Introduction to Optimization

Outline

- Optimization
- Decision-Making
- Basic Definitions
- Classification of Optimization Problems
- Heuristic Techniques

Optimization

- Encountered in many areas, including everyday life
- Important step in decision making
- Aids in determining outcomes of decision-making and design processes
- Tries to arrive at the best possible decision under a given set of circumstances
- What is the *best*?
  - May be unattainable
  - May be different for different people
  - Maybe no clear definition
- So, determine some quantity to be maximized or minimized subject to some constraints.

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Decision-Making

- Decision-making involves all activities necessary to identify the most optimal or preferred choice among the available alternatives.
- Basic requirements of decision-making:
  - set of goals or objectives
  - methods for evaluating alternatives
  - evaluation of consequences of each course of action
- Decision-making phases:
  - formulation of goals and objectives, enumeration of constraints, identification and evaluation of alternatives
  - selection of optimal course of action for a given set of constraints

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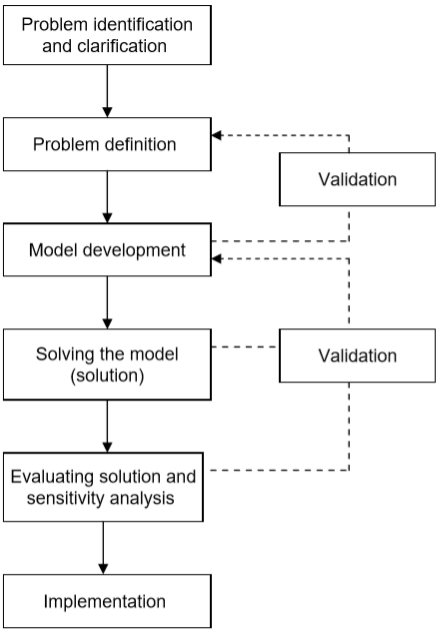
Steps in a Decision Making Process



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Basic Definitions -1

- A *model* is an abstraction or a mathematical representation of a problem.
- Problems that seek to minimize or maximize a mathematical function of a number of variables, subject to constraints are called *optimization problems*.
- Optimization problems: constrained vs. unconstrained
  - Constraints: Conditions that solutions must satisfy

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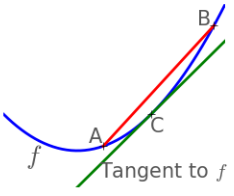
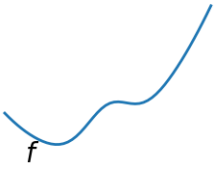
Basic Definitions - 2

- Objective function: single objective vs. multiobjective
- Variables: real, integer (integer or binary), or mixed
- Constraints
  - hard vs. soft
  - equality vs. inequality
- Feasible and infeasible solutions: feasible solutions satisfy the constraints

Convex Functions



A convex function



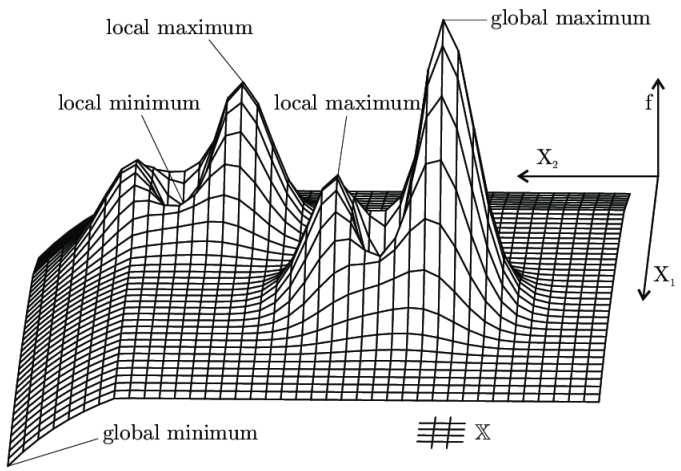
A nonconvex function

- In a convex function:
  - f is above all of its tangents
  - for two points A and B, f(C) lies below the segment [f(A),f(B)] if A<C<B
- Optimizing convex functions is easy

Basic Definitions - 3

- Local and global optima:
  - local optimum: point in the search space which is better than all of its neighboring points
  - global optimum: point in the search space which is better than all of the points in the search space

Global vs. Local Optima



Basic Definitions - 4

- Problem vs. problem instance

EXAMPLE:

Problem: Given any set of points X, find the shortest roundtrip.

Solution: Algorithm that finds shortest roundtrips for any X.

Problem instance: Given a specific set of points P, find a shortest round trip

Solution: Shortest roundtrip for P

⇒ Problems can be formalized as sets of problem instances.

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Classification of Optimization Problems

- Classification based on
  - Type of constraints
  - Nature of the equations involved
  - Deterministic nature of the variables
  - Permissible value of the design variables
  - Separability of the functions
  - Number of objective functions

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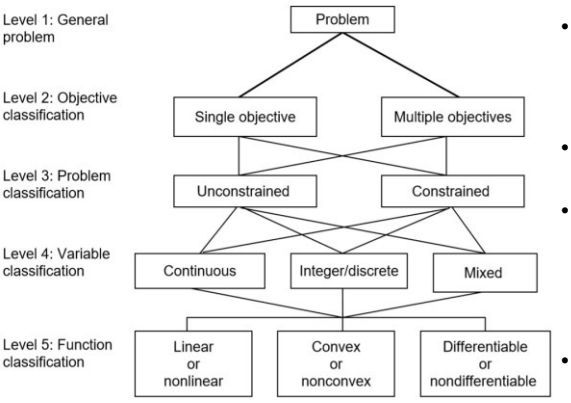
Classification



Note:

- *Discrete optimization* deals with optimization problems with integer or discrete variables.
- Such problems are called *combinatorial problems*.
- In combinatorial problems, the solution is an object from a finite or infinite set—typically an integer, set, permutation, or graph.
- Combinatorial problems involve finding a grouping, ordering, assignment of a discrete, finite set of objects that satisfy given conditions.

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Solution Approaches

- Classical techniques
- Modern heuristics

OR

- Deterministic techniques
- Stochastic techniques

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Classical Optimization Techniques

- Linear programming
- Integer programming
- Goal programming
- Nonlinear programming
- Multiobjective models

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Heuristic Techniques

Heuristic: a rule-of-thumb approach that does not guarantee convergence and optimality, but usually works well and produces solutions of acceptable quality.

Heuristics are usually problem class dependent.

- Hill-climbing techniques
- Simulated annealing
- Tabu search
- Evolutionary algorithms
- Swarm intelligence
- ...

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