CME 2007 –Differential Equations Worksheet-II

Q.1/ A bacteria colony initially has mass μ_0=3.7x10^{-9}. After two hours the colony has mass μ=15x10^{-9}.

- **a)** Find the mass after 5 hours?
- **b)** Find the time it takes for the original mass of the colony to triple?

Q.2/ A metal plate that has been heated cools from 180^0F to 150^0F in 20 minutes when surrounded by air at a temperature of 60^0F.

- **a)** What will the temperature of the plate be after one hour of cooling?
- **b)** When will be the temperature of the plate reach 100^0F?

Q.3/Just before middaythe body of an apparent homicide victim is found in a room that is kept at a constant temperature of 70^0F. At 12 noon the temperature of the body is 80^0Fand 1 P.M. it is 75^0F. Assume that the temperature of the body at the time of death was 98.6^0F. What is the time of death?

Q.4/Solve the Bernoulli DE:$x\frac{dy}{dx} + 6y = 3xy^{\frac{4}{3}}$

Q.5/Solve IVP:$\frac{dy}{dx} - y = \frac{11}{8}e^{-\frac{x}{3}}, \qquad y(0) = -1$

Q.6/First, verify that the given function y(x) is a solution of the given DE, for any value of A. Then, solve for A, so that y(x) satisfies the given initial condition.

$$y' + 6y = 0, \qquad y(x) = Ae^{-6x}, \qquad y(4) = -1$$

Q.7/Solve initial-value problem.

$$\frac{dy}{dx} = 4x^3y - y, \qquad y(1) = -3$$

Q.8/A 4_lb roast, initially at 50^0F is placed in a 375^0F oven at 5.00 P.M. After 75 min. it is found that the temperature T(t) of the roast is 125^0F. When will be the roast be 150^0F (medium rate)?

Q.9/A certain city had a population of 25000 in 1960 and population of 30000 in 1970. Assume that its population will continue to grow exponentially at a constant rate. What population can its city planners expect in the year 2000?

Q.10/Solve $y' = 2(x-1)e^{-y}, \quad y(1) = 2.$

Q.11/Consider the equation:

$$e^x dx + (xe^x - siny)dy = 0$$

a) Show that whether it is exact or not.
b) Find a general solution.

Q.12/An integrating factor for the differential equation:

$\frac{dy}{dx} = x^2y + sinx$ is $I(x) = e^{\int x^2 dx}$

Q.13/Find the orthogonal trajectories to the given family $y = \frac{c}{x}$

Q.14/At 4 P.M. a hot coal was pulled out of a furnace and allowed to cool at room temperature 75^0F. If after 10 minutes in the temperature of the coal was 415^0F, and after 20 minutes its temperature was 347^0F, find the following:

a) The temperature of the furnace.
b) The time when the temperature of the coal was 100^0F.

Q.15/Solve the IVP.

$$y' - y = e^{2x}, \qquad y(0) = 3$$

Q.16/Solve DE (First order DE):

$$x\frac{dy}{dx} + 2y = cosx, \qquad x > 0$$

Q.17/Solve IVP:

$$\frac{dy}{dx} = 4x^3y - y, \qquad y(1) = -3$$

Q.18/An object whose temperature is 615^0F is placed in a room whose temperature is 75^0F. At 4 P.M. (t=0) the temperature of the object is 135^0F, whereas an hour later its temperature is 95^0F. At what time was the object placed in the room?

Q.19/An animal sanctuary had an initial population of 50 animals. After 2 years the population was 62, while after 4 years it was 16. Using the logistic population model, determine the carrying capacity and the number of animals in the sanctuary after 20 years?

Q.20/Find the general solution of $\frac{dx}{dt} - xsint = 2te^{-Cost}$ and the particular solution that satisfies x(0)=1.

Q.21/Solve exact DE:

$$(1 + ye^{xy})dx + (2y + xe^{xy})dy = 0$$

Q.22/$xy\left(\frac{dy}{dx}\right) = y^2 + x\sqrt{4x^2 + y^2}$

Q.23/Suppose that at time t = 0.3 alligators in a lake with population c = 10 alligators have been infected. After 1 month the number of P(t) of alligators that have been infected has increased to P(1)=6. Assuming that $\mathrm{P(t)} = \frac{cP_0}{(P_0 + (\mathrm{c} - P_0)\mathrm{e}^{-\mathrm{rt}})}$ satisfies the logistic equation. What will the percentage of the infected alligators population be in time 2 months?

Q.24/Show that the equation $xy' + y = e^{-x}$ has a general solution as $\mathrm{y} = \frac{\mathrm{c}-\mathrm{e}^{-\mathrm{x}}}{\mathrm{x}}$

Q.25/Find all solution to the equations:

a) $\frac{dy}{dx} + \frac{4}{x}y = 3x^2$.

b) Find the unique solution that satisfies y(1) = 2.

Q.26/Solve the exact DE:$(6xy - y^3)dx + (4y + 3x^2 - 3xy^2)dy = 0$

Q.27/Solve DE (change of variable):

$$2xydy - \left(x^2 e^{-\frac{y^2}{x^2}} + 2y^2\right)dx = 0$$

Q.28/Solve the following equation by applying Bernoulli Equation:

$$x\frac{dy}{dx} + 6x = 3xy^{\frac{4}{3}} \qquad \text{(Hint: } e^{-2lnx}\text{)}$$

Q.29/Scientist have observed that a small colony of penguins on a remote Antarctic Island obeys the population grown low $(\mathrm{P}(t) = P_0 e^{kt})$. There were 2000 penguins initially and 3000 penguins 4 years later.

a) How many penguins will there be after 10 years?

b) How long will it take for the number of penguins to double?