

1.

| P | Q | $P \Rightarrow \neg Q$ | $Q \Rightarrow \neg P$ | $P \Leftrightarrow \neg Q$ | $((P \wedge \neg Q) \vee (\neg P \wedge Q))$ |
|---|---|------------------------|------------------------|----------------------------|--|
| T | T | F | F | F | F |
| T | F | T | T | T | T |
| F | T | T | T | T | T |
| F | F | T | T | F | F |

$\therefore P \Rightarrow \neg Q$ is equivalent to $Q \Rightarrow \neg P$

$P \Leftrightarrow Q$ is equivalent to $((P \wedge \neg Q) \vee (\neg P \wedge Q))$

2. ① = $(\text{Smoke} \Rightarrow \text{Fire}) \Rightarrow (\neg \text{Smoke} \Rightarrow \neg \text{Fire})$

② = $(\text{Smoke} \Rightarrow \text{Fire}) \Rightarrow ((\text{Smoke} \vee \text{Heat}) \Rightarrow \text{Fire})$

③ = $((\text{Smoke} \wedge \text{Heat}) \Rightarrow \text{Fire}) \Leftrightarrow ((\text{Smoke} \Rightarrow \text{Fire}) \vee (\text{Heat} \Rightarrow \text{Fire}))$

| Smoke | Fire | Heat | ① | ② | ③ |
|-------|------|------|---|---|---|
| T | T | T | T | T | T |
| T | T | F | T | T | T |
| T | F | T | T | T | F |
| T | F | F | T | T | T |
| F | T | T | F | T | T |
| F | T | F | F | T | T |
| F | F | T | T | T | F |
| F | F | F | T | T | T |

\therefore ② is valid, ① and ③ are neither

KB:

3. a) $\text{mythical} \Rightarrow \neg \text{mortal}$

$$\neg \text{mythical} \Rightarrow (\text{mortal} \wedge \text{mammal})$$

$$(\neg \text{mortal} \vee \text{mammal}) \Rightarrow \text{horned}$$

$$\text{horned} \Rightarrow \text{magical}$$

b) ① $\neg \text{mythical} \vee \neg \text{mortal}$

② $\text{mythical} \vee (\text{mortal} \wedge \text{mammal})$

$$(\text{mythical} \vee \text{mortal}) \wedge (\text{mythical} \vee \text{mammal})$$

③ $\neg (\neg \text{mortal} \vee \text{mammal}) \vee \text{horned}$

$$(\text{mortal} \wedge \neg \text{mammal}) \vee \text{horned}$$

$$(\text{mortal} \vee \text{horned}) \wedge (\neg \text{mammal} \vee \text{horned})$$

④ $\neg \text{horned} \vee \text{magical}$

$$\text{KB}: (\neg \text{mythical} \vee \neg \text{mortal}) \wedge (\text{mythical} \vee \text{mortal}) \wedge (\text{mythical} \vee \text{mammal})$$

$$\wedge (\text{mortal} \vee \text{horned}) \wedge (\neg \text{mammal} \vee \text{horned}) \wedge (\neg \text{horned} \vee \text{magical})$$

c) To prove $\text{KB} \models \text{mythical}$, try to find contradiction in $\text{KB} \wedge \neg \text{mythical}$

1. $\neg \text{mythical} \vee \neg \text{mortal}$

2. $\text{mythical} \vee \text{mortal}$

3. $\text{mythical} \vee \text{mammal}$

4. $\text{mortal} \vee \text{horned}$

5. $\neg \text{mammal} \vee \text{horned}$

6. $\neg \text{horned} \vee \text{magical}$

*7. $\neg \text{mythical}$

8. mortal (2, 7)

9. mammal (3, 7)

10. horned (5, 9)

11. magical (6, 10)

There is no more resolution.

$\therefore \text{KB} \not\models \text{mythical}$

To prove $KB \models \text{magical}$, try to find contradiction in $KB \wedge \neg \text{magical}$

1. $\neg \text{mythical} \vee \neg \text{mortal}$
 2. $\text{mythical} \vee \text{mortal}$
 3. $\text{mythical} \vee \text{mammal}$
 4. $\text{mortal} \vee \text{horned}$
 5. $\neg \text{mammal} \vee \text{horned}$
 6. $\neg \text{horned} \vee \text{magical}$
 - *7. $\neg \text{magical}$
 8. $\neg \text{horned}$ (6, 7)
 9. mortal (4, 8)
 10. $\neg \text{mammal}$ (5, 8)
 11. $\neg \text{mythical}$ (1, 9)
 12. mythical (3, 10)
- contradiction $\therefore KB \models \text{magical}$

To prove $KB \models \text{horned}$, try to find contradiction in $KB \wedge \neg \text{horned}$

1. $\neg \text{mythical} \vee \neg \text{mortal}$
 2. $\text{mythical} \vee \text{mortal}$
 3. $\text{mythical} \vee \text{mammal}$
 4. $\text{mortal} \vee \text{horned}$
 5. $\neg \text{mammal} \vee \text{horned}$
 6. $\neg \text{horned} \vee \text{magical}$
 - *7. $\neg \text{horned}$
 8. mortal (4, 7)
 9. $\neg \text{mammal}$ (5, 7)
 10. $\neg \text{mythical}$ (1, 8)
 11. mythical (3, 9)
- contradiction $\therefore KB \models \text{horned}$