

1	2	3	4	Total

Name: _____
Number: Answers

BLG560E - Statistics and Estimation in Computer Science

Midterm 2

09.05.2018

Rules:

- Duration is 90 min.
- Show your work, do not write the result directly.
- Use the attached distribution lookup tables if required.
- Solve each question within the corresponding frame. Anything outside the frame **will not be** graded.

Questions:

1. (25 pts) Assume that you would like to report about execution duration of an algorithm. For this purpose, you executed the algorithm for 25 times and measured its execution times. Assume that you obtained the following values for sample mean and sample standard deviation.

Sample mean (\bar{x})	100 sec.
Sample std. dev. (s)	15 sec.
Sample size (n)	25

- 15 a) Find the 95% confidence interval for the algorithm execution.

$$s.e = \frac{s}{\sqrt{n}} = \frac{15}{\sqrt{25}} = 3$$

from t-distr table (dof = 25 - 1 = 24)

$$t_{0.025, 24} = 2.064 \text{ (two-tailed)}$$

Hence 95% CI is

$$[100 - 2.064 \times 3, 100 + 2.064 \times 3]$$

$$\approx [93.8, 106.19]$$

- 10 b) Find the 95% confidence interval for the upper limit of algorithm execution.

$$t_{0.05, 24} = 1.711 \text{ (one-tailed)}$$

95% CI upper limit is

$$100 + 1.711 \times 3 \approx 105.13$$

2. (25 pts) Assume that you proposed a novel classifier and claim its accuracy ratio higher than 80%. For this purpose, you performed tests on 64 datasets and obtained the mean accuracy of 77%.

a) Write your null and alternative hypothesis explicitly.

$$H_0: p \geq 0.8$$

$$H_1: p < 0.8$$

b) Calculate your p-value.

From H_0

$$\begin{aligned}\sigma^2 &= p(1-p) \\ &= 0.8(1-0.8) = 0.16\end{aligned}$$

$$s.e. = \frac{\sigma}{\sqrt{n}} = \frac{0.4}{\sqrt{64}} = 0.05$$

$$z\text{-value} = \frac{0.77 - 0.8}{0.05} = -0.6$$



From z-table

$$p\text{-value} = 1 - 0.7257 = 0.2743$$

c) Should you reject (or retain) null hypothesis for significance level of $\alpha = 0.05$. Why?

$$\text{As } p\text{-value} > \alpha$$

retain H_0

$$\frac{\quad}{z_{0.025} = 1.645 \text{ (one-tailed)}}$$

$$\text{as } -1.645 < -0.6 \text{ retain } H_0$$

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3. (25pts)

Assume that you made 1000 observations of a continuous variable (X) and wonder if the observation data come from a normal distribution of mean 20 and variance 25 ($N(20, 25)$). Hence, you made the following list of observations that are smaller than 10, 15, 25, 30. Note that the list is **cumulative**.

	# of obs.
$X < 15$	130
$X < 25$	852

- (a) Find the approximate p-value of this sample for the following null hypothesis:
 $H_0: X \sim N(20, 25)$

20

	$X < 15$	$15 < X < 25$	$25 < X$
Observed	130	722	148
Expected	158.7	682.6	158.7

As 15 and 25 are one std. dev. away from mean,

$$P(X < \mu - \sigma) = P(X > \mu + \sigma) = 1 - 0.8413 = 0.1587$$

$$P(\mu - \sigma < X < \mu + \sigma) = 0.6826$$

↑
from z table

$$\chi^2 = \frac{(130 - 158.7)^2}{158.7} + \frac{(722 - 682.6)^2}{682.6} + \frac{(148 - 158.7)^2}{158.7} \approx 8.19$$

From χ^2 table with $\text{dof} = 3 - 1 = 2$

$$p \approx 0.01$$

$$p\text{-value} = 0.0167 \text{ (exact from software)}$$

- (b) Should you reject (or retain) null hypothesis for significance level of 0.05? Why?

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As $p < \alpha$ H_0 should be rejected

or

$$\text{dof} = 2 \quad \alpha = 0.05 \quad \chi_{\text{critical}} = 5.991$$

as $\chi^2 > \chi_{\text{critical}}$ reject H_0 .

4. (25 pts) Assume that you would like to test the following hypothesis with significance level of 0.05.

$$H_0: \mu \leq 100$$

$$H_1: \mu > 100$$

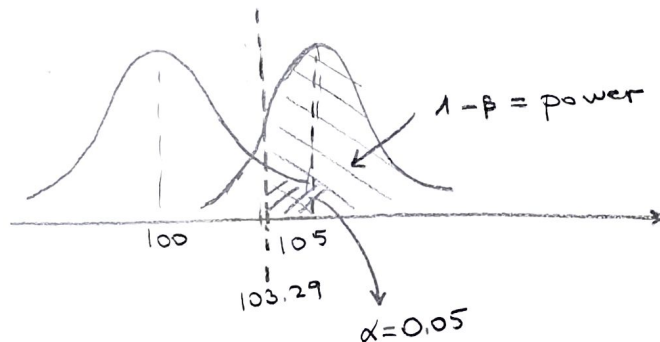
The standard deviation of the population is known to be 20, and the sample size is 100.

Find the power of this test at $\mu = 105$ (with same std. dev).

$$s.e. = \frac{\sigma}{\sqrt{n}} = \frac{20}{\sqrt{100}} = 2$$

$$z_{0.05} = 1.645$$

$$z_{critical} = 100 + 1.645 \times 2 = 103.29$$



$$z = \frac{103.29 - 105}{2} = -0.855$$

$$1 - \beta \approx 0.8023 \text{ from the } z\text{-table}$$