DL-Exercise 1

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1 Eigendecomposition

Card 1: Black-Black - BB
Card 2: White-White - WW
Card 3: Black-White - BW

$$P(BB) = P(WW) = P(BW) = \frac{1}{3}$$

1.1 What are the probabilities that the card on the table shows a black side? What are the probabilities it shows a white side?

$$P(seeBlack) = 1 \cdot \frac{1}{3} + 0 \cdot \frac{1}{3} + \frac{1}{3} \cdot \frac{1}{2} = \frac{2}{6} + \frac{1}{6} = \frac{1}{2}$$

$$P(seeWhite) = \frac{1}{2}$$

1.2 If we draw a card and it shows black, compute the probability that the other side of the card is also black.

Too see Black on the second time we needed to draw Card 1- BB

$$P(seeBlack|BB) = 1$$

The Probability to see black on the first time given we have drawn card 1(BB) is obviously 100%

$$P(BB|seeBlack) = \frac{P(seeBlack|BB)P(BB)}{P(seeBlack)} = \frac{1 \cdot \frac{1}{3}}{\frac{1}{2}} = \frac{2}{3}$$

1.3 Find the probability that the other side of the card is black if the card shows a white side.

Too see white on the first and black on the second time we needed to draw Card 3- BW

$$P(seewhite|BW) = \frac{1}{2}$$

The Probability to see white on the first time given we have drawn card 3(BW) is obviously 50%

$$P(BW|seewhite) = \frac{P(seewhite|BW)P(BW)}{P(seewhite)} = \frac{\frac{1}{2} \cdot \frac{1}{3}}{\frac{1}{2}} = \frac{2}{6} = \frac{1}{3}$$

2 Distributions and Central Limit Theorem

2.1 Central Limit Theorem

As depicted in figure 1, the approximation increases in precision with growing sample sizes. With sample size 1, it is the original distribution. With 1024 samples, both match the normal distribution.

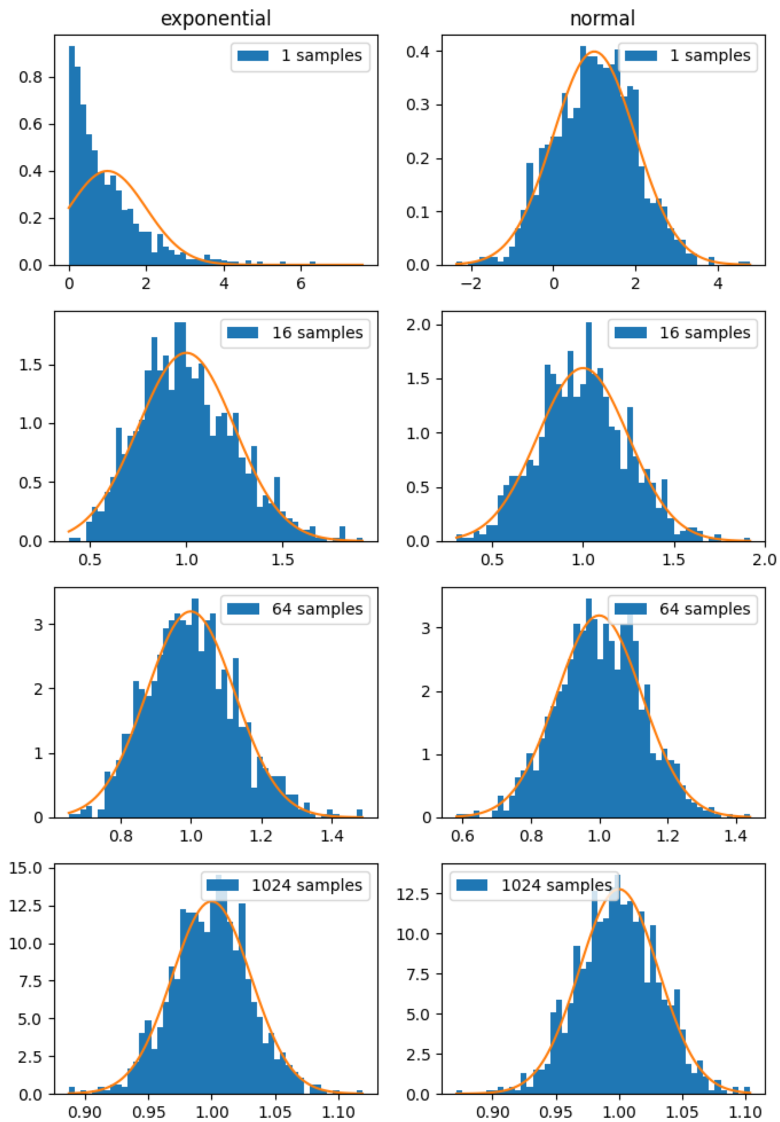

Figure 1: Plot of the Central limit theorem code.

2.2 Bayesian Linear Regression

In figure 2, the prior and posterior gaussian distributions are visualized. As expected, the posterior is more certain than the prior, with lower variance and higher bias.

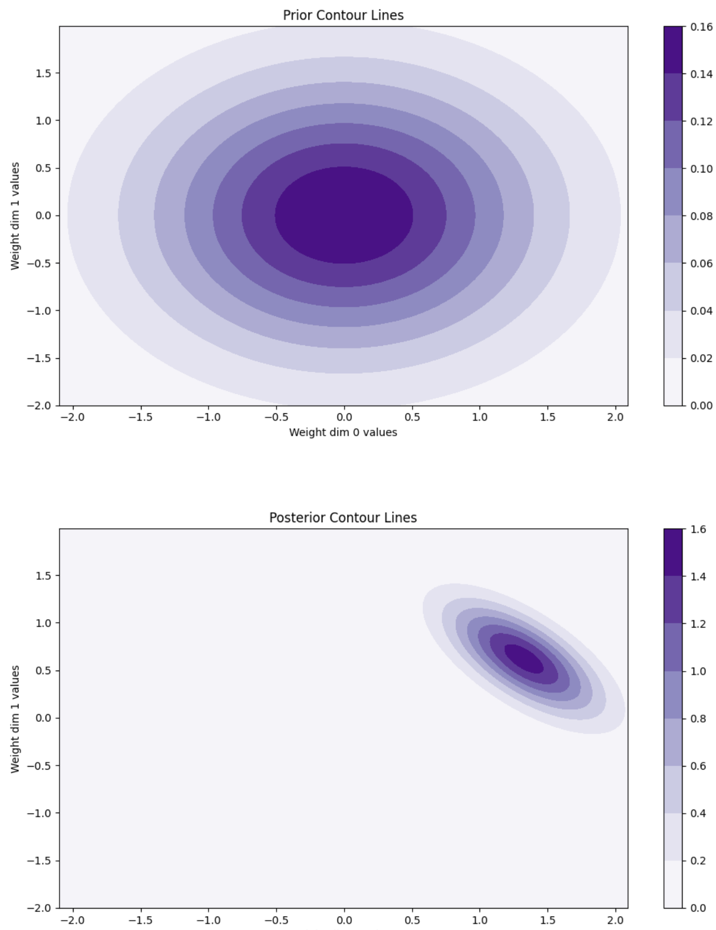

Figure 2: Plot of the prior and posterior gaussian distributions.