lista - 8

1) a- $\vec{u} = (1/2, 1, 1)$

$s: \begin{cases} 3x = z \\ 2y - 16 = z \end{cases} \Rightarrow \begin{cases} x = \lambda/3 \\ y = \lambda/2 + 8 \\ z = \lambda \end{cases} \rightarrow s: (0, 8, 0) + \lambda(1/3, 1/2, 1)$

$\vec{v} = (1/3, 1/2, 1)$ $\quad \|\vec{u}\| = \sqrt{1/4 + 1 + 1} = \sqrt{9/4} = 3/2$

$\|\vec{v}\| = \sqrt{1/9 + 1/4 + 1} = \sqrt{49/36} = 7/6$

$\cos\alpha = \dfrac{|1/6 + 1/2 + 1|}{\frac{3}{2} \cdot \frac{7}{6}}$ $\quad \cos\alpha = \dfrac{10/6}{\frac{21}{12}} = \dfrac{10}{6} \cdot \dfrac{12}{21} = \dfrac{20}{21}$

$\operatorname{sen}\alpha = \sqrt{1 - \dfrac{400}{441}} = \sqrt{\dfrac{41}{441}}$

b- $\vec{u} = (0, -1, 1)$

$s: \begin{cases} x - y + 3 = 2 \\ z = 4 \end{cases} \Rightarrow \begin{cases} x = -3 + \lambda \\ y = \lambda \\ z = 4 \end{cases}$ $\quad \vec{v} = (1, 1, 0)$

$\cos\alpha = \dfrac{|-1|}{\sqrt{2}\sqrt{2}} = \dfrac{1}{2}$ $\quad \operatorname{sen}\alpha = \dfrac{\sqrt{3}}{2}$

c- $r: \begin{cases} x + 3z = 7 \\ y = 0 \end{cases} \rightarrow \begin{cases} x = 7 - 3\lambda \\ y = 0 \\ z = \lambda \end{cases}$ $\quad \vec{u} = (-3, 0, 1)$

$s: \begin{cases} x - 4y - 2z = 5 \\ y = 0 \end{cases} \Rightarrow \begin{cases} x = -1/7 + ? \\ y = 3/7 + 2\lambda \\ z = \lambda \end{cases}$ $\quad \vec{v} = (2, 0, 1).$

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3) a- $r: \begin{cases} x = 0 \\ y - z = 0 \end{cases} \Rightarrow \begin{cases} x = 0 \\ y = \lambda \\ z = \lambda \end{cases}$ $\vec{u} = (0, 1, 1)$

$\vec{n} = (0, 0, 1)$ $\quad sen\,\theta = \dfrac{|1|}{\sqrt{2} \cdot 1} = \dfrac{1}{\sqrt{2}} \Rightarrow \theta = \dfrac{\pi}{4}$ rad

b- $\vec{u} = (-1, 1, 2)$

$sen\,\theta = \dfrac{|-2-1|}{\sqrt{6}\,\sqrt{5}} = \dfrac{3}{\sqrt{30}} = \dfrac{\sqrt{30}}{10}$ [illegible]

$\vec{n} = (2, -1, 0)$

$\Rightarrow arc\,sen\left(\dfrac{\sqrt{30}}{10}\right)$

c- $\vec{u} = (1, 1, -2)$ $\quad sen\,\theta = \dfrac{|1+1+2|}{\sqrt{6}\,\sqrt{3}} = \dfrac{4}{\sqrt{18}} = \dfrac{4}{3\sqrt{2}} = \dfrac{2\sqrt{2}}{3}$ [illegible]

$\vec{n} = (1, 1, -1)$

$\Rightarrow \theta = arcsen\left(\dfrac{2\sqrt{2}}{10}\right)$

4) $\vec{u} = (x, y, z)$ $\quad \dfrac{|\vec{u} \cdot \vec{n_1}|}{\|\vec{u}\|\|\vec{n_1}\|} = 1 \Rightarrow \dfrac{|x+y+z|}{\sqrt{3}} = 1 \Rightarrow |x+y+z| = \sqrt{3}$

$\|\vec{u}\| = 1$

$\dfrac{|\vec{u} \cdot \vec{n_2}|}{\|\vec{u}\|\|\vec{n_2}\|} = \dfrac{1}{\sqrt{2}} \Rightarrow \dfrac{|x-y|}{\sqrt{2}} = \dfrac{1}{\sqrt{2}} \Rightarrow |x-y| = 1$

$\vec{n_1} = (1, 1, 1)$

$\vec{n_2} = (1, -1, 0)$

$\begin{cases} x + y + z = \pm\sqrt{3} \\ x - y = \pm 1 \end{cases} \Rightarrow \begin{cases} x = (\pm\sqrt{3} \pm 1) - \lambda/2 \\ y = (\pm\sqrt{3} \pm 1) - \lambda/2 \\ z = \lambda \end{cases}$

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2) $\begin{cases} x = (\sqrt{3}+1)/2 - z/2 \\ y = (\sqrt{3}-1)/2 - z/2 \\ z = z \end{cases}$ $\quad$ $\begin{cases} x = (\sqrt{3}-1)/2 - z/2 \\ y = (\sqrt{3}+1)/2 - z/2 \\ z = z \end{cases}$

3. $\begin{cases} x = (-\sqrt{3}+1)/2 - z/2 \\ y = (-\sqrt{3}-1)/2 - z/2 \\ z = z \end{cases}$ $\quad$ 4. $\begin{cases} x = (-\sqrt{3}-1)/2 - z/2 \\ y = (-\sqrt{3}+1)/2 - z/2 \\ z = z \end{cases}$

5) a- $\vec{n_1} = (2,1,-1)$ $\quad$ $\cos\theta = \frac{|2-1-3|}{\sqrt{6}\sqrt{11}} = \frac{2}{\sqrt{66}} = \frac{\sqrt{66}}{33}$

$\vec{n_2} = (1,-1,3)$

$\theta = \arccos\left(\frac{\sqrt{66}}{33}\right)$

b- $\vec{n_1} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 1 & 0 & 1 \\ -1 & 0 & 0 \end{vmatrix} = (0,-1,0)$ $\quad$ $\cos\theta = \frac{|-1|}{\sqrt{3}} = \frac{\sqrt{3}}{3}$

$\theta = \arccos\left(\frac{\sqrt{3}}{3}\right)$

$\vec{n_2} = (1,1,1)$

c- $\vec{n_1} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 1 & 0 & 0 \\ 1 & 1 & 1 \end{vmatrix} = (0,-1,1)$ $\quad$ $\cos\theta = \frac{|-1|}{\sqrt{2}} = \frac{\sqrt{2}}{2}$

$\vec{n_2} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ -1 & 0 & 0 \\ 0 & 1 & 0 \end{vmatrix} = (0,0,-1)$ $\quad$ $\theta = \frac{\pi}{4}$ rad.

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6) $\vec{n_1} = (2, -1, 1)$ $\qquad \cos\theta = \dfrac{|2+2+1|}{\sqrt{6}\,\sqrt{6}} = \dfrac{1}{2} \Rightarrow \theta = \dfrac{\pi}{3}$

$\vec{n_2} = (1, -2, 1)$

7) a - $\begin{cases} x - 1 = z \\ 2y = z \end{cases} \Rightarrow \begin{cases} x = 1 + \lambda \\ y = \lambda/2 \\ z = \lambda \end{cases}$ $\qquad P = (1+\lambda, \lambda/2, \lambda)$

$\sqrt{(1+\lambda-1)^2 + (\lambda/2 - 1)^2 + (\lambda)^2} = \sqrt{(1+\lambda)^2 + (\lambda/2 - 1)^2 + (\lambda - 1)^2}$

$\Rightarrow \lambda^2 = \lambda^2 + 1 + 2\lambda + \lambda^2 + 1 - 2\lambda$

$0 = 2$ não existe ponto

b - $P = (4\lambda, 2\lambda, 4 - 3\lambda)$

$(4\lambda - 2)^2 + (2\lambda - 2)^2 + (4 - 3\lambda - 5)^2 = (4\lambda)^2 + (2\lambda)^2 + (4 - 3\lambda - 1)^2$

$16\lambda^2 + 4 - 16\lambda + 4\lambda^2 + 4 - 8\lambda + 1 + 9\lambda^2 + 6\lambda = 16\lambda^2 + 4\lambda^2 + 9 + 9\lambda^2 - 6\lambda$

$-18\lambda + 9 = 9 - 6\lambda$

$\lambda = 0$

$P = (0, 0, 4)$

c - $P = (2+\lambda, 3+\lambda, -3+2\lambda)$

$(2+\lambda-1)^2 + (3+\lambda-1)^2 + (\lambda - 3)^2 = (2+\lambda-2)^2 + (3+\lambda-2)^2 + (2\lambda - 3 - 4)^2$

$4 + \cancel{\lambda^2} + 1 + 9 + \lambda^2 + 1 + \cancel{\lambda^2} + 3 = 4 + \lambda^2 + 4 + 9 + \lambda^2 + 2 + 25$

$-2\lambda + 13 = -28\lambda + 43$ $\qquad P = (44/13, 57/13, -21/13)$

$\lambda = \dfrac{30}{26} = \dfrac{18}{13}$

8) a - $\vec{AP} = (-3,-2,1)$

$$\vec{AP} \times \vec{v} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -3 & -2 & 1 \\ 3 & 2 & 1 \end{vmatrix} = (-4,6,0)$$

$\|\vec{AP} \times \vec{v}\| = \sqrt{16+36} = \sqrt{52}$

$\|\vec{v}\| = \sqrt{9+4+1} = \sqrt{14}$

$dist(P,r) = \dfrac{\sqrt{52}}{\sqrt{14}}$

b - $\vec{AP} = (-1,-1,3)$

$$\vec{AP} \times \vec{v} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -1 & -1 & 3 \\ 4 & -3 & 2 \end{vmatrix} = (11,19,7)$$

$\|\vec{AP} \times \vec{v}\| = \sqrt{270}$

$\|\vec{v}\| = \sqrt{16+9+4} = \sqrt{29}$

$dist(P,r) = \dfrac{\sqrt{270}}{\sqrt{29}}$

c - $r: \begin{cases} x = 2z - 1 \\ 2y - 3 = 2z - 1 \end{cases} \Rightarrow \begin{cases} x = -1 + 2\lambda \\ y = 1 + \lambda \\ z = \lambda \end{cases}$

$\vec{AP} = (-1,-2,0)$

$$\vec{AP} \times \vec{v} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -1 & -2 & 0 \\ 2 & 1 & 1 \end{vmatrix} = (-2,1,3)$$

$\|\vec{AP} \times \vec{v}\| = \sqrt{14}$

$\|\vec{v}\| = \sqrt{6}$

$dist(P,r) = \sqrt{\dfrac{14}{6}}$

9) $r: \begin{cases} x + y = 2 \\ x = y + z \end{cases} \Rightarrow \begin{cases} x = 2 - y \\ x = y + z \end{cases} \Rightarrow \begin{cases} x = 1 + \lambda/2 \\ y = 1 - \lambda/2 \\ z = \lambda \end{cases}$

$P = (1 + \lambda/2, 1 - \lambda/2, \lambda)$

$s: \begin{cases} x = z + 1 \\ y = z + 1 \end{cases} \Rightarrow \begin{cases} x = 1 + \alpha \\ y = 1 + \alpha \\ z = \alpha \end{cases}$

$dist(P,s) = \dfrac{\sqrt{14}}{3}$

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$$\vec{AP} \times \vec{v} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ \lambda/2 & -\lambda/2 & \lambda \\ 1 & 1 & 1 \end{vmatrix} = 1/2\,(-3\lambda, \lambda, 2\lambda)$$

$$\frac{\|\vec{AP} \times \vec{v}\|}{\|\vec{v}\|} = \frac{\sqrt{14}}{\sqrt{3}} \qquad \vec{AP} = (1+\lambda/2, 1-\lambda/2, \lambda) - (1,1,0)$$

$$\vec{AP} = (\lambda/2, -\lambda/2, \lambda)$$

$$\|\vec{AP} \times \vec{v}\| = \frac{1}{2} \cdot \sqrt{9\lambda^2 + \lambda^2 + 4\lambda^2} = \frac{\sqrt{14}}{2}\lambda$$

$$\|\vec{v}\| = \sqrt{3} \qquad \frac{\frac{\sqrt{14}}{2}\lambda}{\sqrt{3}} = \frac{\sqrt{14}}{\sqrt{3}} \Rightarrow \lambda = 2$$

$$P = (2, 0, 2)$$

10) a) $\vec{n} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 1 & 0 & 0 \\ -1 & 0 & 3 \end{vmatrix} = (0,-3,0) \qquad \vec{AP} = (0,3,4)$

$$dist(P, \pi) = \frac{|+9|}{3} = 3$$

b) $dist(P, \pi) = \frac{|0-6|}{3} = 2$

c) $dist(P, \pi) = \frac{|2-(1+2-1)|}{3} = 0$

11) $r: \begin{cases} x = y+3 \\ z-y = y+3 \end{cases} \Rightarrow \begin{cases} x = 1 + \lambda/2 \\ y = 1 - \lambda/2 \\ z = \lambda \end{cases} \qquad P = (1+\lambda/2, 1-\lambda/2, \lambda)$

$$\therefore \pi: x - 2y - z - 1 = 0$$

$$dist(P, \pi) = \sqrt{6} \Rightarrow \frac{|1 + \lambda/2 - 2 + \lambda - \lambda - 1|}{\sqrt{6}} = \sqrt{6}$$

$$|\lambda/2 - 2| = 6$$

$$\begin{cases} P_1 = (9, -7, 16) \\ P_2 = (-3, 5, -8) \end{cases}$$

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(2) a- $s: \begin{cases} x+y+z=0 \\ 2x-y-1=0 \end{cases} \Rightarrow \begin{cases} x = 1/3 - z/3 \\ y = -1/3 - 2z/3 \\ z = z \end{cases}$

$\vec{v} = (-1/2, -2/3, 1)$
$\vec{u} = (1, -1, 1)$
$\vec{AB} = (-9/3, -4/3, 0)$

$\vec{u} \times \vec{v} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 1 & 1 & 1 \\ 1/2 & -2/3 & 1 \end{vmatrix} = (10/6, -3/6, -$

b- $\vec{u} = (3, 4, -2)$
$\vec{v} = (6, -4, 1)$

$\|\vec{u} \times \vec{v}\| = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 3 & 4 & -2 \\ 6 & -4 & 1 \end{vmatrix} = (-4, -15, -36)$

$\vec{AB} = (21, -9, 2) - (-4, 0, -5) = (25, -5, 7)$

$dist(r, s) = \dfrac{|(25, -5, 7) \cdot (-4, -15, -36)|}{\sqrt{1537}} = \dfrac{277}{\sqrt{1537}}$

14) a - $\vec{n_1} = (2,-1,2)$ $\quad \{\vec{n_1}, \vec{n_2}\}$ é L.D $\Rightarrow$ paralelos
$\vec{n_2} = (4,-2,4)$

$P = (0,0,0) \in \pi_1$, $\quad dist(P, \pi_2) = \dfrac{|-21|}{\sqrt{34}} = \dfrac{21}{\sqrt{34}}$

b - $\vec{n_1} = (2,2,2)$

$$\vec{n_2} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ -1 & 0 & 3 \\ 1 & 1 & 0 \end{vmatrix} = (-3,3,-1)$$

$\{\vec{n_1}, \vec{n_2}\}$ é L.I

$dist(\pi_1, \pi_2) = 0$

c - $\vec{n_1} = (1,1,1)$ $\quad \{\vec{n_1}, \vec{n_2}\}$ é L.I $\therefore dist(\pi_1, \pi_2) = 0$

$\vec{n_2} = (2,1,1)$

15) $r: \begin{cases} x + z = 5 \\ y + 4 = 5 \end{cases} \Rightarrow \begin{cases} x = 5 - \lambda \\ y = 1 \\ z = \lambda \end{cases}$

$\pi: X = (5,1,0) + \lambda(-1,0,1) + \beta(4,2,-2)$

$$n = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ -1 & 0 & 1 \\ 4 & 2 & -2 \end{vmatrix} = (-2,2,-2)$$

$-10 + 2 + d = 0$
$d = 8$

$\pi: -2x + 2y - 2z + 8 = 0$

$$\frac{|-10+2+d|}{2\sqrt{3}} = 2$$

$|d - 8| = 4\sqrt{3}$

$\begin{cases} d_1 = 8 + 4\sqrt{3} \\ d_2 = -8 - 4\sqrt{3} \end{cases}$

$\pi_1: -2x + 2y - 2z + 8 + 4\sqrt{3} = 0$

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$\pi_2: -2x + 2y - 2z + 8 - 4\sqrt{3} = 0$

c) $r: \frac{x-1}{-2} = 2y = z \Rightarrow \frac{x-1}{-2} = \frac{y}{\frac{1}{2}} = \frac{z}{1}$

$\vec{AB} = (-1, 0, 2)$

$\vec{u} = (-2, 1/2, 1)$

$\vec{v} = (-4, 1, 2)$

$\{\vec{u}, \vec{v}\}$ é L.D.

$$\vec{AB} \times \vec{v} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -1 & 0 & 2 \\ -4 & 1 & 2 \end{vmatrix} = (-2, -6, -1)$$

$\|\vec{AB} \times \vec{v}\| = \sqrt{4+36+1} = \sqrt{41}$

$\|\vec{v}\| = \sqrt{16+1+4} = \sqrt{21}$

$dist(r, s) = \sqrt{\frac{41}{21}}$.

13) a - $$\vec{n} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{vmatrix} = (0, 0, 1)$$

$(3, 3, 3) \cdot (0, 0, 1) = 3 \neq 0 \therefore dist(r, \pi) = 0$

b - $r: \begin{cases} x - y + z = 0 \\ 2x + y - z = 3 \end{cases} \Rightarrow \begin{cases} x = 1 \\ y = 1 + t \\ z = t \end{cases}$ $\vec{v} = (0, 1, 1)$, $\vec{n} = (0, 1, -1)$

$(0, 1, -1) \cdot (0, 1, 1) = 0 \Rightarrow$ paralelo

$dist(r, \pi) = \frac{|1-4|}{\sqrt{2}} = \frac{3}{\sqrt{2}}$

c - $r: \begin{cases} x = z + 3 \\ y - 1 = z + 3 \end{cases} \Rightarrow \begin{cases} x = 3 + z \\ y = 4 + z \\ z = t \end{cases}$ $\vec{v} = (3, 4, 0)$, $\vec{n} = (2, 1, -3)$

$6 + 4 = 10 \neq 0 \therefore dist(r, \pi) = 0$

$\cos\alpha = \dfrac{|1-6+1|}{\sqrt{10}\sqrt{5}} = \dfrac{5}{\sqrt{50}}$ $\cos\alpha = \dfrac{\sqrt{50}}{10}$ -1

d. $\vec{u} = (1,-2,3)$

$S: \begin{cases} 3x + y - 5z = 0 \\ x - 2y + 3z + 1 = 0 \end{cases} \Rightarrow \begin{cases} x = -1/7 + \lambda \\ y = 3/7 + 2\lambda \\ z = \lambda \end{cases}$ $\vec{v} = (1,2,1)$

$\cos\alpha = \dfrac{|1-4+3|}{\sqrt{1+4+9}\sqrt{1+4+1}} \Rightarrow \cos\alpha = 0$

2) $P = (0, 2+\lambda, 0)$ $\vec{PQ} = (1, -\lambda, u)$

$Q = (1,2,u)$ $\|\vec{PQ}\| = \sqrt{1+\lambda^2+u^2}$

$\dfrac{|\vec{PQ}\cdot\vec{u}|}{\|\vec{PQ}\|\|\vec{u}\|} = \dfrac{1}{\sqrt{2}} \Rightarrow \dfrac{|(1,-\lambda,u)\cdot(0,1,0)|}{\sqrt{1+\lambda^2+u^2}\cdot 1} = \dfrac{1}{\sqrt{2}} \Rightarrow$ -1

$\dfrac{\lambda}{\sqrt{1+\lambda^2+u^2}} = \dfrac{1}{\sqrt{2}} \Rightarrow \sqrt{2}\lambda = \sqrt{1+\lambda^2+u^2}$

$\dfrac{|\vec{PQ}\cdot\vec{v}|}{\|\vec{PQ}\|\|\vec{v}\|} = \dfrac{1}{2} \Rightarrow \dfrac{u}{\sqrt{1+\lambda^2+u^2}} = \dfrac{1}{2} \Rightarrow \sqrt{1+\lambda^2+u^2} = 2u$

$2u = \sqrt{2}\lambda$ $\sqrt{1+2u^2+u^2} = 2u^2$ $\lambda = \sqrt{2}$ -2

$\dfrac{2}{\sqrt{2}}u = \lambda$ $1+2u^2+u^2 = 4u^2$

$1 = u^2 \Rightarrow u = 1$

$P = (0, 2+\sqrt{2}, 0)$

$Q = (1,2,1)$.

ORONI: