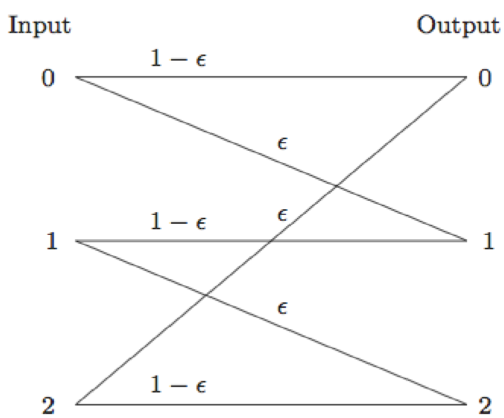
CS112 HOMEWORK 2 FALL 2013

1. Express each of the following events in terms of the events A, B and C as well as the operations of complementation, union and intersection:
(a) at least one of the events A, B, C occurs
(b) at most one of the events A, B, C occurs
(c) none of the events A, B, C occurs
(d) all three events A, B, C occur
(e) exactly one of the events A, B, C occurs
(f) events A and B occur, but not C
(g) either event A occurs or, if not, then B also does not occur

2. Roll two fair dice independently. We define events A, B, C as follows:
A = {First die is 1, 2 or 3}
B = {First die is 2, 3 or 6}
C = {Sum of outcomes is 9} = {(3, 6), (4, 5), (5, 4), (6, 3)}
Are A, B, C independent? Prove your answer.

3. A ternary communication channel is shown in the following figure.



Ternary Communication Channel

The input symbols 0, 1, and 2 occur with probabilities $\frac{1}{2}$, $\frac{1}{4}$ and $\frac{1}{4}$ respectively.
(a) Find the probabilities of the output symbols, in terms of ϵ.
(b) Suppose the observed output is 0. Find the probabilities that the input was 0, 1, 2 respectively.
(c) Find the probability of error, i.e. the probability that the output symbol is different from the input symbol, in terms of ϵ.

4. Suppose that you are the manager in a manufacturing plant and there are large lots of products that need to be tested for quality. It is perhaps impractical to test each one in the lot, so we sample. In each lot of 100 items, 5 are tested. The lot is rejected if any of the tested item is found defective.
(a) Find the probability of accepting a lot with 5 defective items.
(b) Find the probability of accepting a lot with 10 defective items.

5. A system consists of n identical components, each of which is operational with probability p, independent of other components. The system is operational if at least k out of n components are operational. What is the probability that the system is operational?

6. A cellular phone system services a population of n_1 voice users and n_2 data users. We estimate that at given time, each user will need to be connected to the system with probability p_1 for voice users and p_2 for data users, independent of other users. The data rate for a voice user is r_1 bit/sec, and for a data user is r_2 bit/sec. The system has a capacity of c bit/sec. Use the above notations $(n_1, n_2, p_1, p_2, r_1, r_2, c)$ to express the probability that more users want to use the system than the system can accommodate. (Hint: supplementary notations may be needed, e.g. use i to denote the number of voice users who want to connect at the moment, and j denote the number of data users who want to connect at the moment, where $0 \leq i \leq n_1$ and $0 \leq j \leq n_2$).

7. Let A, B, C be independent events with $P(C) > 0$. Prove that A and B are conditionally independent given C.

8. Two dice are rolled. What is the probability that at least one is a six? If the two faces are different, what is the probability that at least one is a six?

9.Two cards are randomly selected from a deck of 52 playing cards.
(a) What is the probability they constitute a pair (that is, that they are of the same denomination)?
(b) What is the conditional probability they constitute a pair given that they are of different suits?

10.Consider two urns. The first contains two white and seven black balls, and the second contains five white and six black balls. We flip a fair coin and then draw a ball from the first urn or the second urn depending on whether the outcome was heads or tails. What is the conditional probability that the outcome of the toss was heads given that a white ball was selected?

11. In answering a question on a multiple-choice test a student either knows the answer or guesses. Let p be the probability that she knows the answer and $1 - p$ the probability that she guesses. Assume that a student who guesses at the answer will be correct with probability $1/m$, where m is the number of multiple-choice alternatives. What is the conditional probability that a student knew the answer to a question given that she answered it correctly?