + Y_{t+1} \text{ for } t < K \tag{2}$$

$$X_{t+1} = (R^f(1-\alpha_t) + \alpha_t R^s_{t+1})(X_t - C_t) + \overline{Y} \text{ for } t \geq K \tag{3}$$

First, households derive a rate of return from their investments X (normalized wealth) in the previous period. Then, they decide to allocate the risky share α and the level of consumption C. Before retirement age K, stochastic labor income Y is realized. After retirement, labor income follows purely deterministic $\overline{Y}$. The focus of this paper is household portfolio allocation decisions before retirement.[6]

3.1.2 Labor income process

Following standard labor income process from literature[7], before retirement age K, income at age t (Y_t) follows a deterministic function of age $f(t)$, temporary income (p_t) and transitory income shock (u_t):

$$logY_t = f(t) + p_t + u_t \text{ where } t < K \tag{4}$$

$$p_t = p_{t-1} + z_t \tag{5}$$

[6] Gomes (2020) shows that life cycle portfolio choice models fit the data better for before retirement and after younger ages.

[7] see: Gourinchas and Parker (2002); Gomes and Michaelides (2005); Cocco *et al.* (2005); Davis *et al.* (2006)

Transitory and permanent income shocks are normally distributed with $z_t \sim N(0, \sigma_z^2)$ and $u_t \sim N(0, \sigma_u^2)$. After retirement, labor income process is a purely deterministic function of age, which is the pension income.

$$log\overline{Y} = \lambda\left[f(t) + p_K\right] \text{ where } t \geq K \tag{6}$$

A low correlation between permanent labor income and stock returns is allowed in the model following Campbell *et al.* (2001), Gomes and Michaelides (2005) and Wachter and Yogo (2010).

3.1.3 Financial assets and participation cost

There are two asset classes in the model: risky and riskless assets. The risky asset corresponds to holding stocks, public equity and the riskless asset class is considered as saving accounts, cash, bank deposits and bonds. The gross return on the riskless asset is denoted as R^f, which is fixed entire lifetime and not correlated with income or other parameters. Log return on stocks at time t is shown as r_t^s:

$$r_t^s = log(1 + R_t^s) \tag{7}$$

$$\text{where } r_t^s = \mu_s + \sigma_s \tag{8}$$

μ_s is equity premium and σ_s is time varying shock of equities that follow normal distribution with mean zero and standard deviation σ_s. Following Fagereng *et al.* (2017), stock return is subject to tail event risk.

$$R_{t+1}^s \sim \begin{cases} N(\mu, \sigma_s^2) & \text{with probability } 1 - \pi_R, \\ R_{\text{low}} & \text{with probability } \pi_R \end{cases}$$

π_R is the probability of the tail event. Tail event risk decreases the expected return from the risky asset, thus affecting the model estimation of the optimal percentage share of risky assets.

3.2 Heterogeneous per-period participation costs

I introduce per-period participation costs into the benchmark model. Every period, conditional on participation, households pay a fixed cost for participating in the stock market that reflects the portfolio monitoring cost and paid fees. Per-period participation cost is commonly used in literature, but not in a continuous heterogeneous setting (Gomes and Michaelides (2005)).[8] This paper utilizes the per-period participation cost to decrease proportionally to the agents' wealth to income ratio. Equation 9 shows the budget constraint of households before retirement:

$$X_{t+1} = (R^f(1-\alpha_t) + \alpha_t R^s_{t+1})(X_t - C_t) - F_t(X_t)I^{\alpha_t>0} + Y_{t+1} \tag{9}$$

I is a dummy variable that is equal to 1 when the household allocates a positive percentage of wealth to risky assets at age t, and F is the per-period cost the agent pays. F depends on the normalised wealth level at each time period (X_t) and it is a proportion of the average permanent income level at age t. A decreasing per-period participation cost increases expected returns from the risky investment, therefore resulting in a higher percentage allocation to risky assets for wealthier households.

3.3 Heterogeneous riskiness

In the model, as households accumulate wealth, they estimate equity returns more accurately with better information, and they diversify more effectively. These two mechanisms result in lower riskiness of the returns from equity investment for wealthier households. The model is modified to incorporate the standard deviation of the risky investment (σ_R) that decreases proportionally with the wealth level (X_t). Therefore, the return from risky assets becomes:

$$R^s_t(X_t) = \mu + R^f + \varepsilon_t(X_t) \text{ where } \varepsilon_t \sim N(0, \sigma^2_R(X_t)) \tag{10}$$

[8] Catherine (2022) suggests model's high estimation of optimal percentage allocation rate for younger ages may be eliminated by introducing heterogeneous per-period participation costs.

The budget constraint becomes:

$$X_{t+1} = (R^f(1-\alpha_t) + \alpha_t R^s_{t+1}(X_t))(X_t - C_t) + Y_{t+1} \tag{11}$$

A decreasing standard deviation leads to a higher Sharpe ratio, which in turn results in a higher estimated percentage allocation to risky assets for wealthier households.

3.4 Heterogeneous returns

Following the results of Fagereng *et al.* (2020)that show the wealthy obtain higher returns than their less wealthy counterparts, this model uses higher expected returns for wealthier households. In this model specification, the expected equity risk premium μ_t depends on the wealth on hand at age t.

$$r^s_t(X_t) = log(1 + R^s_t(X_t)) \tag{12}$$

$$\text{where } r^s_t(X_t) = \mu_s(X_t) + \sigma_s \tag{13}$$

Since μ_t depends on wealth at hand X_t, the return on financial assets R^s_t also depends on X_t, causing the value function to depend on X_t. Because wealth on hand (X) is already a state variable in the model, incorporating this dependency does not require additional computational resources. Value function for model solution is given in Equation 14.

$$V_t = \max_{C_t, \alpha_t} \left\{ (1-\beta)\, C_t^{1-1/\psi} + \beta E_t \left(\pi_{t+1} V_{t+1}(X_{t+1})^{1-\gamma} \right)^{\frac{1-1/\psi}{1-\gamma}} \right\}^{\frac{1}{1-1/\psi}} \tag{14}$$

3.5 Heterogeneous risk, returns and per-period participation costs

The final specification combines heterogeneity in risk, returns and per-period participation costs. As households accumulate wealth, both the risk of equity goes down, expected returns increase, and the per-period participation cost is reduced. Therefore, the budget constraint features a risky return that varies with the level of wealth X_t, and a participation cost representing per-period portfolio monitoring cost, which declines as X_t increases.

$$X_{t+1} = (R^f(1-\alpha_t) + \alpha_t R^s_{t+1}(X_t))(X_t - C_t - F_t(X_t)I_t^{\alpha>0}) + Y_{t+1} \tag{15}$$

Where:

$$R^s_t(X_t) = \mu(X_t) + R^f + \varepsilon_t(X_t) \text{ where } \varepsilon_t \sim N(0, \sigma^2_R(X_t)) \tag{16}$$

Wealth effect is now stronger: as households accumulate wealth, both equity risk decreases and the portfolio monitoring cost that is paid period by period goes down. Section 5 shows the model solutions, and Section 6 demonstrates how the model simulation of the final specification brings the model estimates closer to data observations.

4 Calibration

4.1 Incorporating heterogeneous risk, returns and costs

The unique feature of the model is incorporating heterogeneous equity risk, returns, and per-period participation cost into the framework of Gomes and Michaelides (2005). In the model, all parameters are normalized with the level of permanent income to reduce complexity. Thus, wealth to income ratio is a state variable, and equity risk, returns and per-period participation costs are decreasing functions of wealth to income ratio. One challenge in this analysis is determining the extent to which equity risk, returns and participation cost decrease in relation to the observed wealth to income levels.

There are three things to consider for calibrating decreasing equity risk, returns, and cost in the model: the lower bound, the upper bound and the decreasing function between the lower and upper bounds. Even though some risk and per-period participation costs can be eliminated with effective diversification and reduced costs of access to better information and portfolio monitoring costs with wealth, the risk and cost can not be eliminated perfectly. Similarly, the equity premium cannot rise to levels not observed in the data. Consequently, risk, returns, and costs are all constrained to a certain threshold for the wealthiest investors.

[Figure 4 about here.]

To identify the wealth to income ratio of the wealthiest households, Figure 4 plots wealth percentile groups against their corresponding wealth to income ratios. The horizontal axis groups households into five wealth percentile groups, while the vertical axis shows the average wealth to income ratio for each group. Figure 4 illustrates the increase in wealth to income ratios for higher wealth percentile groups. Households in the top 20 percent of the wealth distribution have a wealth to income ratio of about 13. Reflecting this empirical finding, the model calibration sets wealth to income ratio of 13 as the threshold for the equity risk, return, and participation cost functions.

The lowest equity risk is selected as 0.16 to reflect the historical average of standard deviation of returns in the S&P500, with the highest equity premium of 4.5%. The lowest per-period portfolio monitoring cost is selected as 0.1% of the permanent average yearly labor income. The model is calibrated to have these fixed levels of risk, returns and participation costs for wealth to income ratios higher than 13.

[Figure 5 about here.]

[Figure 6 about here.]

[Figure 7 about here.]

Figures 5, 6, and 7 display the calibrated values for equity risk (standard deviation of returns), portfolio monitoring cost as a percentage of the yearly average labor income, and risk premium for different levels of wealth. For wealth to income levels above 13, the standard deviation of equity returns is fixed at 0.16, and per-period participation cost is fixed at 0.1% of the labor income and the risk premium is 4.5%. For wealth to income ratios below 13, calibrations for risk, cost, and returns have the functional form that is shown in Equations 17, 18 and 19, respectively. The results indicate that these functional forms provide the best fit between the model estimates and the observed data.

$$y = (1 - x^a) \times b_1 + 0.16 \tag{17}$$

$$y = (1 - x^a) \times b_2 + 0.001 \tag{18}$$

$$y = 0.045 - (1 - x^a) \times b_3 \tag{19}$$

Where x is uniformly distributed between 0 and 1. As wealth increases, both risk and cost decrease, whereas the risk premium rises. The equations involve two free parameters, a and b. Parameter a determines the curvature, meaning the magnitude of the change from the lowest to the highest levels of risk, return, and cost. Households are subject to similar levels of risk, returns and cost with respect to their level of wealth, thus the same curvature parameter a is chosen for both calibrations.

Parameter b captures the difference between the lowest and highest levels, b_1, b_2 and b_3 determine the difference in risk, cost, and returns, respectively. Results show that $a =$

15 combined with $b_1 = 0.10$, $b_2 = 0.013$ and $b_3 = 0.015$ yield model estimates that align closely with the observed data. While these parameters are chosen to ensure the model aligns with observed data, to improve the accuracy and precision of these estimates, future research will use the simulated method of moments (SMM) to directly estimate a, b_1, b_2 and b_3 by matching moments of the model and the data.

[Figure 8 about here.]

[Figure 9 about here.]

Fagereng *et al.* (2020) show a 2.5% difference in stock returns between the lowest and highest wealth groups.[9] Xavier (2021) extends their findings to dividend yield from the stock market in the US and shows that wealthier households have a higher dividend yield from the stock market. By calculating the returns from capital gains, I show the wealth effect for returns at the capital gains and demonstrate wealthier have higher total returns from the stock market in the US. Results are shown in Figures 8 and 9. Figures show that higher wealth percentiles obtain larger total returns and capital gains. Detailed information on return calculations is given in the Appendix A.4. Following Fagereng *et al.* (2020), I assume a 1.5% difference in the expected returns from the stock market in the calibrations.

4.2 Parameters

The calibrated values of the model are shown in Table 2. In the baseline model, I set relative risk aversion to 6, which falls within the range used in studies addressing life-cycle portfolio choice problems. Catherine (2022) estimates relative risk aversion at 6 using countercyclical skewness in labor income shocks, and Meeuwis (2020) estimates a baseline risk aversion for DRRA preferences between 6 and 6.5. Following Gomes

[9]Their results show the wealth effect from the stock market both at the risk-adjusted and non risk-adjusted returns.

and Michaelides (2005), I set the elasticity of intertemporal substitution (EIS) to 0.5 and the time discount rate to 0.97.

In the model, agents start at age 20 and immediately start receiving labor income. Labor income is calibrated using the estimated values of Cocco *et al.* (2005).[10] Correlation between permanent income shock and stock returns is set at 0.15 following Campbell *et al.* (2001), Gomes and Michaelides (2005) and Wachter and Yogo (2010). The risk-free rate is set at 1.5% and the risk premium is equal to 4%. The standard deviation of stock returns is set at 0.18, reflecting the historical average of the US stocks. Finally, following Fagereng *et al.* (2017), the probability of a tail event is equal to 2%, and in the case of a rare disaster, agents lose 45% of the realised return.

[Table 2 about here.]

5 Policy functions

There are two state variables in the model: age and wealth level, which is normalised with the income, and there are two choice variables: consumption and percentage allocation to risky assets. The variable of interest in this research is the choice of percentage allocation to risky assets (α). Policy functions represent the optimal level of α for each age and wealth level. Results are given in Figure 10a for the benchmark case with normalised wealth to income. The optimal percentage of the risky share starts from a hundred percent for low wealth levels and decreases as households obtain more wealth. As discussed by Cambell and Viceira (2002), Gomes and Michaelides (2005) and Cocco *et al.* (2005), labor income acts as an implicit bond. Therefore, at low levels of wealth, most of the household's total wealth is derived from labor income (implicit bond), making it optimal to allocate a hundred percent of wealth to risky assets. As a result, as the household gets wealthier, the optimal percentage allocation to risky assets decreases.

[10] Cocco *et al.* (2005) estimate labor income process and standard deviations of labor income of US households using Panel Study of Income Dynamics (PSID).

[Figure 10 about here.]

Figure 10b shows the α policy function of the final specification. The addition of per-period participation cost makes the optimal α start from zero for low levels of cash-at-hand. When a household has more wealth, it allocates a higher percentage of its wealth to risky assets due to the lower risk of equity with wealth and reduced per-period costs paid for participation in the equity market. Labor income continues to act as an implicit bond, this reverse effect drives the optimal percentage allocation to risky assets downward for high levels of normalized wealth level.

6 Simulations

The model is estimated with three main specifications: heterogeneous risk, heterogeneous returns, and heterogeneous costs, and it is compared with the baseline model. Finally, combinations of the three dimensions are quantified. Each specification produces a policy function that gives optimal consumption and risky share decisions. Using these policy functions, I simulate 10,000 households that are subject to idiosyncratic risks and create an economy to compare model estimates with the observations in the data. Initial wealth levels are calibrated using the SCF with ages between 26 and 28. Detailed information about initial wealth estimation using the SCF is given in the Appendix A.8. For each model specification, I rank individuals in the simulated economy based on their average wealth. I then identify wealth percentiles and create quartiles of the wealth distribution, and compare these groups with the corresponding observations in the data.

6.1 Baseline model

[Figure 11 about here.]

The benchmark model is without per-period participation costs and assumes a constant level of risk for equity investments for all wealth levels. As discussed in Section 5, labor income acts as an implicit bond, consequently estimating a hundred percent allocation to risky assets because of low wealth levels at early ages. Simulation results of the benchmark model are given in Figure 11. The lines represent wealth percentile groups, with the wealthiest shown in the darkest blue. Results show that the average risky share starts at nearly a hundred percent and then decreases with age as households accumulate wealth. Also, the benchmark model estimates a higher percentage allocation to risky assets for the bottom wealth percentiles, which is inconsistent with data observations.

6.2 Heterogeneous returns

In the second specification, following the empirical findings in Fagereng *et al.* (2020), wealthier households are subject to higher expected returns. Figure 12 shows the simulation results.

[Figure 12 about here.]

In this model, agents with higher wealth to income ratios have a higher expected return, which would increase the estimated percentage allocation to risky assets of wealthier households relative to their less wealthy counterparts. Consequently, the higher percentage allocation to the risky asset for less wealthy households seen in the baseline model is not present in this specification. Instead, the results show approximately similar percentage allocations to the risky asset across different wealth groups. Even though higher expected returns for wealthier households bring the model results closer to matching the data observations, they are still not sufficient to generate a higher percentage allocation of wealth to the risky asset among the wealthiest households.

6.3 Heterogeneous per-period participation costs

In this specification, I introduce per-period participation costs that decrease with the household's wealth level, reflecting lower suggested fees for wealthier individuals, reduced costs of monitoring portfolios due to better access to information, and higher financial sophistication (Gomes *et al.* (2021), Campbell (2006), Guiso and Sodini (2013)).

[Figure 13 about here.]

Using the SCF, Catherine (2022) estimates the optimal fixed per-period participation cost as 0.8% of yearly income. In this model, instead of assuming a fixed cost for all wealth levels, per-period participation cost is calibrated to decrease from 1.4% of yearly income to 0.1% as wealth to income ratio rises. Simulation results are given in Figure 13. Introducing per-period cost decreases the percentage allocation for young ages and estimates a higher percentage allocation to risky assets for the top wealth percentile. However, once sufficient wealth is accumulated after the early years, heterogeneity in per-period participation costs no longer significantly impacts households at older ages.

6.4 Heterogeneous riskiness

Two main factors contribute to the lower risk from equity for wealthier households: better diversification and more accurate estimation. Following this idea, I incorporate heterogeneous riskiness into the model. In India, Campbell *et al.* (2019) empirically show that less wealthy households experience twice the standard deviation in stock market returns compared to wealthier households due to less effective diversification. Similarly, for the US, Figure 2 shows the increase in the number of different companies in household stock portfolios with an increase in wealth percentiles. Overall stock market volatility in the US is around 18% historically. To reflect this, the calibrated standard deviation of equity returns in the model decreases from 0.26 to 0.16 as wealth levels increase, capturing the 0.18 standard deviation of returns.

[Figure 14 about here.]

Simulation results are given in Figure 14. The results suggest that heterogeneous returns and heterogeneous risk have a similar impact; they both yield approximately similar percentage allocations to the risky asset across different wealth groups. Although heterogeneous risk brings the model results closer to the data observations relative to the baseline, it is still insufficient to match the observations in the SCF.

6.5 Heterogeneous risk and per-period participation costs

Results show that heterogeneous costs explain the higher percentage allocation to risky assets of wealthier households at younger ages. Therefore, in the next model specifications I combine heterogeneous costs with risks and returns. In this section I introduce heterogeneous risks and per-period participation costs into the baseline model. The standard deviation of equity returns decreases from 0.21 to 0.16, and per-period participation costs fall from 1.4% of yearly income to 0.1% from the least wealthy to the most wealthy. The results are presented in Figure 15.

[Figure 15 about here.]

The results indicate that once agents accumulate wealth exceeding the participation cost, a threshold reached at relatively young ages, the model predicts an unrealistically high allocation to risky assets. This finding suggests that heterogeneity in costs and risks by itself cannot align the model with the data. Appendix A.7 reports the results for the case with heterogeneous risk and fixed (wealth-invariant) costs with a per-period portfolio monitoring cost of 0.8% of yearly income as estimated by Catherine

(2022). The evidence suggests that this combination does not generate a higher share of risky assets in the portfolios of wealthier households.

6.6 Heterogeneous return and per-period participation costs

Next, I consider the specification with heterogeneous return and per-period participation costs. The risk premium rises from 3% to 4.5%, and per-period participation costs reduce from 1.4% of yearly income to 0.1% from the least wealthy to the most wealthy. The results are presented in Figure 16.

[Figure 16 about here.]

Results show that the 3% risk premium for the lowest wealth level significantly decreases the optimal percentage allocation to risky assets for the least wealthy. Therefore, with heterogeneous risks and costs, the model can capture the increasing pattern of risky share with age. However, the model's results do not align with the SCF data, suggesting that heterogeneous returns and costs are not sufficient to match the data observations.

6.7 Heterogeneous risk, returns and costs

In the final specification I combine both heterogeneous risk, returns and costs. In this model specification the standard deviation of equity returns decreases from 0.21 to 0.16, per-period participation costs fall from 1.4% of yearly income to 0.1% from the least wealthy to the most wealthy and the risk premium increases from 3% to 4.5% for the least to the most wealthy. Results are given in Figure 17.

[Figure 17 about here.]

Figure 17 presents the model results on the left and the corresponding SCF data on the right. In the final specification, which incorporates heterogeneity in risk, returns, and costs, the model successfully generates a higher percentage allocation to risky assets for wealthier households.

7 Aggregate effects

Heterogeneous risk, returns and per-period participation costs combined with type dependence have important implications at the aggregate level, mainly for explaining wealth inequality and estimating preference heterogeneity.

Matching the wealth inequality of model estimates and data is not achievable without modifications in the model because of the higher wealth inequality that is observed in the US data. Even with a highly realistic life cycle portfolio choice model setting that relies on machine learning methods, at the aggregate, the simulated population can not match the observed wealth inequality dynamics of the US (Duarte *et al.* (2021)). Meeuwis (2020) shows that the addition of DRRA preferences to the model is not enough to generate enough estimated wealth inequality on its own. To address this issue, he introduces heterogeneous time preferences that vary with the permanent income level of the household. The main idea he adopts is that high-income households become more patient. As a result, they save a higher percentage of their wealth, which results in elevated wealth inequality.

This research achieves the same percentage allocations to risky assets without DRRA preferences. Consequently, the model used here also fails to generate wealth inequality as high as observed in the data, similar to the findings in Meeuwis (2020). Instead of utilizing heterogeneous preferences to capture wealth inequality dynamics, I follow the macroeconomics literature, which suggests that the high observed wealth inequality in the US arises from heterogeneous mean returns across households. This concept, known as *type dependence*, asserts that different household types experience varying returns, a phenomenon empirically documented by Bach *et al.* (2020) and Fagereng *et al.* (2017).[11]

In the model, heterogeneity in household types arises from the presence of private wealth among a subset of households. Standard life-cycle models typically include only public equity as the risky asset. To better capture the empirical dynamics of wealth inequality, I extend the model by allowing households with private wealth to include it in their risky asset portfolio. These households, referred to as high-type agents, earn higher mean returns on their risky investments, as private wealth has been shown to yield higher returns than public equity (Kartashova (2014); Gocmen *et al.* (2025); Balloch and Richers (2023); Xavier (2021); Fagereng *et al.* (2020)).

For high-type households, the risky asset thus comprises both private and public equity, resulting in a portfolio with a higher expected return and greater risk. Using the SCF from 1989 to 2022, the weighted average of entrepreneurs in the sample is 12%. Thus, the model is modified to include 12% of the population as high type. The percentage of high-type households is given exogenously, as the decision to obtain private wealth is outside the scope of this paper. Following empirical findings of Xavier (2021), returns from business wealth in the US are calibrated as 13.4%. Finally, I calibrate the standard deviation of returns of private wealth following Fagereng *et al.* (2020). Their granular Scandinavian administrative data indicate that the cross-sectional standard deviation of returns on private wealth is about twice that of public equity. Taking the historical U.S. equity standard deviation to be 18%, I therefore set the standard deviation of returns from private business wealth to 36%.

7.1 Wealth inequality

Results show that the benchmark model simulations with type dependence estimate that the top 1% wealth percentile holds 12% of the total wealth in the population.

[11]Theoretically, Benhabib *et al.* (2011), Gabaix *et al.* (2016) and Saez and Zucman (2016) show the impact of heterogeneous returns on explaining wealth inequality.

Adding heterogeneous riskiness and per-period costs increases the top 1% wealth share to 21%. Lower risk, higher risk premium, and reduced per-period participation costs increase the demand for risky assets for the wealthy, consequently increasing their return on their portfolio and resulting in an increase in the top 1% wealth share. Similarly, Meeuwis (2020) shows that CRRA preferences estimate the top 1% wealth share as 18% and DRRA preferences bring up the value to 25%.

While type dependence is necessary to explain the effects of heterogeneous risk, returns and participation costs on wealth inequality at the aggregate level, it does not influence the ex-ante results, where the model still estimates a higher percentage allocation to risky assets for the top wealth percentile, except for the top 25th wealth percentile. Figure 18 illustrates this finding. Type dependence is only used to capture the aggregate level effects of heterogeneous risk, returns and costs on wealth inequality dynamics.

[Figure 18 about here.]

7.2 Preference heterogeneity

Heterogeneous preferences are used in asset pricing literature for matching the observed moments in the data and the model estimates. Using administrative Swedish data, Calvet *et al.* (2021) estimate high preference heterogeneity across households, especially in time preferences, assuming constant returns for all households in the economy.

Preferences, specifically time preferences, which determine the patience levels of households, are directly linked with the consumption-saving decisions in portfolio choice models. Therefore, heterogeneity in time preferences is the main driver of estimated wealth inequality in life-cycle portfolio choice models when all households are subject to the same level of returns.

In this paper, I show that when the model incorporates type dependence, which allocates a higher mean return resulting from private wealth, it can account for the observed wealth inequality without requiring heterogeneity in time preferences. The estimated inequality is primarily driven by differences in mean returns rather than differences in discount factors. Therefore, when heterogeneous returns that arise from private wealth are included in the life cycle portfolio choice model, instead of assuming the only risky asset for all households is public equity, estimated preference heterogeneity across households is lower.

8 Conclusion

This paper proposes an alternative explanation for the higher percentage allocation to risky assets among the wealthy. One possible explanation is the DRRA preferences that achieve a higher percentage allocation to risky assets by decreasing relative risk aversion with the level of wealth. However, DRRA preferences fail to explain the observed long-term consumption and equity premium.

I propose that the higher percentage allocation to equity of higher wealth percentiles results from the two main mechanisms: a higher Sharpe ratio due to lower riskiness of equity investments with wealth through more effective diversification and more accurate estimation of returns, and higher expected returns resulting from lower per-period participation costs with wealth driven by reduced portfolio monitoring costs and decreased fees. Fagereng *et al.* (2020) show higher realized returns of wealthier, which may affect their expectations and lead to higher expected returns. I incorporate these three mechanisms into the portfolio choice life cycle model and show that heterogeneous risk, returns, and participation costs can explain the higher percentage allocation to risky assets among the wealthy without requiring DRRA preferences.

A risk aversion of 6, standard deviations of equity decreasing from 0.21 to 0.16, per-period participation costs reduced from 1.4% to 0.1% of yearly income and the risk premium rising from 3% to 4.5% can explain observed patterns in the data. At the aggregate level, heterogeneous risks and participation costs bring the top 1% wealth share in total wealth from 12% to 21%, by increasing the demand for risky assets for wealthier households with higher expected returns through higher Sharpe ratios, and consequently result in elevated wealth inequality.

References

AMERIKS, J. and ZELDES, S. P. (2004). *How do household portfolio shares vary with age*. Tech. rep., working paper, Columbia University.

BACH, L., CALVET, L. E. and SODINI, P. (2020). Rich pickings? risk, return, and skill in household wealth. *American Economic Review*, **110** (9), 2703–2747.

BAKSHI, G. S. and CHEN, Z. (1996). The spirit of capitalism and stock-market prices. *The American Economic Review*, pp. 133–157.

BALLOCH, C. and RICHERS, J. (2023). *Asset Allocation and Returns in the Portfolios of the Wealthy*. LSE Financial Markets Group.

BENHABIB, J., BISIN, A. and ZHU, S. (2011). The distribution of wealth and fiscal policy in economies with finitely lived agents. *Econometrica*, **79** (1), 123–157.

BENZONI, L., COLLIN-DUFRESNE, P. and GOLDSTEIN, R. S. (2007). Portfolio choice over the life-cycle when the stock and labor markets are cointegrated. *The Journal of Finance*, **62** (5), 2123–2167.

CALVET, L. E., CAMPBELL, J. Y., GOMES, F. and SODINI, P. (2021). The cross-section of household preferences.

— and SODINI, P. (2014). Twin picks: Disentangling the determinants of risk-taking in household portfolios. *The Journal of Finance*, **69** (2), 867–906.

CAMBELL, J. Y. and VICEIRA, L. M. (2002). Strategic asset allocation: Portfolio choice for long-term investors.

Campbell, J. Y. (2006). Household finance. *The journal of finance*, **61** (4), 1553–1604.

—, Cocco, J. F., Gomes, F. J. and Maenhout, P. J. (2001). Investing retirement wealth: A life-cycle model. In *Risk aspects of investment-based Social Security reform*, University of Chicago Press, pp. 439–482.

—, Ramadorai, T. and Ranish, B. (2019). Do the rich get richer in the stock market? evidence from india. *American Economic Review: Insights*, **1** (2), 225–240.

Carroll, C. D. (1998). Why do the rich save so much?

— (2000). Portfolios of the rich.

Catherine, S. (2022). Countercyclical labor income risk and portfolio choices over the life cycle. *The Review of Financial Studies*, **35** (9), 4016–4054.

Choukhmane, T. and de Silva, T. (2024). *What Drives Investors' Portfolio Choices? Separating Risk Preferences from Frictions*. Tech. rep., National Bureau of Economic Research.

Cocco, J. F. (2005). Portfolio choice in the presence of housing. *The Review of Financial Studies*, **18** (2), 535–567.

—, Gomes, F. J. and Maenhout, P. J. (2005). Consumption and portfolio choice over the life cycle. *The Review of Financial Studies*, **18** (2), 491–533.

Davis, S. J., Kubler, F. and Willen, P. (2006). Borrowing costs and the demand for equity over the life cycle. *The Review of Economics and Statistics*, **88** (2), 348–362.

Deaton, A. and Paxson, C. (1994). Intertemporal choice and inequality. *Journal of political economy*, **102** (3), 437–467.

Dimmock, S. G., Kouwenberg, R., Mitchell, O. S. and Peijnenburg, K. (2016). Ambiguity aversion and household portfolio choice puzzles: Empirical evidence. *Journal of Financial Economics*, **119** (3), 559–577.

Duarte, V., Fonseca, J., Goodman, A. S. and Parker, J. A. (2021). *Simple allocation rules and optimal portfolio choice over the lifecycle*. Tech. rep., National Bureau of Economic Research.

Fagereng, A., Gottlieb, C. and Guiso, L. (2017). Asset market participation and portfolio choice over the life-cycle. *The Journal of Finance*, **72** (2), 705–750.

—, Guiso, L., Malacrino, D. and Pistaferri, L. (2020). Heterogeneity and persistence in returns to wealth. *Econometrica*, **88** (1), 115–170.

Gabaix, X., Lasry, J.-M., Lions, P.-L. and Moll, B. (2016). The dynamics of inequality. *Econometrica*, **84** (6), 2071–2111.

Gocmen, A., Martínez-Toledano, C. and Mittal, V. (2025). Private capital markets and inequality. *Available at SSRN 5166981*.

Gomes, F. (2020). Portfolio choice over the life cycle: A survey. *Annual Review of Financial Economics*, **12** (1), 277–304.

—, Haliassos, M. and Ramadorai, T. (2021). Household finance. *Journal of Economic Literature*, **59** (3), 919–1000.

— and Michaelides, A. (2003). Portfolio choice with internal habit formation: A life-cycle model with uninsurable labor income risk. *Review of Economic Dynamics*, **6** (4), 729–766.

— and — (2005). Optimal life-cycle asset allocation: Understanding the empirical evidence. *The Journal of Finance*, **60** (2), 869–904.

— and Smirnova, O. (2021). Stock market participation and portfolio shares over the life-cycle. *Available at SSRN 3808350*.

Gourinchas, P.-O. and Parker, J. A. (2002). Consumption over the life cycle. *Econometrica*, **70** (1), 47–89.

Guiso, L. and Sodini, P. (2013). Household finance: An emerging field. In *Handbook of the Economics of Finance*, vol. 2, Elsevier, pp. 1397–1532.

Hubmer, J., Krusell, P. and Smith Jr, A. A. (2021). Sources of us wealth inequality: Past, present, and future. *NBER Macroeconomics Annual*, **35** (1), 391–455.

Kacperczyk, M., Nosal, J. and Stevens, L. (2019). Investor sophistication and capital income inequality. *Journal of Monetary Economics*, **107**, 18–31.

Kartashova, K. (2014). Private equity premium puzzle revisited. *American Economic Review*, **104** (10), 3297–3334.

Lusardi, A., Michaud, P.-C. and Mitchell, O. S. (2017). Optimal financial knowledge and wealth inequality. *Journal of political Economy*, **125** (2), 431–477.

— and Mitchell, O. S. (2014). The economic importance of financial literacy: Theory and evidence. *American Economic Journal: Journal of Economic Literature*, **52** (1), 5–44.

Meeuwis, M. (2020). Wealth fluctuations and risk preferences: Evidence from us investor portfolios. *Available at SSRN 3653324*.

Merton, R. (1971). Optimum consumption and portfolio rules in a continuous-time model. *Journal of Economic Theory*, **3** (4), 373–413.

Moskowitz, T. J. and Vissing-Jørgensen, A. (2002). The returns to entrepreneurial investment: A private equity premium puzzle? *American Economic Review*, **92** (4), 745–778.

Parker, J. A., Schoar, A., Cole, A. T. and Simester, D. (2022). *Household portfolios and retirement saving over the life cycle*. Tech. rep., National Bureau of Economic Research.

Poterba, J. M., Samwick, A. A. *et al.* (1997). *Household portfolio allocation over the life cycle*, vol. 6185. National Bureau of economic research Cambridge, Mass., USA.

Saez, E. and Zucman, G. (2016). Wealth inequality in the united states since 1913: Evidence from capitalized income tax data. *The Quarterly Journal of Economics*, **131** (2), 519–578.

Shen, J. (2024). Countercyclical risks, consumption, and portfolio choice: Theory and evidence. *Management Science*, **70** (5), 2862–2881.

Viceira, L. M. (2001). Optimal portfolio choice for long-horizon investors with nontradable labor income. *The Journal of Finance*, **56** (2), 433–470.

Vissing-Jorgensen, A. (2002). Towards an explanation of household portfolio choice heterogeneity: Nonfinancial income and participation cost structures.

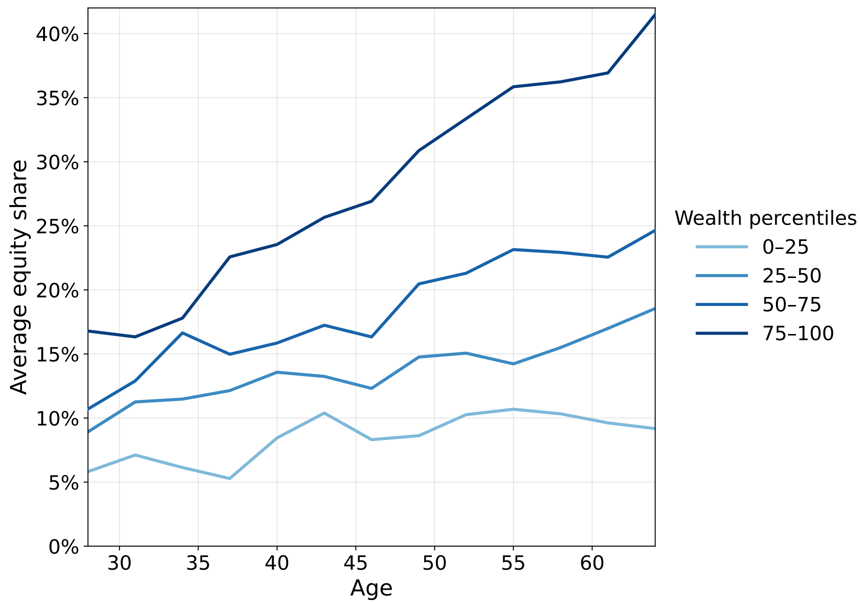
Wachter, J. A. and Yogo, M. (2010). Why do household portfolio shares rise in wealth? *The Review of Financial Studies*, **23** (11), 3929–3965.

Xavier, I. (2021). Wealth inequality in the us: the role of heterogeneous returns. *Available at SSRN 3915439*.

Yao, R. and Zhang, H. H. (2005). Optimal consumption and portfolio choices with risky housing and borrowing constraints. *The Review of Financial Studies*, **18** (1), 197–239.

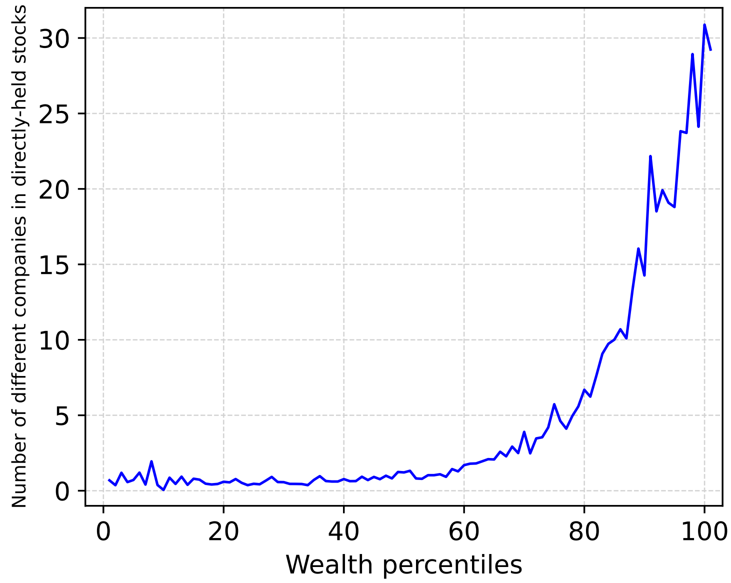
Figures

Figure 1
Equity profiles with age and wealth



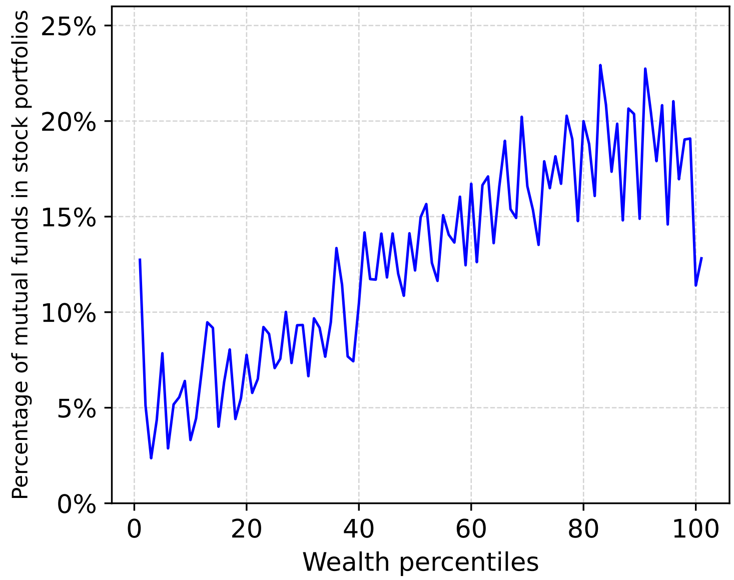
This figure shows estimated equity shares with age and wealth that are calculated from the Survey of Consumer Finances (SCF) from 1989 to 2022. Wealth is recorded as *networth* in the SCF and represents the total value of assets minus debt in dollar amount. The sample is restricted to households that have a net worth greater than $1,000, who are not business owners and receive wage income (*wageinc*) greater than $100. Following Ameriks and Zeldes (2004), 3-year age and cohort groups are constructed and Deaton and Paxson (1994) methodology is used to decompose age, time and cohort effects. Wealth percentiles are calculated using net worth for each age group. Household weights are normalised to give each survey year an equal weight.

Figure 2
Number of different companies in household stock portfolios



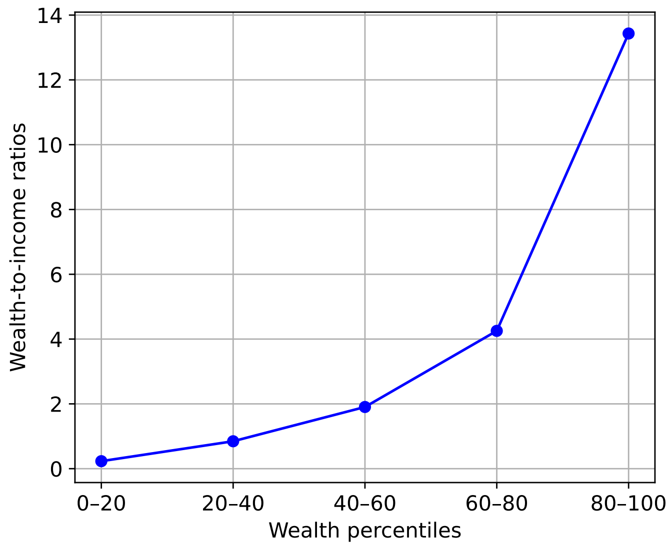
This figure shows the weighted average number of different companies in household stock portfolios in the SCF from 1989 to 2022. Household weights are normalised to give each survey year an equal weight. For each survey year, *networth* is used to determine wealth percentiles.

Figure 3
Percentage of stock mutual funds



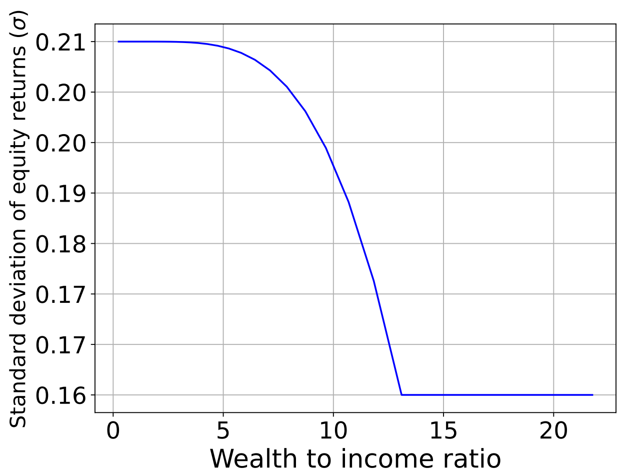
This figure shows the weighted average percentage of stock mutual funds in stock portfolios where stock portfolio = directly-held stocks + stock mutual funds, in the SCF from 1989 to 2022. Household weights are normalised to give each survey year an equal weight. For each survey year, *networth* is used to determine wealth percentiles.

Figure 4
Wealth percentiles and wealth to income ratios



This figure shows the average wealth to income ratios for each wealth percentile group in the SCF from 1989 to 2022. The X-axis represents households grouped into deciles based on wealth percentiles. The Y-axis shows the corresponding wealth to income ratios for each group. The sample is restricted to households with wealth to income ratios between 0.01 and 100. *networth* is used to calculate wealth percentiles for each survey year. Wealth to income ratio is calculated by dividing $networth$ of the household to the average labor income of households before age 65 in that survey year.

Figure 5
Calibrated equity risk

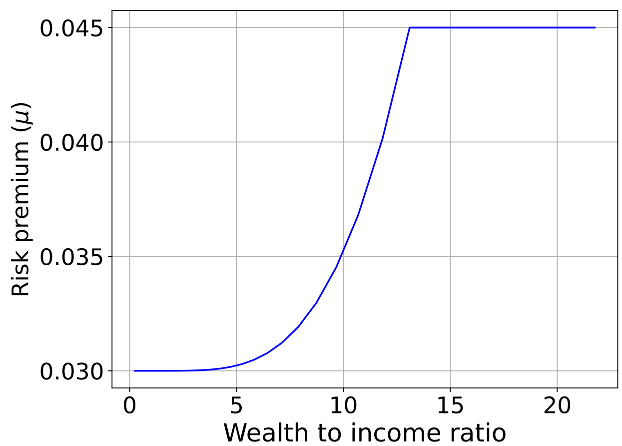


This figure illustrates calibration for reducing equity risk with increasing wealth. The X-axis shows wealth to income ratios. The Y-axis demonstrates calibrated values for the standard deviation of equity returns (σ).

Figure 6
Calibrated participation cost

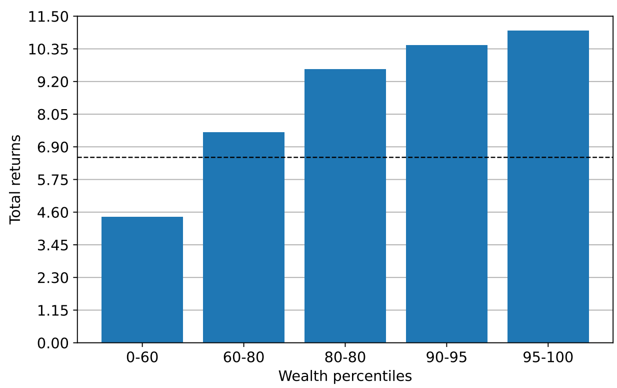
This figure illustrates calibration for decreasing per-period participation costs with increasing wealth. The X-axis shows wealth to income ratios. The Y-axis demonstrates calibrated values for per-period participation cost (F) as a percentage of yearly average labor income.

Figure 7
Calibrated risk premium



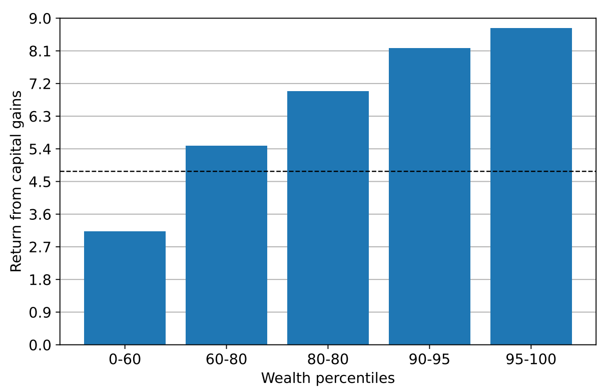
This figure illustrates calibration for increasing risk premium with increasing wealth. The X-axis shows wealth to income ratios. The Y-axis demonstrates calibrated values for risk premium (μ).

Figure 8
Wealth effect on total returns



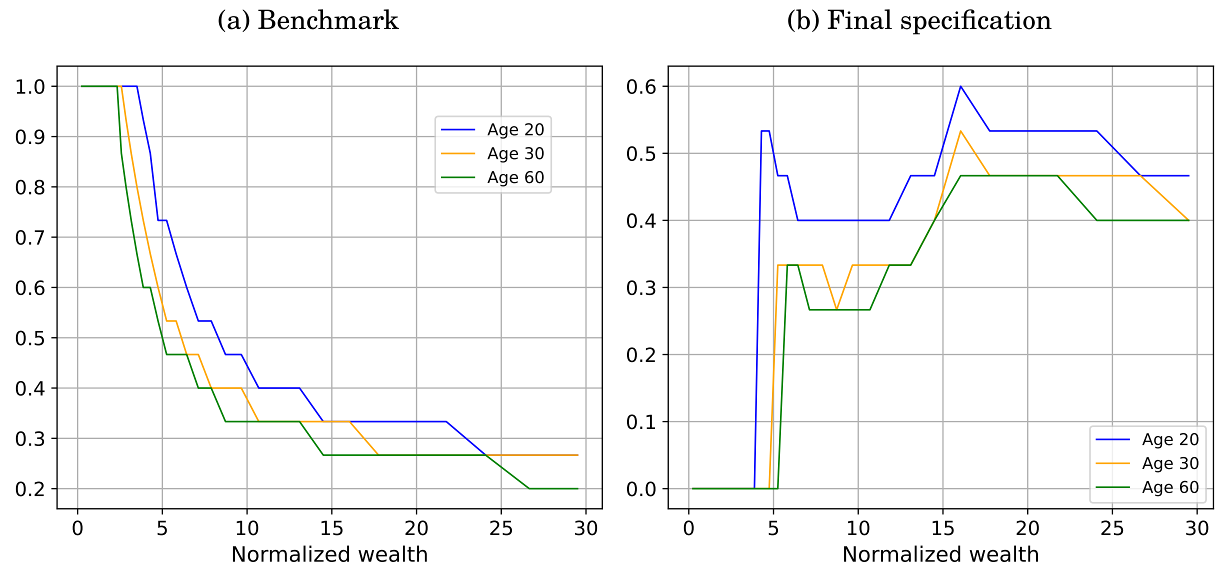
This figure shows return estimations using the SCF from 1989 to 2022 at total return for different wealth percentile groups. For each survey wave *networth* is used to calculate wealth percentiles. The X-axis shows selected wealth percentile groups.

Figure 9
Wealth effect on capital gains



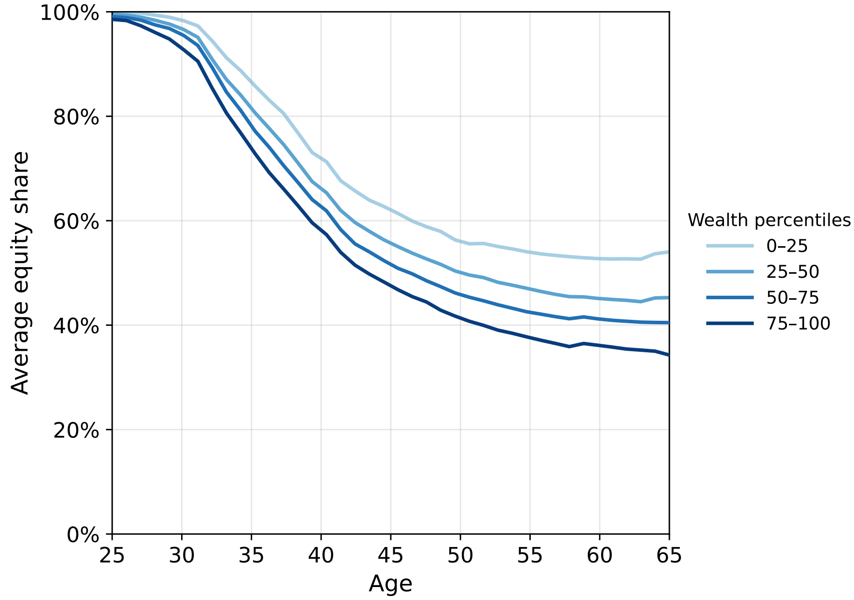
These figure shows return estimations using the SCF from 1989 to 2022 from capital gains for different wealth percentile groups. For each survey wave *networth* is used to calculate wealth percentiles. The X-axis shows selected wealth percentile groups.

Figure 10
Optimal percentage allocation to the risky asset (α)



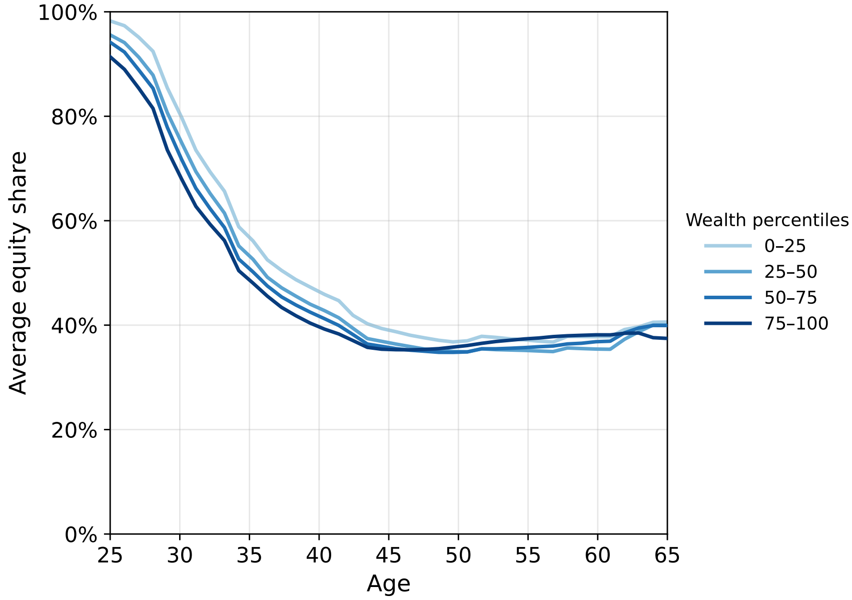
Panel (a) and panel (b) show the policy functions for optimal percentage allocation to the risky asset of the benchmark model and final specification with heterogeneous risk, returns and participation cost by wealth for different wealth to income ratios, respectively.

Figure 11
Baseline model



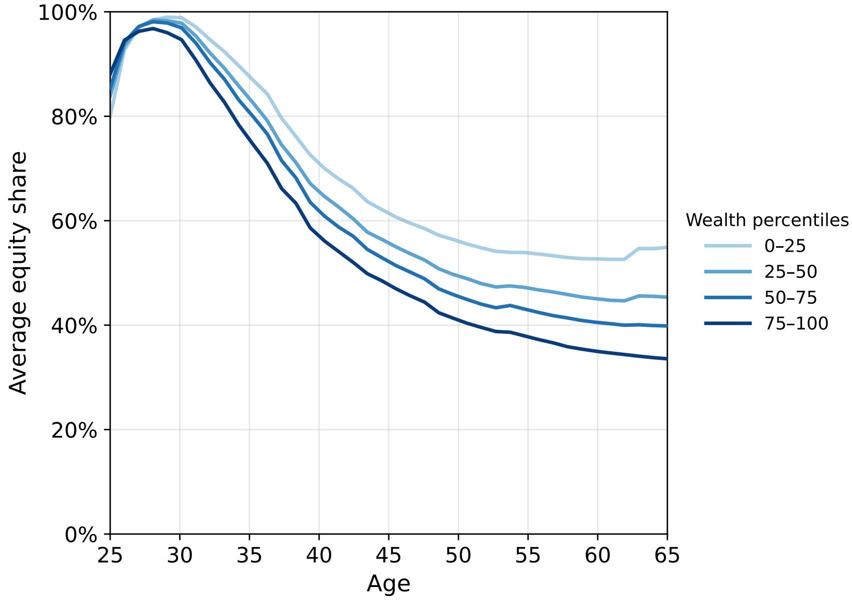
This figure represents the estimated average equity share of the model without heterogeneous risks and participation cost for different wealth percentile groups. The simulation is run 10,000 times. The initial wealth of agents is calibrated using the SCF waves from 1989 to 2022. Simulated households are ranked by their average wealth.

Figure 12
Heterogeneous returns



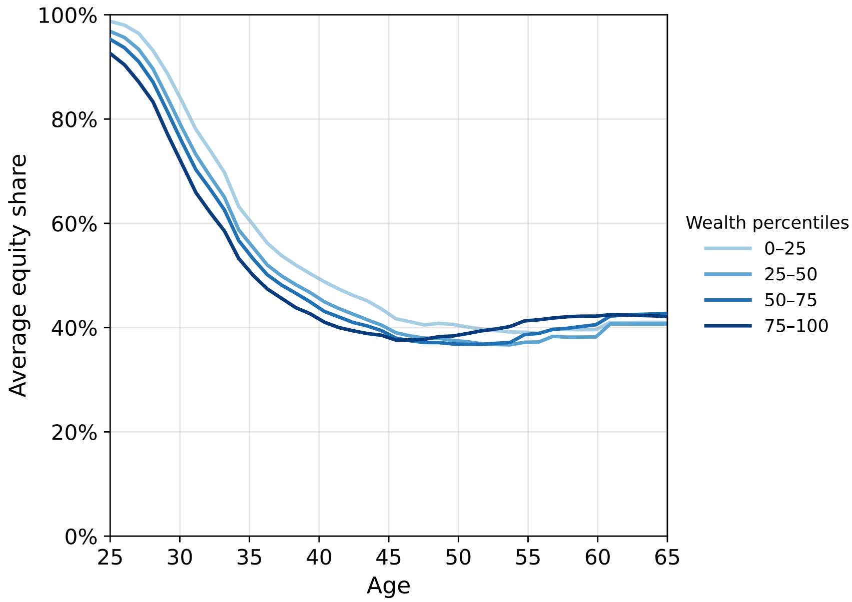
This figure represents the estimated average equity share of the model with heterogeneous expected returns. The simulation is run 10,000 times. The initial wealth of agents is calibrated using the SCF waves from 1989 to 2022. Simulated households are ranked by their average wealth.

Figure 13
Heterogeneous costs



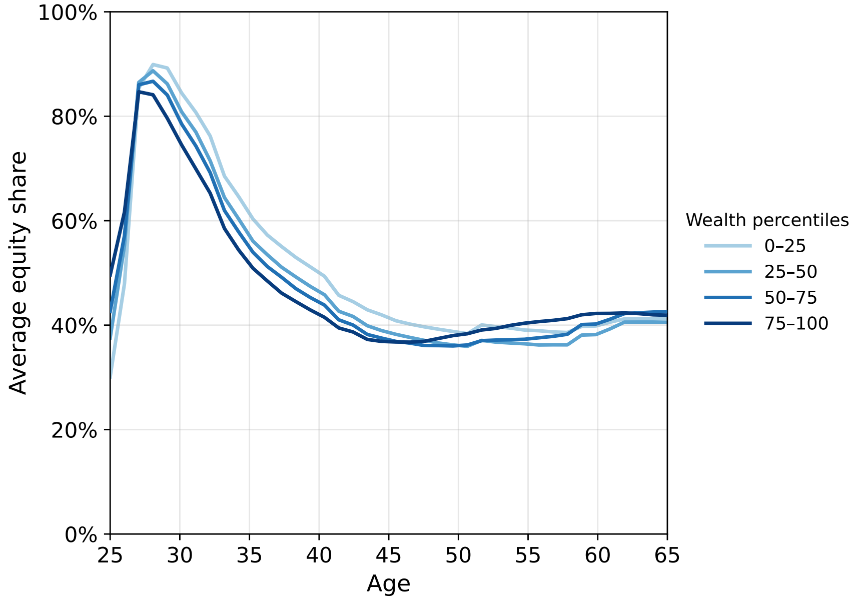
This figure represents the estimated average equity share of the model with only heterogeneous per period costs for different wealth percentile groups. The simulation is run 10,000 times. The initial wealth of agents is calibrated using the SCF waves from 1989 to 2022. Simulated households are ranked by their average wealth.

Figure 14
Heterogeneous risk



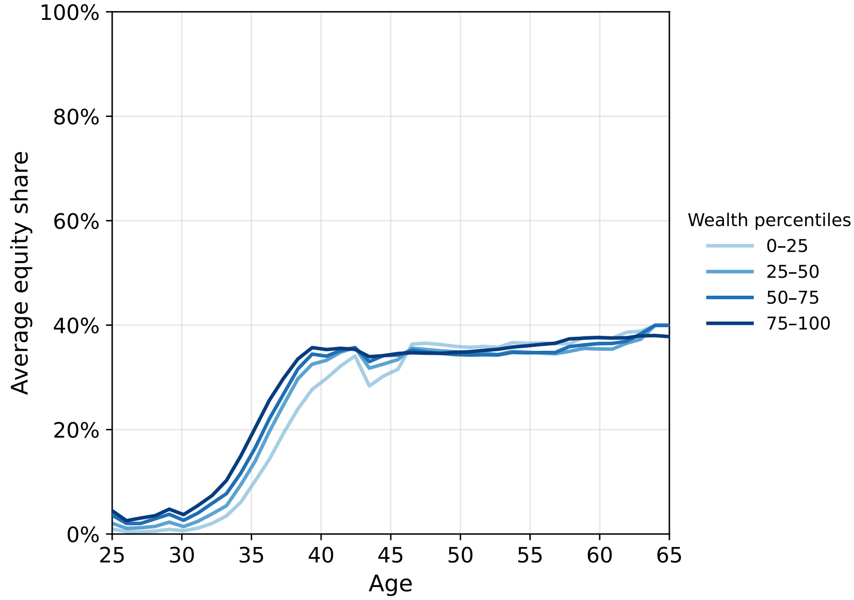
This figure represents the estimated average equity share of the model with only heterogeneous risks for different wealth percentile groups. The simulation is run 10,000 times. The initial wealth of agents is calibrated using the SCF waves from 1989 to 2022. Simulated households are ranked by their average wealth.

Figure 15
Heterogeneous risk and costs



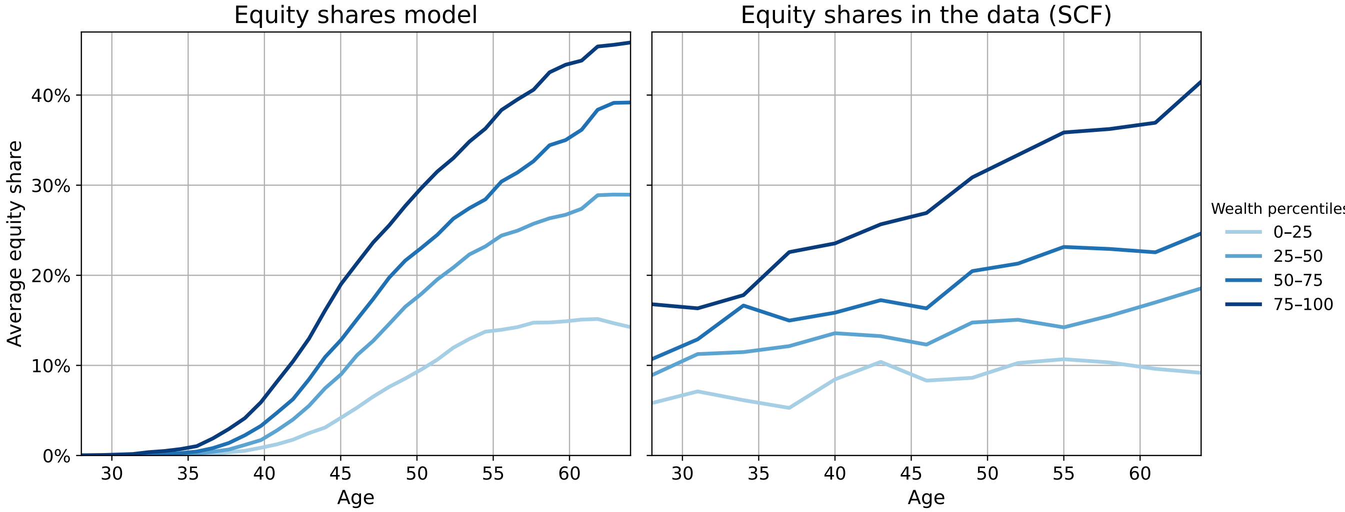
This figure represents the estimated average equity share of the model with heterogeneous risks and costs for different wealth percentile groups. The simulation is run 10,000 times. The initial wealth of agents is calibrated using the SCF waves from 1989 to 2022. Simulated households are ranked by their average wealth.

Figure 16
Heterogeneous return and costs



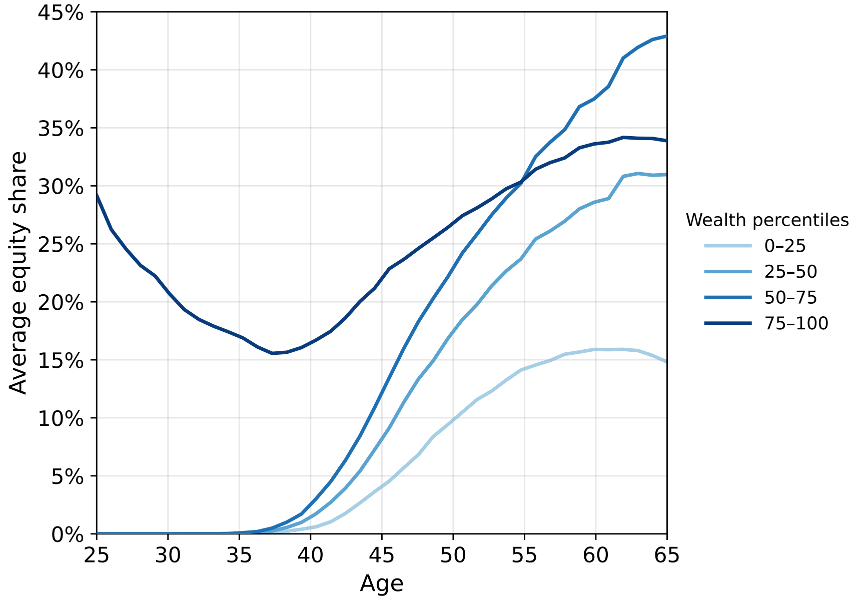
This figure represents the estimated average equity share of the model with heterogeneous return and costs for different wealth percentile groups. The simulation is run 10,000 times. The initial wealth of agents is calibrated using the SCF waves from 1989 to 2022. Simulated households are ranked by their average wealth.

Figure 17
Final specification with the SCF data



This figure represents the estimated average equity share of the model with both heterogeneous risks and participation costs for different wealth percentile groups and corresponding SCF data side by side. The simulation is run 10,000 times. The initial wealth of agents is calibrated using the SCF waves from 1989 to 2022. Simulated households are ranked by their average wealth.

Figure 18
Final specification with type dependence



This figure represents the estimated average equity share of the model with heterogeneous risks, returns and costs for different wealth percentile groups with the addition of returns from private wealth. The simulation is run 10,000 times. The initial wealth of agents is calibrated using the SCF waves from 1989 to 2022. Simulated households are ranked by their average wealth.

Tables

Table 1
Summary statistics

	Mean	Std Deviation	Observations
Whole sample			
Age	42.7	12.1	23,406
Wealth	330,091	1,152,002	23,406
Stock market participation	0.52		23,406
Equity share	0.15	0.22	23,406
Conditional equity share	0.25	0.25	14,353
Top 25 wealth percentiles			
Age	51.28	12.27	5,845
Wealth	1,315,396	3,220,575	5,845
Stock market participation	0.90		5,845
Equity share	0.27	0.31	5,845
Conditional equity share	0.30	0.32	5,822
Bottom 25 wealth percentiles			
Age	36.01	13.55	5,857
Wealth	13,801	11,807	5,857
Stock market participation	0.27		5,857
Equity share	0.07	0.22	5,857
Conditional equity share	0.27	0.26	1,524

This table shows the summary statistics of the SCF sample from 1989 to 2022. Wealth is recorded as *networth* in SCF and represents total value of assets minus debt in dollar amount. Sample is restricted to households that are aged between 26 to 64 and have a net worth greater than $1,000, who are not business owners and have wage income (*wageinc*) greater than $100. Household weights are normalised to give each survey year an equal weight.

Table 2
Calibrated parameters of the baseline model

	Value	Source
Preferences		
Relative risk aversion (γ)	6	Catherine (2022)
EIS (ψ)	0.5	Gomes (2020)
Time discount rate (β)	0.97	Gomes (2020)
Labor income process		
St. dev. of transitory shock (σ_z)	0.10	Gomes (2020)
St. dev. of permanent shock (σ_u)	0.10	Gomes (2020)
Corr. btw. perm. shock and stock returns (ρ)	0.15	Gomes and Michaelides (2005)
Financial market		
Risk-free rate (r_f)	0.015	Gomes (2020)
Risk premium (μ)	0.04	Gomes and Michaelides (2005)
St. dev. of stock returns	0.18	Gomes and Michaelides (2005)
Tail event probability	0.02	Fagereng *et al.* (2017)
Return at tail event	0.51	Fagereng *et al.* (2017)

This table represents calibrated values for the benchmark model. Survival probabilities are taken from Cocco *et al.* (2005).

A Appendix

A.1 SCF Data

In this paper, the Survey of Consumer Finances (SCF) is used from the 1989 to 2022 survey waves. SCF is a triannual survey that collects information on household wealth and portfolios. SCF overweights the wealthy; the reported averages throughout the paper are weighted averages using survey weights to make the results representative of all US households. For constructing equity share, SCF variables *equity* and *networth* are used.

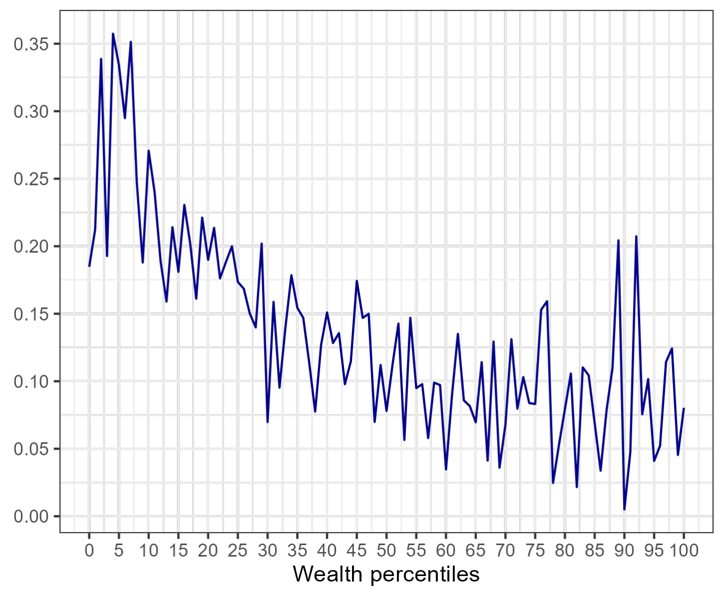
- *Equity:* directly-held stocks, stock mutual funds, 1/2 value of combination mutual funds, IRAs (full value if mostly invested in stock, 1/2 value if invested between bonds, 1/3 value if invested between bonds and money market), annuities (full value if mostly invested in stock, 1/2 value if invested between bonds), retirement and saving accounts invested in stocks.[12]
- *Wealth:* variable *networth* is used which represents total net worth, assets of the households including housing wealth minus debt.
- *Labor income:* variable *wageinc* is used for calculating labor income before retirement age 65.
- Wealth to income ratio is calculated by dividing $networth$ of the household by the average $wageinc$ in that survey year for households under age 65. For households that are above 65, pension income (coded as $pension$ in the SCF documentation) is used.
- Equity share is calculated as $equity/networth$.

[12] Information taken from https://www.federalreserve.gov/econres/files/bulletin.macro.txt.

A.2 Financial literacy

In the SCF, respondents are asked: ”Do you think that the following statement is true or false: buying a single company’s stock usually provides a safer return than a stock mutual fund?”. The sample is restricted to only include equity participants who answer the question with true or false. The sample is sorted by their $networth$ for each survey year, and wealth percentiles are constructed. Figure A.19 shows the weighted average of answering the question incorrectly.

Figure A.19
Answering equity risk question incorrectly

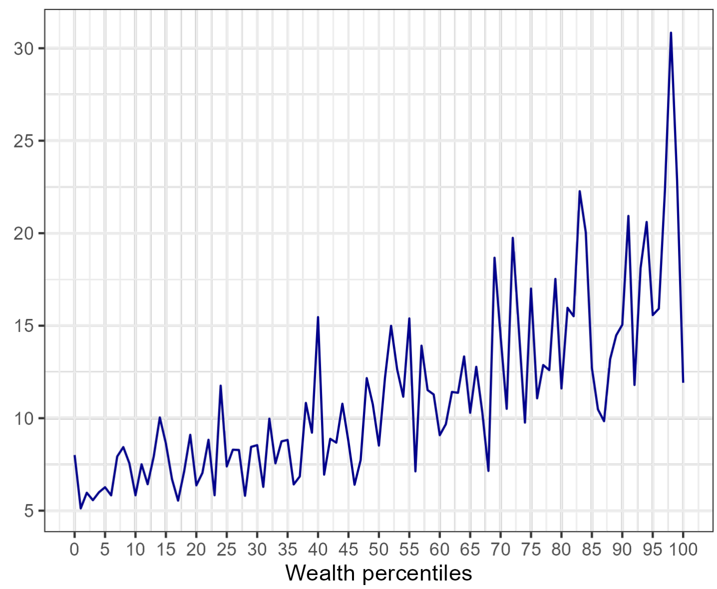


Values show the weighted average number of trades in the SCF from 1989 to 2022 using household weights. For each survey year, *networth* is used to find wealth percentiles.

A.3 Number of trades

In the SCF, respondents are asked: ”Over the past year, about how many times did you buy or sell stocks or other securities through a brokerage?”. Values are coded as $ntrad$ in the SCF documentation. The sample is restricted to only include equity participants who answer the question with more than zero trades. Households that reported more than 1000 trades in a year are removed from the sample. The sample is sorted by their $networth$ for each survey year, and wealth percentiles are constructed.

Figure A.20
Number of trades



Values show the weighted average number of trades in the SCF from 1989 to 2022 using household weights. For each survey year, *networth* is used to find wealth percentiles.

A.4 Calculating stock returns

For calculating stock returns, households that do not have direct stock holdings or mutual funds are removed from the sample.[13] Total number of observations after removing households that do not have direct stock holdings or mutual funds is 11,617.[14] Equation 20 shows the calculation of stock returns. The total value of directly held stocks and mutual funds is shown as *Equity* in Equation 20. SCF is conducted every

[13]Total direct stock holdings are recorded as *stocks* and total directly held mutual funds are recorded as *nmmf* in the SCF.

[14]Sample includes business owners and is restricted to ages from 25 to 82.

three years and dividend income is reported for the preceding year. Therefore, following Kartashova (2014), Moskowitz and Vissing-Jørgensen (2002) and Xavier (2021), I multiply the reported dividend income by three.

$$R = \left(\frac{Equity_{i,t} + 3 \times Dividend_{i,t}}{Equity_{i,t} - Capitalgains_{i,t-3:t}} \right)^{\frac{1}{3}} \tag{20}$$

$$R_{i,t} = (R - 1) \times 100 \tag{21}$$

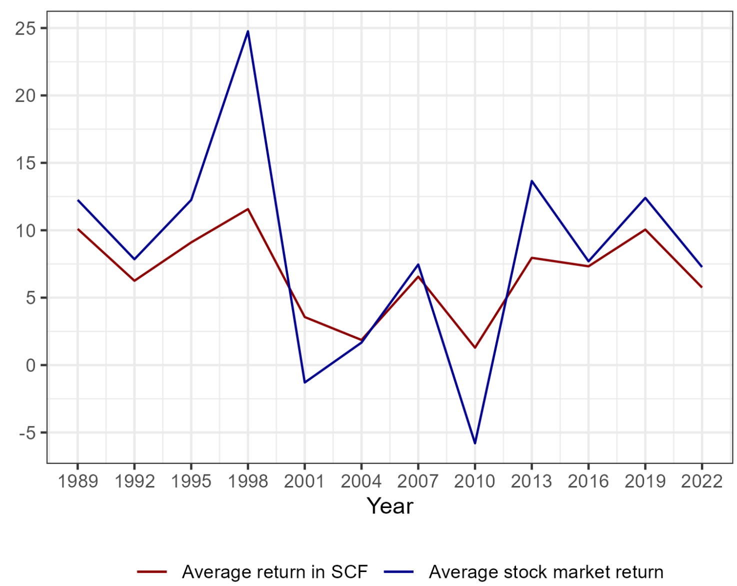
In the SCF, first households are asked to report the total value of their asset holdings in directly held stocks and mutual funds. Next, they are asked: ”Overall, has there been a gain or loss in the value of stock/fund since you obtained it?”. I select the sample that only gives an answer to this question.[15] SCF doesn't collect information on the stock obtaining time. Therefore, I assume that respondents hold it for n years, then I estimate n by trying to make the SCF return fit best with the realised gains of the stock market over the years between 1989 to 2022. Results show that the assumption of holding period of 3 years fits the data best with macroeconomic observations, which also aligns with the fact that the SCF is triannual. Robustness for return estimation is given in Appendix A.5.

Figure A.21 shows the comparison between the average stock return from the US stock market and SCF return estimates. Triannual average stock market return is calculated using Shiller (2015) data.[16] It is observed that even though return estimates can capture the overall trend, return estimates from the SCF are more centred around the mean than the actual stock market returns. The reason for this result may be because of the survey design, respondents give an average estimate of their gain/loss over the three years. One way to test the average return estimates is to use macroeconomic US stock market returns as a proxy for time effects. In an OLS regression with age and cohort dummies, adding the average US stock market return as a proxy for the time

[15] Respondents who answered this question as there is no change are recorded as having zero capital gains.

[16] http://www.econ.yale.edu/ shiller/data.htm.

Figure A.21
Average return estimates and US stock market returns



This figure shows average estimated stock returns from the SCF (red line) and average stock returns in the US (blue line) from Shiller (2015) data taken from http://www.econ.yale.edu/ shiller/data.htm.

effects shows that the time proxy is highly statistically significant, with slight changes in the coefficient estimates of age and cohort dummies.

A.5 Robustness

The number of years that households hold the stock is denoted as n. Assuming that households enter the stock market at age 24 and stay until the survey date t, n is equal to $age_{i,t} - 24$. With this choice of n, the average capital gain is equal to approximately 1%, which is much lower than the actual observed value, indicating that n should be higher. Table A.3 presents different choices of n with confidence intervals. Results show that $n = 3$ is the best estimate with 5% of returns attributed to capital gains, aligning most closely with macroeconomic observations. This is further supported by the fact that the SCF is a triennial survey, implying that the assumption aligns with respondents acquiring the stock during the prior survey year.

Table A.3
Estimating n

Choice of n	**Lower CI (%)**	**Average (%)**	**Upper CI (%)**
2	5.49	5.89	6.23
3	4.46	4.78	5.09
4	2.13	2.34	2.55

Values show aggregate averages of SCF stock returns from capital gains between 1989 and 2022. Columns 2 and 4 represent 5% and 95% confidence intervals, respectively.

A constant n is assumed for every individual for capital gains in the analysis. One of the concerns that may be raised would be that different age groups or cohorts might have different stock holding periods, and taking this into account will cause a decline in the wealth effect. Mainly, it may be argued that a higher choice of n for older cohorts, or older age groups might result in a diminished wealth effect, because older age and cohorts are at the top of the wealth percentiles and assuming that their stock holding period is higher will reduce their returns from capital gains.

Results show that a higher choice of n for older cohorts and age groups still results in a persistent wealth effect. The reason is presented in Figures A.22, A.23 and A.24. Figures show how returns from capital gains evolve with wealth for different cohort, age and time groups, respectively. It is observed that within each group, categorized by age, cohort, and time, the wealth effect remains persistent, indicating that returns from capital gains consistently increase with wealth.

A different choice of n for each group adjusts the average return from capital gains for that group, meaning that the choice of n only changes the average level of returns from capital gains, but the increasing pattern of wealth is not affected. Therefore, assuming different n values for different age, cohort or time groups doesn't result in deterioration of the wealth effect at returns from capital gains.

Table A.4 shows the summary statistics of the calculated returns for different cohorts. Calculated return values align with macroeconomic observations of the US

Figure A.22
Wealth effect at cohort groups

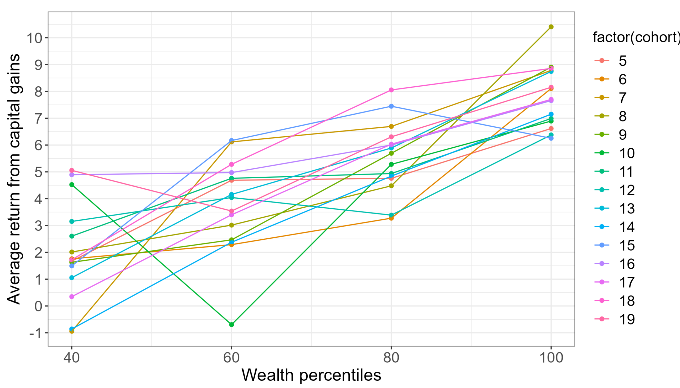


Figure A.23
Wealth effect at age groups

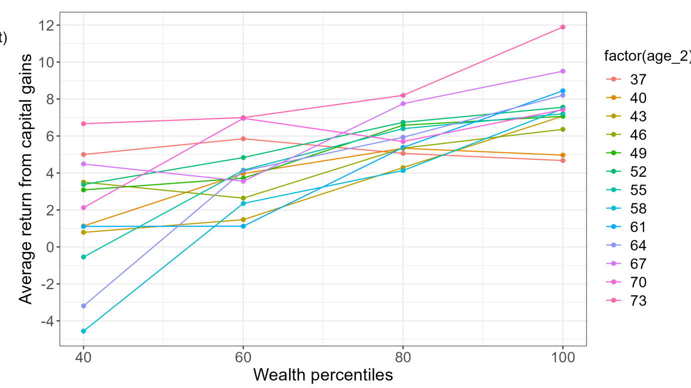


Figure A.24
Wealth effect at time groups

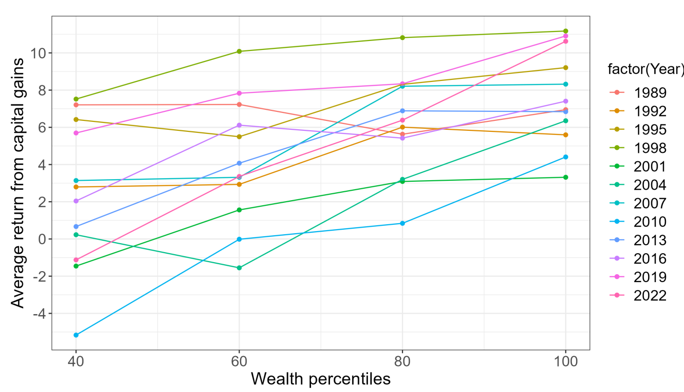
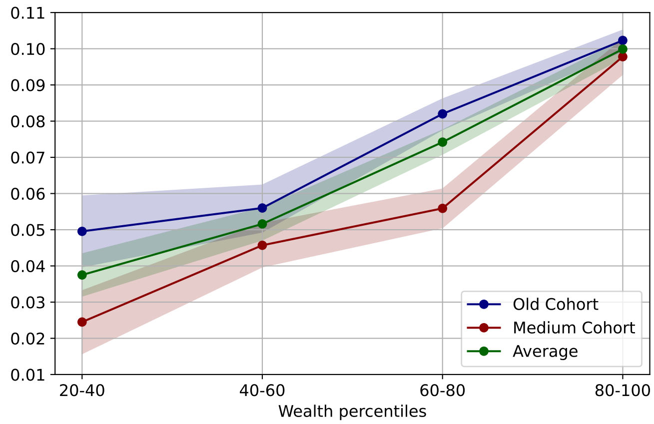


Table A.4
Summary statistics

	Total return (%)	**Dividend yield (%)**	**Capital gains (%)**
Old cohort	7.60	2.40	5.48
Medium cohort	5.04	1.48	3.74
Young cohort	4.69	0.93	3.83
Average	6.53	1.99	4.77

Total number of observations is 11,617. Medium cohort has approximately 3,700 observations, old cohort 7,400, and young cohort 500.

Figure A.25
Wealth and cohort effects at total return



stock market for all cohorts. Figure A.25 divides the sample into three main cohorts; old, medium and young and demonstrates that the wealth effect is persistent for all cohorts.

A.6 Life-cycle profiles

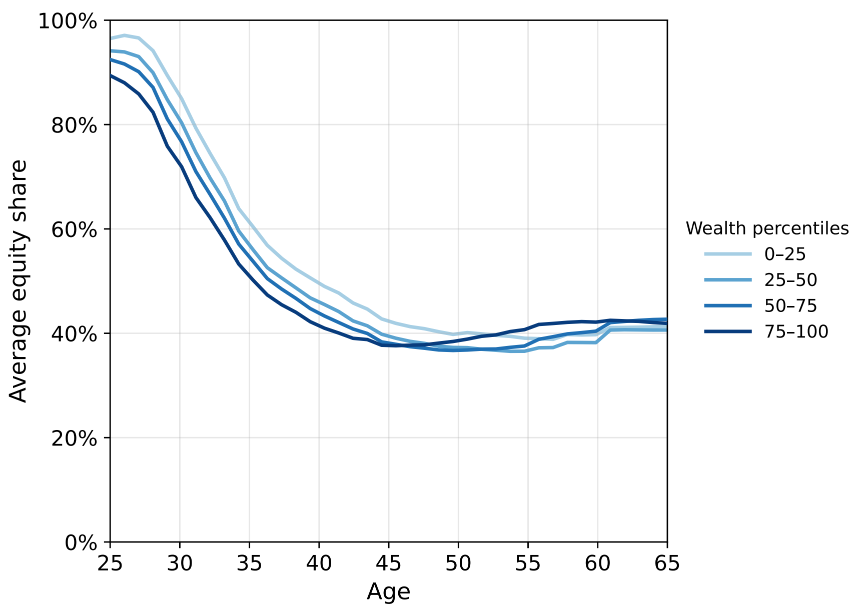
Ameriks and Zeldes (2004) show the perfect multicollinearity between age, time and cohort (age = time - cohort), which makes it difficult to disentangle the three effects and identify the age effect separately. First, following their approach, I construct three-year age and cohort groups between ages 26 and 64. The age groups are: 26-28, 29-31, and so on, until 62-64, with corresponding cohort groups. In addition, household weights in each survey wave are normalised so that all waves have an equal weight.

$$y = A\alpha + C\gamma + Y\psi + c + \varepsilon \tag{22}$$

The identification of the age effect is achieved by adopting the methodology of Deaton and Paxson (1994). Their strategy eliminates the multicollinearity by imposing two constraints on the time dummies: that they sum to zero and are orthogonal to a time trend. I then implement the regression specified in Equation 22, where A, C, and Y represent dummy variables for age group, cohort group, and year. The life-cycle profile of equity shares is then constructed by taking the estimated age effect for each group and adding the corresponding average cohort effect along with level c.

A.7 Heterogeneous risks and fixed per-period costs

Figure A.26
Heterogeneous risk and fixed cost



This figure represents the estimated average equity share of the model with heterogeneous risks and a fixed cost of 0.8% of the average yearly labor income. The simulation is run 10,000 times. The initial wealth of agents is calibrated using the SCF waves from 1989 to 2022. Simulated households are ranked by their average wealth.

Section 6.6 shows that when portfolio monitoring costs fall with wealth, heterogeneity in equity risk reproduces the empirical fact that richer households devote a larger share of their portfolios to risky assets. In this section, I investigate whether the same pattern emerges when participation costs are instead constant across wealth levels. Following the empirical estimate in Catherine (2022), I set the per-period cost equal to 0.8% of average annual labour income, while leaving all other parameters unchanged from the baseline calibration in Section 6.6. As Figure A.26 illustrates, a realistic fixed cost combined with heterogeneous risk is not sufficient to generate the steeper risky asset share observed among wealthier households.

A.8 Simulations

Each model specification is simulated 10,000 times. Agents' initial wealth is set using the SCF, matching the wealth-to-income ratios observed in the data. For the subsample of agents aged 26–28, households are ranked by their wealth to income ratios from lowest to highest to create percentiles. Initial wealth of each simulated decile is given in Table A.5.

Table A.5
Calibrated initial wealth

Percentiles	**Wealth to income ratios**
0 - 10	0.018
10 - 20	0.040
20 - 30	0.066
30 - 40	0.101
40 - 50	0.164
50 - 60	0.252
60 - 70	0.423
70 - 80	0.814
80 - 90	1.227
90 - 100	3.199

Values are calibrated from the SCF between 1989 and 2022. The sample is restricted to ages between 26 and 28. Income represents average labor income in the corresponding survey year. Wealth is net worth of the household in the corresponding survey year, coded as *networth* in the SCF.

A.9 Model solution

The state and control variables in the model are discretized as follows:

- Scaled wealth: Normalized by average labor income, scaled wealth ranges from 0.25 to 40. The grid consists of 51 points, uniformly distributed in logarithmic space between these endpoints.
- Income shocks: Both transitory and permanent income shocks are represented by 5 values, equally spaced between minus two and plus two standard deviations.
- Equity share: The share of wealth allocated to equities is discretized into 11 equally spaced points between 0 and 1.

The model is solved by dynamic programming using backward induction. First, the agent receives income with a permanent and transitory income shock, then decides how much to consume and how to invest remaining wealth between the risky and the risk-free asset. Shocks to equity are realized. Lastly, the agent receives a return from her investment in the previous period.

A.10 Model estimation

In the model calibrations, relative risk aversion is set at 6, the time discount rate is set at 0.97, with the highest standard deviation of stock returns equal to 0.21 and the highest per-period portfolio monitoring cost set at 1.4% of the average annual labor income. As shown in Section 6.6, these parameter values enable the model to closely align with observed data. However, in an ideal setting, the simulated method of moments (SMM) would be employed to estimate these parameters, a methodology that will be implemented in this paper.

Firstly, after running 10,000 model simulations, the selected moments are calculated to match the model estimates with the data observations. Given the model parameter θ for each moment, the simple absolute error is minimized.

$$\min_{\theta} ||m(x|\theta) - m(x)|| \tag{23}$$

Where x is the data and m is the chosen moment. In order to calculate the optimal parameters, first, the criterion function is calculated:

$$\hat{\theta}_{SMM} = \min_{\theta} e(\tilde{x})^T W e(\tilde{x}) \tag{24}$$

Where the initial weighting matrix W is equal to the identity matrix and e is the error amount, where weighting matrix controls how each moment is weighted. In order to find the optimal weighting matrix, first, the error $e(\tilde{x})$ is calculated. Then, its variance-covariance matrix is determined:

$$\Omega = \frac{1}{N} e(\tilde{x})^T e(\tilde{x}) \tag{25}$$

The second step involves setting the weighting matrix equal to Ω^{-1}. This procedure is iterated until the weighting matrix from the previous step (W) converges to the current W. The estimation results will be reported once available.