CME4422, Graph Theory Midterm Exam

Name, Surname:

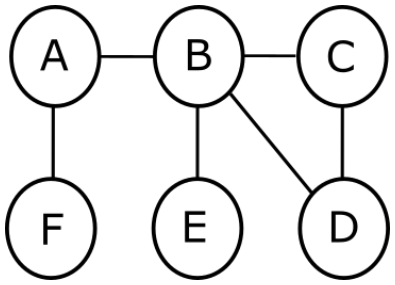
Student No :

Q1	Q2	Q3	Q4	Q5	Q6	Q7	Total

1. Give an example to a both directed and weighted graph in one sentence. (10 pts)

Exchange of gold: a person can buy gold from 395TL, but sell it for 410TL, also banks will buy and sell at different rates, so there is a direction(buy/sell) and a weight(price).

2. Find the bridges and hub in the graph below. Circle on the figure below. (20 pts)



B is a hub, AF, AB and BE are bridges.

3. For the graph above:
   a. Find the degrees of all the nodes. (3 pts)

deg(A):2, deg(B):4, deg(C) =2, deg(D) = 2, deg(E)=1, deg(F)=1

   b. What is the most central node according to Degree Centrality measure? (Don't use normalization). (2 pts)

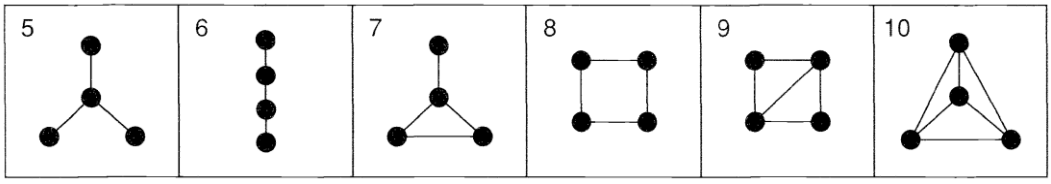
B.

   c. Eccentricity of a node is the maximum of the shortest paths from that node to every other node. Find the Eccentricity of A. (5 pts)

max(AB:1,AC:2,AD:2,AE:2,AF:1)=2

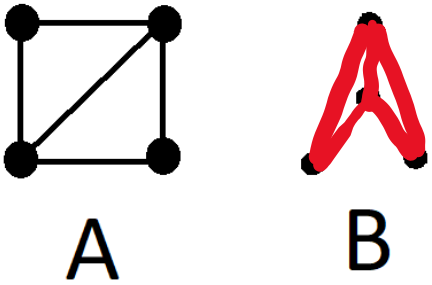
4. Reciprocity measure is only defined for (directed/undirected) graphs. (5 pts)

5. In the 4-vertex unlabelled graphs below, find those which are bi-partite. (20 pts)



5,6,8

6. In the figure below, draw the edges on B so that A and B are isomorphic. (10 pts)



7. Alexander G. Bell runs the Bell Telephone Company. For any house that wants a phone connection, they lay telephone cables between that house and every other house. But this process is very costly. John, an engineer proposes that they continue connecting all the houses together as before, like a complete graph. Another engineer, Jane proposes that they build a central switching station where all houses can connect, like a star topology network. But John opposes this idea, telling Jane that building a switching station costs $5150. Alexander G. Bell calls you and asks this question: when are the costs of John's and Jane's proposal will be equal? (20 pts) After that number, which project is more cost-effective? (5 pts)
Hint: 10300 = 100x103. Assume distances between all the houses are the same and the distances between the houses and the switching station are the same.

We want to find John=Jane.

John: $n(n-1)/2$
Jane: $n+5150$

$n(n-1)/2 = n+5150$
$n^2-n = 2n+10300$
$n^2-3n-10300 = 0$
$(n+100)(n-103)=0$ so $n=103$. At 103 houses, both methods have the same cost. After that, at n=104: John(104)=104(103)/2=5356 and Jane(104)=104+5140 = 5244 so Jane's method is cheaper for all n>103.

Good Luck!