

CS180 Spring 2013

Homework 2

The following homework is due Thursday, April 18 at the beginning of lecture.

When submitting your homework, please include your name at the top of each page. If you submit multiple pages, please staple them together. We also ask you to indicate which name is your last name on the first page, such as underlining it.

Please provide complete arguments and time complexity analysis for all solutions

1. Design an algorithm of constructing a heap with linear time complexity $O(n)$.
2. In class, we discussed the CELEBRITY PROBLEM. Among n people, a *celebrity* is defined as somebody whom everyone knows, but who knows no one else. You are given a $n \times n$ Boolean matrix whose ij th entry is 1 if the i th person knows the j th person and 0 otherwise. Give an *iterative* (not recursive) algorithm with complexity $O(n)$ to find the celebrity, if one exists.
3. In class, we discussed the INDEPENDENT SET. Given a graph $G' = (V', E')$, we say a set of nodes $S \subseteq V'$ is independent if no two nodes in S are joined by an edge. Given a directed graph $G = (V, E)$, design an *iterative* (not recursive) algorithm to find a set of nodes I such that 1) the set of nodes I is independent in the underlying undirected graph obtained from the directed graph G by ignoring the edge direction; 2) $\forall w \in V, \exists v \in I, v$ can reach w in at most two steps.
4. Given a tree $T = (V, E)$, design an algorithm of time complexity $O(|E|)$ that pairs up odd degree nodes of the tree such that the nodes in these pairs are connected by paths that are edge disjoint. Prove the correctness of your algorithm.