CS180 Winter 2011

Homework 5

Due: 2/9

When submitting your homework, please include your name at the top of each page. If you submit multiple pages, *please staple them together.* We also ask that you do something to indicate which name is your last name on the first page, such as underlining it.

Please submit your solutions with the problems solved in the order they are given.

1. From book, Exercise 28 on page 203.
2. From book, Exercise 5 on pg 317.
3. From book, Exercise 13 on pg 324.
4. We are given a sequence $(A_1, A_2, \ldots A_n)$ of n matrices to be multiplied, and we wish to compute the product

$$A_1 A_2 \ldots A_n.$$

We can evaluate the expression using standard algorithm for multiplying pairs of matrices as a subroutine once we have parenthesized it to resolve all ambiguities in how the matrices are multiplied together. A product of matrices is *fully parenthesized* if it is either a single matrix or the product of two fully parenthesized matrix products, surrounded by parentheses. Matrix multiplication is associative and so all parenthesizations yield the same product. For example, sequence (A_1, A_2, A_3, A_4) can be parenthesized in the following ways-

$$(A_1(A_2(A_3A_4)))$$

$$(A_1((A_2A_3)A_4))$$

$$((A_1A_2)(A_3A_4))$$

$$((A_1(A_2A_3))A_4)$$

$$(((A_1A_2)A_3)A_4)$$

The way we parenthesize a chain can have a dramatic impact on the cost of evaluating the product.

Problem Given a sequence $(A_1, A_2, \ldots A_n)$ of n matrices, where $\forall i \in \{1, 2 \ldots n\}$, matrix A_i has dimensions $p_{i-1} \times p_i$, fully parenthesize the product $(A_1 A_2 \ldots A_n)$ in a way that minimizes the number of scalar multiplication when evaluating the matrix product.