STAT 420 Spring 2017
2GR, 2UG

Homework 1

(Due Friday, February 3, by 4:00 p.m.)

Please submit your assignment *on paper*, following the Guidelines for Homework Write-Ups and Submissions. Please include your name (with your last name underlined), and your NetID at the top of the first page. (Even if correct, answers might not receive credit if they are too difficult to read. No credit will be given without supporting work.)

1. The covariance matrix of a *n*-dimensional random vector $\mathbf{X} = (X_1, X_2, ..., Xn)^T$ is given by

$\boldsymbol{\Sigma}_{\mathbf{X}} = E[(\mathbf{X} - E(\mathbf{X}))(\mathbf{X} - E(\mathbf{X}))^T]$.

(a) Show or prove that $\boldsymbol{\Sigma}_{\mathbf{X}} = E[\mathbf{X}\mathbf{X}^T] - E(\mathbf{X})E(\mathbf{X})^T$.

(b) Let $n = 3$. Show or prove that $\boldsymbol{\Sigma}_{\mathbf{X}} = \begin{pmatrix} \mathrm{Var}(X_1) & \mathrm{Cov}(X_1,X_2) & \mathrm{Cov}(X_1,X_3) \\ \mathrm{Cov}(X_2,X_1) & \mathrm{Var}(X_2) & \mathrm{Cov}(X_2,X_3) \\ \mathrm{Cov}(X_3,X_1) & \mathrm{Cov}(X_3,X_2) & \mathrm{Var}(X_3) \end{pmatrix}$.

2. Let Z be a standard normal random variable, i.e. $Z \sim N(0,1)$

(a) Show or prove that the moment generating function of Z is

$M_Z(t) = \exp\left\{\frac{1}{2}t^2\right\}$, for $-\infty < t < \infty$.

(b) Derive the first two derivatives of $M_Z(t)$.

(c) Calculate the mean and variance of Z by evaluating the two derivatives in (b) at $t = 0$.

3. In a grocery store in Pawnee, IN, the price of a pound of bacon (U) and the price of a dozen of eggs (V) vary from day to day and jointly follow a bivariate normal with

$\mu_U = \$4.34$, $\sigma_U = \$0.10$, $\mu_V = \$2.22$, $\sigma_V = \$0.03$, $\rho_{UV} = 0.60$.

(a) Find the probability that on a given day, a pound of bacon costs more than \$4.30. That is, find $P(U > 4.30)$.

(b) Suppose that on a given day, a dozen of eggs costs \$2.26. Find the probability that a pound of bacon costs more than \$4.30. That is, find $P(U > 4.30 \mid V = 2.26)$.

(c) Find the probability that on a given day, a pound of bacon costs more than two dozen of eggs. That is, find $P(U > 2V)$.

(d) Ron Swanson buys 5 pounds of bacon and 4 dozens of eggs. Find the probability that he paid more than \$30. That is, find $P(5U + 4V > 30)$.

3. (continued)
Suppose that the price of a pound of bacon (U), the price of a dozen of eggs (V), and the price of a pound of ham (W) [in dollars] jointly follow $N_3(\mu, \Sigma)$ distribution with

$$\mu = \begin{pmatrix} 4.34 \\ 2.22 \\ 3.31 \end{pmatrix} \quad \text{and} \quad \Sigma = \begin{pmatrix} \alpha & \beta & 0.0024 \\ \gamma & \delta & 0.0009 \\ 0.0024 & 0.0009 & 0.0036 \end{pmatrix}.$$

(e) What are the values of α, β, γ, and δ?

(f) What are the values of ρ_{UW} and ρ_{VW}?

(g) Find the probability that on a given day, a pound of ham costs more than \$3.34. That is, find $P(W > 3.34)$.

(h) Ron Swanson buys 5 pounds of bacon, 4 dozens of eggs, and 3 pounds of ham. Find the probability that he paid more than \$40. That is, find $P(5U + 4V + 3W > 40)$.

4. Ron Swanson orders a new menu item at Charles Mulligan's Steakhouse – Tic-Tac-Toe – nine "8-ounce" steaks. After photographing and weighing his nine steaks, he obtains the following results (in ounces):

7.5 8.2 8.1 7.8 7.6 8.0 7.8 7.9 8.2

Assume the weights are normally distributed.

(a) Compute the sample mean $\bar{x}$ and the sample standard deviation s.

(b) Construct a 90% (two-sided) confidence interval for the overall average weight of "8-ounce" steaks at Charles Mulligan's Steakhouse.

(c) The manager at Charles Mulligan's Steakhouse claims that the overall average weight of "8-ounce" steaks is at least 8 ounces. Use $\alpha = 0.10$ to perform the appropriate test. State the null and alternative hypothesis for this test in terms of the relevant parameter, report the value of the test statistic, the critical value(s), and state your decision.

(d) Using the *t* distribution table only, what is the p-value of the test in part (c)? (You may give a range.)

(e) Use a computer (e.g. R or R Studio) to find the p-value of the test in part (c).

Hint: When using R or R Studio,

```
> pt( t , degrees of freedom )
```

gives area to the left of t.

[Type `? pt` in the R console or R Studio console window for the help file.]