

# CS180 Winter 2011

## Homework 6

The following homework was due Wednesday, February 23.

1. Consider two number  $a$  and  $b$  of length  $n$ , i.e. each of them is  $n$  bits long. A very simple solution for multiplying the two numbers takes  $O(n^2)$  time. We want to improve the time taken. Give a better algorithm, and the time complexity. Hint: Lets say  $a = a_1 2^{\frac{n}{2}} + a_2$  and  $b = b_1 2^{\frac{n}{2}} + b_2$ . Now  $a \cdot b = (a_1 \cdot b_1) 2^n + (a_1 \cdot b_2 + b_1 \cdot a_2) 2^{\frac{n}{2}} + a_2 \cdot b_2$ . Now to evaluate  $a \cdot b$  we need to solve four problems of smaller size. Can we evaluate  $a \cdot b$  by solving fewer sub-problems?
2. Consider  $n$  nuts and  $n$  screws. Assume that for each screw there exists only one nut that fits (is of the same size). You can try fitting a nut with a screw. After one such trial either the nut turns out to be bigger than the screw, or they fit nicely or the nut turns out to be smaller than the screw. Give an algorithm to pair the nuts with the corresponding matching (of the same size) screws making as few fitting trials (asymptotically) as possible.
3. Problem 5 Pg 248 from book.