Centrality Measures for Simple Graphs

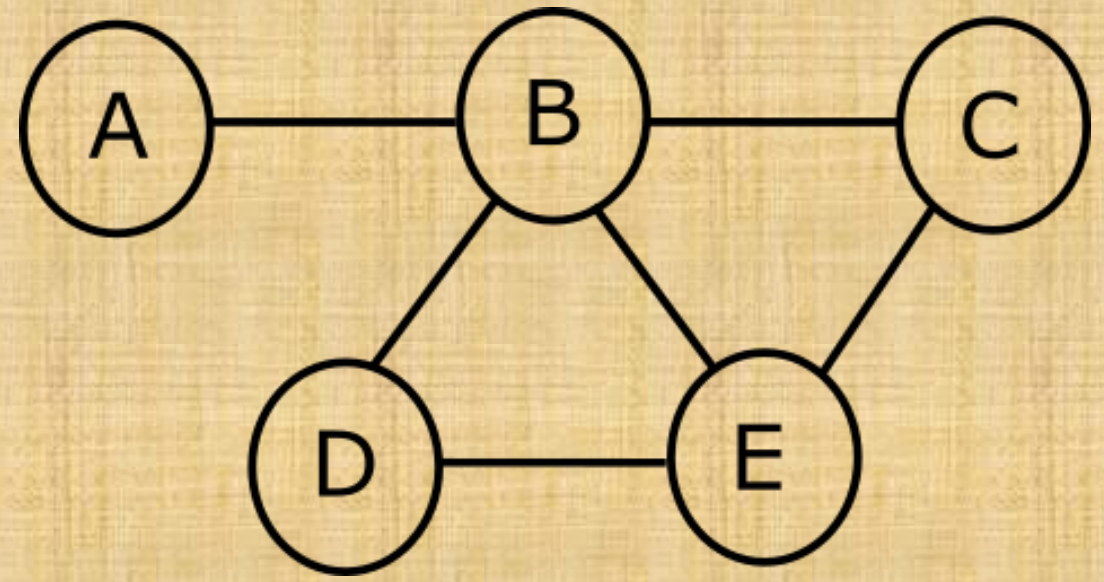
CME4422

Outline

- Degree Centrality
- Closeness Centrality
- Eccentricity Centrality
- Betweenness Centrality
- Eigenvector Centrality
- Other Centrality Measures

Degree Centrality

- Deg(A)=1
- Deg(B)=4
- Deg(C)=Deg(D)=2
- Deg(E)=3



According to degree centrality, degree=centrality score.

Normalization

- The degree centrality for each node is usually normalized by the following three methods:
  - Divide degree by max. Possible degree=# of nodes-1.
  - Divide by the highest degree.
  - Divide by sum of all degrees.

Divide degree by max. Possible degree=# of nodes-1

- Max. Possible degree: 5-1=4
- Deg(A)=1/4 = 0.25
- Deg(B)=4/4 = 1
- Deg(C)=Deg(D)=2/4=0.5
- Deg(E)=3/4=0.75

Divide by the highest degree.

- In our example the highest degree (4) is the maximum possible degree.
- The result is the same.

Divide by sum of all degrees.

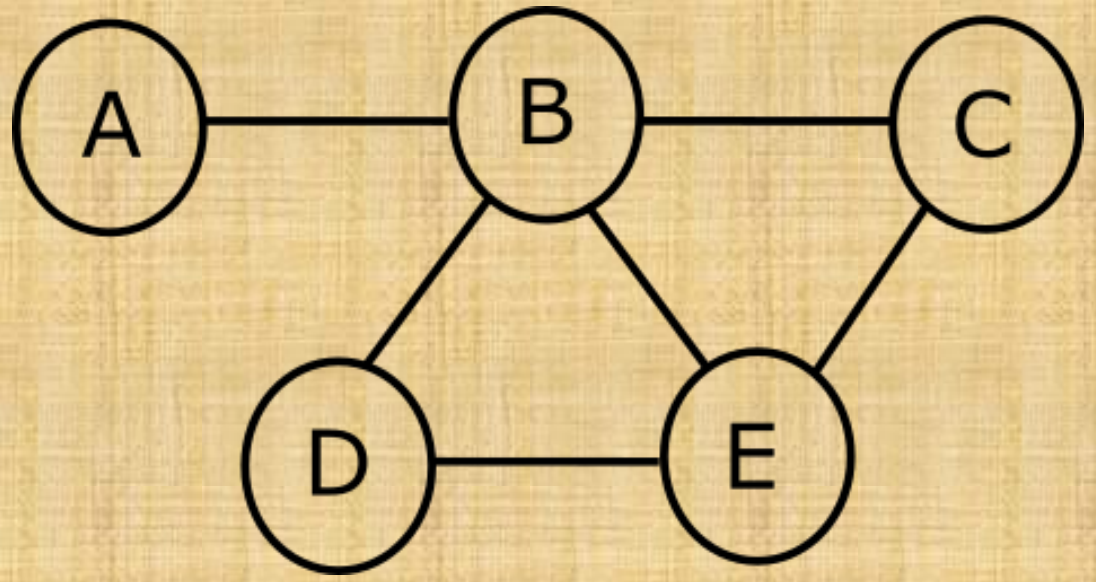
- Sum of all degrees is 1+2+2+3+4=12.
- Deg(A)=1/12
- Deg(B)=4/12
- Deg(C)=Deg(D)=2/12
- Deg(E)=3/12

Closeness Centrality

- Calculate the shortest path from a given node to every other node,
- Add them,
- Normalize by N-1 where N is the total # of nodes,
- Take the inverse of that value.

Example

	A	B	C	D	E	∑	∑/4	1/(∑/4)
A	-	1	2	2	2	7	1.75	0.57
B	1	-	1	1	1	4	1	1
C	2	1	-	2	1	6	1.5	0.67
D	2	1	2	-	1	6	1.5	0.67
E	2	1	1	1	-	5	1.25	0.8

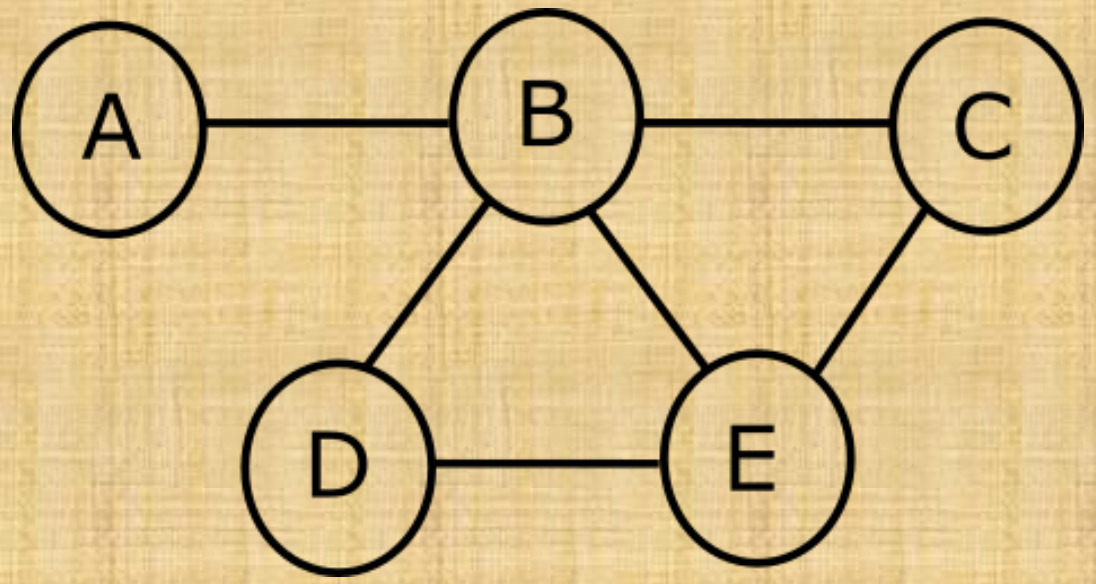


Eccentricity Centrality

- Eccentricity is the maximum of the shortest paths from a given node to every other node.

Example

	A	B	C	D	E	max	1/max
A	-	1	2	2	2	2	0.5
B	1	-	1	1	1	1	1
C	2	1	-	2	1	2	0.5
D	2	1	2	-	1	2	0.5
E	2	1	1	1	-	2	0.5

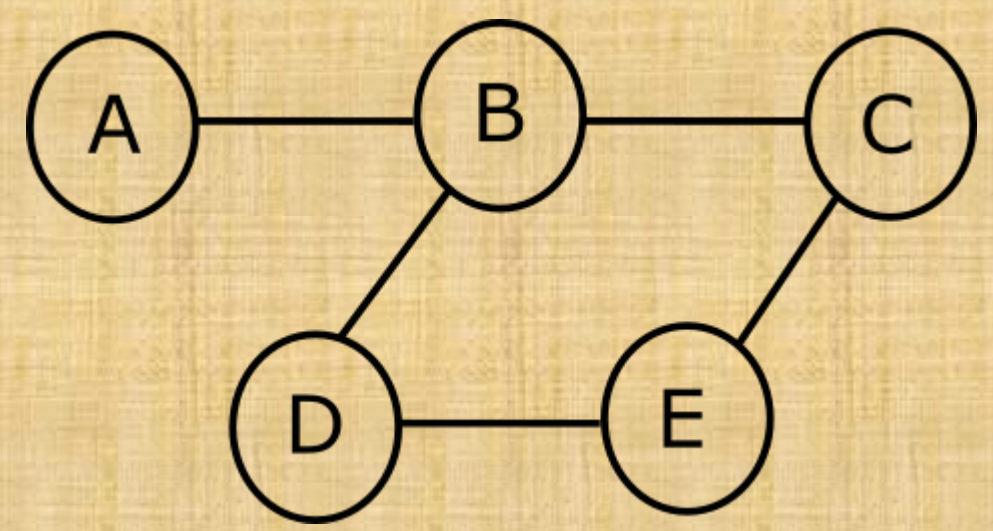


Betweenness Centrality

- For a given node, we are interested in the # of shortest paths that go through it.
- We are no longer interested in the path length!

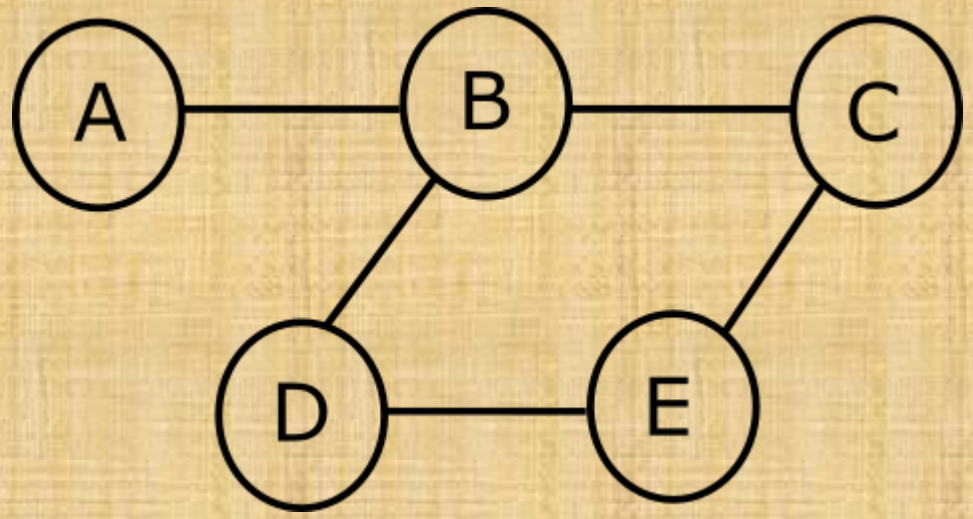
Example (1)

Let's consider node B. List all the shortest paths that go through it(except B):



Example (2)

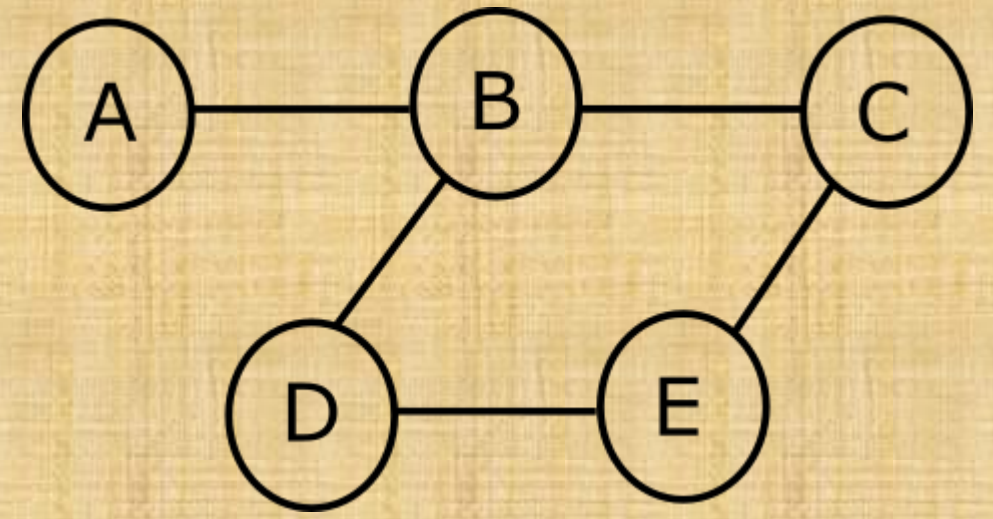
- A-C: A<u>B</u>C (1/1) C-D: C<u>B</u>D,CED (1/2)
- A-D: A<u>B</u>D (1/1) C-E: CE (0)
- A-E: A<u>B</u>CE,A<u>B</u>DE (2/2) D-E: DE (0)



$C_{bw}(B)=(1/1)+(1/1)+(1/2)+(2/2)$
$=3.5$

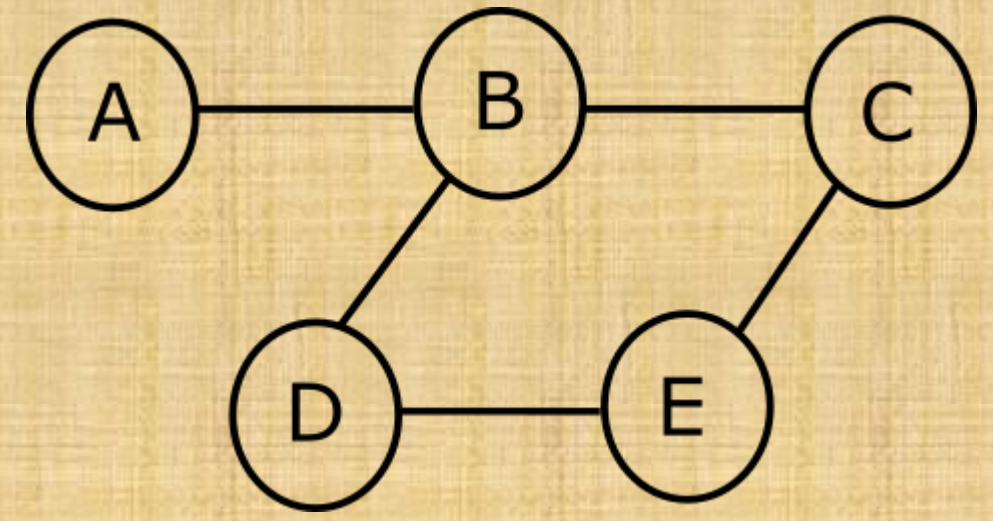
Example (3)

- For node *A*: B-C:BC; B-D:BD; B-E:BCE,BDE
- C-D:CBD,CED; C-E:CE; D-E:DE
- $C_{bw}(A)=0$



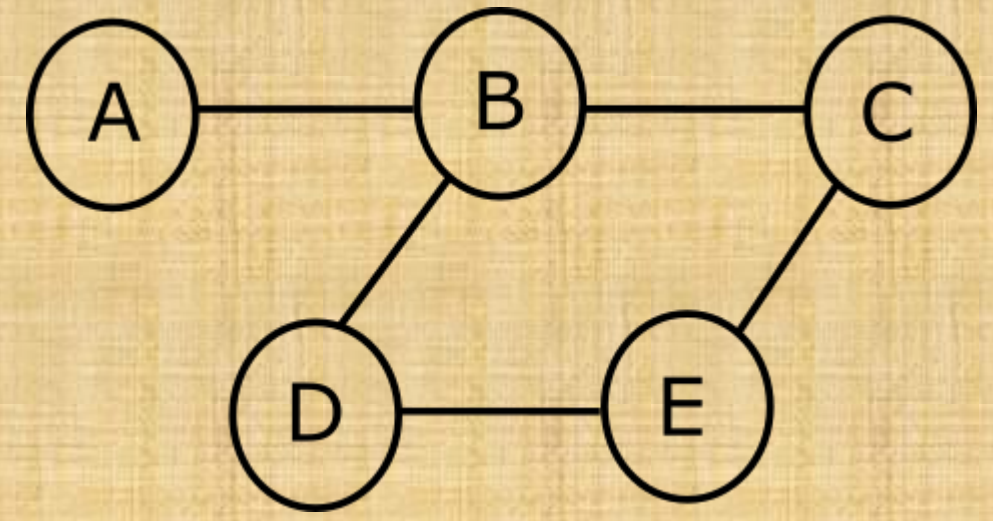
Example (4)

- For node C: A-B:AB; A-D:ABD;
- A-E:ABCE,ABDE; B-D:BD; B-E:BCE,BDE
- D-E:DE
- $C_{bw}(C)=(1/2)+(1/2) = 1$



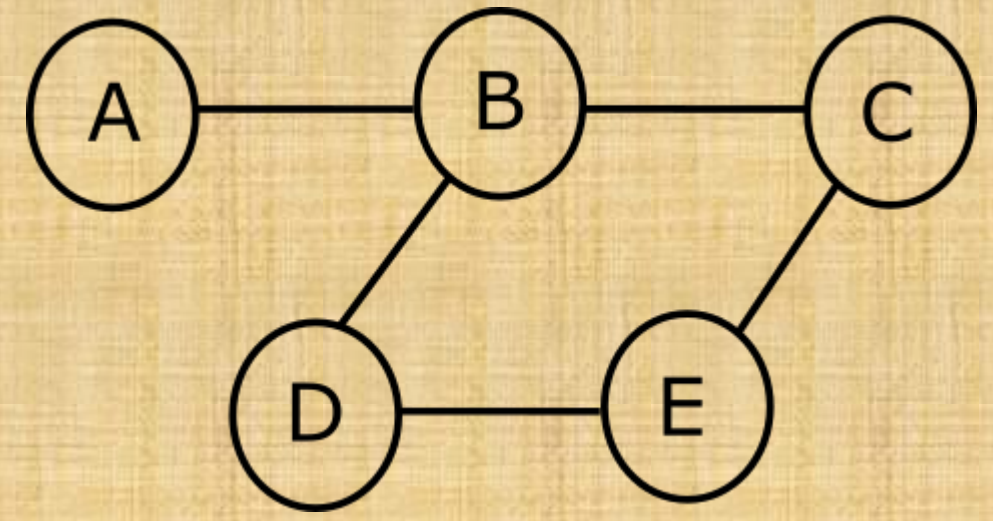
Example (5)

- For node D: A-B:AB; A-C:ABC;
- A-E:ABCE,ABDE; B-C:BC; B-E:BCE,BDE
- $C_{bw}(D)=(1/2)+(1/2) = 1$



Example (6)

- For node E: A-B:AB; A-C:ABC;
- A-D:ABD; B-C:BC; C-D: CBD,CED;
- $C_{bw}(E)=(1/2) = 0.5$



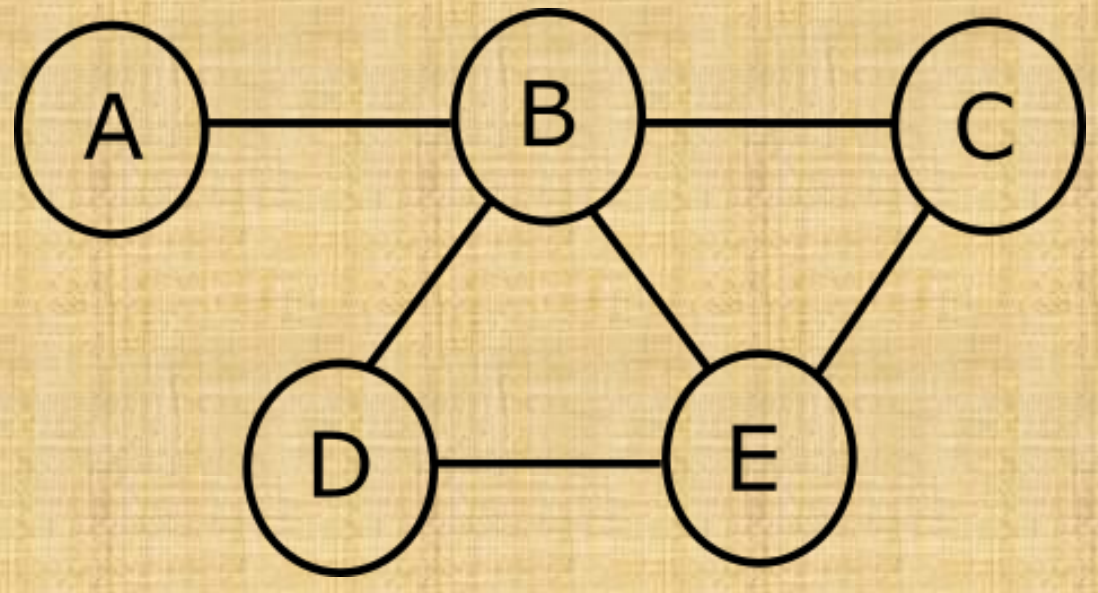
Eigenvector Centrality

- Form the adjacency matrix
- Find the maximum Eigenvalue
- Find the corresponding Eigenvector

Example

	A	B	C	D	E
A	0	1	0	0	0
B	1	0	1	1	1
C	0	1	0	0	1
D	0	1	0	0	1
E	0	1	1	1	0

$$\begin{bmatrix} 0.22 & A \\ 0.58 & B \\ 0.41 & C \\ 0.41 & D \\ 0.52 & E \end{bmatrix}$$



Max. Eigenvalue is 2.69. The corresponding Eigenvector is:

Other Centrality Measures

- Katz Centrality
- Pagerank Centrality