b- $r: C + \lambda \vec{CM}$ $M = C + \vec{CA} + 1/2\,\vec{AB}$

$M = (4,-7,-6)+(-1,13,-1)+(-4,-2,5)$

$M = (-1,4,-2)$

$\vec{CM} = (-1,4,-2)-(4,-7,-6) = (-5,11,4)$

$r: X = (4,-7,-6) + \lambda(-5,11,4)$

4) a- $\vec{AB} = (-3,0,9)-(0,1,8) = (-3,-1,1)$

$\vec{BC} \perp \vec{AB} \Rightarrow [(1+\lambda, 2+\lambda, -3\lambda)-(-3,0,9)]\cdot(-3,-1,1)=0$

$(4+\lambda, 2+\lambda, -9-3\lambda)\cdot(-3,-1,1)=0$

$-12-3\lambda-2-\lambda-9-3\lambda=0$

$23-7\lambda=0$

$\lambda=23/7$

$C = (1,2,0)+23/7(1,1,-3) = (1+23/7, 2+23/7, -23/7)$

$\therefore C = (30/7, 37/7, -23/7)$

b- $C = (1+\lambda, \lambda, \lambda)$

$\sqrt{(1-1-\lambda)^2+(1-\lambda)^2+(1-\lambda)^2} = \sqrt{(-1-\lambda)^2+(-\lambda)^2+(1-\lambda)^2}$

$1+\lambda^2-2\lambda = 1+\lambda^2+2\lambda \Rightarrow \lambda=0$

$C = (1,0,0)$

2- a) $(x, y, z) = (1, 0, 4) + \lambda(-1, 1, 2) : r$

$P_1 = (1, 0, 4)$ $\quad \vec{v_1} = (-1, 1, 2)$

$P_2 = (1, 0, 4) + 2(1, 0, 4)$ $\quad \vec{v_2} = (-2, 2, 4) \Rightarrow \vec{v_1} \cdot 2$

$P_2 = (3, 0, 12)$

b) P: $\begin{cases} 1 = 1 - \lambda \\ 3 = \lambda \\ -3 = 4 + 2\lambda \end{cases} \Rightarrow \nexists\ \lambda \in \mathbb{R}$ $\therefore P \notin r$

$\lambda = 3$ não resolve o sistema

Q: $\begin{cases} -3 = 1 - \lambda \\ 4 = \lambda \\ 12 = 4 + 2\lambda \end{cases} \Rightarrow \lambda = 4 \in \mathbb{R} \therefore Q \in r$

c) $s : X = A + \lambda \vec{v}$

$s : (a, b, c) = (1, 4, -7) + \lambda \vec{v}$

$s \parallel r \rightarrow$ logo $(-1, 1, 2) = \vec{v}$

$s : x = (1, 4, -7) + \lambda(-1, 1, 2)$

3- a) $\vec{AB} + \vec{BC} + \vec{CA} = \vec{0}$

$\vec{AB} = (-5, 2, 3) - (3, 6, -7) = (-8, -4, 10)$
$\vec{BC} = (4, -7, -6) - (-5, 2, 3) = (9, -9, -9)$
$\vec{CA} = (3, 6, -7) - (4, -7, -6) = (-1, 13, -1)$

$(-8, -4, 10) + (9, -9, -9) + (-1, 13, -1) = (0, 0, 0)$

FORONI

7) a- $z = 4x + 2y + 5$

$$\begin{cases} x = \alpha \\ y = \beta \\ z = 4\alpha + 2\beta + 5 \end{cases}$$

b- $y = 5x - 1$

$$\begin{cases} x = \alpha \\ y = 5\alpha - 1 \\ z = \beta \end{cases}$$

c- $z = 3$

$$\begin{cases} x = \alpha \\ y = \beta \\ z = 3 \end{cases}$$

d- $y = z + 2$

$$\begin{cases} x = \alpha \\ y = \beta + 2 \\ z = \beta \end{cases}$$

8) a- $\vec{v} = (1, 2, 0)$

$\vec{u} = (-1, 1, -1)$

$P = (1, 0, 3)$

$$\begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 1 & 2 & 0 \\ -1 & 1 & -1 \end{vmatrix} = -2\vec{i} + \vec{j} + 3\vec{k}$$

$-2x + y + 3z + d = 0$

$-2 + 9 + d = 0$

$d = -7$

$\pi: -2x + y + 3z - 7 = 0$

FORON:

5) a- $\pi : X = A + \alpha \vec{u} + \beta \vec{v}$

$(x,y,z) = (1,2,0) + \alpha(1,1,0) + \beta(2,3,-1)$

$$\begin{cases} x = 1 + \alpha + 2\beta \\ y = 2 + \alpha + 3\beta \\ z = -\beta \end{cases}, \quad \alpha, \beta \in \mathbb{R}$$

b- $\vec{AB} = (1,-1,-1) - (1,1,0) = (0,-2,-1)$

$\pi : (x,y,z) = (1,1,0) + \alpha(0,-2,-1) + \beta(2,1,0)$

$$\begin{cases} x = 1 + 2\beta \\ y = 1 - 2\alpha + \beta \\ z = -\alpha \end{cases}, \quad \alpha, \beta \in \mathbb{R}$$

c- $\vec{AB} = (1,1,-3)$
$\vec{AC} = (0,-1,-1)$

$\pi : (x,y,z) = (1,0,1) + \alpha(1,1,-3) + \beta(0,-1,-1)$

$$\begin{cases} x = 1 + \alpha \\ y = \alpha - \beta \\ z = 1 - 3\alpha - \beta \end{cases}, \quad \alpha, \beta \in \mathbb{R}$$

6) a-

$$\det \begin{pmatrix} x-1 & y-2 & z \\ 0 & 1 & 0 \\ 1 & 1 & 1 \end{pmatrix} = 0$$

FORON:

$1(x-1)+0(y-2)-1(z)=0$

$x-1-z=0 \Rightarrow x+0y-z-1=0 : \pi$

b) $\vec{AB}=(-1,0,1)-(1,0,1)=(-2,0,0)$

$\vec{AP}=(2,1,2)-(1,0,1)=(1,1,1)$

$$\det\begin{pmatrix} x-1 & y & z-1 \\ -2 & 0 & 0 \\ 1 & 1 & 1 \end{pmatrix}=0$$

$0(x-1)+2(y)-2(z-1)=0$

$\pi: 2y-2z+2=0$

c) $\vec{AB}=(1,-1,-1)-(1,1,0)=(0,-2,-1)$

$$\det\begin{pmatrix} x-1 & y-1 & z \\ 0 & -2 & -1 \\ 2 & 1 & 0 \end{pmatrix}=0$$

$1(x-1)-2(y-1)+4(z)=0$

$x-1-2y+2+4z=0$

$\pi: x-2y+4z+1=0$

d) $\vec{PX}=(0,2,2)-(1,-1,1)=(-1,3,1)$

$\pi:(1,-1,1)+\alpha(-1,3,1)+\beta(1,1,-1)$

$$\begin{vmatrix} i & j & k \\ -1 & 3 & 1 \\ 1 & 1 & -1 \end{vmatrix} = -4i-4k$$

$(-4,0,-4)$

$-4-4+d=0$

$d=8$

$\pi: -4x-4z+8=0$

c) $\vec{v} = (-4, 5, 0)$
$\vec{u} = (2, -2, 1)$
$\vec{AB} = (0, 0, 3) - (2, 4, 11) = (-2, -4, -8)$

$$\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -4 & 5 & 0 \\ 2 & -2 & 1 \end{vmatrix} = (5, 4, -2)$$

$\vec{AB} \cdot (\vec{v} \times \vec{u}) = -2\ 9 - 4 \cdot 4 + 16 = -10 \neq 0 \Rightarrow$ reversas

d) $\vec{v} = (3, 4, 1)$
$\vec{u} = (4, 2, 2)$
$\vec{AB} = (0, 0, 3) - (2, -2, 1) = (-2, 2, 2)$

$$\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & 4 & 1 \\ 4 & 2 & 2 \end{vmatrix} = (6, -2, -10)$$

$\vec{AB} \cdot (\vec{v} \times \vec{u}) = -12 - 4 - 20 = -36 \Rightarrow$ reversa

10) a. $\begin{cases} x + 2y = -3\lambda + 1 \\ x - y = -2\lambda \\ z = \lambda \end{cases} \Rightarrow \begin{cases} x + 2y = 1 - 3\lambda \\ -3y = -1 + \lambda \\ z = \lambda \end{cases} \Rightarrow$

$\Rightarrow \begin{cases} x + 2y = 1 - 3\lambda \\ y = 1/3 - 1/3\lambda \\ z = \lambda \end{cases} \Rightarrow \begin{cases} x = 1/3 - 7/3\lambda \\ y = 1/3 - 1/3\lambda \\ z = \lambda \end{cases}$

$r: (1/3, 1/3, 0) + \lambda(-7/3, -1/3, 1)$

b. $\begin{cases} x + y + z - 1 = 0 \\ x + y - z = 0 \end{cases} \Rightarrow \begin{cases} x + y + z = 1 \\ -2z = -1 \end{cases} \Rightarrow \begin{cases} x = 1/2 - \lambda \\ y = \lambda \\ z = 1/2 \end{cases}$

$r: (1/2, 0, 1/2) + \lambda(-1, 1, 0)$

c- $\begin{cases} x = 3 \\ 2x - z + 1 = 0 \end{cases} \Rightarrow \begin{cases} x = 3 \\ y = \lambda \\ z = -7 \end{cases}$ $\quad r: (3,0,-7) + \lambda(0,1,0)$

d- $\begin{cases} y = 2 \\ z = 0 \\ x = \lambda \end{cases} \Rightarrow \begin{cases} x = \lambda \\ y = 2 \\ z = 0 \end{cases}$ $\quad r: (0,2,0) + \lambda(1,0,0)$

11) a- $s: \begin{cases} y + z = 3 \\ x + y - z = 6 \end{cases} \Rightarrow \begin{cases} x = 3 + 2\lambda \\ y = 3 - \lambda \\ z = \lambda \end{cases}$ $\quad s: (3,3,0) + \lambda(2,-1,1)$

$\vec{u} = (-2,1,-1)$
$\vec{v} = (2,-1,1)$
$\vec{AB} = (3,3,0) - (1,-1,1) = (2,4,-1)$

$\{\vec{u}, \vec{v}\}$ é L.D.
$\{\vec{AB}, \vec{u}\}$ é L.I.
$\to$ retas distintas paralelas.

b- $\vec{u} = (2,3,2)$
$\vec{v} = (1,2,0)$
$\vec{AB} = (0,0,0) - (-1,0,-1) = (1,0,1)$

$\vec{AB} \cdot (\vec{u} \times \vec{v}) = \begin{vmatrix} 1 & 0 & 1 \\ 2 & 3 & 2 \\ 1 & 2 & 0 \end{vmatrix} = -3$ $\neq 0$ retas reversas.

c- $\vec{u} = (2,-1,3)$
$\vec{v} = (1,-2,2)$
$\vec{AB} = (3,-4,4) - (9,1,9) = (-9,3,-9)$

$\vec{AB} \cdot (\vec{u} \times \vec{v}) = \begin{vmatrix} -9 & -9 & -3 \\ 2 & -1 & 3 \\ 1 & -2 & 2 \end{vmatrix} = -20 \neq 0 \Rightarrow$ reversas

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b- $\vec{v} = (1, 0, -1)$
$\vec{n} = (0, 0, 1)$
$P = (1, 2, 3)$

$$\begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 1 & 0 & -1 \\ 0 & 0 & 1 \end{vmatrix} = -\vec{j}$$

$-y + d = 0$
$-2 + d = 0$
$d = 2$

$\pi : -y + 2 = 0$

c- $\vec{v} = (1, 2, 1)$
$\vec{u} = (-1, 2, 1)$
$P = (-2, 0, 0)$

$$\begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 1 & 2 & 1 \\ -1 & 2 & 1 \end{vmatrix} = -2\vec{j} + 4\vec{k}$$

$-2y + 4z + d = 0$
$d = 0$

$\pi : -2y + 4z = 0$

9) a- $\vec{v} = (2, 1, 3)$
$\vec{u} = (4, 2, 6)$

$\vec{AB} = (-1, -1, -2) - (1, 0, 1) = (-2, -1, -3)$

$\{\vec{u}, \vec{v}\} = LD$, paralela

$$\vec{v} \times \vec{AB} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 1 & 2 & 3 \\ -2 & -2 & 1 \end{vmatrix} = -4\vec{i} + 8\vec{j} - 4\vec{k} = (-4, 8, -4)$$

b- $\{\vec{AB}, \vec{v}\} \in LI$

$\vec{AB} = (2, 3, 3) - (1, 1, 0) = (1, 2, 3)$ $\quad \vec{n} \cdot (\vec{AB}) = -4 + 16 - 12 = 0$

$d = -4$

$-4x + 8y - 4z - d = 0$

$\pi : -4x + 8y - 4z - 4 = 0$

14) a) $\vec{n_1} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 1 & 2 \\ 3 & 3 & 2 \end{vmatrix} = (-4, 4, 0)$

$\vec{n_2} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 1 & 0 \\ 0 & 1 & 4 \end{vmatrix} = (4, -4, 1)$

$\begin{cases} 4x - 4y + z = 12 \\ -4x + 4y + 8 = 0 \end{cases} \Rightarrow$

$\begin{cases} x = 8/3 + \lambda \\ y = \lambda \\ z = 4/3 \end{cases}$

$-4 \cdot 4 + 4 \cdot 2 + d = 0$ $\quad$ $4 \cdot 3 + d = 0$

$d = 8$ $\quad$ $d = -12$

$r: X = (8/3, 0, 4/3) + \lambda(1, 1, 0)$

$\pi_1: -4x + 4y + 8 = 0$ $\quad$ $\pi_2: 4x - 4y + z - 12 = 0$

$\{\vec{n_1}, \vec{n_2}\}$ é L.I $\Rightarrow$ planos transversais

b- $\vec{n_1} = (1, -1, 2)$

$\vec{n_2} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 0 & 3 \\ -1 & 1 & 1 \end{vmatrix} = (-3, -4, 1)$

$1 + d = 0$

$d = 1$ $\quad$ $\pi_2: -3x - 4y + z - 1 = 0$

$\{\vec{n_1}, \vec{n_2}\}$ é L.I $\Rightarrow$ planos transversais

$\begin{cases} x - y + 2z = 2 \\ -3x - 4y + z = 1 \end{cases} \Rightarrow \begin{cases} x = 1 - \lambda \\ y = -1 + \lambda \\ z = \lambda \end{cases}$

$\pi: (1, -1, 0) + \lambda(-1, 1, 1)$

c- $\vec{n_1} = (2, -1, 1)$ $\quad$ $\{\vec{n_1}, \vec{n_2}\}$ é L.D $\Rightarrow$ planos paralelos distintos

$\vec{n_2} = (4, -2, 2)$

FORONI:

d- $\begin{cases} x-2y=3-2z+4 \\ x-2y=2x-z \end{cases} \Rightarrow \begin{cases} x-3y-2z=3 \\ -x-2y-z=0 \end{cases} \Rightarrow \begin{cases} x=6/5+\lambda/5 \\ y=-3/5-3\lambda/5 \\ z=\lambda \end{cases}$

$\vec{u}\cdot(\vec{v}\times\vec{w})=(1/5,-3/5,1)\cdot(1,1,0)=1/5-3/5=-2/5\neq 0$

transversal

$6/5+\lambda/5-3/5-3\lambda/5=2$

$-2\lambda/5=2-3/5$

$\lambda=-7/2$

$P=(6/5,-3/5,0)-7/2(1/5,-3/5,1)$

$P=(1/2,3/2,-7/2)$

13) a- $\vec{AB}=(0,0,0)-(1,1,1)=(-1,-1,-1)$

$\vec{AB}\cdot(\vec{u}\times\vec{v})\neq 0 \Rightarrow \begin{vmatrix} 2 & m & 1 \\ 1 & 2 & 0 \\ 1 & 0 & 1 \end{vmatrix}\neq 0$

$-m+2\neq 0$

$m\neq 2$

b- $\begin{cases} m-6=1 \\ (2,m,m)\cdot(1,-3,1)=0 \end{cases} \Rightarrow \begin{cases} m=7 \\ 2-3m+m=0 \end{cases} \Rightarrow \begin{cases} m=7 \\ m=1 \end{cases}$

c- $\vec{u}=(m,2,m)$

$\vec{u}\cdot(\vec{v}\times\vec{w})\neq 0 \Rightarrow (m,2,m)\cdot(1,m,1)\neq 0$

$m+2m+m\neq 0$

$m\neq 0$

FORONI

d- $r: \begin{cases} x + y - 3z = 1 \\ 2x - y - 2z = 0 \end{cases} \Rightarrow \begin{cases} x = 1/3 + 9/3\,\lambda \\ y = 2/3 + 4/3\,\lambda \\ z = \lambda \end{cases}$

$\vec{u} = (2, 1, 1)$

$\vec{v} = (9/3, 4/3, 1)$

$\vec{AB} = (1/3, 2/3, 0) - (-1, 0, 0) = (-2/3, 2/3, 0)$

$\vec{AB} \cdot (\vec{u} \times \vec{v}) = \begin{vmatrix} -2/3 & 2/3 & 0 \\ 2 & 1 & 1 \\ 9/3 & 4/3 & 1 \end{vmatrix} = 0 \Rightarrow$ retas concorrentes.

12) a- $\vec{u} \cdot (\vec{v} \times \vec{w}) = (0, 1, 1) \cdot (1, -1, -1) = -2 \neq 0$ = transversas

$1 - (1 + \lambda) - \lambda = 2$

$\lambda = -2$

$p = (1, 1, 0) - 2(0, 1, 1) = (1, -1, -2)$

b- $\vec{u} = (2, 1, 1)$ $\quad \vec{u} \cdot (\vec{v} \times \vec{w}) = \begin{vmatrix} 2 & 1 & 1 \\ 1 & 0 & 1 \\ 2 & 2 & 0 \end{vmatrix} = 0$

não é transversal

c- $r: \begin{cases} x - y = 1 \\ x - 2y = 0 \end{cases} \Rightarrow \begin{cases} x = 2 \\ y = 1 \\ z = \lambda \end{cases} \Rightarrow \vec{v} = (0, 0, 1)$

$\vec{u} \cdot (\vec{v} \times \vec{w}) = (0, 0, 1) \cdot (1, 1, 0) = 0$ não é transversal

FORON:

b) $\vec{n_1} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ m & 1 & 1 \\ 1 & 1 & m \end{vmatrix} = (m-1, 1-m^2, m-1)$

$\vec{n_1} = \alpha \vec{n_2}$

$$\begin{cases} m-1 = 2\alpha \\ 1-m^2 = 3\alpha \\ m-1 = 2\alpha \end{cases}$$

$\Rightarrow 1 - m^2 = 3/2\,(m-1)$

$-m^2 = 3/2\,m - 3/2 - 1$

$m^2 + 3/2\,m - \frac{5}{2} = 0$

$$\frac{-\frac{3}{2} \pm \sqrt{\frac{9}{4} + \frac{20}{2}}}{2} \nearrow m_1 = 1 \quad m_2 = -\frac{5}{2}$$

$(m-1)x + (1-m)y + (m-1)z + d = 0$

$(x, y, z) = (1, 1, 0)$

$-\frac{9}{2} + 1 - \frac{25}{4} + d = 0$

$d = 35/4$

$m = 1$ e $m \neq 0$

$m = -9/2$ e $m \neq -2{,}15/16$

$-(...)$

d- $\vec{AB} = (5,0,1) - (0,1,6) = (5,0,6)$

$\vec{AC} = (4,0,0) - (0,1,6) = (4,-1,-6)$

$\vec{n_2} = (4,10,-4)$

$$\vec{n_1} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 5 & 0 & 6 \\ 4 & -1 & -6 \end{vmatrix} = (6,54,-5)$$

$\{\vec{n_1}, \vec{n_2}\}$ é L.I $\Rightarrow$ planos transversais

$6.4 + d = 0$ $\quad \pi: 6x + 54y - 5z - 24 = 0$

$d = -24$

$$\begin{cases} 6x + 54y - 5z = 24 \\ 4x + 40y - 4z = 16 \end{cases} \Rightarrow \begin{cases} x = 4 - 213\lambda \\ y = 116\lambda \\ z = \lambda \end{cases}$$

$r: X = (4,0,0) + \lambda(-213, 116, 1)$

15) a- $$\vec{n_1} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ -1 & m & 1 \\ 2 & 0 & 1 \end{vmatrix} = (m, 3, -2m)$$

$\vec{n_1} \cdot \vec{n_2} = (m, 3, -2m) \cdot (m, -m^2, -1)$

$$\vec{n_2} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ m & 1 & 0 \\ 1 & 0 & m \end{vmatrix} = (m, m^2, -1)$$

$\vec{n_1} = \alpha \vec{n_2}$

$$\begin{cases} m = \alpha m \Rightarrow \alpha = 1 \\ 3 = \alpha m^2 \Rightarrow \alpha < 0 \\ 2m = \alpha 1 \end{cases}$$

$\nexists\, \alpha \in \mathbb{R}$

$\{\vec{n_1}, \vec{n_2}\}$ é L.I $\Rightarrow$ planos serão transversais

Lista - 7

1- a) $\vec{AB} = (-2, 3, 2) - (3, 5, 1) = (-5, -2, 1)$

$r: X = A + \alpha \vec{u}$

$(x, y, z) = (-2, 3, 2) + \alpha(-5, -2, 1)$

$$\begin{cases} x = -2 - 5\alpha \\ y = 3 - 2\alpha \quad , \alpha \in \mathbb{R} \\ z = 2 + \alpha \end{cases}$$

paramétrica

$$\underbrace{\frac{x+2}{-5} = \frac{y-3}{-2} = \frac{z-2}{1}}_{\text{simétrica}} = \alpha$$

b) $\vec{AB} = (1, 0, 0) - (0, 1, 0) = (1, -1, 0)$

$$\begin{cases} x = \alpha \\ y = 1 - \alpha \quad , \alpha \in \mathbb{R} \\ z = \alpha \end{cases}$$

$$x = z = \frac{y-1}{-1}$$

c) $\vec{AB} = (0, 0, 0) - (0, 1, 1) = (1, -1, 0)$

$(x, y, z) = (0, 1, 0) + \lambda(1, -1, 0)$

$$\begin{cases} x = \lambda \\ y = 1 - \lambda \quad , \lambda \in \mathbb{R} \\ z = \lambda \end{cases}$$

$$x = z = \frac{y-1}{-1}$$

d) $\vec{AB} = (6, 1, 4) - (3, 2, 1) = (3, -1, 3)$

$$\begin{cases} x = 6 + 3\lambda \\ y = 1 - \lambda \quad , \lambda \in \mathbb{R} \\ z = 4 - 3\lambda \end{cases}$$

$$\frac{x-6}{3} = \frac{y-1}{1} = \frac{z-4}{-3}$$