

Submission - Assignment 2 - Deep Learning

Marta Gulida - Mn: 5585808 - mg776

Erik Bode - Mn: 4505193 - kb301

Tillman Heisner - Mn: 4517815 - th273

Task 1.: $X = \begin{bmatrix} 1 & 3 \\ 2 & 4 \\ 3 & 5 \end{bmatrix}$

~~W~~ $W^{(1)T} = \begin{bmatrix} -2 & 2 & -3 \\ 1 & 0 & 1 \end{bmatrix}$

~~H~~ $H^{(1)} = g^{(1)} \left(\begin{bmatrix} -2 & 2 & -3 \\ 1 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 3 \\ 2 & 4 \\ 3 & 5 \end{bmatrix} + \begin{bmatrix} 3 \\ 0 \end{bmatrix} \right)$

~~W~~ $= g^{(1)} \left(\begin{bmatrix} -7 & -13 \\ 4 & 8 \end{bmatrix} + \begin{bmatrix} 3 & 3 \\ 0 & 0 \end{bmatrix} \right)$

$= g^{(1)} \left(\begin{bmatrix} -4 & -10 \\ 4 & 8 \end{bmatrix} \right)$

$= \text{Relu} \left(\begin{bmatrix} -4 & -10 \\ 4 & 8 \end{bmatrix} \right) = \begin{bmatrix} 0 & 0 \\ 4 & 8 \end{bmatrix}$

~~H~~ $H^{(2)} = g^{(2)} \left(W^{(2)T} H^{(1)} + b^{(2)} \right)$

$= g^{(2)} \left(\begin{bmatrix} -1 & 1 \end{bmatrix} \begin{bmatrix} 0 & 0 \\ 4 & 8 \end{bmatrix} + \begin{bmatrix} 3 \end{bmatrix} \right)$

$= g^{(2)} \left(\begin{bmatrix} 4 & 8 \end{bmatrix} + \begin{bmatrix} -3 & -3 \end{bmatrix} \right) = g^{(2)} \left(\begin{bmatrix} 1 & 5 \end{bmatrix} \right)$

~~W~~ $= \begin{bmatrix} \frac{1}{1+e^{-1}} & \frac{1}{1+e^{-5}} \end{bmatrix}$

$$\Rightarrow \hat{y}_1 = \frac{1}{1 + \frac{1}{e}} = \frac{e}{1+e} \quad ; y_1 = 0$$

$$\hat{y}_2 = \frac{1}{1 + \frac{1}{e^5}} = \frac{e^5}{1+e^5} \quad ; y_2 = 1$$

Cross-Entropy-Loss:

~~$$\frac{1}{2} \sum_{n=1}^2 y_n \log(\hat{y}_n) + (1-y_n) \log(1-\hat{y}_n)$$~~

~~$$\frac{1}{2} \sum_{n=1}^2 y_n \log(\hat{y}_n) + (1-y_n) \log(1-\hat{y}_n)$$~~

$$= -\frac{1}{2} \sum_{n=1}^2 y_n \log(\hat{y}_n) + (1-y_n) \log(1-\hat{y}_n)$$

$$= -\frac{1}{2} \left(\log\left(1 - \frac{e}{1+e}\right) + \log\left(\frac{e^5}{1+e^5}\right) \right)$$

$$= -\frac{1}{2} \left(\log\left(\frac{1}{1+e}\right) + \log(e^5) - \log(1+e^5) \right)$$

$$= -\frac{1}{2} \left(\log(1) - \log(1+e) + 5 - \log(1+e^5) \right)$$

$$= -\frac{1}{2} \left(0 + 5 - \log(1+e) - \log(1+e^5) \right)$$

$$\approx 0.66$$

The total Cross-Entropy-Loss is ≈ 0.66

I think it is not necessary to do it for each datapoint individually as the information is included in my calculation 😊

$$\text{Loss for datapoint } (x_1, y_1): -1 \log\left(1 - \frac{e}{1+e}\right) \approx 1.3133$$

$$\text{Loss for datapoint } (x_2, y_2): -1 \log\left(\frac{e^5}{1+e^5}\right) \approx 0.0067$$

Task 3.1:

The best accuracy we can achieve in practise using Logistic Regression is $\frac{3}{4}$. We have to draw a line, where everything right of it is 1, everything left of it is 0, so we would miss-classify one of the 0 as 1 and the rest would work out