CS180 Winter 2011
Due: 2^{nd} March
Homework 7

When submitting your homework, please include your name at the top of each page. If you submit multiple pages, *please staple them together*. We also ask that you do something to indicate which name is your last name on the first page, such as underlining it.

Please submit your solutions with the problems solved in the order they are given.

1. In a school there are n boys and n girls. Each boy knows exactly k girls ($1 \leq k \leq n$) and each girl knows exactly k boys. In this problem, "knowing" is mutual.

   (a) Prove that all the boys and girls can participate in one dance, where each pair of dancers (a boy and a girl) know each other.

   (b) Show that it is always true that k consecutive dances can be organized so that everyone will dance once with everyone he or she knows.

2. We define the *Escape Problem*[1] as follows. We are given a directed graph $G = (V, E)$ (picture a network of roads). A certain collection of nodes $X \subset V$ are designated as *populated nodes*, and a certain other collection $S \subset V$ are designated as *safe nodes*. Assume that X and S are disjoint. In case of an emergency, we want evacuation routes from the populated nodes to the safe nodes. A set of evacuation routes is defined as a set of paths in G so that (i) each node in X is the tail of one path, (ii) the last node on each path lies in S, and (iii) the paths do not share any edges. Such a set of paths gives a way for the occupants of the populated nodes to "escape" to S, without overly congesting any edge in G.

   (a) Given G, X, and S, show how to decide in polynomial time whether such a set of evacuation routes exists.

   (b) Suppose we have exactly the same problem as in (a), but we want to enforce an even stronger version of the "no congestion" condition (iii). Thus we change (iii) to say "the paths do not share any *nodes*."
   With this new condition, show how to decide in polynomial time whether such a set of evacuation routes exists.
   Also, provide an example with the same G, X, and S, in which the answer is yes to the question in (a) but no to the question in (b).

3. Back in the euphoric early days of the Web, people liked to claim that much of the enormous potential in a company like Yahoo! was in the "eyeballs" – the simple fact that millions of people look at its pages every day[2]. Further, by convicing people to register personal information with the site, a site like Yahoo! can show each user an extremely targeted advertisement whenever he or she visits the site, in a way that TV networks or magazines couldn't hope to match. So if a user has told Yahoo! that he or she is a 20-year-old computer science major from Cornell University, the iste can present a banner ad for apartments in

[1] This is problem 7.14 from your textbook.
[2] This is problem 7.16 from your textbook.

Ithaca, New York; on the other hand, if he or she is a 50-year-old investment banker from Greenwich, Connecticut, the site can display a banner ad pitching Lincoln Town Cars instead.

But deciding on which ads to show which people involves some serious computation behind the scenes. Suppose that the managers of a popular Web site have identified k distinct *demographic groups* $G_1, G_2, \ldots, G_k$. These groups can overlap; for example, G_1 can be residents of New York State, and G_2 ca be people with a degree computer science. The site has contracts with M different *advertisers*, to show a certain number of copies of their ads to users of the website. Here's what the contract with the ith advertiser looks like.

- For a subset $X_i \subseteq \{G_1, \ldots, G_k\}$ of the demographic groups, advertiser i wants its ads shown only to users who belong to at least one of the demographic groups in the set X_i.
- For a number $_i$, advertiser i wants its ads shown to at least r_i users each minute

Now consider the problem of designing a good *advertising policy* – a way to show a single ad to each user of the site. Suppose at a given minute, there are n users visiting the site. Because we have registration information on each of the users, we know that user j belongs to a subset U_j of the demographic groups. The problem is: Is there a way to show a single ad to each user so that the site's contracts with each of the m advertisers is satsified for this minute?

Give an efficient algorithm to decide if this is possible, and if so, to actually choose an ad to show each user.

4. Problem 23 (Pg 428) from book. (Also try Problem 24. Problem 24 is an extension of Problem 23, however it is not a part of this homework.)