e- Área do triângulo BCD = $\dfrac{\|\vec{BE} \times \vec{BD}\|}{2}$

$$\frac{\|\vec{BE} \times \vec{BD}\|}{2} = \frac{1}{2}\begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 2 & 2 & 2 \\ -1 & 3 & 2 \end{vmatrix} \Rightarrow$$

$\vec{BD} = -\vec{AB} + \vec{AD}$

$\vec{BD} = (-1, 0, -1) + (0, 3, 3)$

$\vec{BD} = (-1, 3, 2)$

$\Rightarrow -2\vec{i} - 6\vec{j} + 8\vec{k} = (-2, -6, 8)$

$\vec{BE} = -\vec{AB} + \vec{AE}$

$\vec{BE} = (-1, 0, -1) + (3, 2, 3)$

$\vec{BE} = (2, 2, 2)$

$$\frac{\|\vec{BE} \times \vec{BD}\|}{2} = \frac{\sqrt{4 + 36 + 64}}{2} = \frac{\sqrt{104}}{2}$$

$$= \frac{\cancel{2}\sqrt{26}}{\cancel{2}} = \sqrt{26}$$

$$V = \frac{A_B \cdot H}{3} \Rightarrow 1 = \frac{\sqrt{26} \cdot H}{3} \Rightarrow H = \frac{3}{\sqrt{26}}$$

$\Rightarrow a^2 + 4 + (2+a)^2 = 6$

$a^2 + 8 + a^2 + 4a = 6$

$2a^2 + 4a + 2 = 0$

$\frac{-4 \pm \sqrt{16-16}}{4} = a = -1$

$$\begin{cases} a = -1 \\ b = 2 \\ c = 1 \end{cases} \qquad \vec{x} = -\hat{i} + 2\hat{j} + \hat{k}$$

13) a) $\vec{AD} = D - A = (5,3,3) - (3,2,-1) = (2,1,4)$

$$\vec{AB} \times \vec{AD} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 1 & 1 & -1 \\ 2 & 1 & 4 \end{vmatrix} = 5\vec{i} - 6\vec{j} - \vec{k} = (5, -6, -1)$$

$A = \sqrt{25 + 36 + 1} = \sqrt{62}$

b) $\vec{AB} \times \vec{AC} = \begin{vmatrix} \hat{i} & \hat{j} & \vec{k} \\ -1 & 1 & 0 \\ 0 & 1 & 3 \end{vmatrix} = 3\vec{i} + 3\vec{j} - \vec{k} = (3, 3, -1)$

$A = \frac{\sqrt{9+9+1}}{2} = \frac{\sqrt{19}}{2}$

$H = \frac{\frac{\sqrt{19}}{2}}{\|\vec{BC}\|} = \frac{\sqrt{19}}{2 \cdot \sqrt{10}}$

$\vec{BC} = \vec{BA} + \vec{AC} \rightarrow \vec{BC} = -\vec{AB} + \vec{AC} = (1,-1,0) + (0,1,3) = (1,0,3)$

14) a) $\vec{u} \cdot (\vec{v} \times \vec{w}) = \begin{vmatrix} x_1 & y_1 & z_1 \\ x_2 & y_2 & z_2 \\ x_3 & y_3 & z_3 \end{vmatrix}$

$(\vec{u} \times \vec{v}) \cdot \vec{w} = \begin{vmatrix} x_3 & y_3 & z_3 \\ x_1 & y_1 & z_1 \\ x_2 & y_2 & z_2 \end{vmatrix} = (-1)^2 \cdot \begin{vmatrix} x_1 & y_1 & z_1 \\ x_2 & y_2 & z_2 \\ x_3 & y_3 & z_3 \end{vmatrix}$

FORON:

11) a) $\|\vec{u}\times\vec{v}\|^2 = \|\vec{u}\|^2\|\vec{v}\|^2 \operatorname{sen}^2\theta$

$\|\vec{u}\times\vec{v}\|^2 = \|\vec{u}\|^2\|\vec{v}\|^2(1-\cos^2\theta)$

$\|\vec{u}\times\vec{v}\|^2 = \|\vec{u}\|^2\|\vec{v}\|^2 - \|\vec{u}\|^2\|\vec{v}\|^2\cos^2\theta$

$\|\vec{u}\times\vec{v}\|^2 = \|\vec{u}\|^2\|\vec{v}\|^2 - (\vec{u}\cdot\vec{v})^2$

b) $\|\vec{u}\times\vec{v}\| = \sqrt{1\cdot 25-9} = \sqrt{16} = 4$

c) $\|\vec{AB}\times\vec{AC}\| = l\cdot l\cdot \operatorname{sen}60°$

$\|\vec{AB}\times\vec{AC}\| = l^2\frac{\sqrt{3}}{2}$

12) a) $\vec{x}\times(-\vec{i}+\vec{j}-\vec{k}) = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ a & b & c \\ -1 & 1 & -1 \end{vmatrix} = (-b-c)\hat{i} + (a-c)\hat{j} + (a+b)\vec{k}$

$$\begin{cases} 2a+3b+4c=9 \\ (-b-c)\hat{i}+(a-c)\hat{j}+(a+b)\hat{k} = -2\hat{i}+2\hat{k} \end{cases} \Rightarrow \begin{cases} 2a+3b+4c=9 \\ a-c=0 \\ -b-c=-2 \\ a+b=2 \end{cases}$$

$$\Rightarrow \begin{cases} 2a+3b+4c=9 \\ c=2-b \\ a=2-b \end{cases}$$

$2(2-b)+3b+4(2-b)=9$

$4-2b+3b+8-4b=9$

$-3b=-3$

$b=1$

$$\begin{cases} a=1 \\ b=1 \\ c=1 \end{cases}$$

b) $\vec{x}\times(1,0,1) = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ a & b & c \\ 1 & 0 & 1 \end{vmatrix} = b\hat{i}+(c\hat{j}-(a\hat{j}+b\hat{k}) =$

$b\hat{i}+(c-a)\hat{j}-b\hat{k}$

$$\begin{cases} b\hat{i}+(c-a)\hat{j}-b\hat{k} = 2\hat{i}+2\hat{j}-2\hat{k} \\ a^2+b^2+c^2=6 \end{cases} \Rightarrow \begin{cases} b=2 \\ c-a=2 \\ a^2+b^2+c^2=6 \end{cases}$$

FORONI

b) $[\vec{u}, \vec{v}, \vec{w}] = \begin{vmatrix} 1 & 3 & 2 \\ 0 & 1 & -2 \\ -1 & 2 & 0 \end{vmatrix} = 6 - (-2 - 4) = 12$

$[\vec{v}, 2\vec{w}, \vec{u}] = 24$

$[\vec{u}, 3\vec{v} - 2\vec{u}, \vec{w} + 3\vec{u}] = 3[\vec{u}, \vec{v}, \vec{w} + 3\vec{u}] - 2[\vec{u}, \vec{u}, \vec{w} + 3\vec{u}]$

$= 3([\vec{u}, \vec{v}, \vec{w}] + 3[\vec{u}, \vec{v}, \vec{u}])$

$= 3 \cdot 12 = 36$

15) a- $\vec{AE} = \vec{AB} + \vec{BE} = (1,0,1) + (2,2,2) = (3,2,3)$

$\vec{AD} = \vec{AF} - \vec{AE} = (3,5,6) - (3,2,3) = (0,3,3)$

Área paralelogramo ABCD $= \|\vec{AB} \times \vec{AD}\|$

$\vec{AB} \times \vec{AD} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 0 & 1 \\ 0 & 3 & 3 \end{vmatrix} = 3\vec{k} - (3\vec{i} + 3\vec{j}) = (-3, -3, 3)$

$\|\vec{AB} \times \vec{AD}\| = \sqrt{(-3)^2 + (-3)^2 + 3^2} = \sqrt{27}$ = Área

b- $[\vec{AD}, \vec{AB}, \vec{AE}] = \begin{vmatrix} 0 & 3 & 3 \\ 1 & 0 & 1 \\ 3 & 2 & 3 \end{vmatrix} = 9 + 6 - (9) = 6$

c- $V = A_B \cdot H \Rightarrow \sqrt{27} = 6 \cdot H \rightarrow \frac{\sqrt{27}}{6}$

d- $6/6 = 1$

d) $\vec{v} = (1, 2, 4)$, $\vec{u} = (-2, -4, -8)$

$\vec{u} \cdot \vec{v} = -2 - 8 - 32 = -42$

$\| \vec{u} \| = \sqrt{(-2)^2 + (-4)^2 + (-8)^2} = \sqrt{4 + 16 + 64} = (\sqrt{84})^2 = 84$

$proj_{\vec{u}} \vec{v} = \frac{\vec{u} \cdot \vec{v}}{\| \vec{u} \|^2} \cdot \vec{u} = \frac{-42}{84} \cdot \vec{u} = -\frac{1}{2} \cdot (-2, -4, -8) = (1, 2, 4)$

9) a- projeção ortogonal de $\vec{v}$ sobre $\vec{u}$

$\vec{u} = (2, -2, 1)$, $\vec{v} = (3, -6, 0)$

$\vec{u} \cdot \vec{v} = 6 + 12 + 0 = 18$

$\| \vec{u} \| = \sqrt{2^2 + (-2)^2 + 1^2} = \sqrt{4 + 4 + 1} = \sqrt{9}$

$\| \vec{v} \| = \sqrt{3^2 + (-6)^2 + 0^2} = \sqrt{9 + 36} = \sqrt{45}$

$proj_{\vec{u}} \vec{v} = \frac{\vec{u} \cdot \vec{v}}{\| \vec{u} \|^2} \cdot \vec{u} = \frac{18}{9} \cdot \vec{u} = 2 \cdot (2, -2, 1) = (4, -4, 2)$

$proj_{\vec{v}} \vec{u} = \frac{\vec{u} \cdot \vec{v}}{\| \vec{v} \|^2} \cdot \vec{v} = \frac{18}{45} \cdot \vec{v} = \frac{2}{5} \cdot (3, -6, 0) = \left(\frac{6}{5}, \frac{-12}{5}, 0\right)$

b- $\vec{p} = (a, b, c)$
$\vec{q} = (x, y, z)$

$p = \lambda \vec{u} \rightarrow \begin{cases} a = 2\lambda \\ b = -2\lambda \\ c = \lambda \end{cases}$

$\vec{q} \cdot \vec{v} = 0 \rightarrow 3x - 6y = 0$

$\begin{cases} a = 90/18 \\ b = -90/18 \\ c = 45/18 \end{cases}$

$\vec{q} + \vec{p} = \vec{v}$

$\rightarrow \begin{cases} a + x = 3 \\ b + y = -6 \\ c + z = 0 \end{cases} \Rightarrow \begin{array}{l} x = 3 - 2\lambda \\ y = -6 + 3\lambda \\ z = -\lambda \end{array}$

8) a- $\vec{v} = (1, -1, 2)$, $\vec{u} = (3, -1, 1)$

$$proj_{\vec{u}}(\vec{v}) = \frac{\vec{u} \cdot \vec{v}}{\|\vec{u}\|^2} \cdot \vec{u}$$

$$\vec{u} \cdot \vec{v} = (3 \cdot 1) + ((-1) \cdot (-1)) + (2 \cdot 1) = 3 + 1 + 2 = 6$$

$$\|\vec{u}\| = \sqrt{3^2 + (-1)^2 + 1^2} = \sqrt{9 + 1 + 1} = \sqrt{11}$$

$$\|\vec{u}\|^2 = (\sqrt{11})^2 = 11$$

$$proj_{\vec{u}}(\vec{v}) = \frac{6}{11} \cdot (3, -1, 1) = \left(\frac{18}{11}, \frac{-6}{11}, \frac{6}{11}\right)$$

b- $\vec{v} = (1, 3, 5)$, $\vec{u} = (-3, 1, 0)$

$$proj_{\vec{u}}(\vec{v}) = \frac{\vec{u} \cdot \vec{v}}{\|\vec{u}\|^2} \cdot \vec{u} \Rightarrow \frac{0}{\|\vec{u}\|^2} \cdot \vec{u} = 0 \cdot (-3, 1, 0) = (0, 0, 0)$$

$$\vec{u} \cdot \vec{v} = -3 + 3 + 0 = 0$$

c- $\vec{v} = (-1, 1, 1)$, $\vec{u} = (-2, 1, 2)$

$$\vec{u} \cdot \vec{v} = 2 + 1 + 2 = 5$$

$$\|\vec{u}\| = \sqrt{(-2)^2 + 1^2 + 2^2} = \sqrt{4 + 1 + 4} = 3$$

$$proj_{\vec{u}}(\vec{v}) = \frac{\vec{u} \cdot \vec{v}}{\|\vec{u}\|^2} \cdot \vec{u} \Rightarrow \frac{5}{3^2} \cdot \vec{u} = \frac{5}{9} \cdot (-2, 1, 2) = \left(\frac{-10}{9}, \frac{5}{9}, \frac{10}{9}\right)$$

$3\lambda - 6\lambda = 0$

$3(3-2\lambda) - 6(-6+2\lambda) = 0$

$9 - 6\lambda + 36 - 2\lambda = 0$

$-3\lambda = -45$

$\lambda = 45/8$

c) $A = \sqrt{5}\ \sqrt{\frac{8145}{64}} = \frac{\sqrt{10725}}{8}$

Se $\vec{p}$ é paralelo a $\vec{u}$, então $\vec{q}$ deve ser a altura do paralelogramo para que $\vec{p} + \vec{q} = \vec{v}$. Então a área é igual a $\|\vec{u}\|\ \|\vec{q}\|$

10) a- $\vec{u} \times \vec{v} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & 3 & 0 \\ 5 & 4 & 0 \end{vmatrix} = 12 - 15 = -3\hat{k}$

$\|\vec{u} \times \vec{v}\| = 3$

b- $\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 7 & 0 & -5 \\ 1 & 2 & -1 \end{vmatrix} = 10\hat{i} + 2\hat{j} + 14\hat{k} = (10, 2, 14)$

$\|\vec{u} \times \vec{v}\| = \sqrt{100+4+196} = \sqrt{300} = 10\sqrt{3}$

c- $\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & -3 & 1 \\ 1 & 1 & 4 \end{vmatrix} = -13\hat{i} - 3\hat{j} + 4\hat{k} = (-13, -3, 4)$

$\|\vec{u} \times \vec{v}\| = \sqrt{169+9+16} = \sqrt{194}$

d- $\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 1 & 2 \\ 4 & 2 & 4 \end{vmatrix} = (0, 0, 0)$ $\|\vec{u} \times \vec{v}\| = 0$

FORON:

b- $\vec{u} = (x, x, 4)$, $\vec{v} = (4, x, 1)$

$\vec{u} \cdot \vec{v} = 0 \rightarrow (x.4) + (x.x) + (4.1) = 0$

$4x + x^2 + 4 = 0$

$\rightarrow x^2 + 4x + 4 = 0$

$\Delta = b^2 - 4ac$

$\Delta = 16 - 4.1.4$

$\Delta = 16 - 16$

$\Delta = 0$

$x = \frac{-b \pm \sqrt{\Delta}}{2a} \rightarrow x = \frac{-4 \pm 0}{2} = \frac{-4}{2} = -2$

$\vec{u} \cdot \vec{v} = 0 \Leftrightarrow x = -2$

7) a- $\vec{u} = \vec{v} \times \vec{w}$

$\vec{u} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 4 & -1 & 5 \\ 1 & -2 & 3 \end{vmatrix} = 7\hat{i} + 7\hat{j} - 7\hat{j} \Rightarrow \vec{u} = (7, -7, -7)$

$\vec{u} \cdot (1,1,1) = -1 \rightarrow (7, -7, -7) \cdot (1, 1, 1) = 7 - 7 - 7 = -7$

$\vec{u} \cdot \frac{-1}{-7} = \frac{1}{7} \rightarrow \vec{u} = \frac{1}{7} \cdot (7, -7, -7) = (1, -1, -1)$

$\vec{u} = (1, -1, -1)$

b- $\vec{u} = \vec{v} \times \vec{w}$

$\vec{u} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 3 & -1 \\ 2 & -4 & 6 \end{vmatrix} = (18 - 4), (-2 - 12), (-8 - 6)$

$\vec{u} = 14\hat{i} + 14\hat{j} - 14\hat{k}$

$\vec{u} = (14, -14, -14)$

FORON:

c) $\vec{u} = (3,3,0)$, $\vec{v} = (2,1,-2)$

1º $\vec{u} \cdot \vec{v} = (3.2) + (3.1) + 0 = 9$

$\|\vec{u}\| = \sqrt{3^2 + 3^2} = \sqrt{18}$

$\|\vec{v}\| = \sqrt{2^2 + 1^2 + (-2)^2} = \sqrt{4+1+4} = \sqrt{9} = 3$

$$\cos\theta = \frac{\vec{u} \cdot \vec{v}}{\|\vec{u}\|\|\vec{v}\|} = \frac{9}{3\sqrt{18}} = \frac{3}{\sqrt{18}} = \arccos\left(\frac{3}{\sqrt{18}}\right) \text{ (rad)}$$

d) $\vec{u} = (\sqrt{3},1,0)$, $\vec{v} = (\sqrt{3},1,2\sqrt{3})$

2º $\vec{u} \cdot \vec{v} = (\sqrt{3}.\sqrt{3}) + (1.1) + 0 = 4$

$\|\vec{u}\| = \sqrt{(\sqrt{3})^2 + 1^2} = \sqrt{3+1} = \sqrt{4}$

$\|\vec{v}\| = \sqrt{(\sqrt{3})^2 + 1^2 + (2\sqrt{3})^2} = \sqrt{3+1+12} = \sqrt{16} = 4$

$$\cos\theta = \frac{\vec{u} \cdot \vec{v}}{\|\vec{u}\|\|\vec{v}\|} = \frac{4}{4\sqrt{4}} = \frac{1}{\sqrt{4}} = \arccos\frac{1}{2} \text{ (rad)}$$

6) a- $\vec{u} = (x+1, 1, 2)$, $\vec{v} = (x-1, -1, -2)$

$\vec{u} \cdot \vec{v} = 0 \rightarrow [(x+1).(x-1)] + (1.-1) + (2.-2) = 0$

$[x^2 - x + x - 1] + (-1) + (-4) = 0$

$x^2 - 1 - 1 - 4 = 0$ para que $\vec{u} \cdot \vec{v} = 0 \rightarrow x = \sqrt{6}$

$x^2 - 6 = 0$

$x^2 = 6 \rightarrow x = \sqrt{6}$

FORONI

$\|\vec{u}\| = \sqrt{14^2 + (-14)^2 + (-14)^2} = \sqrt{588} = 14\sqrt{3}$

$\rightarrow$ para $\|\vec{u}\| = 3\sqrt{3}$

$\vec{u} = \frac{3\sqrt{3}}{14\sqrt{3}} \cdot (14, -14, -14) = \frac{3}{14} \cdot (14, -14, -14)$

$\vec{u} = \left(\frac{3 \cdot \cancel{14}}{\cancel{14}}, \frac{3 \cdot (\cancel{-14})}{\cancel{14}}, \frac{3 \cdot (\cancel{-14})}{\cancel{14}} \right) = (3, -3, -3)$

$\vec{i} = (1, 0, 0)$ $\rightarrow 3 > 0$ log o ângulo entre $\vec{u}$ e $\vec{i}$ é agudo.

$\vec{u} \cdot \vec{i} = 3$

$\cos\theta = \frac{3}{3\sqrt{3} \cdot 1} = \frac{3}{3\sqrt{3}} = \frac{1}{\sqrt{3}}$

$\theta \approx 55^\circ$

c - $\cos\alpha = \frac{(\vec{u}+\vec{v}) \cdot (\vec{u}-\vec{v})}{\|(\vec{u}+\vec{v})\| \, \|(\vec{u}-\vec{v})\|} =$

$= \frac{\|\vec{u}\|^2 - \cancel{\vec{u} \cdot \vec{v}} + \cancel{\vec{u} \cdot \vec{v}} - \|\vec{v}\|^2}{\sqrt{\|\vec{u}\|^2 + 2\vec{u} \cdot \vec{v} + \|\vec{v}\|^2} \sqrt{\|\vec{u}\|^2 - 2\vec{u} \cdot \vec{v} + \|\vec{v}\|^2}}$

$= \frac{5-1}{\sqrt{5 + 2\sqrt{5} \cdot \frac{\sqrt{2}}{2} + 1} \cdot \sqrt{5 - \sqrt{10} + 1}} = \frac{4}{\sqrt{6+\sqrt{10}} \sqrt{6-\sqrt{10}}} = \frac{4}{\sqrt{26}} = \frac{2\sqrt{26}}{13}$

$\theta = \arccos \frac{2\sqrt{26}}{13}$

Se for triângulo retângulo:

$a^2 = b^2 + c^2$

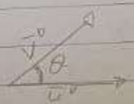
$a^2 = \left(\frac{\sqrt{174}}{2}\right)^2 + \left(\frac{\sqrt{54}}{2}\right)^2 = \frac{174}{4} + \frac{54}{4} = \frac{228}{4} = 57$

$a^2 = 57 \rightarrow a = \sqrt{57}$

Portanto, provamos que $\vec{AP}$, $\vec{PC}$ e $\vec{CA}$ formam um triângulo retângulo, por isso, o ângulo entre $\vec{PC}$ (a mediana) e $\vec{AB}$ é reto, que é a definição de mediatriz

d- Essa soma deve ser zero, pois ela forma um polígono fechado, resultando, então, em zero.

4) a- $|\vec{u} \cdot \vec{v}| \leq \|\vec{u}\| \, \|\vec{v}\|$



$\rightarrow \vec{u} \cdot \vec{v} = \|\vec{u}\| \, \|\vec{v}\| \cos\theta$

$= \|\vec{u}\| \, \|\vec{v}\| \, |\cos\theta| \leq \|\vec{u}\| \, \|\vec{v}\| \cdot 1$ $\quad \begin{cases} -1 \leq \cos\theta \leq 1 \\ 0 \leq |\cos\theta| \leq 1 \end{cases}$

b- $\|\vec{u} + \vec{v}\| \leq \|\vec{u}\| + \|\vec{v}\|$

$\|\vec{u} + \vec{v}\|^2 = \|\vec{u}\| + 2\vec{u} \cdot \vec{v} + \|\vec{v}\|^2$

$\|\vec{u}\| + 2\vec{u} \cdot \vec{v} + \|\vec{v}\|^2 \leq \|\vec{u}\| + 2\|\vec{u}\| \, \|\vec{v}\| + \|\vec{v}\|^2$

$= (\|\vec{u}\| + \|\vec{v}\|)^2$

c – $\vec{HB} = \vec{HD} + \vec{DA} + \vec{AB}$

$\vec{HB} = -\vec{DH} + \vec{DA} + \vec{DC}$

$\vec{HB} = (-1,1,1)_E = (0,\sqrt{2},1)_F$

$$\begin{pmatrix} 0 & -1/\sqrt{2} & 1/\sqrt{2} \\ 0 & 1/\sqrt{2} & 1/\sqrt{2} \\ -1 & 0 & 0 \end{pmatrix} \begin{pmatrix} -1 \\ 1 \\ 1 \end{pmatrix} = \begin{pmatrix} 0 \\ \sqrt{2} \\ 1 \end{pmatrix}$$

3 – a – $\cdot\ \vec{AB} = B - A = (5,1,-3) - (2,4,3) = (3,-3,-6)$

$\cdot\ \vec{BC} = C - B = (0,-3,1) - (5,1,-3) = (-5,-4,4)$

$\cdot\ \vec{CA} = A - C = (2,4,3) - (0,-3,1) = (2,7,2)$

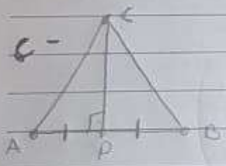
b – $\|\vec{AB}\| = \sqrt{3^2 + (-3)^2 + (-6)^2} = \sqrt{9+9+36} = \sqrt{54}$

$\|\vec{BC}\| = \sqrt{(-5)^2 + (-4)^2 + 4^2} = \sqrt{25+16+16} = \sqrt{57}$

$\|\vec{CA}\| = \sqrt{2^2 + 7^2 + 2^2} = \sqrt{4+49+4} = \sqrt{57}$

Conclusão: Visto que dois lados possuem o mesmo comprimento ($\sqrt{57}$), o triângul é isósceles.

c –



$\vec{PC} = C - P = (0,-3,1) - (7/2, 5/2, 0) = (-7/2, -\frac{11}{2}, 1)$

$\|\vec{PC}\| = \sqrt{\frac{49}{4} + \frac{121}{4} + 1} = \sqrt{\frac{174}{4}} = \frac{\sqrt{174}}{2}$

$\vec{AP} = P - A = (7/2, 5/2, 0) - (2,4,3) = (3/2, -3/2, -3)$

$\|\vec{AP}\| = \sqrt{9/4 + 9/4 + 9} = \frac{\sqrt{54}}{2}$

$\vec{CA} = (2,7,2)$

$\|\vec{CA}\| = \sqrt{57}$

FORONI:

c - $4\vec{u}\cdot\vec{v} = \|\vec{u}+\vec{v}\|^2 - \|\vec{u}-\vec{v}\|^2$

1º $\|\vec{u}+\vec{v}\|^2 = (\vec{u}+\vec{v})\cdot(\vec{u}+\vec{v})$
$= \vec{u}\cdot\vec{u} + 2\vec{u}\cdot\vec{v} + \vec{v}\cdot\vec{v}$

2º $\|\vec{u}-\vec{v}\|^2 = (\vec{u}-\vec{v})\cdot(\vec{u}-\vec{v})$
$= \vec{u}\cdot\vec{u} - 2\vec{u}\cdot\vec{v} + \vec{v}\cdot\vec{v}$

Logo: $\|\vec{u}+\vec{v}\|^2 - \|\vec{u}-\vec{v}\|^2 =$

$= (\vec{u}\cdot\vec{u} + 2\vec{u}\cdot\vec{v} + \vec{v}\cdot\vec{v}) - (\vec{u}\cdot\vec{u} - 2\vec{u}\cdot\vec{v} + \vec{v}\cdot\vec{v})$
$= \vec{u}\cdot\vec{u} + 2\vec{u}\cdot\vec{v} + \vec{v}\vec{v} - \vec{u}\cdot\vec{u} + 2\vec{u}\cdot\vec{v} - \vec{v}\cdot\vec{v}$
$= 4\vec{u}\cdot\vec{v}$

5) a- $\vec{u} = (1,0,1)$, $\vec{v} = (-2,10,2)$

$\cos\theta = \dfrac{\vec{u}\cdot\vec{v}}{\|\vec{u}\|\cdot\|\vec{v}\|}$ $\quad\cdot\ \vec{u}\cdot\vec{v} = (1\cdot-2)+(0\cdot10)+(1\cdot2) = 0$

$\cos\theta = \dfrac{0}{\|\vec{u}\|\cdot\|\vec{v}\|} = \cos\theta = 0$

conclusão: $\theta = 90°$ ou $\dfrac{\pi}{2}$

b - $\vec{u} = (-1,1,1)$, $\vec{v} = (1,1,1)$ $\quad \cos\theta = \dfrac{\vec{u}\cdot\vec{v}}{\|\vec{u}\|\,\|\vec{v}\|}$

$\cdot\ \vec{u}\cdot\vec{v} = (-1\cdot1)+(1\cdot1)+(1\cdot1) = 1$

$\cos\theta = \dfrac{1}{\sqrt{3}\sqrt{3}} = \dfrac{1}{3}$

$\|\vec{u}\| = \sqrt{(-1)^2+1^2+1^2} = \sqrt{3}$

$\theta = \arccos\left(\dfrac{1}{3}\right)$ (rad)

$\|\vec{v}\| = \sqrt{1^2+1^2+1^2} = \sqrt{3}$

Lista 6.

$\|\vec{v}\| = \sqrt{\vec{v}\cdot\vec{v}}$

1) a- $\|\vec{u}\| = \sqrt{\vec{u}\cdot\vec{u}}$

$\|\vec{u}\| = \sqrt{1^2+1^2+1^2}$

$\|\vec{u}\| = \sqrt{3}$

b- $\vec{u} = 3\vec{i} + 4\vec{k}$ $\qquad E = (\vec{i}, \vec{j}, \vec{k})$

$\vec{u} = (3, 0, 4)_E$

$\|\vec{u}\| = \sqrt{3^2+0^2+4^2} = \sqrt{9+16} = 5$

c- $\vec{u} = -\vec{i} + \vec{j}$

$\vec{u} = (-1, 1, 0)_E$

$\|\vec{u}\| = \sqrt{(-1)^2+0^2+1^2} = \sqrt{1+1} = \sqrt{2}$

d- $\vec{u} = 4\vec{i} + 3\vec{j} - \vec{k}$

$\vec{u} = (4, 3, -1)_E$

$\|\vec{u}\| = \sqrt{4^2+3^2+(-1)^2} = \sqrt{16+9+1} = \sqrt{26}$

2) a- A base $E = (\vec{e_1}, \vec{e_2}, \vec{e_3})$ é ortonormal visto que as arestas do cubo são unitárias, portanto:

$\|\vec{e_1}\| = \|\vec{DH}\| = 1$ $\|\vec{e_2}\| = \|\vec{DC}\| = 1$ $\|\vec{e_3}\| = \|\vec{DA}\| = 1$

Além disso, $\vec{DH}$ é perpendicular a $\vec{DC}$, $\vec{DC}$ é perpendicular a $\vec{DA}$, e $\vec{DH}$ é perpendicular a $\vec{DA}$. Atingindo assim, os requisitos para que $E = (\vec{e_1}, \vec{e_2}, \vec{e_3})$ seja uma base ortonormal.

b- • $\vec{u} = \vec{CD} + \vec{CB}$ • $\vec{CD} = -\vec{DC} = -\vec{e_2}$

• $\vec{u} = -\vec{e_2} + \vec{e_3}$ • $\vec{CB} = \vec{DA} = \vec{e_3}$

• $\vec{v} = \vec{DC} + \vec{CB}$ • $\vec{w} = \vec{GC}$

• $\vec{v} = \vec{e_2} + \vec{e_3}$ $\vec{w} = \vec{GC} = -\vec{CG} = -\vec{DH} = -\vec{e_1}$

c- $\vec{f_1} = \frac{\vec{u}}{\|\vec{u}\|} = \frac{(0,-1,1)}{\|(0,-1,1)\|} = \frac{(0,-1,1)}{\sqrt{0^2+(-1)^2+1^2}} = \frac{(0,-1,1)}{\sqrt{2}} = \frac{1}{\sqrt{2}}(0,-1,1)$

$= \left(0, \frac{-1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right)$

$\vec{f_2} = \frac{\vec{v}}{\|\vec{v}\|} = \frac{(0,1,1)}{\|(0,1,1)\|} = \frac{(0,1,1)}{\sqrt{0^2+1^2+1^2}} = \frac{1}{\sqrt{2}} \cdot (0,1,1) = \left(0, \frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right)$

$\vec{f_3} = -(1,0,0) = (-1,0,0)$

Base ortogonal:

• Normas:

$\|\vec{f_1}\| = \left\|\left(0, \frac{-1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right)\right\| = \sqrt{0^2 + \left(-\frac{1}{\sqrt{2}}\right)^2 + \left(\frac{1}{\sqrt{2}}\right)^2} = \sqrt{\frac{1}{2}+\frac{1}{2}} = 1$

$\|\vec{f_2}\| = \|(0, \frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}})\| = \sqrt{0^2 + (\frac{1}{\sqrt{2}})^2 + (\frac{1}{\sqrt{2}})^2} = \sqrt{\frac{1}{2} + \frac{1}{2}} = 1$

$\|\vec{f_3}\| = \|(-1, 0, 0)\| = \sqrt{(-1)^2 + 0^2 + 0^2} = \sqrt{1} = 1$

conclusão: os vetores são unitários

• Ortogonalidade

$$\begin{cases} \vec{f_1} \cdot \vec{f_2} = 0 \\ \vec{f_1} \cdot \vec{f_3} = 0 \\ \vec{f_2} \cdot \vec{f_3} = 0 \end{cases}$$

d- mudança de E para F

$$(\vec{f_1}\ \vec{f_2}\ \vec{f_3}) = (\vec{e_1}\ \vec{e_2}\ \vec{e_3})M$$

$$\begin{cases} \vec{f_1} = \vec{u} / \|\vec{u}\| \\ \vec{f_2} = \vec{v} / \|\vec{v}\| \\ \vec{f_3} = \vec{w} / \|\vec{w}\| \end{cases} \qquad \begin{cases} f_1 = (0, -1/\sqrt{2}, 1/\sqrt{2}) \\ f_2 = (0, 1/\sqrt{2}, 1/\sqrt{2}) \\ \vec{f_3} = (-1, 0, 0)_E \end{cases}$$

$$M = \begin{pmatrix} 0 & 0 & -1 \\ -1/\sqrt{2} & 1/\sqrt{2} & 0 \\ 1/\sqrt{2} & 1/\sqrt{2} & 0 \end{pmatrix} \qquad M^{-1} = \begin{pmatrix} 0 & -1/\sqrt{2} & 1/\sqrt{2} \\ 0 & 1/\sqrt{2} & 1/\sqrt{2} \\ -1 & 0 & 0 \end{pmatrix}$$

$$M^{-1} \begin{pmatrix} x \\ y \\ z \end{pmatrix}_E = \begin{pmatrix} x' \\ y' \\ z' \end{pmatrix}_F$$