# CS180 Winter 2011

Homework 1

The following homework is due Wednesday, January 12 at the beginning of lecture.

When submitting your homework, please include your name at the top of each page. If you submit multiple pages, please staple them together. We also ask that you do something to indicate which name is your last name on the first page, such as underlining it.

**Please provide complete arguments and time complexity analysis for all solutions, unless otherwise stated.**

**In order to have any work graded in this class, you must first sign and return the academic honesty agreement.**

1. Prove the following by induction:

$$\begin{bmatrix} 0 & 1 \\ 1 & 1 \end{bmatrix}^n = \begin{bmatrix} f_{n-2} & f_{n-1} \\ f_{n-1} & f_n \end{bmatrix}$$

   where $f_i$ is the $i$th fibonacci number. As a reminder, $f_0 = f_1 = 1$ and $f_{n>1} = f_{n-1} + f_{n-2}$.

2. In class, we discussed the game of Nim. This game begins with a placement of $n$ rows of matches on a table. Each row $i$ has $m_i$ matches. Players take turns selecting a row of matches and removing any or all of the matches in that row. Whoever claims the final match from the table wins the game. We also discussed a strategy based on writing the count for each row in binary, and noted that a table was *favorable* if there was a column with an odd number of ones in it, and the table was *unfavorable* if all columns have an even number of ones.
   Prove that, for any favorable table, there exists a move that makes the table unfavorable for one's opponent. Prove also that, for any unfavorable table, any move makes the table favorable for one's opponent.

3. Suppose you are given a sorted array $A$ of distinct integers. Your goal is to determine if there is an index $i$ in the array such that $A[i] == i$. Show how to make this determination using at most $O(\log n)$ accesses to the array.

4. Consider $n$ players- $P_1, P_2 \dots P_n$ who play chess with each other. For every $i, j \in \{1 \dots n\}$ a game is held between players $P_i$ and $P_j$ and *exactly* one of them wins (no game ends in a draw). A player $P_i$ is said to *beat* $P_j$ if $P_i$ wins the game between $P_i$ and $P_j$. A *king* is defined as a player $P_i$ for $i \in \{1 \dots n\}$ such that $\forall j \neq i$ and $j \in \{1 \dots n\}$ $P_i$ beats $P_j$ or there exists $k \in \{1 \dots n\}$ such that $k \neq i$ and $k \neq j$ such that $P_i$ beats $P_k$ and $P_k$ beats $P_j$. Intuitively, every player is either directly beaten by the king or a player who was directly beaten by the king. Argue that-

   (a) There exists a king. (Do not use the theorem on independent sets proved in class.)

   (b) Give an efficient (as efficient as you can) algorithm to find au king. Argue the correctness of your algorithm.

   (c) What is the time complexity?