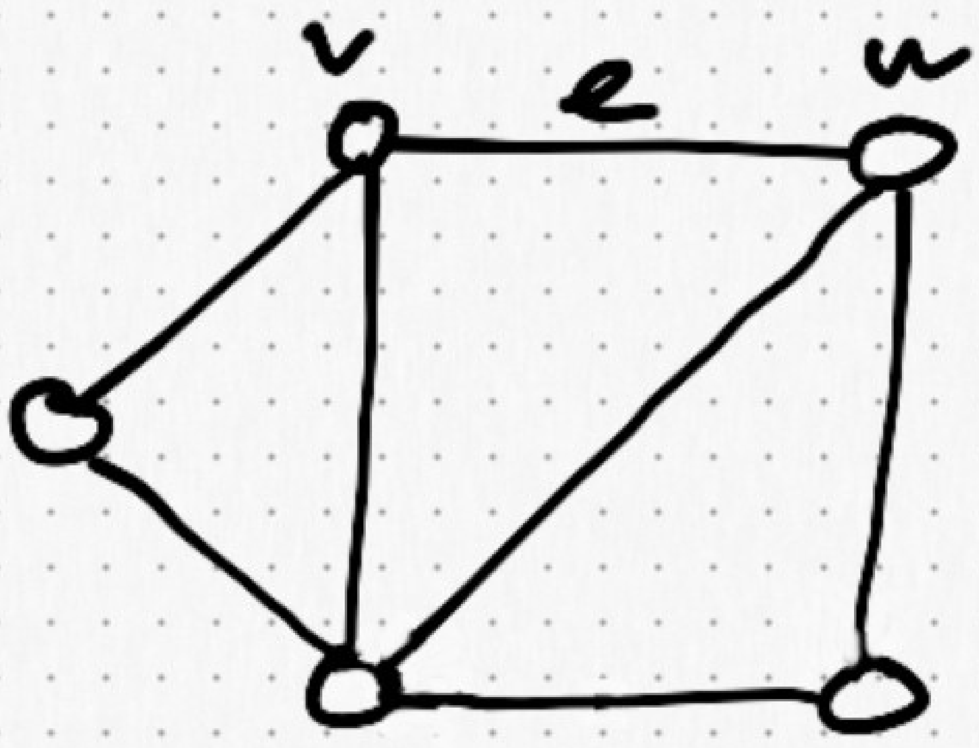
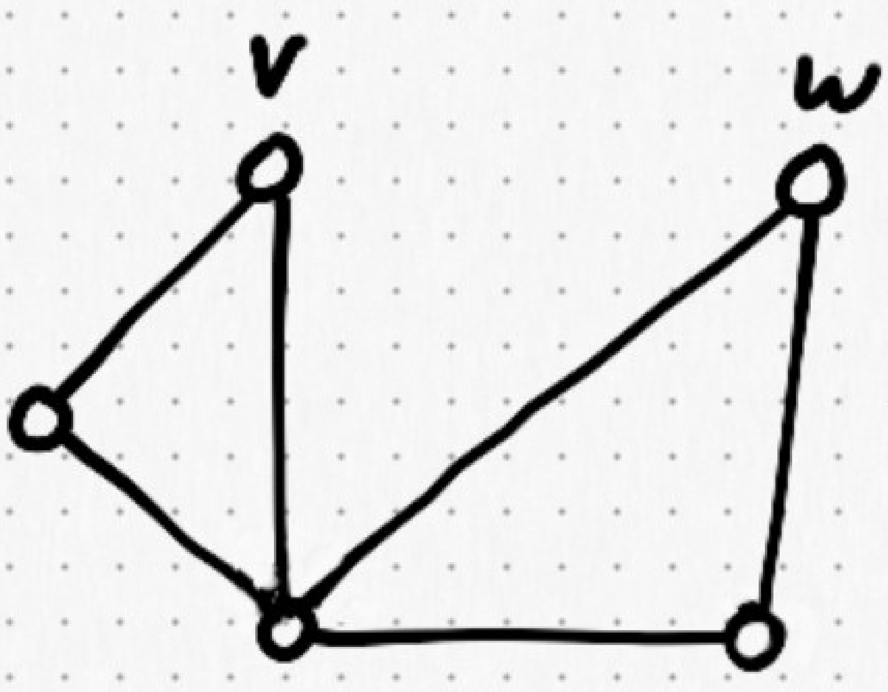
The Handshaking Lemma

- In any graph the sum of degrees is even.
- Each handshake takes an even # of hands.
- Each edge adds 2 to vertex degrees.
- There are an even # of odd degree nodes in any graph, or all degrees are even.

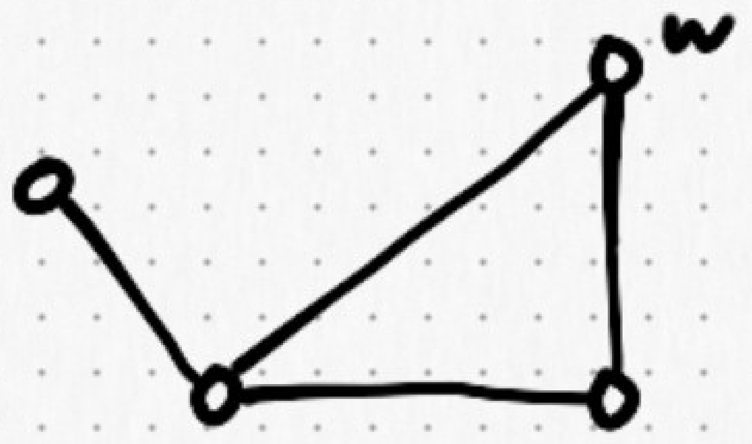
Deleting Edges and Vertices



G



G-e



G-v

← When a vertex is deleted all edges incident on that vertex are also deleted.

A subgraph is found by deleting edges and vertices.

Complete Graph

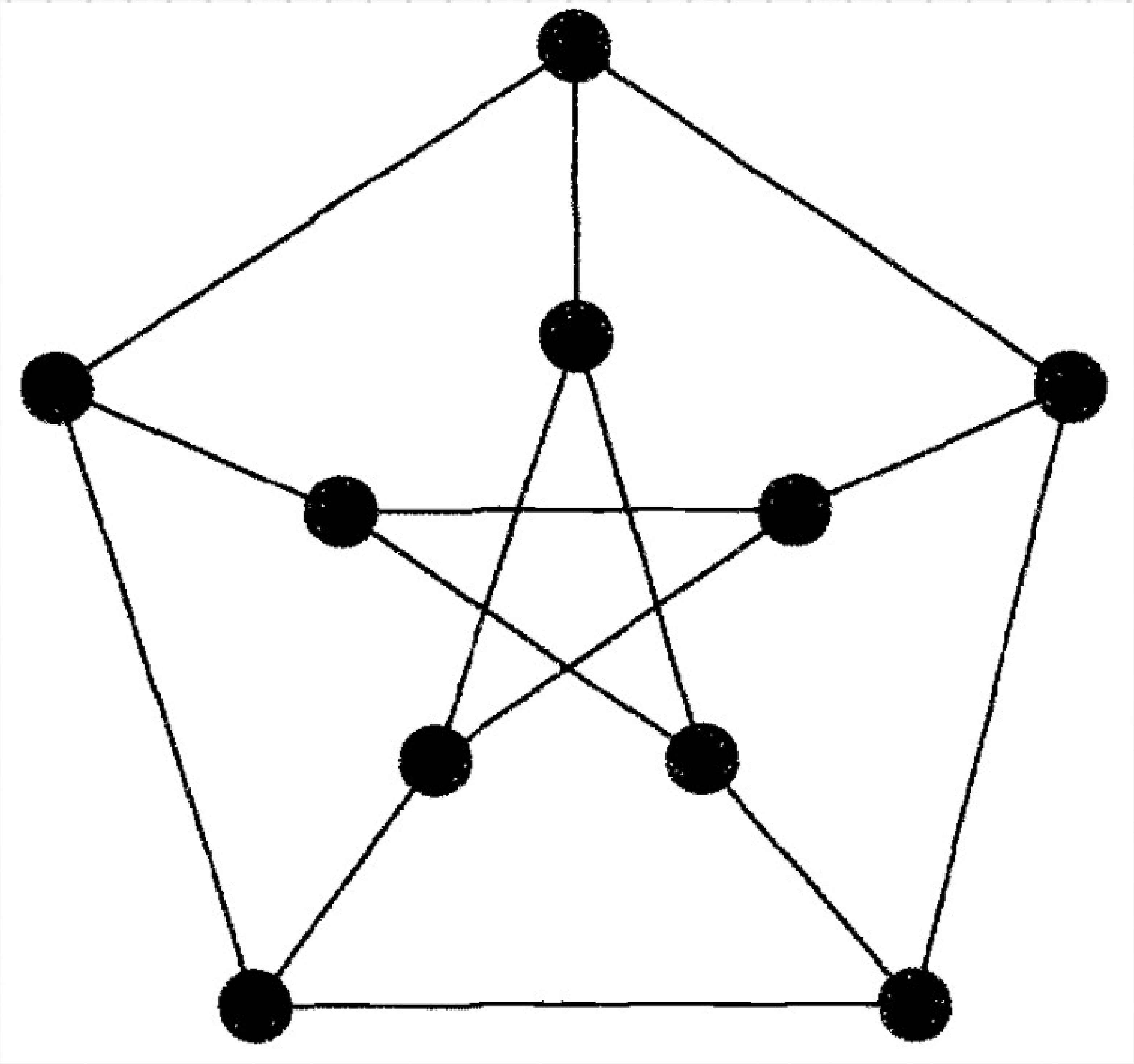
All nodes are connected.
Let $|V| = n$
A complete graph with n vertices has $n(n-1)/2$ undirected edges. $C\binom{n}{2}$.

A complete graph with n vertices is K_n
K_5 has $5 \cdot 4/2 = 10$ undirected edges.

Regular Graph

A graph where each vertex has degree r is called r-regular.

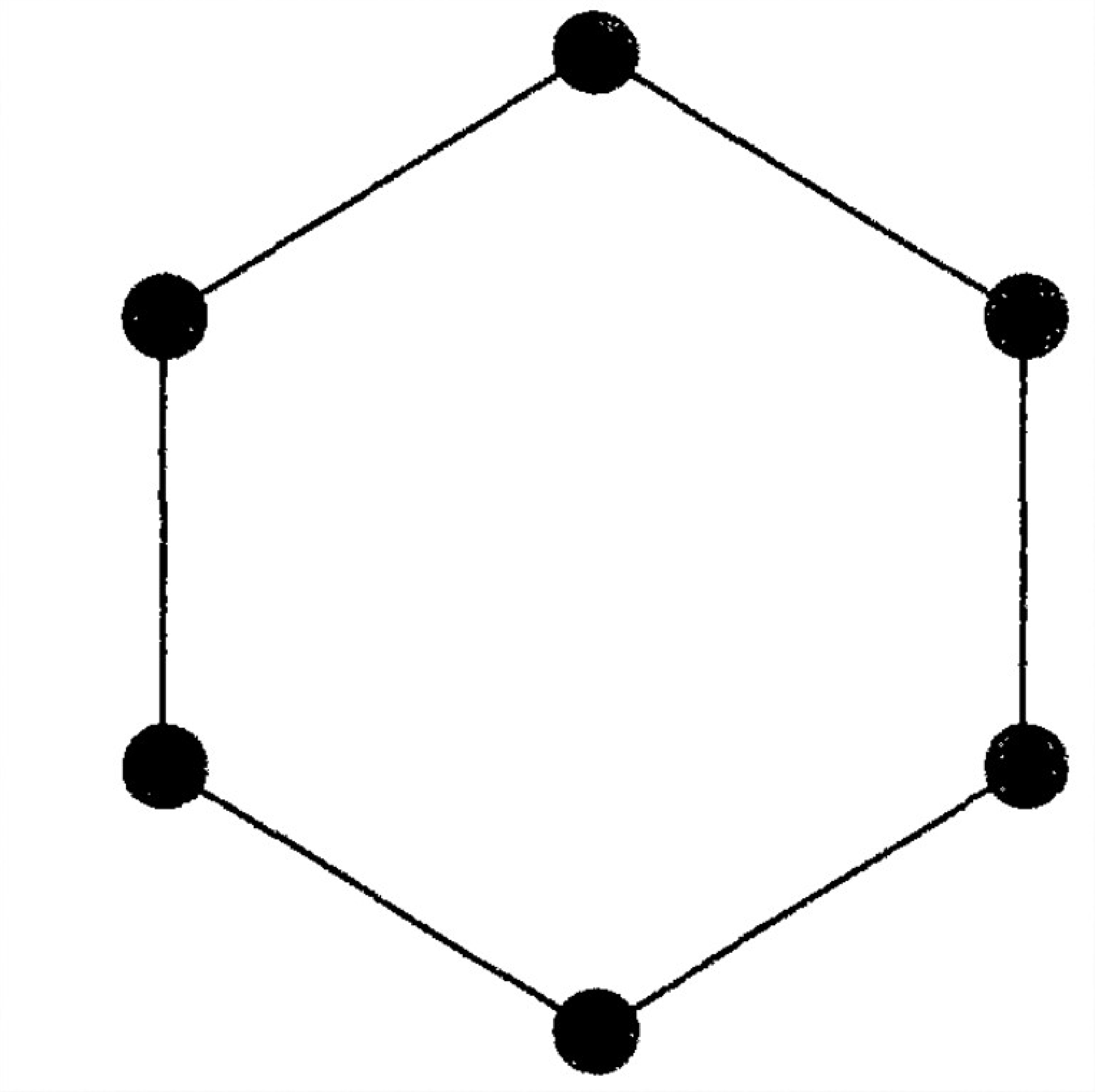
Example: Peterson's Graph



3-regular

Cycle Graph

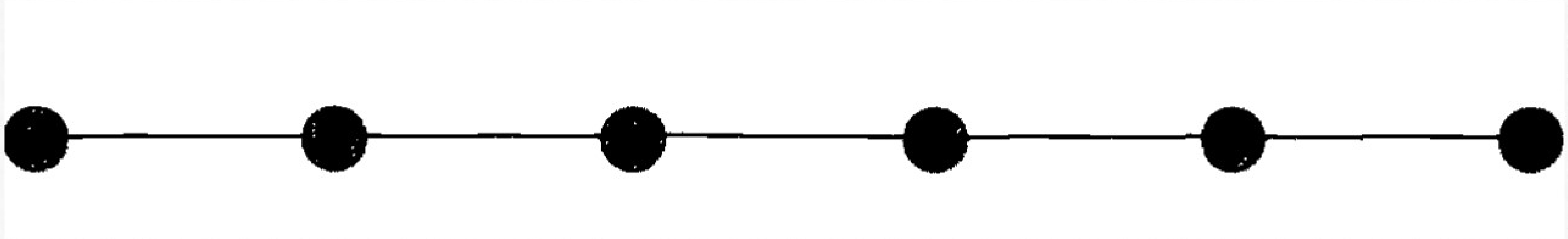
A connected 2-regular graph with n nodes is a cycle graph (C_n).



C_6

Path Graph

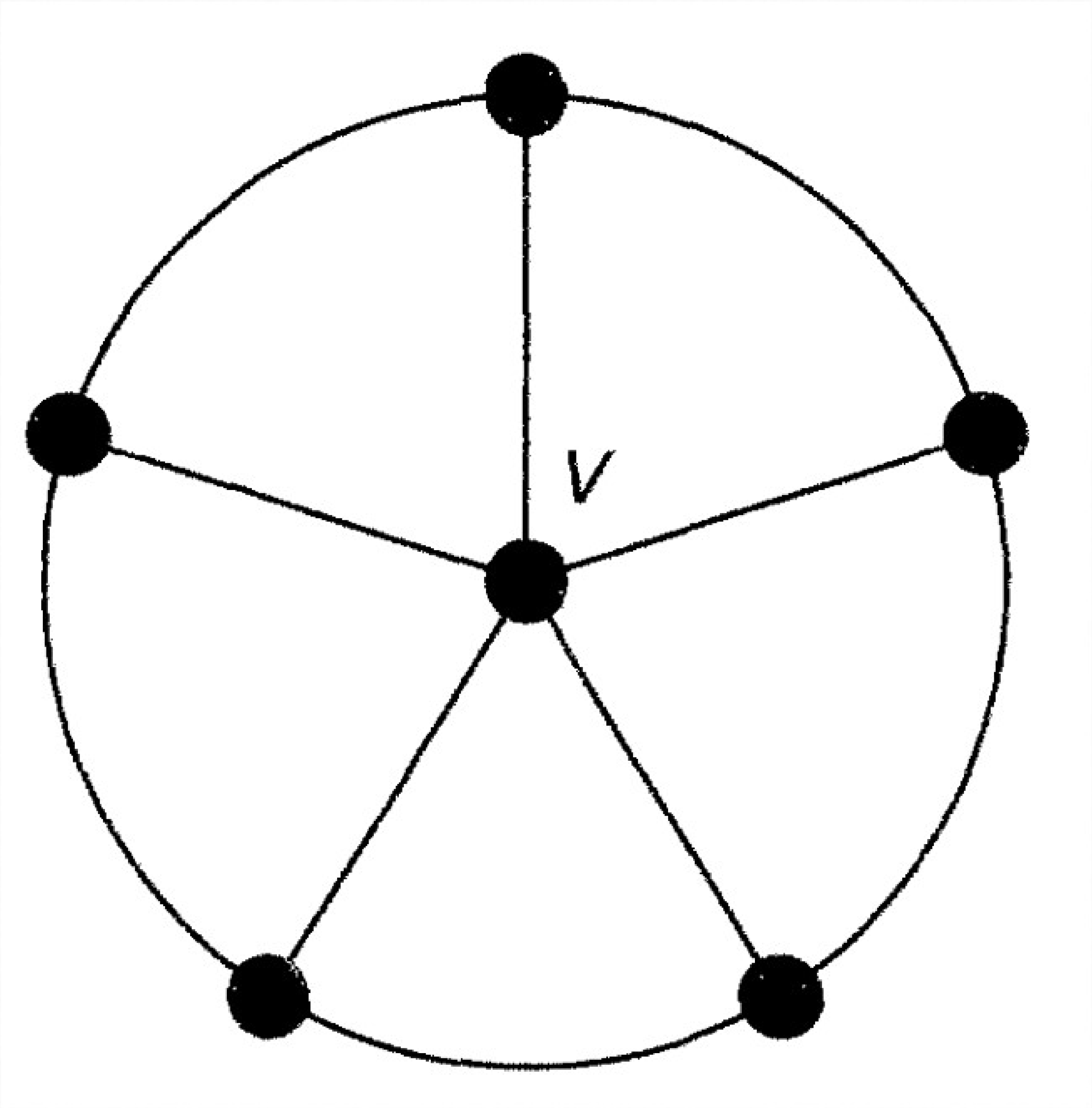
A path graph is a cycle graph with one edge removed.



P_6

Wheel Graph

Its the graph obtained from C_{n-1} by connecting each vertex to a new vertex v.

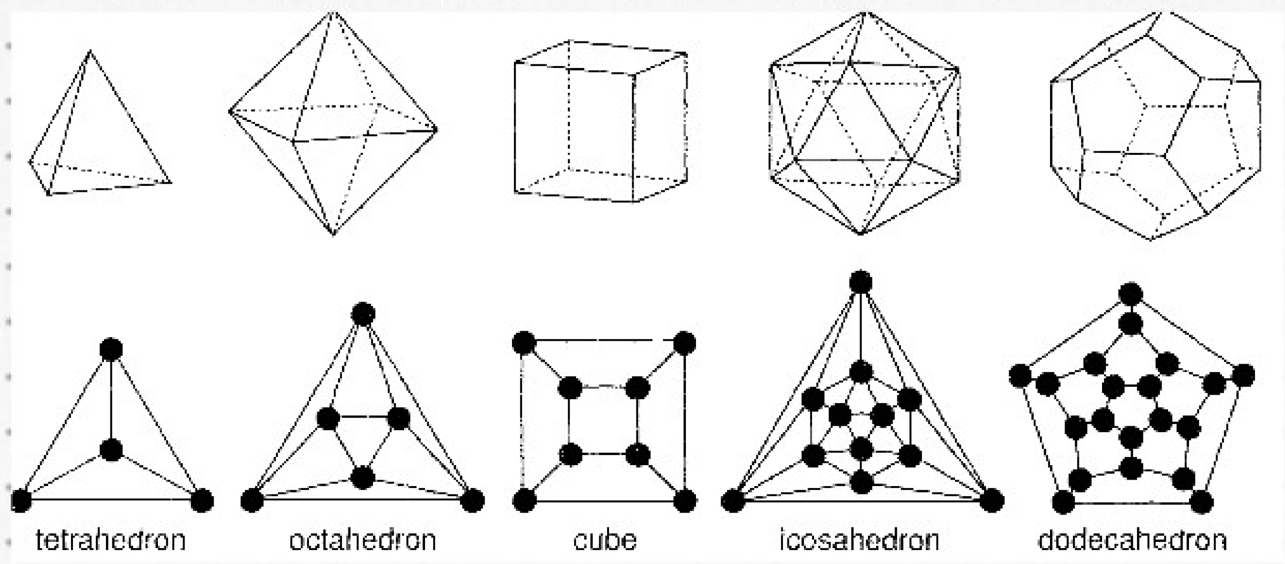


W_6

Platonic Graphs

(|V|, |E|)

(4,6) (6,12) (8,12) (12,24) (20,30)



Bi-partite Graphs

We can identify a bi-partite graph as follows: We can color the vertices either black or white in such a way that all edges are connected to one black and one white vertex.

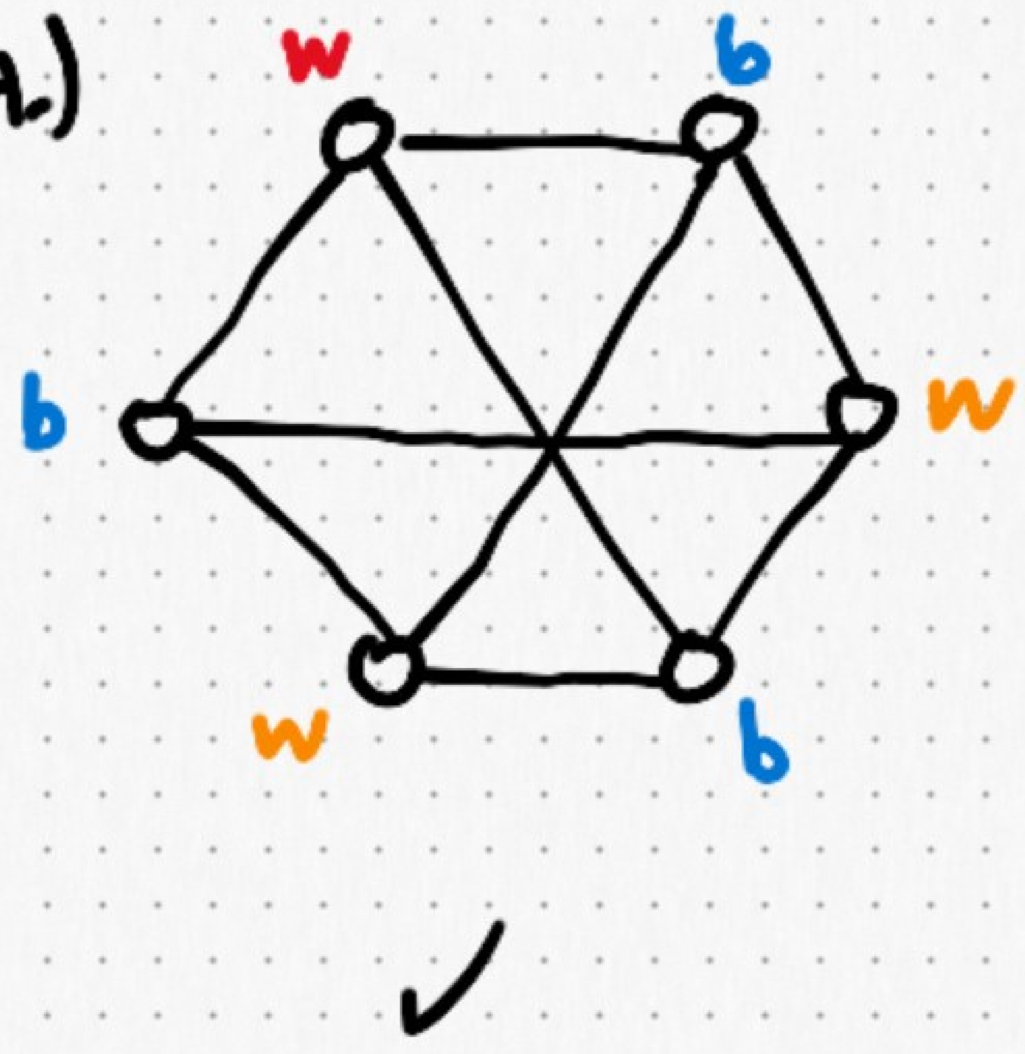
A graph with r black and s white vertices is denoted $K_{r,s}$.

A complete bi-partite graph has rs edges.

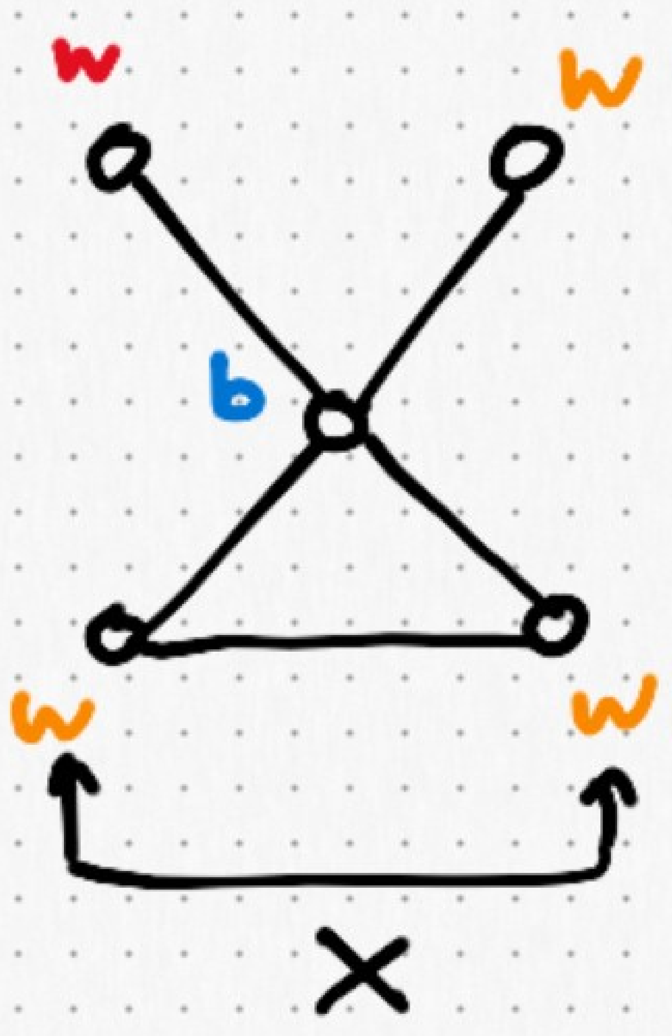
Example

Are the following graphs bi-partite?

A.)



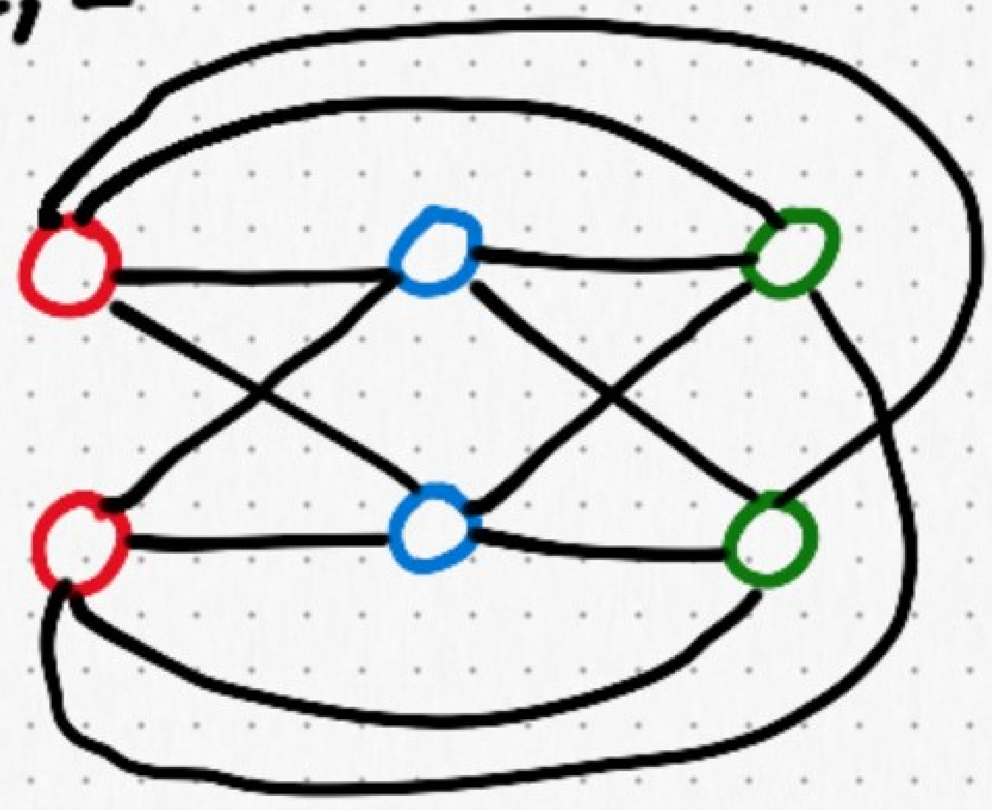
B.)



Example

Draw the following complete tri-partite graph:

$K_{2,2,2}$



If it was NOT tri-partite we would have

$$\frac{n(n-1)}{2} = \frac{6(5)}{2} = 15 \text{ edges.}$$

But we can't have edges between vertices in the same set so we subtract 3:

$15 - 3 = 12$ edges.

In general

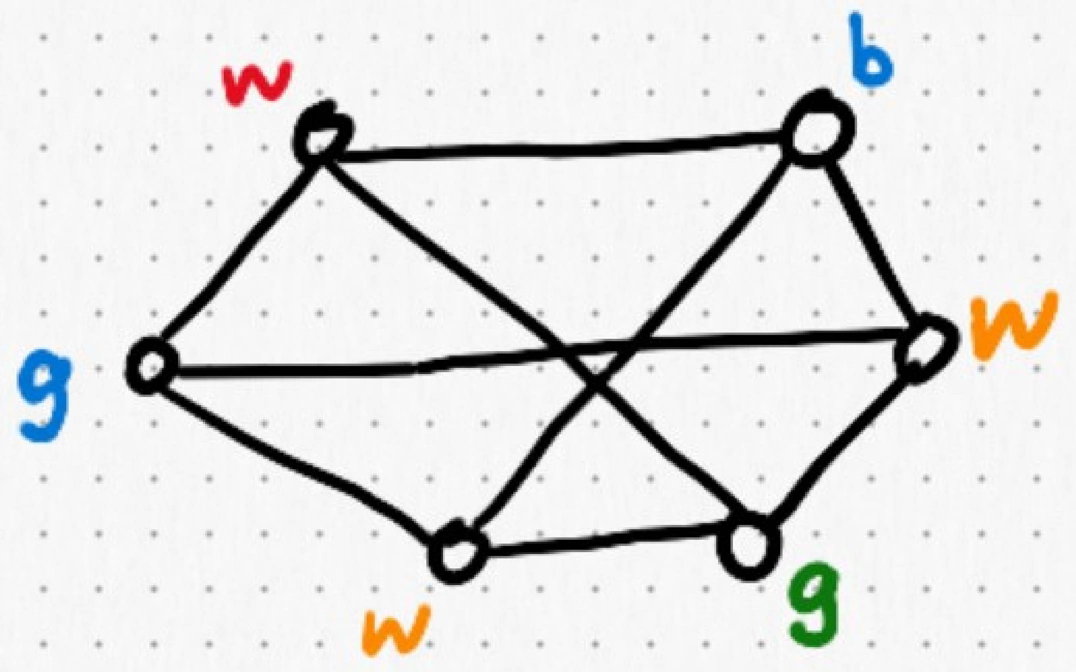
For a k-partite where each set has n_i vertices, a complete k-partite graph has.

$$\boxed{|E| = \frac{n(n-1)}{2} - \sum_{i=1}^{k} \frac{n_i(n_i-1)}{2}}$$

$$\Rightarrow |E| = \frac{6 \cdot 5}{2} - \left[\frac{2 \cdot 1}{2} + \frac{2 \cdot 1}{2} + \frac{2 \cdot 1}{2}\right] = 12$$

Example

Is the graph below tri-partite?



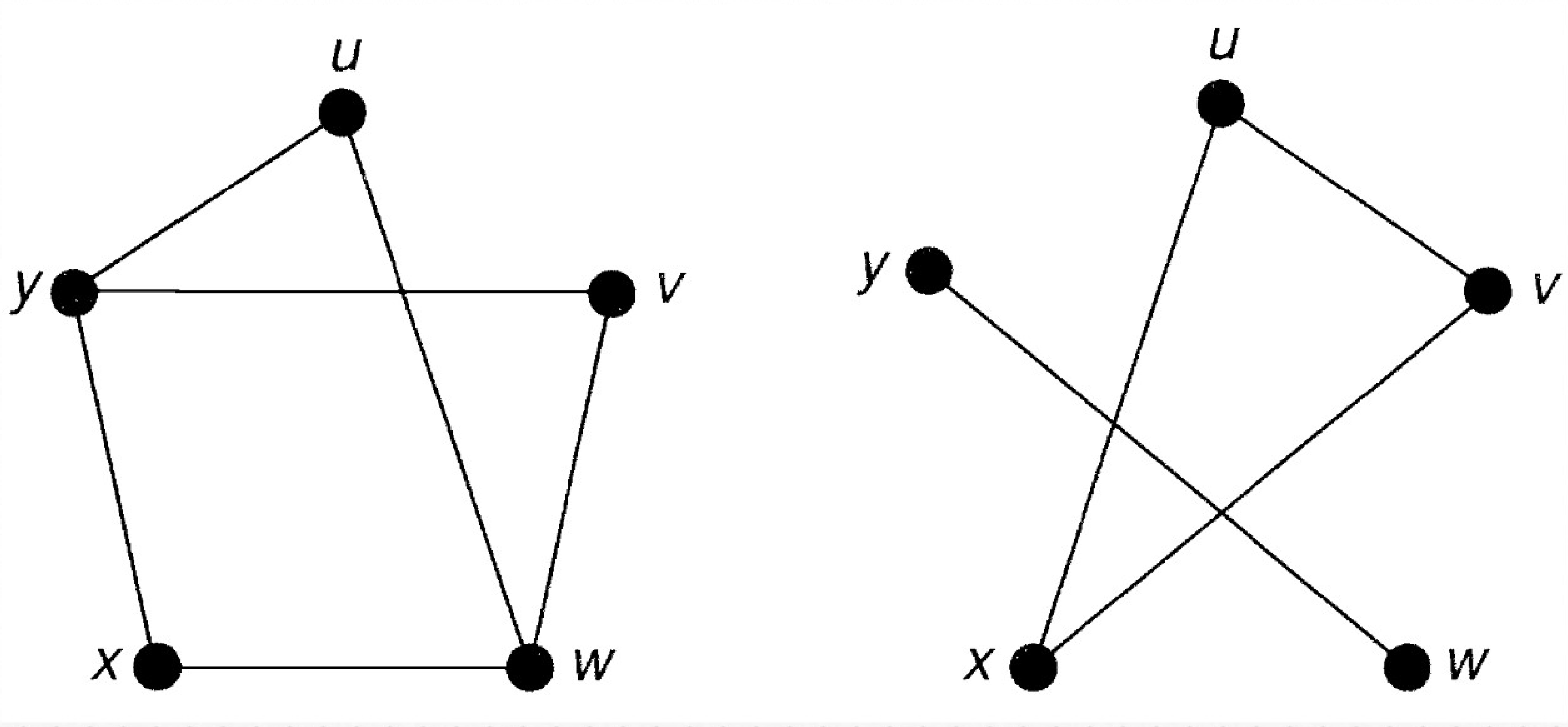
✓

But not complete.

$K_{3,2,1}$
w, g, b

Complement of a Graph

If G is a simple graph, then its complement $\overline{G}$ is such that any two vertices are adjacent in $\overline{G}$ only if they are not adjacent in G.



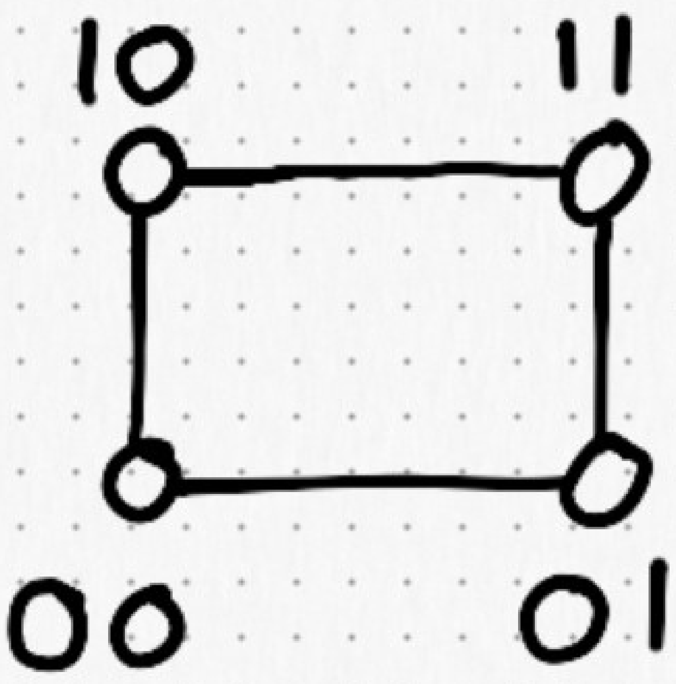
Cubes

A k-cube (Q_k) is a graph whose vertices correspond to sequences $(a_1, a_2, \ldots a_k)$ where $a_i = 0$ or 1.
The edges connect vertices with sequences that differ only one bit.

$\underline{Q_1}$

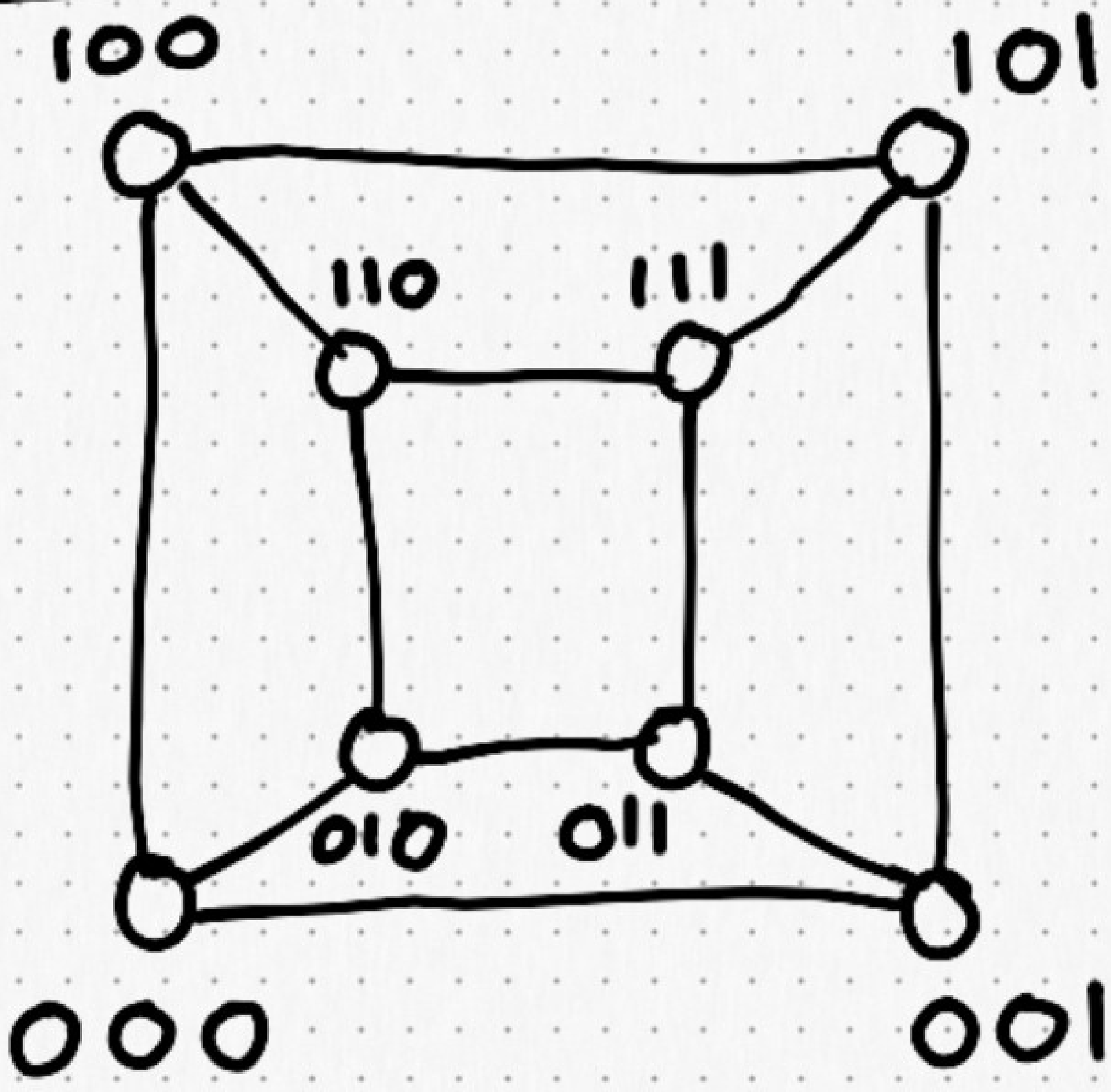
1-regular
(2,1)

$\underline{Q_2}$



2-regular
(4,4)

$\underline{Q_3}$



3-regular
(8,12)

In general Q_k is

a.) k-regular

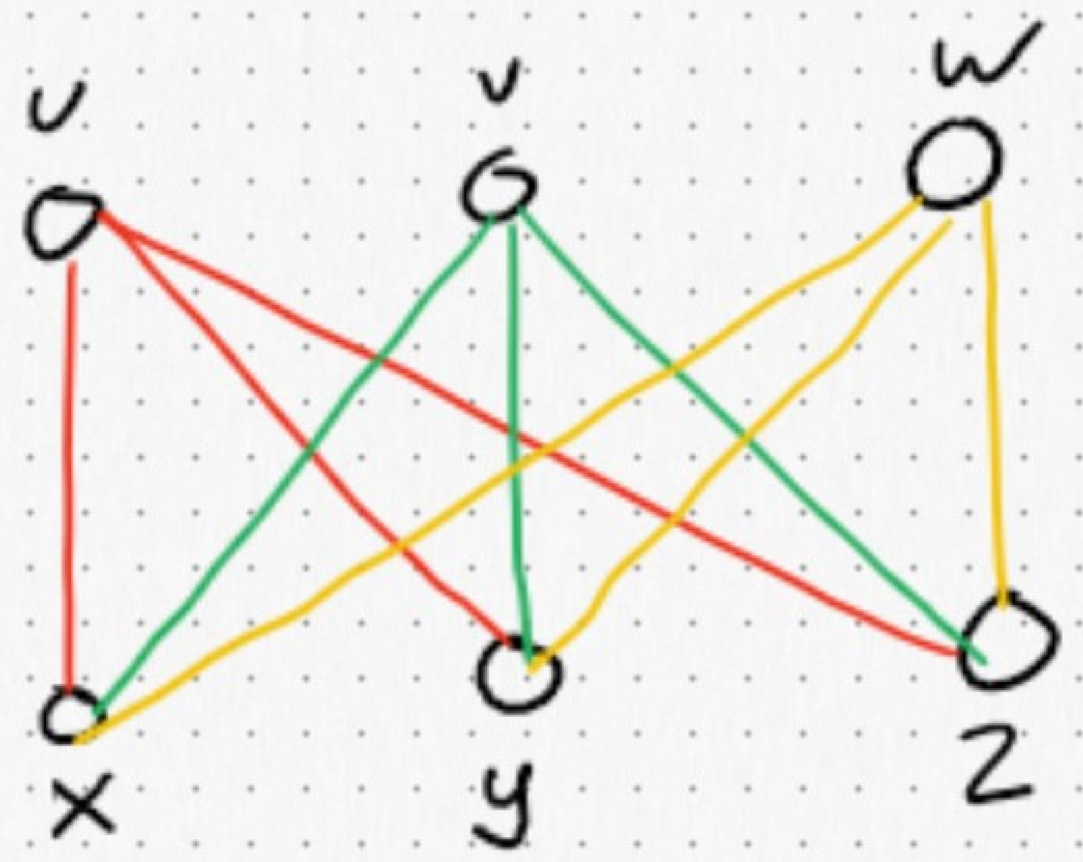
b.) 2^k vertices

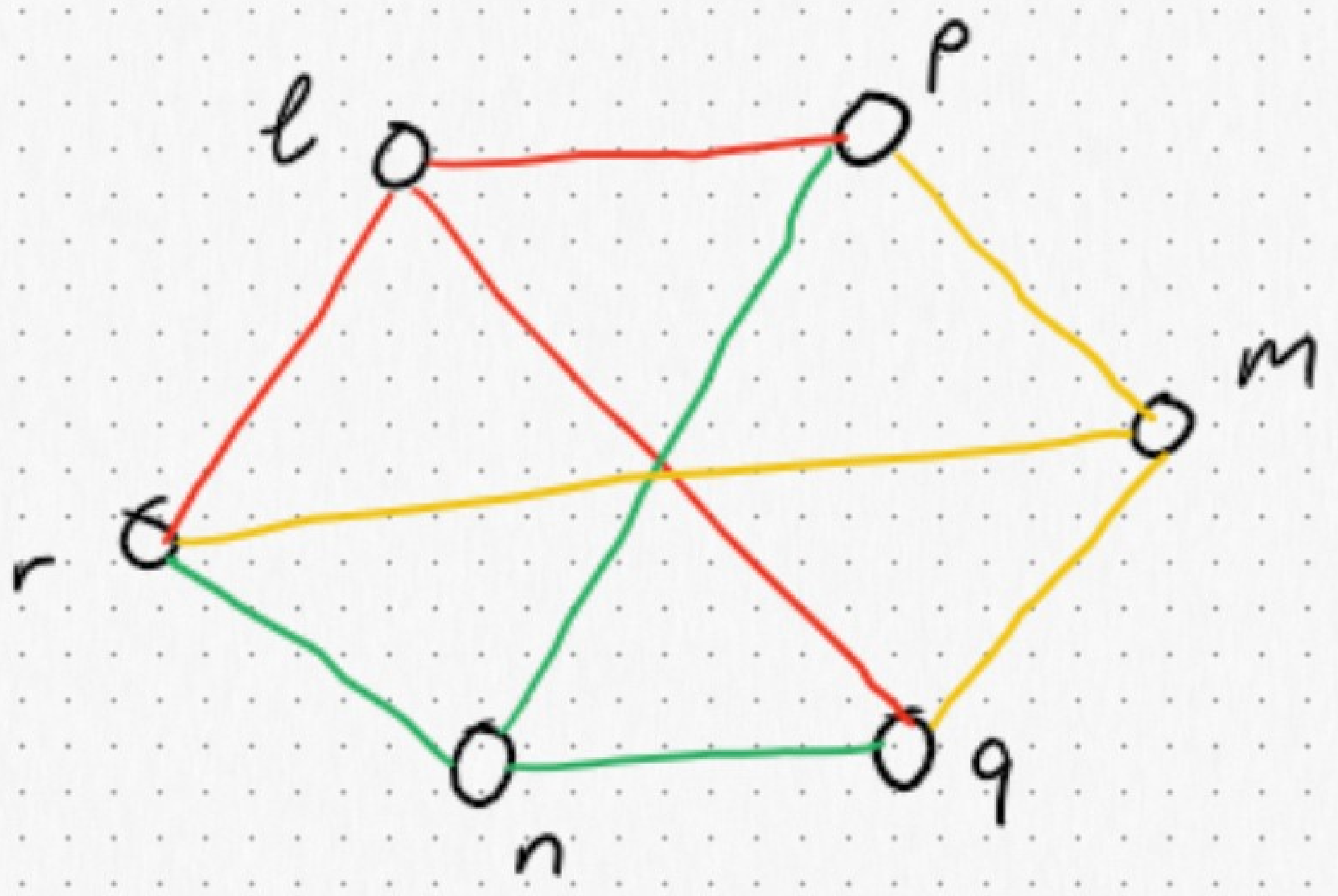
c.) $k2^{k-1}$ edges

Isomorphism

Iso: equal
morph: shape

Two graphs are isomorphic if
a.) They have the same # of vertices
b.) The relationships between the vertices are the same.





$u \to l$

$v \to n$

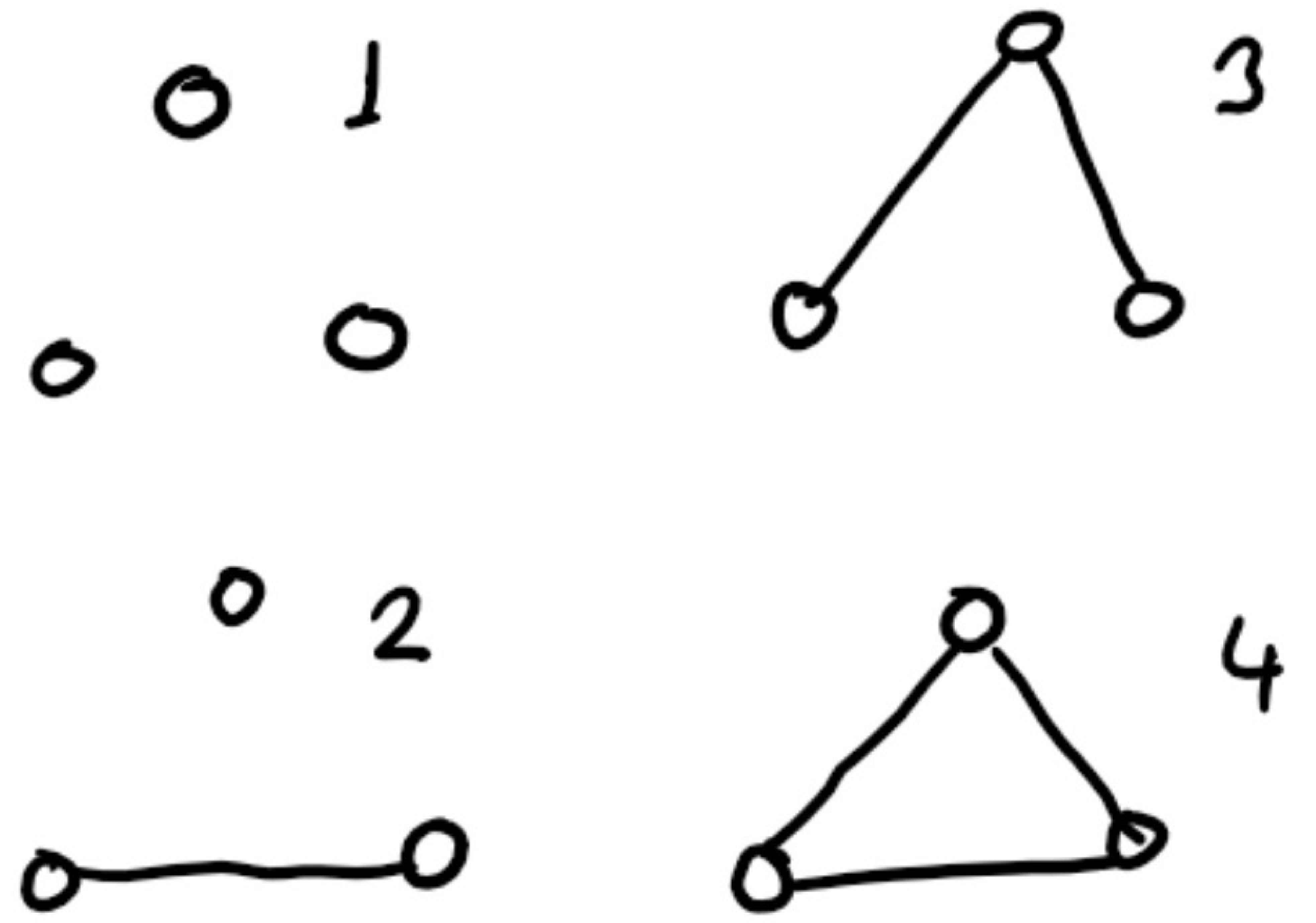
$w \to m$

$x \to q$

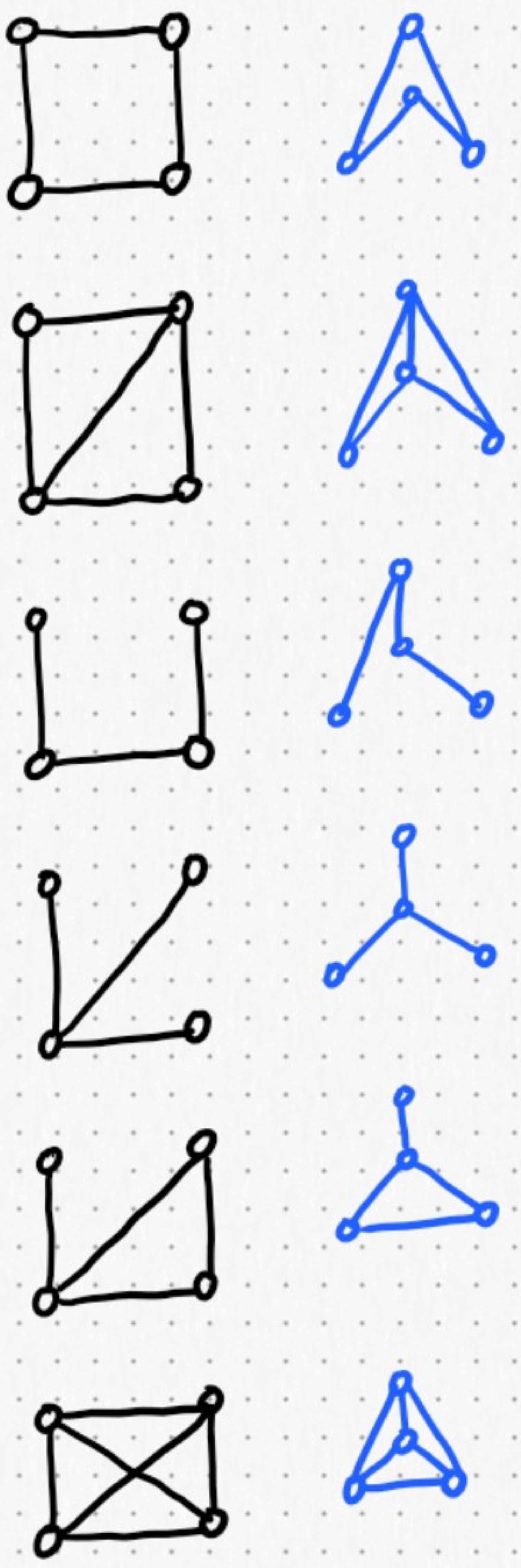
$y \to p$

$z \to r$

Example 1 | Draw all simple graphs with 3 vertices.



ex. Draw all connected graphs with 4 vertices.



ex. Show that the two graphs below are isomorphic.