Discrete Optimization Assignment 3

Warehouse Location Problem

1 Problem Statement

In this assignment, you are asked to design an algorithm to solve the Warehouse Location Problem (WLP). A distribution company uses warehouses to provide goods to many different customers. The goal of this problem is to determine which warehouses will be the most cost-effective for serving the customers. The complexity of the problem comes from the fact that each warehouse has different costs and storage capabilities.

2 Assignment

Write an algorithm to solve the WLP. The problem is mathematically formulated in the following way: The set of warehouses to choose from is $N = \{0, \ldots, n-1\}$, and the set of customers that need to be served is $M = \{0, \ldots, m-1\}$. Each warehouse, $w \in N$ has a capacity cap_w and a setup cost s_w. Each customer, $c \in M$, has a demand d_c and travel cost t_{cw} based on which warehouse, $w \in N$, serves it. Lastly, all customers must be served by exactly 1 warehouse. Let a_w be a set variable denoting the set of customers assigned to warehouse w. Then, the warehouse location problem is formalized as the following optimization problem:

Minimize:

$$\sum_{w \in N} \left((|a_w| > 0) s_w + \sum_{c \in a_w} t_{cw} \right)$$

subject to:

$$\sum_{c \in a_w} d_c \leq cap_w \qquad (w \in N)$$

$$\sum_{w \in N} (c \in a_w) = 1 \qquad (c \in M)$$

3 Input and Output Data Format

The input file consists of $|N| + 2|M| + 1$ lines. The first line contains two numbers, $|N|$ followed by $|M|$. The first line is followed by $|N|$ lines, where each line represents a warehouse capacity cap_w and setup cost s_w. The last $2|M|$ capture the customer information. Each customer block begins with a line with one number, the customer's demand, d_c. The following line has $|N|$ values, one for each warehouse. These values capture the cost of serving that customer from each warehouse, t_{cw}.

Input format:

```
|N| |M|
cap_0 s_0
cap_1 s_1
...
cap_|N|-1 s_|N|-1
d_0
t_0_0 t_0_1 t_0_2 ... t_0_|N|-1
d_1
t_1_0 t_1_1 t_1_2 ... t_1_|N|-1
...
d_|M|-1
t_|M|-1_0 t_|M|-1_1 t_|M|-1_2 ... t_|M|-1_|N|-1
```

The output has two lines. The first line contains one value: obj. This is the cost of the customer warehouse assignment (i.e., the objective value) as a real number. The next line is a list of $|M|$ values that are selected from the set N (so, the values are not necessarily unique). This is the mapping of customers to warehouses.

Output Format:

```
obj
c_0 c_1 c_2 ...  c_|M|-1
```

Examples

Input:

```
3 4
100 100.123
100 100.456
500 100.789
50
100.1 200.2 2000.3
50
100.4 200.5 2000.6
75
200.7 100.8 2000.9
75
200.10 200.11 100.12
```

Output:

```
1002.888
1 1 0 2
```

This output represents the assignment of customers to warehouses, $a_0 = \{2\}$, $a_1 = \{0, 1\}$, $a_2 = \{3\}$. That is, customers 0 and 1 are assigned to warehouse 1, customer 2 is assigned to warehouse 0, and customer 3 is assigned to warehouse 2.

4 Instructions

For now, please start to work on your computer locally. For uploading to the test system, see the file "instructions.pdf".