#homework (3 questions, A-C. Make sure to expand all bullets!)

- A) Complete all the exercises in: https://stephens999.github.io/fiveMinuteStats/MH-examples1.html (There are 4 exercises, including the Gibbs sampler at the end).
- B) The Gibbs sampler in https://stephens999.github.io/fiveMinuteStats/gibbs_structure_simple.html assumes that $Pr(z_i = k) = 0.5$ for each of the two groups/clusters. That is it sets the mixture component weights to 0.5. Your task here is to generalize this to estimate the mixture component weights from the data rather than fixing them to 0.5. Before attempting this you should read carefully through the related example in https://stephens999.github.io/fiveMinuteStats/gibbs2.html.
  - 1. Write out the generalized model (likelihood and priors), introducing a parameter \(\pi\) to denote the mixture component weights \(\Pr(z_i = k) = \pi_k\). (Note: in specifying a prior for \(\pi\) you will want to choose it so that you can exploit conjugacy in part 2 below.)
  - 2. Provide a mathematical derivation of the conditional distribution \(\pi | P,Z,X\) under your model and prior in 1. (exploit conjugacy here!).
  - 3. Modify the code of the Gibbs sampler to update \(\pi\) as well as \(Z\) and \(P\).
  - 4. Illustrate your Gibbs sampler on simulated data where \(\pi = (0.2,0.8)\). Compare the posterior distribution you get from the sampled values of \(\pi\) with the true value of \(\pi\).
- C) Follow instructions in https://stephens999.github.io/stat34800/bayes_normal_means.html