BLG560E Exercises

(1) Consider a simple hypothesis

$H_0: \mu = 100$
$H_1: \mu \neq 100$

A sample of size 9 is collected. Sample mean is computed as 106

For significance level of 0.05, should H_0 be rejected if

a) Population std dev is 6

$$T = \frac{\bar{X} - \mu_0}{\sigma/\sqrt{N}} = \frac{106 - 100}{6/\sqrt{9}} = \frac{6}{2} = 3$$

$T \sim N(0,1)$ $\quad z_{0.025} = 1.96$

As $T > z_{0.025}$ H_0 should be rejected

b) Population std dev is 15

$$T = \frac{\bar{X} - \mu_0}{\sigma/\sqrt{N}} = \frac{106 - 100}{15/2} = 0.8$$

As $T < z_{0.025}$ H_0 should be retained

c) Find lower limit of σ such that H_0 is retained

$T \leq z_{0.025}$

$$T = \frac{106 - 100}{\sigma/3} \leq 1.96 \Rightarrow \frac{12}{1.96} \cong 6.32 < \sigma$$

! in part (a) $\sigma = 6$ and H_0 is rejected

d) For $\sigma = 6$, find the critical sample mean $\bar{X}_c$ such that H_0 is rejected.

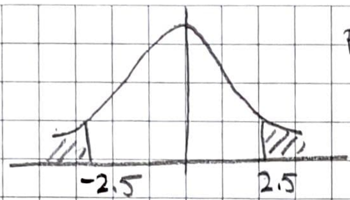
$$|T| = \left| \frac{\bar{X}_c - 100}{6/\sqrt{9}} \right| \geq z_{0.025}$$

$\bar{X}_c' \geq 100 + 1.96 \times 2$ $\qquad$ $\bar{X}_c'' \leq 100 - 1.96 \times 2$

$\bar{X}_c' \geq 103.92$ $\qquad$ $\bar{X}_c'' \leq 96.08$

e) For $\bar{x} = 95$ and $\sigma = 6$, find p-value

$$T = \frac{95 - 100}{6/\sqrt{9}} = -2.5$$



p-value is the shaded area

$$\text{p-value} = 2 \times (1 - F_Z(2.5))$$ ← from z-table = 0.9938

$$= 0.0124$$

(2) In order to test the following hypothesis, a sample of size 16 is collected. Assume population variance is 36.

$H_0: \mu \leq 100$
$H_1: \mu > 100$

Let $\bar{X} = 103$

a) Test H_0 for $\alpha = 0.05$

$$T = \frac{\bar{X} - \mu_0}{\sigma/\sqrt{N}} = \frac{103 - 100}{6/\sqrt{16}} = 2$$

$Z_{0.05} = 1.645$

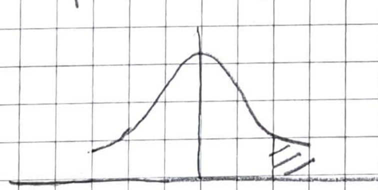
As $T > Z_{0.05}$ reject H_0

b) Test H_0 for $\alpha = 0.01$

$T = 2$ and $Z_{0.01} = 2.326$

$T < Z_{.01}$ retain H_0

c) Find p-value of this sample



$$p = 1 - F_Z(2) = 1 - 0.9772$$
$$= 0.0028$$

d) For the following significance level, check test result

α	Reject	Retain
0.1	✓	
0.05	✓	
0.01	✓	
0.005	✓	
0.002		✓
0.001		✓

If $p > \alpha$ retain H_0
$p \leq \alpha$ reject H_0

e) what is the smallest significance level for rejection of H_0? $p = 0.0028$

(3) Consider

$H_0: \mu = 250$
$H_1: \mu \neq 250$

Let $N = 25$ and $s = 10$

a) Find min. critical sample mean $\bar{X}_c$ that leads to rejection of H_0 for $\alpha = 0.05$

$$T = \frac{\bar{X}_c - 250}{s/\sqrt{25}} = \frac{\bar{X}_c - 250}{2} \geq t_{24,0.025} = 2.064$$

(from t-table)

$$X_c \geq 250 - 2 \times 2.064 = 245.87$$

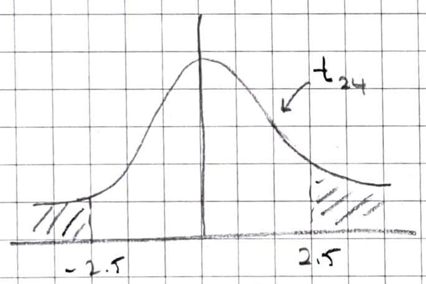
b) Let $\bar{X} = 245$, test H_0 for $\alpha = 0.01$

$$T = \frac{245 - 250}{2} = -2.5 \qquad t_{24,0.005} = 2.797$$

As $T < t_{24,0.005} \Rightarrow$ retain H_0

c) For $\bar{X} = 245$ find p-value.

! From part b
$p > 0.01$



$p\text{-value} = 2 \times (1 - F_t(2.5))$

$\cong 0.0197$

typically not on the table

use software

Excel $= 2*(1 - \text{T.DIST}(2.5; 24; \text{TRUE}))$

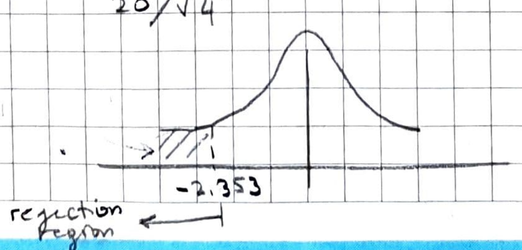
(4) Consider

$H_0: \mu \geq 250$
$H_1: \mu < 250$

Let $N = 4$ and $s = 20$

a) Let $\bar{x} = 230$, test H_0 for $\alpha = 0.05$

$$T = \frac{230 - 250}{20/\sqrt{4}} = -2 \qquad t_{3,0.05} = 2.353$$



$T \notin R \rightarrow$ retain H_0

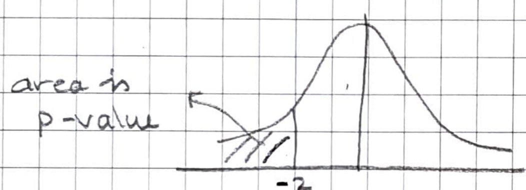
b) Find $\bar{X}_c$, critical sample mean that leads to rejection of Ho

$$T = \frac{\bar{X}_c - 250}{20/\sqrt{4}} \leq -2.353$$

$$\bar{X}_c \leq 250 - 23.53 = 226.47$$

c) Find p-value when $\bar{X} = 230$

$$T = -2$$



p-value = $F_t(-2)$

From part-a $p > 0.05$ (otherwise Ho should be rejected in part(a))

(5) Consider the following hypothesis

$H_0: \sigma^2 = 12$
$H_1: \sigma^2 \neq 12$

Let $s^2 = 10$ and $N = 10$

a) Test Ho with $\alpha = 0.05$

$$T = \frac{(N-1)s^2}{\sigma_0^2} \sim \chi^2_{N-1}$$

$$T = \frac{9 \cdot 10}{12} = 7.5$$

$\chi^2_{9,0.025} \cong 19$

$\chi^2_{9,0.975} \cong 2.7$

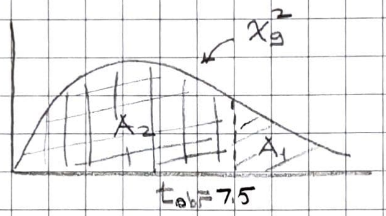
Use software. For example in Excel =CHIINV(0.025,9)

As $\chi^2_{9,0.975} < T < \chi^2_{9,0.025} \longrightarrow$ Ho retained

b) Find critical values of sample variance that leads to rejection of Ho

$$\frac{(N-1)s_L^2}{12} \leq 2.7 \Rightarrow s_L^2 \leq 3.6 \quad \text{and} \quad \frac{9 \cdot s_H^2}{12} \geq 19 \Rightarrow s_H^2 \geq 25.33$$

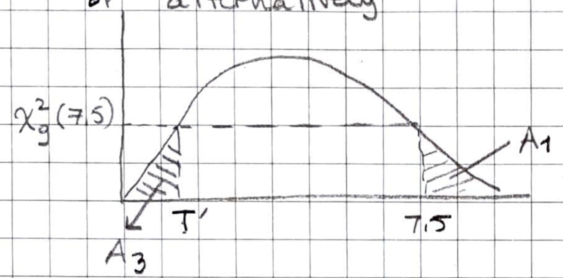
c) Find p-value



asymmetric sampling distr. when H_0 is correct

$$p = 2 \times \min(A_1, A_2)$$

or alternatively



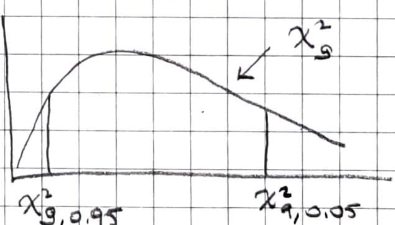
$$p = A_1 + A_3$$

$$T' = \underbrace{\chi^{2\,-1}_{9}}_{\text{chiinv}}\left(\chi^2_9(7.5)\right)$$

⑥ Consider following hypothesis

H_0: $\sigma^2 \geq 12$
H_1: $\sigma^2 < 12$

Let $s^2 = 10$ and $N = 10$



a) Test H_0 with $\alpha = 0.05$

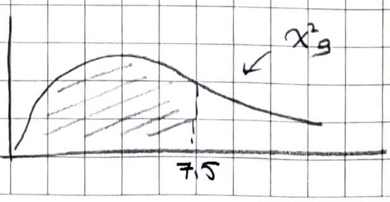
$T = 7.5$ $\qquad \chi^2_{9,0.95} \cong 3.33$

as $T \geq \chi^2_{9,0.95}$ H_0 is retained

b) Find critical value for sample variance for rejection of H_0

$$\chi^2_{9,0.95} \geq \frac{9 \cdot s_L^2}{12} \Rightarrow s_L^2 \leq 4.44$$

c) Find p-value



p-value $= 1 - 0.415$
$= 0.585$

Excel: 1 – CHISQUARE.DIST(7.5;9;TRUE)

⑦ Consider following hypothesis

$H_0: p = 0.7$
$H_1: p \neq 0.7$

Let $N = 100$ and $X = 62$ (# of cases with the condition)

a) Test H_0 with $\alpha = 0.05$

$$T = \frac{X - p_0 N}{\sqrt{N p_0 (1-p_0)}} = \frac{X/N - p_0}{\sqrt{\frac{p_0(1-p_0)}{N}}} = \frac{0.62 - 0.7}{\sqrt{\frac{0.7 \times 0.3}{100}}} = \frac{-0.08}{\sqrt{0.0021}} \cong -1.75$$

as $Np_0 = 100 \times 0.7 > 5$ and $N(1-p_0) = 100 \times 0.3 > 5$

Binomial distr. can be approximated by std. normal distr.

$T \sim N(0,1)$

Since $|T| < z_{0.025} = 1.96$ retain H_0

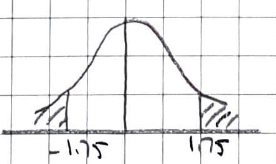
b) Find critical value of X (X_c) that leads to rejection of H_0

$$\left| \frac{X_c - 70}{\sqrt{100 \times 0.7 \times 0.3}} \right| \geq 1.96$$

$X_c \geq 70 + 1.96\sqrt{21} \cong 78.98$ X_c is an integer $= 79$

$X_c \leq 70 - 1.96\sqrt{21} \approx 61.01$ X_c " $= 61$

c) Calculate p-value



$t_{obs} = -1.75$

p-value $= 2 \times (1 - F_Z(1.75))$

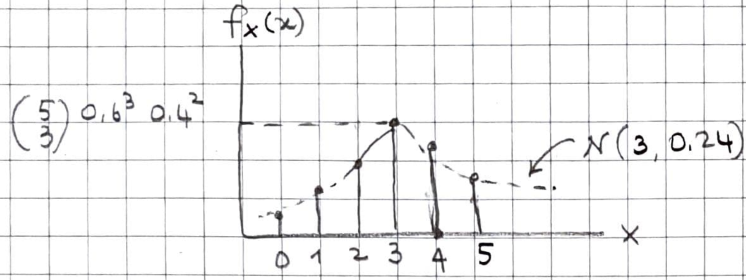
↳ From z table → 0.9599

$\cong 0.08$

Continuity correction

When a discrete distr. is approximated by a cont. distr. a simple correction gives more accurate approximation.

Consider $X \sim \text{Binomial}(5, 0.6)$



Let $Y \sim N(3, 0.24)$ be the cont distr. approximation of $X \sim \text{Binomial}(5, 0.6)$

- $P(X=4) \neq P(Y=4)$

  $P(X=4) \cong P(3.5 \leq Y \leq 4.5)$

- $P(X \leq 4)$ is better approximated by $P(Y \leq 4.5)$ compared to $P(Y \leq 4)$

- $P(X < 4)$ is better appr. by $P(Y \leq 3.5)$ compared to $P(Y \leq 4)$

In part (c), to have a better approximation of p-value

$$\text{p-value} = 2\, P(X \geq 62) \quad \text{where } X \sim \text{Binomial}(100, 0.7)$$

$$= 2(1 - P(X < 62)) \qquad \text{Let } Y \sim N(70, 21)$$

$$= 2(1 - P(Y \leq 61.5)) \leftarrow \text{cont. correction}$$

$$z = \frac{61.5 - 70}{\sqrt{21}} \approx -1.85$$

$$\text{p-value} = 2 \times (1 - \underbrace{F_Z(1.85)}_{0.9678}) \cong 0.064$$

(8) Consider following hypothesis

$$H_0: p \geq 0.2$$
$$H_1: p < 0.2$$

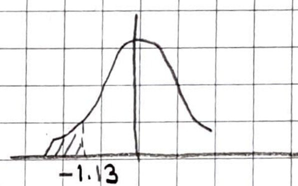
Let $N = 100$ and $x = 15$

a) Test H_0 with $\alpha = 0.10$

$$T = \frac{0.15 - 0.2}{\sqrt{0.0016}} = -1.25 \qquad z_{0.10} \cong 1.28$$

$$|T| < z_{0.10} \longrightarrow \text{retain } H_0$$

b) Calculate p-value with cont. correction



$$\text{p-value} = P(X \leq 15)$$
$$= P(Y \leq 15.5) \quad \text{cont. correction}$$
$$\cong F_Z(-1.13) \qquad z = \frac{15.5 - 20}{4} \cong -1.13$$
$$= 1 - \underbrace{F_Z(1.13)}_{0.8686} \cong 0.13 \qquad \text{Excel } 1 - \text{NORM.DIST}(1.13;0;1;\text{TRUE})$$

From part (a) p-value $\gtrsim \alpha$

c) Find min X_c for $\alpha = 0.10$ that leads to rejection of H_0

$$T = \frac{X_c - 20}{\sqrt{16}} \leq -1.28 \qquad X_c \geq 20 - 4 \times 1.28$$

$$X_c \geq 14.88$$

X_c is an integer $\Rightarrow$ min $X_c = 15$