

STAT33600 Homework 5

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Section 3.4 Forecasting

3.17

Because the ARMA prediction error is:

$$x_{n+m} - \tilde{x}_{n+m} = \sum_{j=0}^{m-1} \psi_j w_{n+m-j}$$

So for a fixed sample size n and for $k \geq 1$, the correlation between the prediction errors are:

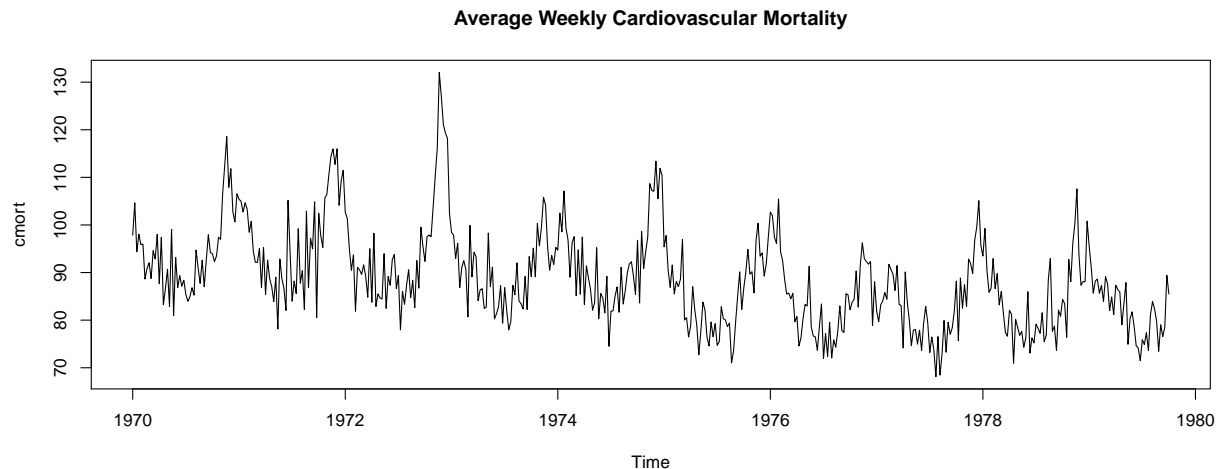
$$\begin{aligned} E[(x_{n+m} - \tilde{x}_{n+m})(x_{n+m+k} - \tilde{x}_{n+m+k})] &= E[(\sum_{j=0}^{m-1} \psi_j w_{n+m-j})(\sum_{i=0}^{m+k-1} \psi_i w_{n+m+k-i})] \\ &= E[(\sum_{j=0}^{m-1} \psi_j w_{n+m-j})(\sum_{j=-k}^{m-1} \psi_{j+k} w_{n+m-j})] = E[(\sum_{j=0}^{m-1} \psi_j w_{n+m-j})(\sum_{j=0}^{m-1} \psi_{j+k} w_{n+m-j})] \\ &= \sum_{j=0}^{m-1} \psi_j \psi_{j+k} E(w_{n+m-j}^2) = \sigma_w^2 \sum_{j=0}^{m-1} \psi_j \psi_{j+k} \neq 0 \end{aligned}$$

Therefore, we've verified the statement (3.87), that for a fixed sample size, the ARMA prediction errors are correlated.

Section 3.5 Estimation

3.18

```
library(astsa)
plot(cmort, main="Average Weekly Cardiovascular Mortality")
```



(a) & (b)

(1) Fitting using Linear Regression

```
# fit an AR(2) model to x_t using linear regression by OLS  
( lr = ar.ols(cmort, order=2) )
```

```
##  
## Call:  
## ar.ols(x = cmort, order.max = 2)  
##  
## Coefficients:  
##      1      2  
## 0.4286 0.4418  
##  
## Intercept: -0.04672 (0.2527)  
##  
## Order selected 2  sigma^2 estimated as  32.32  
lr$asy.se.coef # standard errors of the estimates
```

```
## $x.mean  
## [1] 0.2527231  
##  
## $ar  
## [1] 0.03979433 0.03976163
```

(2) Fitting using Yule-Walker estimation

```
# fit an AR(2) model to x_t using Yule-Walker estimation  
( yw = ar.yw(cmort, order=2) )
```

```
##  
## Call:  
## ar.yw.default(x = cmort, order.max = 2)  
##  
## Coefficients:  
##      1      2  
## 0.4339 0.4376  
##  
## Order selected 2  sigma^2 estimated as  32.84  
sqrt(diag(yw$asy.var.coef)) # standard errors of the estimates
```

```
## [1] 0.04001303 0.04001303
```

```
# compute the standard errors of the estimates by hand  
sample_acf = acf(cmort, type="correlation", plot=FALSE)$acf  
rho_0 = sample_acf[1]  
rho_1 = sample_acf[2]  
rho_2 = sample_acf[3]  
rho = c(rho_1, rho_2)  
R = matrix(c(rho_0, rho_1, rho_1, rho_0), nrow=2, byrow=TRUE)  
n = length(cmort)  
cov_matrix = 1/n * as.numeric(1 - t(rho) %*% solve(R) %*% rho) * solve(R)  
se = sqrt(diag(cov_matrix))  
se
```

```
## [1] 0.03989471 0.03989471
```

Here the computation is from Property 3.10:

$$\sqrt{n}(\hat{\beta} - \beta) \rightarrow N(0, \sigma_w^2 \Gamma_{p,q}^{-1})$$

$$\hat{\beta} \rightarrow N(\beta, \frac{1}{n} \sigma_w^2 \Gamma_{p,q}^{-1})$$

Here for the AR(2) model, we have $p = 2, q = 0$, so :

$$\Gamma_{p,q} = \Gamma_{p=2} = \begin{pmatrix} \gamma(0) & \gamma(1) \\ \gamma(1) & \gamma(0) \end{pmatrix}, \quad \sigma_w^2 = \gamma(0) - \gamma_2^T \Gamma_2^{-1} \gamma_2$$

So the variance-covariance matrix of $\hat{\beta}$ is:

$$\frac{1}{n} \sigma_w^2 \Gamma_2^{-1} = \frac{1}{n} (\gamma(0) - \gamma_2^T \Gamma_2^{-1} \gamma_2) \Gamma_2^{-1} = \frac{1}{n} (1 - \rho_2^T R_2^{-1} \rho_2) R_2^{-1}$$

Results:

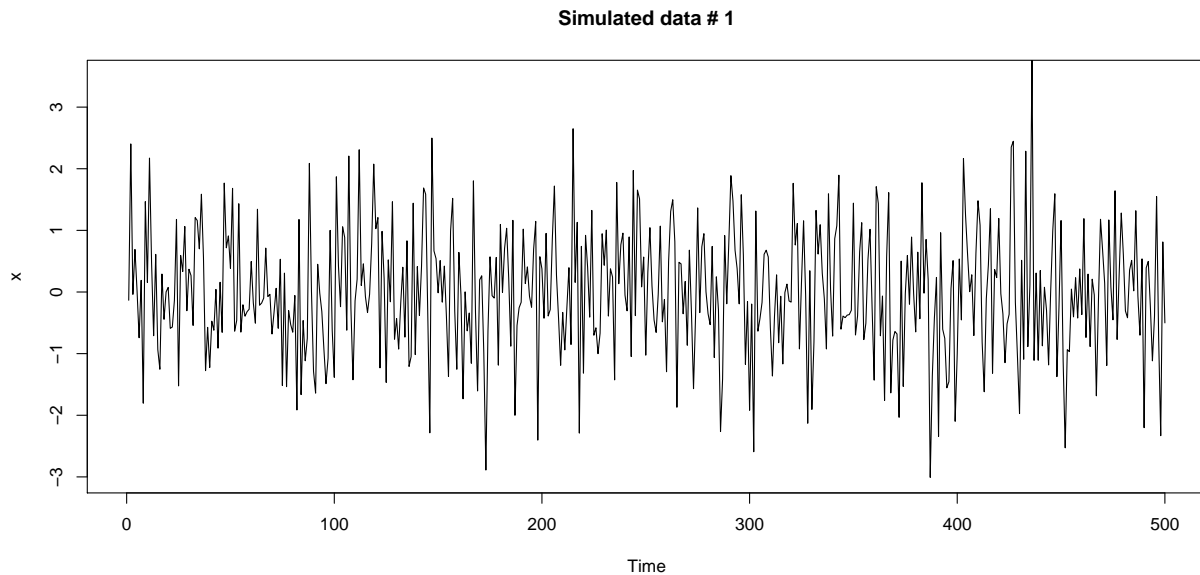
The parameters estimated by linear regression are 0.4286 and 0.4418, respectively, and their corresponding estimated standard errors are 0.03979433 and 0.03976163, respectively.

While the parameters estimated by Yule-Walker methods are 0.4339 and 0.4376, respectively, and their corresponding standard errors are both 0.03989471.

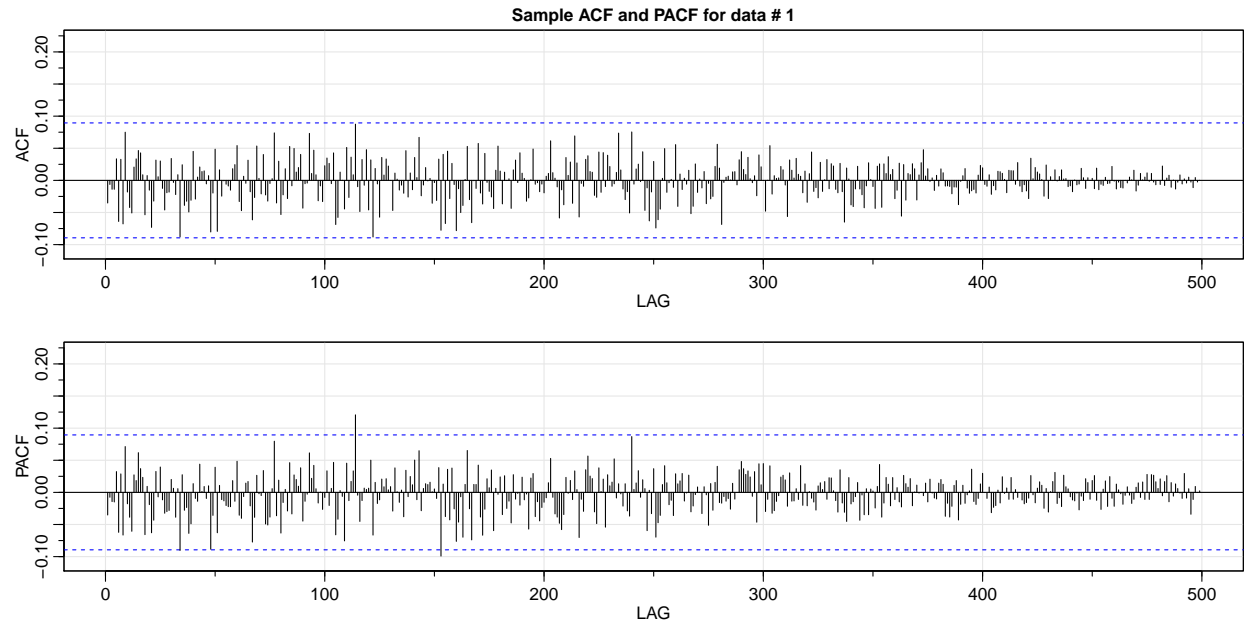
We can see that the two methods produce very similar results for AR(2) model.

3.20

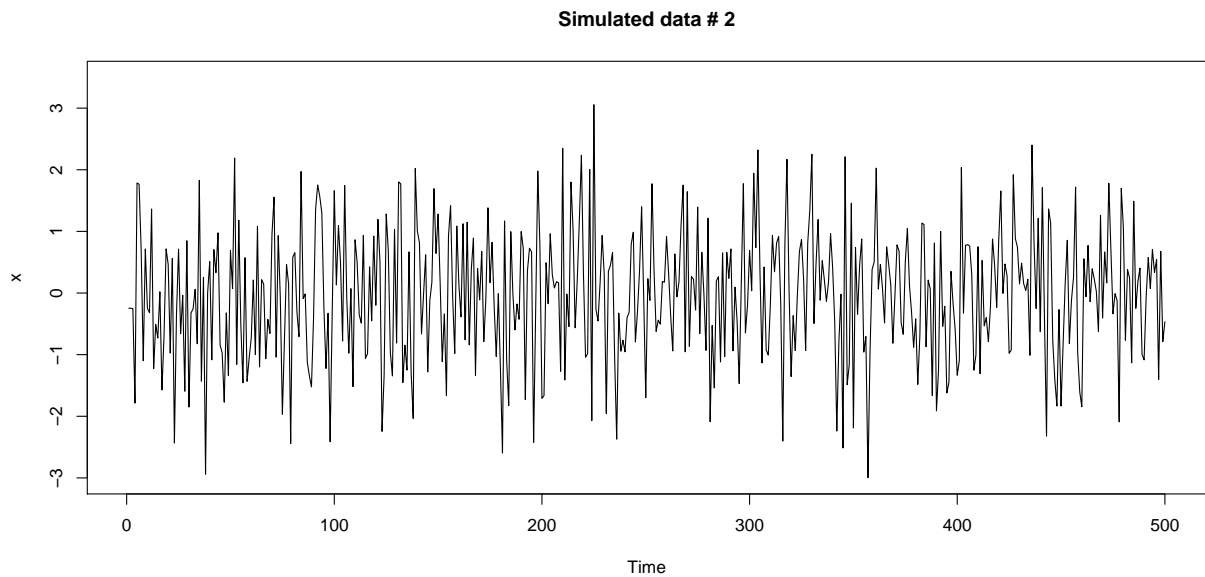
```
set.seed(1)
for (i in 1:3) {
  x = arima.sim(model=list(ar=0.9, ma=-0.9), n=500)
  plot(x, main=paste("Simulated data #",i), ylim=c(-3,3.5))
  acf2(x, max.lag=499, main=paste("Sample ACF and PACF for data #",i))
  print(sarima(x,p=1,d=0,q=1, details=FALSE)$ttable) # fit an ARMA(1,1) model
}
```

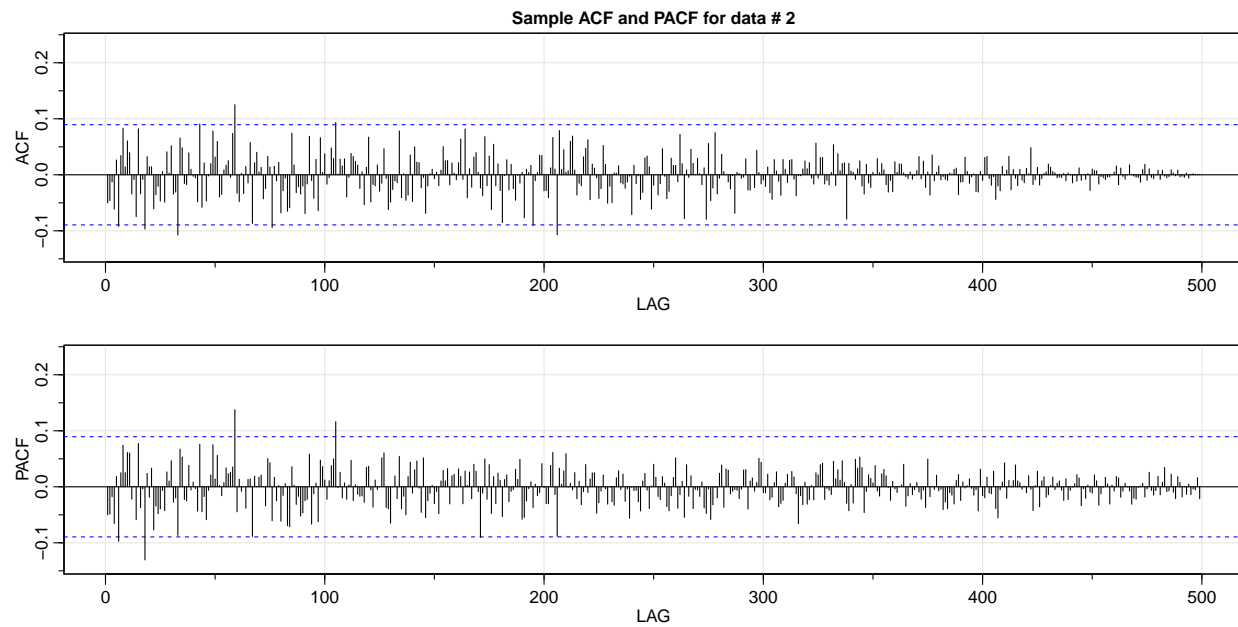


```
## Warning in stats::arima(xdata, order = c(p, d, q), seasonal = list(order =
## c(P, : possible convergence problem: optim gave code = 1
```

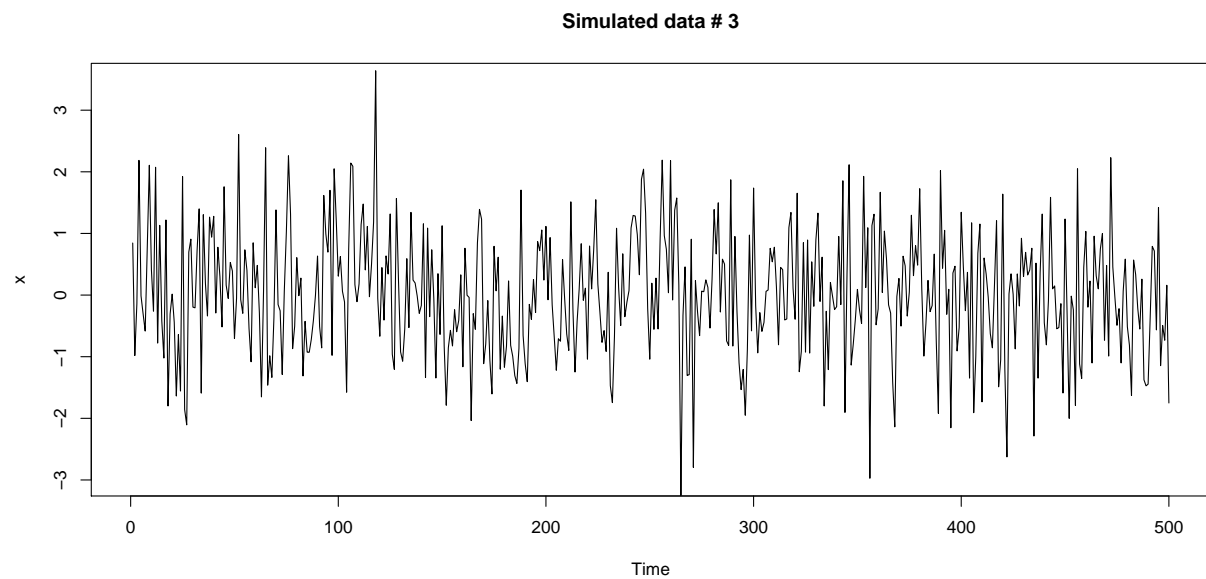


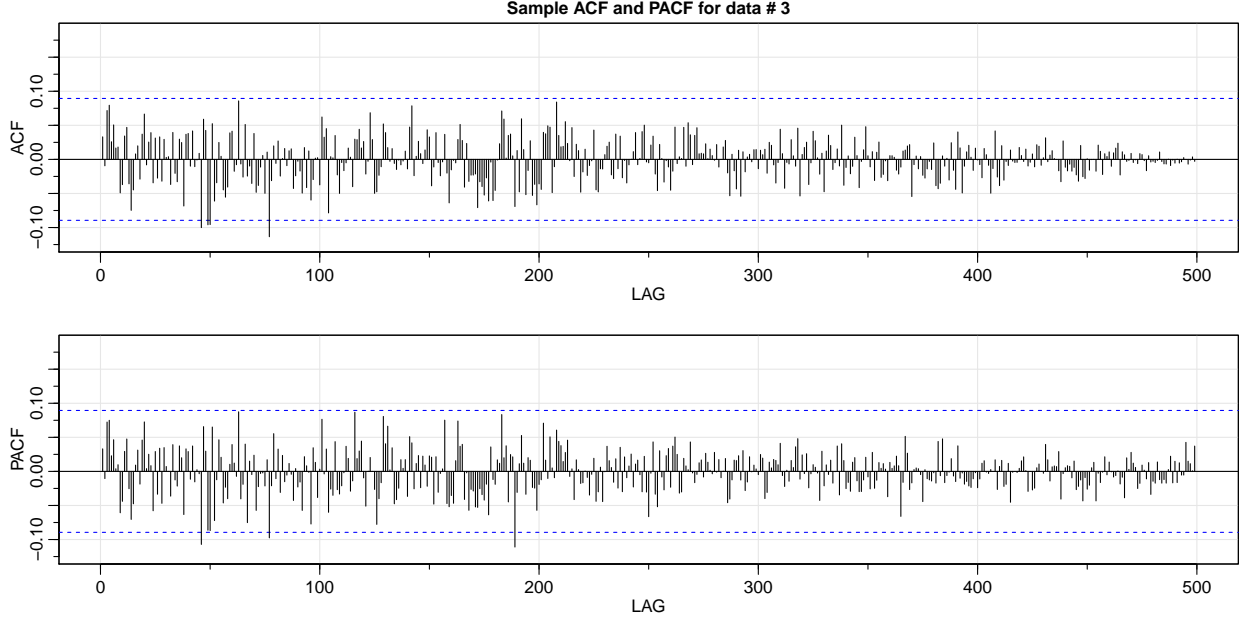
```
##      Estimate      SE t.value p.value
## ar1      0.5313 0.9832  0.5404 0.5891
## ma1     -0.5653 0.9642 -0.5863 0.5579
## xmean   -0.0031 0.0423 -0.0740 0.9411
```





##	Estimate	SE	t.value	p.value
## ar1	0.5997	0.2143	2.7983	0.0053
## ma1	-0.6651	0.1985	-3.3511	0.0009
## xmean	-0.0413	0.0394	-1.0470	0.2956





##		Estimate	SE	t.value	p.value
##	ar1	-0.4217	0.5118	-0.8240	0.4104
##	ma1	0.4590	0.5004	0.9173	0.3594
##	xmean	0.0098	0.0464	0.2122	0.8321

The true model is:

$$\begin{aligned}
 x_t &= 0.9x_{t-1} + w_t - 0.9w_{t-1} \\
 x_t - 0.9x_{t-1} &= w_t - 0.9w_{t-1} \\
 (1 - 0.9B)x_t &= (1 - 0.9B)w_t \\
 x_t &= w_t \sim \text{iid } N(0, 1)
 \end{aligned}$$

So this ARMA(1,1) model is actually a white noise sequence. The plot of the simulated data does look like a series of white noise.

The sample ACF and PACF are not significantly different from 0 (all within the blue lines) for all the three simulated data when lag $h > 0$. This again is consistent with the pattern of a white noises series.

And for the fitted ARMA(1,1) model, the estimated results from three simulations are quite different from each other, and they are all far away from the true setting where $\text{ar} = 0.9, \text{ma} = -0.9$ with high standard errors, resulting in high p-values. This makes sense since we are fitting an ARMA(1,1) model to a white noise sequence by mistakenly ignoring the parameter redundancy, so the fitted models will not be good, and the estimates of the coefficients will not be significant.

3.24

(a)

(1) Determine the mean function

Since $E(x_t) = \mu$, so we have:

$$\mu = E(x_t) = E(\alpha + \phi x_{t-1} + w_t + \theta w_{t-1}) = \alpha + \phi E(x_{t-1}) = \alpha + \phi \mu$$

$$\Rightarrow \mu = \frac{\alpha}{1 - \phi}$$

which is a constant that does not depend on time t .

(2) Find the Autocovariance & ACF

The model can be also written as:

$$x_t = \mu(1 - \phi) + \phi x_{t-1} + w_t + \theta w_{t-1}$$

$$(x_t - \mu) = \phi(x_{t-1} - \mu) + w_t + \theta w_{t-1}$$

Let's set $y_t = x_t - \mu$, then the model is:

$$y_t = \phi y_{t-1} + w_t + \theta w_{t-1}$$

where $|\phi| < 1, |\theta| < 1$.

Since:

$$\gamma_x(h) = Cov(x_{t+h}, x_t) = Cov(y_{t+h} + \mu, y_t + \mu) = Cov(y_{t+h}, y_t) = \gamma_y(h)$$

So we only need to find the autocovariance of series y_t .

The ARMA(1,1) process:

$$y_t = \phi y_{t-1} + \theta w_{t-1} + w_t$$

$$(1 - \phi B)y_t = (1 + \theta B)w_t$$

Because $|\phi| < 1, |\theta| < 1$, so the roots for both polynomials $\phi(z) = 1 - \phi z$ and $\theta(z) = 1 + \theta z$ are outside the unit circle, so series y_t is both causal and invertible.

y_t can be written as:

$$y_t = \sum_{j=0}^{\infty} \psi_j w_{t-j}$$

The autocorrelation function for y_t is:

$$\gamma_y(h) = Cov(y_{t+h}, y_t) = Cov(\phi y_{t+h-1} + \theta w_{t+h-1} + w_{t+h}, y_t)$$

$$= \phi \gamma_y(h-1) + \theta Cov(w_{t+h-1}, \sum_{j=0}^{\infty} \psi_j w_{t-j}) + Cov(w_{t+h}, \sum_{j=0}^{\infty} \psi_j w_{t-j})$$

$$= \phi \gamma_y(h-1) + \theta \psi_{1-h} \sigma_w^2 + \psi_{-h} \sigma_w^2 = \phi \gamma_y(h-1) \quad (h = 2, 3, \dots)$$

So the autocorrelation function satisfies:

$$\gamma_y(h) - \phi \gamma_y(h-1) = 0 \quad (h = 2, 3, \dots)$$

And its general solution is:

$$\gamma_y(h) = \phi \gamma_y(h-1) = \phi^2 \gamma_y(h-2) = \dots = \phi^{h-1} \gamma_y(1) = c \phi^h \quad (h = 1, 2, \dots), \quad \text{where } c = \frac{\gamma_y(1)}{\phi} \quad (*)$$

Since:

$$y_t = \phi y_{t-1} + \theta w_{t-1} + w_t = \sum_{j=0}^{\infty} \psi_j w_{t-j} \quad \text{where } \psi_0 = 1, \psi_j = (\theta + \phi) \phi^{j-1} \quad \forall j \geq 1$$

so we have: $\psi_0 = 1, \psi_1 = \theta + \phi$

Thus the initial conditions are:

$$\gamma_y(1) = \phi\gamma_y(0) + \theta\psi_0\sigma_w^2 + \psi_{-1}\sigma_w^2 = \phi\gamma_y(0) + \theta\sigma_w^2 \quad (1)$$

$$\gamma_y(0) = \phi\gamma_y(-1) + \theta\psi_1\sigma_w^2 + \psi_0\sigma_w^2 = \phi\gamma_y(1) + [\theta(\theta + \phi) + 1]\sigma_w^2 \quad (2)$$

Substitue (1) into (2), we get:

$$\gamma_y(0) = \phi(\phi\gamma_y(0) + \theta\sigma_w^2) + [\theta(\theta + \phi) + 1]\sigma_w^2 = \phi^2\gamma_y(0) + (\theta^2 + 2\phi\theta + 1)\sigma_w^2$$

$$\Rightarrow \gamma_y(0) = \sigma_w^2 \frac{\theta^2 + 2\phi\theta + 1}{1 - \phi^2}$$

$$\Rightarrow \gamma_y(1) = \phi\sigma_w^2 \frac{\theta^2 + 2\phi\theta + 1}{1 - \phi^2} + \theta\sigma_w^2 = \sigma_w^2 \frac{(\theta + \phi)(1 + \theta\phi)}{1 - \phi^2}$$

Take the value of $\gamma_y(1)$ into the function (*), we can get that:

$$\gamma_x(h) = \gamma_y(h) = c \phi^h = \phi^{h-1}\gamma(1) = \phi^{h-1}\sigma_w^2 \frac{(\theta + \phi)(1 + \theta\phi)}{1 - \phi^2} \quad (h = 1, 2, \dots)$$

Therefre, the autocovariance function is:

$$\gamma_x(h) = \gamma_y(h) = \begin{cases} \sigma_w^2 \frac{\theta^2 + 2\phi\theta + 1}{1 - \phi^2} & \text{if } h = 0 \\ \phi^{h-1}\sigma_w^2 \frac{(\theta + \phi)(1 + \theta\phi)}{1 - \phi^2} & \text{if } h \geq 1 \end{cases}$$

And the ACF function is:

$$\rho_x(h) = \rho_y(h) = \frac{\gamma_y(h)}{\gamma_y(0)} = \begin{cases} 1 & \text{if } h = 0 \\ \frac{(\theta + \phi)(1 + \theta\phi)}{\theta^2 + 2\phi\theta + 1} \phi^{h-1} & \text{if } h \geq 1 \end{cases}$$

(3) Prove Weakly & Strictly Stationary

From (1) and (2), we get that the mean function is a constant that does not depend on time t , and the autocovariance function only depends on lag h , so the process x_t is weakly stationary.

To prove the y_t is strictly stationary, we need to prove that the distribution of the collection of values $\{y_1, y_2, \dots, y_n\}$ is identical to that of the time shifted set $\{y_{1+h}, y_{2+h}, \dots, y_{n+h}\}$.

Since:

$$y_t = \phi y_{t-1} + \theta w_{t-1} + w_t = \sum_{j=0}^{\infty} \psi_j w_{t-j} \quad \text{where } \psi_0 = 1, \psi_j = (\theta + \phi)\phi^{j-1} \quad \forall j \geq 1$$

Suppose the characteristic function of iid random variable w_t is $\varphi_w(\lambda) = E(e^{i\lambda w})$, and the joint characteristic function of y_1, \dots, y_n is $\varphi_{y_1, \dots, y_n}(\lambda_1, \dots, \lambda_n)$. Then essential part of the exponent of $\varphi_{y_1, \dots, y_n}(\lambda_1, \dots, \lambda_n)$ is:

$$\sum_{i=1}^n \lambda_i y_i = \sum_{i=1}^n \lambda_i \left(\sum_{j=0}^{\infty} \psi_j w_{i-j} \right) = \sum_{i=1}^n \sum_{j=0}^{\infty} \lambda_i \psi_j w_{i-j} = \sum_{k=0}^{\infty} \left(\sum_{i=1}^n \lambda_i \psi_{i-k} \right) w_k = \sum_{k=0}^{\infty} c_k w_k$$

where $\psi_j = 0 \quad \forall j < 0, \quad \psi_0 = 1, \quad \psi_j = (\theta + \phi)\phi^{j-1} \quad \forall j \geq 1$

So the joint characteristic function is:

$$\begin{aligned}\varphi_{y_1, \dots, y_n}(\lambda_1, \dots, \lambda_n) &= E(e^{i(\sum_{i=1}^n \lambda_i y_i)}) = E(e^{i \sum_{k=0}^n c_k w_k}) = E(\prod_{k=0}^n e^{i c_k w_k}) \\ &= \prod_{k=0}^n E(e^{i c_k w_k}) = \prod_{k=0}^n \varphi_w(c_k) = \prod_{k=0}^n \varphi_w(\sum_{i=1}^n \lambda_i \psi_{i-k})\end{aligned}$$

which only depends on $\lambda_1, \dots, \lambda_n$ and the values of ψ_0, \dots, ψ_n .

So the joint characteristic function for the shifted set $\{y_{1+h}, y_{2+h}, \dots, y_{n+h}\}$ is the same as the that of $\{y_1, y_2, \dots, y_n\}$, that is:

$$\varphi_{y_{1+h}, y_{2+h}, \dots, y_{n+h}}(\lambda_1, \dots, \lambda_n) = \varphi_{y_1, \dots, y_n}(\lambda_1, \dots, \lambda_n)$$

Therefore, the distribution of $\{y_1, y_2, \dots, y_n\}$ is identical to that of the time shifted set $\{y_{1+h}, y_{2+h}, \dots, y_{n+h}\}$, which means the distribution of $\{x_1, x_2, \dots, x_n\}$ is identical to that of the time shifted set $\{x_{1+h}, x_{2+h}, \dots, x_{n+h}\}$, so the series x_t is strictly stationary.

(b)

Since x_t is a linear process of the form:

$$x_t = \mu + y_t = \mu + \sum_{j=0}^{\infty} \psi_j w_{t-j} \quad \text{where } \psi_0 = 1, \psi_j = (\theta + \phi)\phi^{j-1} \quad \forall j \geq 1$$

And:

$$\sum_{j=0}^{\infty} \psi_j = 1 + (\theta + \phi) \sum_{j=1}^{\infty} \phi^{j-1} = 1 + (\theta + \phi) \lim_{n \rightarrow \infty} \frac{1 - \phi^n}{1 - \phi} = 1 + \frac{\theta + \phi}{1 - \phi} = \frac{1 + \theta}{1 - \phi} \neq 0$$

Thus by Theorem A.5, we have that:

$$\bar{x}_n \sim \text{AN}(\mu, n^{-1}V)$$

where

$$V = \sum_{h=-\infty}^{\infty} \gamma_x(h) = \sigma_w^2 \left(\sum_{j=-\infty}^{\infty} \psi_j \right)^2 = \sigma_w^2 \left(\sum_{j=0}^{\infty} \psi_j \right)^2 = \sigma_w^2 \left(\frac{1 + \theta}{1 - \phi} \right)^2$$

So, we get that:

$$\bar{x}_n \sim \text{AN} \left(\frac{\alpha}{1 - \phi}, n^{-1} \sigma_w^2 \left(\frac{1 + \theta}{1 - \phi} \right)^2 \right)$$

Section 3.6 Integrated Models for Nonstationary Data

3.28

The model is:

$$x_t = x_{t-1} + w_t - \lambda w_{t-1} \quad (|\lambda| < 1)$$

Set $y_t = x_t - x_{t-1}$, so the model can be written as:

$$y_t = w_t - \lambda w_{t-1} \quad (|\lambda| < 1)$$

Since $|\lambda| < 1$, then the series y_t is invertible, so we have invertible representation:

$$w_t = y_t + \lambda y_{t-1} = y_t + \lambda(y_{t-1} + \lambda y_{t-2}) = y_t + \lambda y_{t-1} + \lambda^2 y_{t-2} = \sum_{j=0}^{\infty} \lambda^j y_{t-j}$$

$$= \sum_{j=0}^{\infty} \lambda^j (x_{t-j} - x_{t-j-1}) = \sum_{j=0}^{\infty} \lambda^j x_{t-j} - \sum_{j=0}^{\infty} \lambda^j x_{t-j-1} = x_t + \sum_{j=1}^{\infty} (\lambda^j - \lambda^{j-1}) x_{t-j}$$

$$\Rightarrow x_t = w_t - \sum_{j=1}^{\infty} (\lambda^j - \lambda^{j-1}) x_{t-j} = w_t + \sum_{j=1}^{\infty} (1 - \lambda) \lambda^{j-1} x_{t-j}$$

3.29

(a)

The ARIMA(1,1,0) model with drift:

$$(1 - \phi B)(1 - B)x_t = \delta + w_t$$

Let $y_t = (1 - B)x_t = x_t - x_{t-1} = \nabla x_t$, which is a AR(1) model, so the model can be converted to the AR(1):

$$(1 - \phi B)y_t = \delta + w_t$$

$$y_t = \phi y_{t-1} + \delta + w_t$$

$$\Rightarrow y_{n+j} = \phi(\phi y_{n+j-2} + \delta + w_{n+j-1}) + \delta + w_{n+j} = \phi^2 y_{n+j-2} + (1 + \phi)\delta + \phi w_{n+j-1} + w_{n+j}$$

$$= \dots = \phi^j y_n + (1 + \phi + \dots + \phi^{j-1})\delta + \sum_{k=0}^{j-1} \phi^k w_{n+j-k}$$

So the forecasts y_{n+j} based on $\{y_1, \dots, y_n\}$ is:

$$\begin{aligned} y_{n+j}^n &= E(y_{n+j} | y_1, \dots, y_n) = E\left(\phi^j y_n + (1 + \phi + \dots + \phi^{j-1})\delta + \sum_{k=0}^{j-1} \phi^k w_{n+j-k} \mid y_1, \dots, y_n\right) \\ &= \phi^j y_n + (1 + \phi + \dots + \phi^{j-1})\delta \end{aligned}$$

(b)

From (a), we have that:

$$x_{n+j}^n - x_{n+j-1}^n = y_{n+j}^n = \phi^j y_n + (1 + \phi + \dots + \phi^{j-1})\delta = \phi^j (x_n - x_{n-1}) + \frac{1 - \phi^j}{1 - \phi} \delta$$

$$\Rightarrow x_{n+m}^n = x_n + \sum_{j=1}^m (x_{n+j}^n - x_{n+j-1}^n) = x_n + \sum_{j=1}^m \left(\phi^j (x_n - x_{n-1}) + \frac{1 - \phi^j}{1 - \phi} \delta \right)$$

$$= x_n + (x_n - x_{n-1}) \frac{\phi(1 - \phi^m)}{1 - \phi} + \frac{\delta}{1 - \phi} \left[m - \frac{\phi(1 - \phi^m)}{1 - \phi} \right]$$

(c)

As shown in (3.145), the mean-squared prediction error can be approximated by:

$$P_{n+m}^n = \sigma_w^2 \sum_{j=0}^{m-1} \psi_j^{*2}$$

where ψ_j^* is the coefficient of z^j in $\psi^*(z)$, that is:

$$\begin{aligned} \psi^*(z) &= \frac{\theta(z)}{\phi(z)(1-z)^d} = \frac{1}{(1-\phi z)(1-z)} = \frac{1}{1-(1+\phi)z+\phi z^2} = \sum_{j=0}^{\infty} \psi_j^* z^j \\ \Rightarrow \quad 1 &= \psi^*(z)(1-(1+\phi)z+\phi z^2) = \left(\sum_{j=0}^{\infty} \psi_j^* z^j \right) (1-(1+\phi)z+\phi z^2) \\ &= (\psi_0^* + \psi_1^* z + \psi_2^* z^2 + \dots) (1-(1+\phi)z+\phi z^2) \\ &= \psi_0^* + [\psi_1^* - (1+\phi)\psi_0^*] z + \sum_{j=2}^{\infty} [\psi_j^* - (1+\phi)\psi_{j-1}^* + \phi\psi_{j-2}^*] z^j \end{aligned}$$

Therefore, we can get that:

$$\psi_0^* = 1, \quad \psi_1^* = 1 + \phi, \quad \psi_j^* - (1 + \phi)\psi_{j-1}^* + \phi\psi_{j-2}^* = 0 \quad \forall j \geq 2$$

To solve for sequence ψ_j^* , set polynomial $\phi(z) = 1 - (1 + \phi)z + \phi z^2 = (1 - z)(1 - \phi z) = 0$, get solutions $z_1 = 1, z_2 = 1/\phi$, so the general form of ψ_j^* is: $\psi_j^* = c_1 z_1^{-j} + c_2 z_2^{-j} = c_1 + c_2 \phi^j$. Using the initial consitions $\psi_0^* = 1, \psi_1^* = 1 + \phi$, we can get that:

$$\psi_j^* = \frac{1}{1-\phi} - \frac{\phi}{1-\phi} \phi^j = \frac{1-\phi^{j+1}}{1-\phi} \quad (\forall j \geq 0)$$

Then,

$$P_{n+m}^n = \sigma_w^2 \sum_{j=0}^{m-1} \psi_j^{*2} = \sigma_w^2 \sum_{j=0}^{m-1} \left(\frac{1-\phi^{j+1}}{1-\phi} \right)^2 = \frac{\sigma_w^2}{(1-\phi)^2} \sum_{j=0}^{m-1} (1-\phi^{j+1})^2$$

3.30

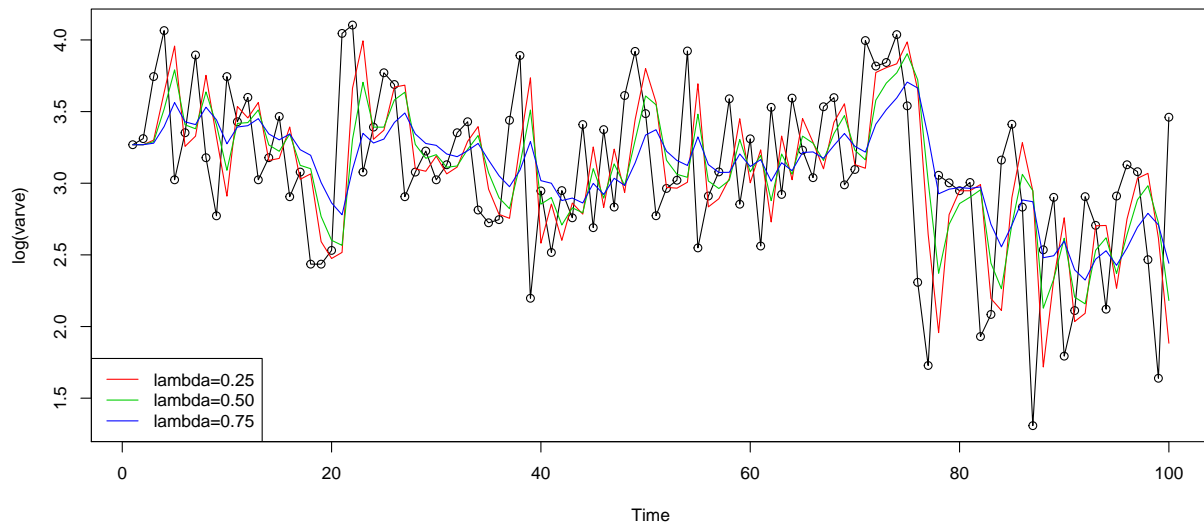
```
library(astsa)
x = log(varve)[1:100]
lambda = c(0.25, 0.50, 0.75)

# computation function
pred = function(lambda){
  x_pred = vector(length=100)
  x_pred[1] = x[1]
  for (i in 2:100) {
    x_pred[i] = (1-lambda)*x[i-1] + lambda*x_pred[i-1]
  }
  return(x_pred)
}
```

```

# plot
plot(x, type="o", xlab="Time" , ylab="log(varve)", col=1)
for (i in 1:length(lambda)) {
  lines(pred(lambda[i]), lty=1, col=i+1)
  # same as:
  #x.ima = HoltWinters(x, alpha=1-lambda[i], beta=FALSE, gamma=FALSE)
  #lines(x.ima$fitted[, "xhat"], lty=1, col=i+1)
}
legend("bottomleft", legend=c("lambda=0.25", "lambda=0.50", "lambda=0.75"), lty=1, col=c(2,3,4))

```



From (3.151), the truncated forecasts are:

$$\tilde{x}_{t+1}^t = (1 - \lambda)x_t + \lambda\tilde{x}_t^{t-1}, \quad t \geq 1 \quad \text{and} \quad \tilde{x}_1^0 = x_1$$

The results are plotted in the above figure. We can see that all the EWMA forecasts are within the oscillation range of the original data, and the EWMA forecasts are all smoother than the original data. Plus, when λ is larger, the EWMA provides smoother forecasts.