BLG560E - Exercises

(1) A lecture is given in two sections: in-class and online

	in-class	online
$\bar{X}$	62	67
N	20	25

The final exam grade statistics and # of students are given in the table

Variance is known to be $\sigma^2 = 25$ for both sections

a) Test for $\alpha = 0.01$

$H_0: \mu_1 = \mu_2$

$H_1: \mu_1 \neq \mu_2$

where μ_1: mean of in-class population

μ_2: " online "

$$T = \frac{\bar{X}_1 - \bar{X}_2}{\sqrt{\frac{\sigma^2}{N_1} + \frac{\sigma^2}{N_2}}} = \frac{-5}{\sqrt{1.25 + 1}} = -3.33$$

For this two-tailed test $z_{0.005} = 2.576$

Since $|t_{obs}| > z_{0.005}$ reject H_0.

b) Find p-value

Since H_0 is rejected in part (a), we know $p < 0.01$

$$p\text{-value} = 2(1 - \underbrace{F_Z(3.33)}_{0.9996}) = 0.0008$$

(2) For exercise 1, assume variance is unknown but equal for both sections. Solve (a) & (b) from exercise 1. Let $s_1^2 = 15$, $s_2^2 = 25$.

a) Compute pooled variance estimator

$$S_p^2 = \frac{20 \times 15 + 25 \times 25}{20 + 25 - 2} = 21.51 \Rightarrow S_p = 4.64$$

$$T = \frac{-5}{S_p \underbrace{\sqrt{\frac{1}{20} + \frac{1}{25}}}_{0.3}} \Rightarrow t_{obs} = \frac{-5}{1.39} = -3.6 \qquad t_{43, 0.005} \cong 2.695$$

$|t_{obs}| > t_{43, 0.005} \Rightarrow$ reject H_0

b) p-value $= 2\, F_t(-3.6) = 2(1 - F_t(+3.6))$

$= 0.0008$

③ Use statistics from exercise (2)

Assume $s_1^2 = 15$ and $s_2^2 = 25$ and section variances are different

Test

$H_0: \mu_1 \geq \mu_2$

$H_1: \mu_1 < \mu_2$

for $\alpha = 0.01$

a) effective dof $= \left\lfloor \frac{(s_1^2/N_1 + s_2^2/N_2)^2}{\left(\frac{(s_1^2/N_1)^2}{N_1 - 1} + \frac{(s_2^2/N_2)^2}{N_2 - 1}\right)} \right\rfloor = \left\lfloor \frac{1.75^2}{(0.03 + 0.04)} \right\rfloor = 43$

$$t_{obs} = \frac{-5}{\sqrt{\frac{15}{20} + \frac{25}{25}}} = \frac{-5}{1.32} \approx -3.79 \qquad t_{43,0.005} \approx 2.695$$

As $t_{obs} < t_{43,0.005}$ H_0 is rejected.

b) p-value $= F_t(-3.79) = 0.0002$

④ Test the following hypothesis for $\alpha = 0.05$

$H_0: \sigma_1^2 \geq \sigma_2^2$

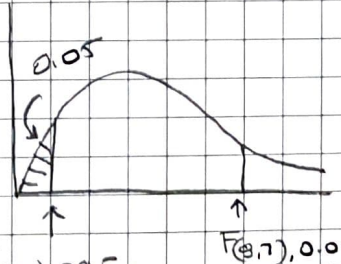
$H_1: \sigma_1^2 < \sigma_2^2$

	Sample 1	Sample 2
N	10	8
s^2	20	22

$$t_{obs} = \frac{s_1^2}{s_2^2} = \frac{20}{22} = 0.9$$

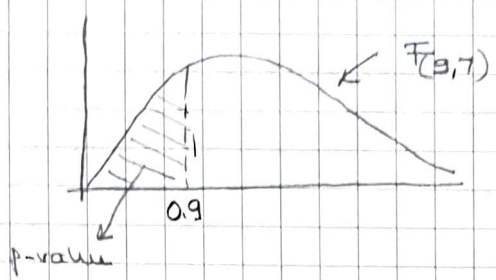
Need the lower critical value

$$F_{(9,7),0.95} = \frac{1}{F_{(7,9),0.05}} = \frac{1}{3.29} = 0.3$$



$0.3 < t_{obs} \Rightarrow$ retain H_0

b) Compute p-value



In Excel

F.DIST(0.9; 9; 7; TRUE)

$p = 0.43$

(5) At the end of the semester, the # of students passing the course is

	In-class	online
N	20	22
# of pass	16	14

a) Test

$H_0: p_1 = p_2$

$H_1: p_1 \neq p_2$

for $\alpha = 0.05$

$$Z = \frac{\hat{p}_1 - \hat{p}_2}{\sqrt{\hat{p}(1-\hat{p})\left(\frac{1}{N_1} + \frac{1}{N_2}\right)}}$$

$\hat{p}_1 = \frac{16}{20} = 0.8$

$\hat{p}_2 = \frac{14}{22} = 0.64$

$\hat{p}$: pooled estimator $= \frac{16+14}{20+22} = 0.71$

$$Z_{obs} = \frac{0.8 - 0.64}{\sqrt{0.71 \times 0.29 \times \underbrace{\left(\frac{1}{20} + \frac{1}{22}\right)}_{0.095}}} = \frac{0.16}{0.14} = 1.14$$

$Z_{obs} < Z_{0.025} = 1.96 \rightarrow$ retain H_0

b) p-value $= 2(1 - \underbrace{F(1.14)}_{0.8729}) = 0.25$

(6) Students who fail online lecture are required to take in-class section. Let X_{1i}, X_{2i} be the score of student i from online and in-class lecture.

Student	Online	In-class
1	40	45
2	30	35
3	30	30
4	20	30
5	40	35

a) Test the hypothesis that

$H_0: \mu_2 \geq \mu_1$

$H_1: \mu_2 < \mu_1$

for $\alpha = 0.05$

Use paired t-test

$d_i = x_{2i} - x_{1i} \Rightarrow$ $H_0: d \geq 0$, $H_1: d < 0$

$d = [5, 5, 0, 10, -5]$

$\bar{d} = 3 \quad s_d^2 \approx 32$

$$t_{obs} = \frac{3}{\sqrt{32/5}} \approx \frac{3}{2.55} = 1.18$$

$t_c = -t_{4, 0.05} = -2.132 \quad t_{obs} > t_c \Rightarrow$ retain H_0

(7) Letter grades for online and in class lecture is given below.

	A	B	C	D	F
In class	3	7	6	1	3
Online	2	8	8	3	4

Test H_0: letter grades and lecture-type are independent

Expected

	A	B	C	D	F	
In-class	2.22	6.67	6.22	1.78	3.11	20
Online	2.78	8.33	7.78	2.22	3.89	25
	5	15	14	4	7	45

$dof = (2-1) \times (5-1) = 4$

$$X^2 = \sum_i \frac{(O_i - E_i)^2}{E_i} = 1.15 \qquad X^2 \sim \chi^2_4$$

The critical value of χ^2_c for $\alpha = 0.05$ is 9.49

As $X^2 < X^2_c$ H_0 is retained.

p-value = 0.89

← 1 − CHISQUARE.DIST(1.15; 4; TRUE) in Excel