Homework 6

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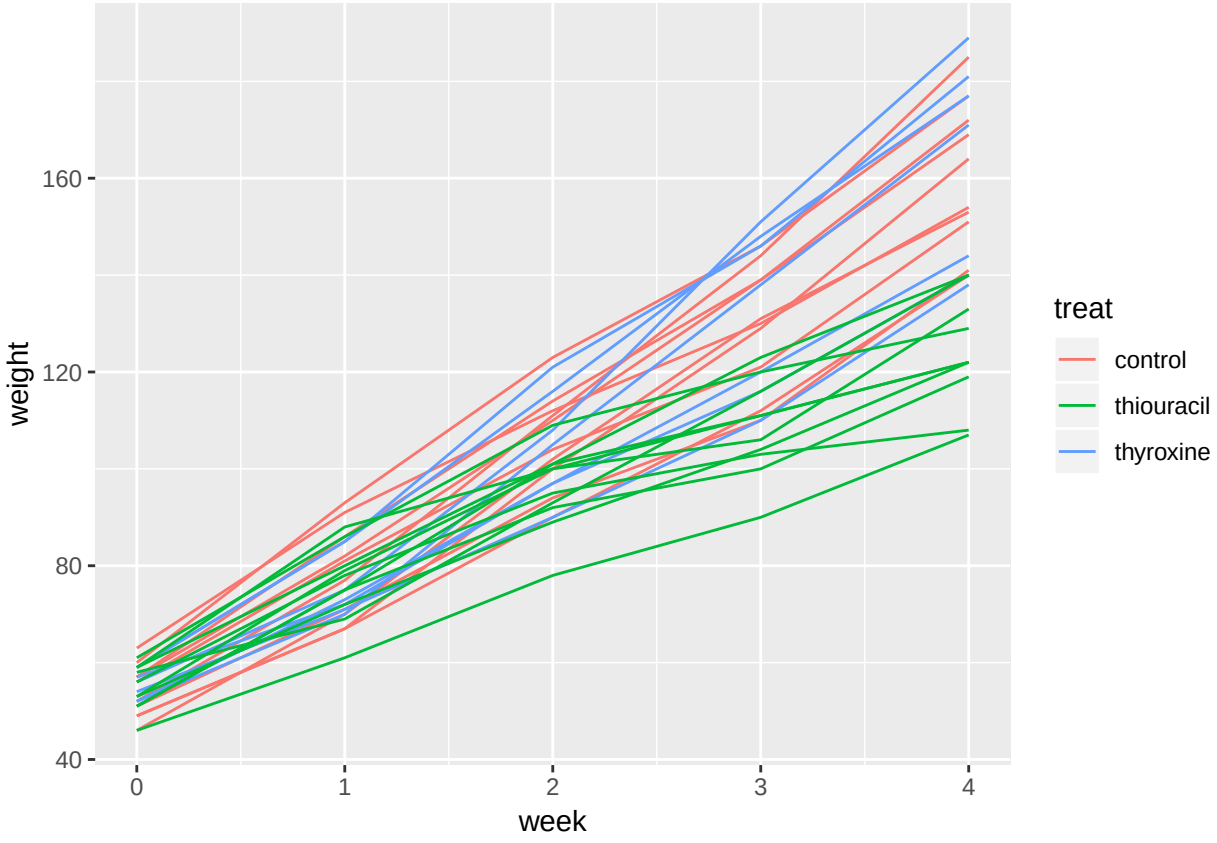
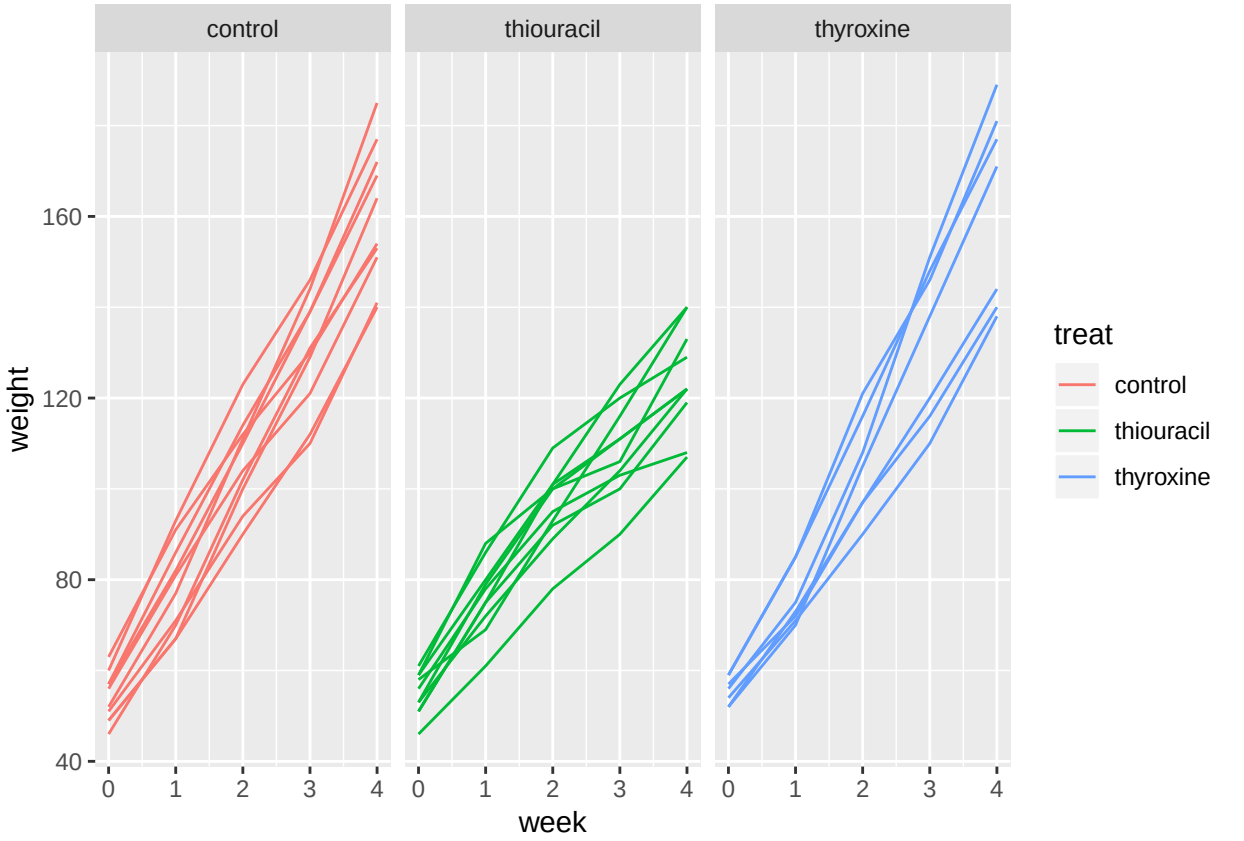
Problem 1

(a)

```
## Warning: package 'ggplot2' was built under R version 3.5.2
```





Answer:

In the first plot, we see that the both intercepts and slopes of how the weights of rats increases with age are different for each rat. And the intercepts within control group is more variable than that within thiouracil group, which are both more variable than the intercepts within thyroxine group. What' more, generally, the weights of thiouracil group are significantly lower than the other two groups, but the difference between the control group and thiouracil group is hard to tell.

In the second plot, we can see clearly that the increasing rates (slopes) of weights over age are different for each treatment group. The slope of the control group is higher than that of thiouracil group. And there seems to have two different slope trends within the thyroxine group, of which one is slightly higher than the control group and the other is smaller than the control group but slightly higher than the thyroxine group.

(b)

The mixed effects model is:
$weight_{ij} = \mu + week_i + treat_j + week_i \times treat_j + \gamma_j^0 + \gamma_j^1 week_i + \epsilon_{ij}$, where i indexes the week and j indexes the individual.
$week_i$ and $treat_j$ are fixed effects. Random effects $(\gamma_k^0 \quad \gamma_k^1)^T$ i.i.d. $\sim N(0, \sigma^2 D)$, error term ϵ_{ij} i.i.d. $\sim N(0, \sigma^2 I)$, and the random effects are independent with the error term.

```
library(lme4)
mmod = lmer(wt ~ weeks*treat + (weeks|subject), ratdrink)
sumary(mmod)
```

```
## Fixed Effects:
##                        coef.est coef.se
```

```
## (Intercept)           52.88     2.09
## weeks                 26.48     1.27
## treatthiouracil        4.78     2.96
## treatthyroxine        -0.79     3.26
## weeks:treatthiouracil -9.37     1.79
## weeks:treatthyroxine   0.66     1.97
##
## Random Effects:
##  Groups   Name        Std.Dev. Corr
##  subject  (Intercept) 5.70
##           weeks       3.76     -0.13
##  Residual             4.35
## ---
## number of obs: 135, groups: subject, 27
## AIC = 898.7, DIC = 912.7
## deviance = 895.7
```

Answer:

The interpretations for the following estimates:

i. The estimate of the fixed effect intercept term is 52.88, which means that the average weight of a rat in the control group at the first week is about 52.88.

ii. The estimate of the fixed effect interaction term between **thiouracil** and **week** is -9.37, which means that the average weight of a rat in the thiouracil group increases about 26.48 - 9.37 = 17.11 a week, which is -9.37 lower than the increasing rate of the control group. Here 26.48 is the estimate of the fixed effect parameter of the predictor "week".

iii. The estimate of the intercept random effect SD is 5.70, which represents the variation (SD) in overall weight between individual rats.

(c)

```
library(pbkrtest)

# significance of interaction term
mmod_ml = lmer(wt ~ weeks*treat + (weeks|subject), ratdrink, REML=FALSE)
nmod_ml = lmer(wt ~ weeks+treat + (weeks|subject), ratdrink, REML=FALSE)
KRmodcomp(nmod_ml, mmod_ml)
```

```
## F-test with Kenward-Roger approximation; computing time: 0.22 sec.
## large : wt ~ weeks + treat + (weeks | subject) + weeks:treat
## small : wt ~ weeks + treat + (weeks | subject)
##         stat    ndf    ddf F.scaling   p.value
## Ftest 18.319  2.000 24.000         1 1.478e-05 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

```
# significance of treatment
mmod_ml = lmer(wt ~ weeks+treat + (weeks|subject), ratdrink, REML=FALSE)
nmod_ml = lmer(wt ~ weeks       + (weeks|subject), ratdrink, REML=FALSE)
KRmodcomp(nmod_ml, mmod_ml)
```

```
## F-test with Kenward-Roger approximation; computing time: 0.13 sec.
## large : wt ~ weeks + treat + (weeks | subject)
## small : wt ~ weeks + (weeks | subject)
```

```
##          stat     ndf     ddf F.scaling p.value
## Ftest  0.1014  2.0000 24.0000         1   0.904
```

```
# significance of both terms
mmod_ml = lmer(wt ~ weeks*treat + (weeks|subject), ratdrink, REML=FALSE)
nmod_ml = lmer(wt ~ weeks       + (weeks|subject), ratdrink, REML=FALSE)
KRmodcomp(nmod_ml, mmod_ml)
```

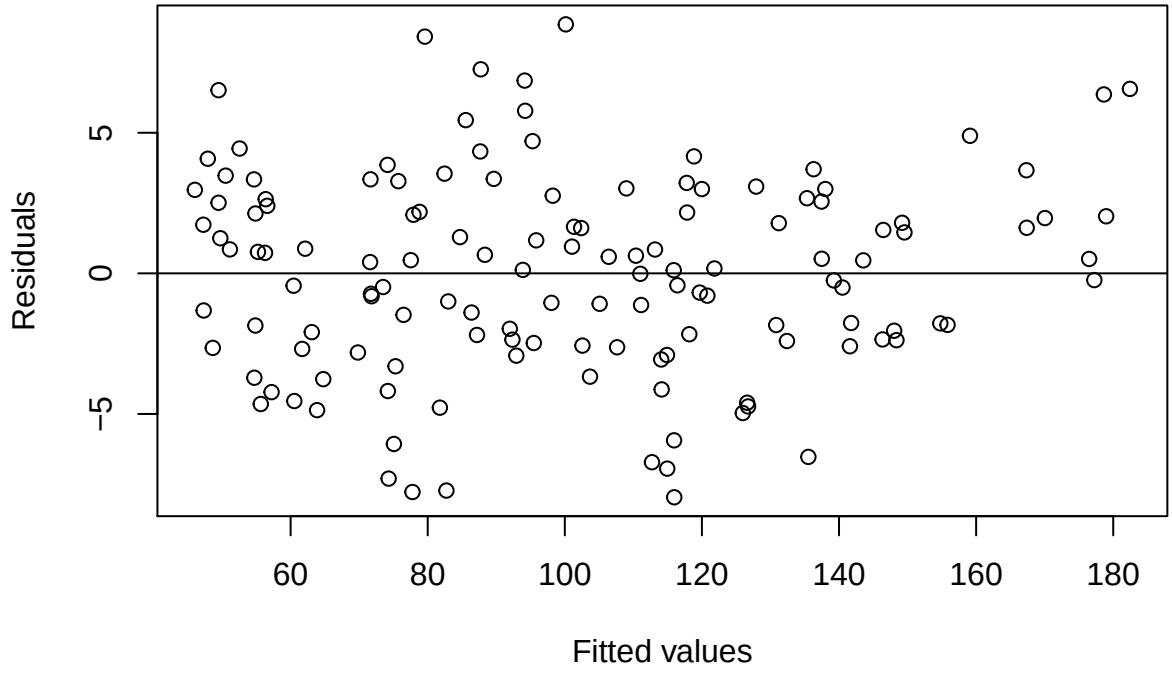
```
## F-test with Kenward-Roger approximation; computing time: 0.11 sec.
## large : wt ~ weeks + treat + (weeks | subject) + weeks:treat
## small : wt ~ weeks + (weeks | subject)
##          stat     ndf     ddf F.scaling   p.value
## Ftest  8.7124  4.0000 26.8141   0.94552 0.0001215 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

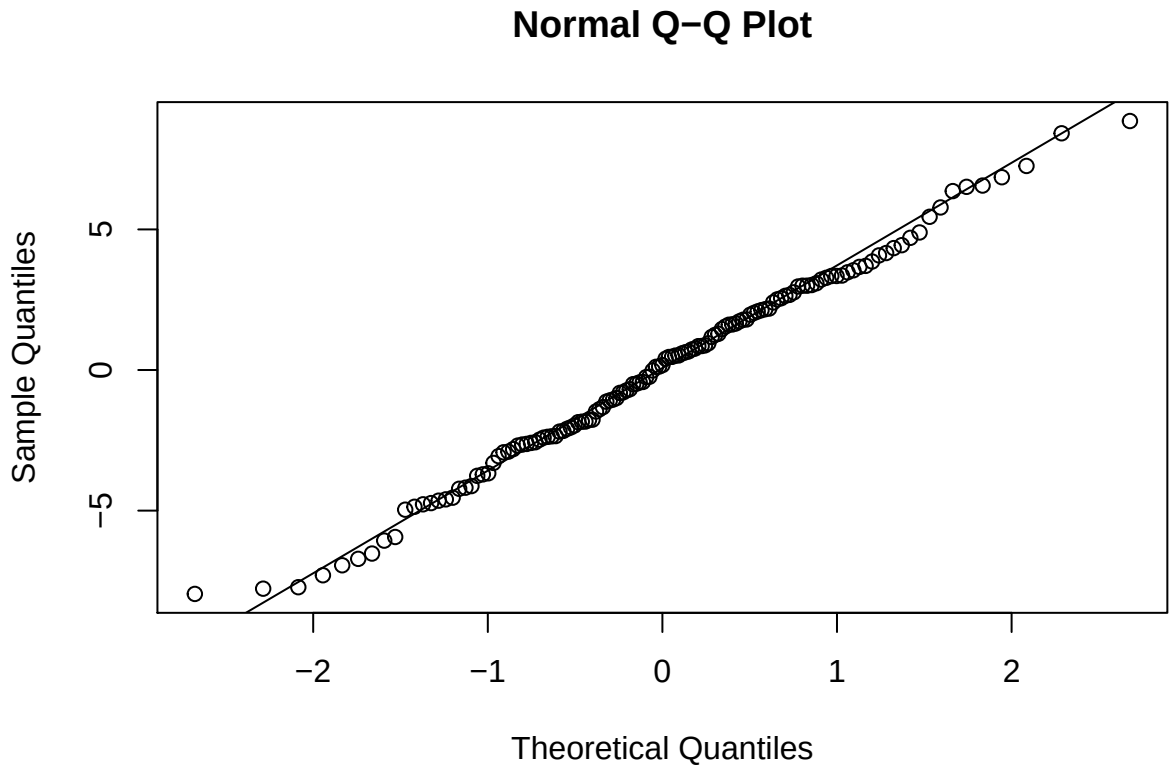
Answer:

Here I used the Kenward-Roger adjusted F-test for testing the significance of the fixed effects terms. The results show that the interaction term between **treatment** and **week** is very significant, but the **treatment** term itself is not. When testing these two terms together, they are overall significant. And we should not remove the main effect term when keeping the interaction term in the model.

Therefore, as a result, we keep both the **treatment** efffect term and its interaction term with week in the model, so we can think it as there is a significant treatment effect.

(d)


Residuals
5
0
−5
60
80
100
120
140
160
180
Fitted values


Normal Q−Q Plot
Sample Quantiles
5
0
−5
−2
−1
0
1
2
Theoretical Quantiles

Answer:

The plot of residuals against the fitted values do not show significant anomalous patterns, thus indicating a roughly constant variance in residuals and independent errors.

The QQ plot of the residuals shows that the residuals are basically normally distributed, though the the most left tail and right tail points are a little bit off the line.

Thus, in general, this model is a good fit for the data.

(e)

```
# confidence intervals of all parameters
CI = confint(mmod, method="boot", oldNames=FALSE); CI
```

```
## Computing bootstrap confidence intervals ...
```

```
##                                      2.5 %     97.5 %
## sd_(Intercept)|subject           3.4780279  7.7910669
## cor_weeks.(Intercept)|subject   -0.5428637  0.4085339
## sd_weeks|subject                 2.6384623  5.0598683
## sigma                            3.6017668  4.9607860
## (Intercept)                     49.0206524 57.0224460
## weeks                           23.8959500 28.7442025
## treatthiouracil                 -1.3743011 10.1458137
## treatthyroxine                  -7.2052728  5.7970014
## weeks:treatthiouracil          -12.7213236 -5.9636395
## weeks:treatthyroxine            -3.1077390  4.5865639
```

```
# effect of thyroxine at each week
thyroxine = matrix(NA,5,2)
for (i in 0:4) {
    thyroxine[i+1,] = CI["treatthyroxine",] + CI["weeks:treatthyroxine",] * i
}
colnames(thyroxine) = colnames(CI)
thyroxine
```

```
##           2.5 %    97.5 %
## [1,]  -7.205273  5.797001
## [2,] -10.313012 10.383565
## [3,] -13.420751 14.970129
## [4,] -16.528490 19.556693
## [5,] -19.636229 24.143257
```

Answer:

Both the 95% confidence intervals of random intercept SD and random slope SD are well above zero, thus they are all significant at 5% significance level. However, the 95% confidence interval of the correlation between random intercept and slope covers zero, thus this term may not be significant. But this correlation term is difficult to interpret and little would be gained from removing it. So it is simpler just to keep it in.

The 95% confidence interval of the fixed effect of **thyroxine** covers zero, and the 95% confidence interval of the interaction term between **thyroxine** and **weeks** also covers zero. As a result, when adding these two terms for each week, we can see that all the 95% confidence intervals at each week cover zeros. Therefore, the thyroxine group is not significantly different from the control group.