Reaction-Diffusion Models on Complex Networks

Guglielmo Reggio[1]

As part of the King's Undergraduate Research Fellowship

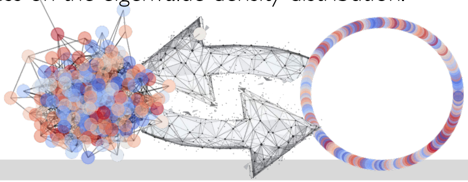
Introduction

Reaction–diffusion (RD) systems are systems involving constituents locally transformed into each other by chemical reactions and transported in space by diffusion [1].

$$\partial_t \overline{\rho} = \underline{\underline{D}} \cdot \underline{\underline{L}} \cdot \overline{\rho} + f(\overline{\rho})$$

Usually RD systems are simulated on a regular mesh. We generalise to an arbitrary network.

The aim is to understand wether the concept of 'space' or 'distance' can be applied to networks and what is the effect of increasing randomness on the eigenvalue density distribution.



Methods

In order to study the Turing instability in reaction-diffusion models defined on complex networks, a 1-dimensional ring graph was generated.

We then studied the effect of adding edges on a 1D ring (spatial) graph on it's Laplacian matrix eigenvalue distribution [2]. This was done in order to study the effect of transforming a spatial ring into a non-spatial random graph on its 'theoretical stability'.
The 'theoretical stability' here is represented by the value of the highest eigenvalue as it is the one that generates instability.

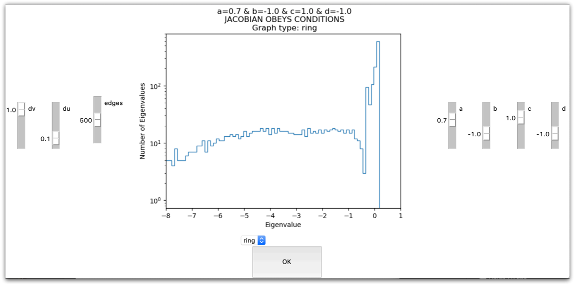


Figure 1 - GUI showing the eigenvalues distribution for a set of parameters.

Results

I. As the maximum eigenvalue does not change with topology modifications, in some cases, stability might be topology invariant.

II. Transforming a random network into a spatial network allows to make it easier to visualise and extrapolate pattern features, e.g. wavelength.

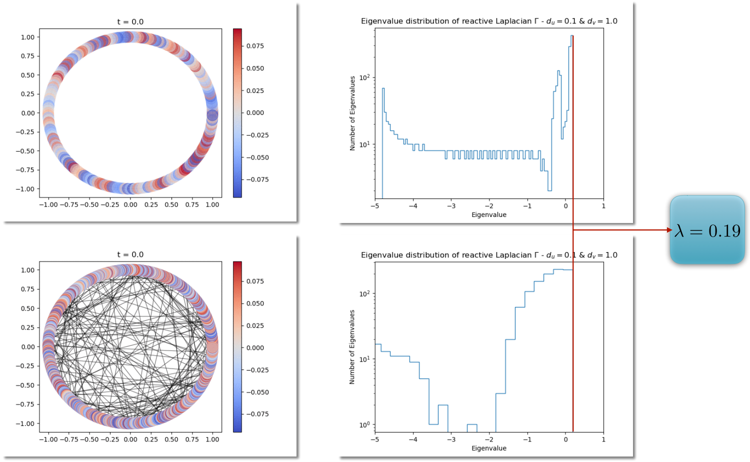


Figure 2 - Adding edges to a ring network alters the eigenvalue distribution But leaves the highest eigenvalue unchanged..

Additional Results

I. Figure 3 shows the bifurcation curve and the asymptotical line of each eigenvalue.
The combination of curves outlines the stable and unstable regions in the diffusivities phase space and the inverse of the eigenvalue defines the asymptote.

II. The diffusivities (du, dv) alter the stability of the system by shifting the eigenvalue distribution. In particular by shifting the maximum eigenvalue above 0, therefore making the system unstable.

Additional Results

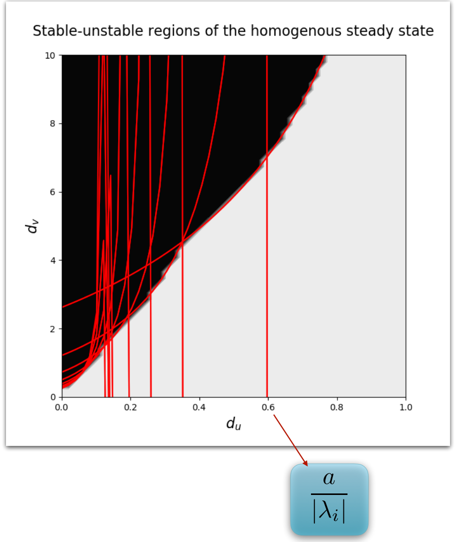


Figure 3 - Stable unstable regions of the homogenous steady state and eigenvalue asymptotes.

Future Investigations

I. Is it possible to mimic non-spatial networks with spatial networks?

II. Is it possible to control the pattern's wavelength?

References

1. Ide Y., Izuhara H., Machida T. (2016). Turing instability in reaction-diffusion models on complex networks. *Physica A: Statistical Mechanics and its Applications, Volume 457, p. 331-347.*
2. Silvester J.R. (2000). Determinants of block matrices. *The Mathematical Gazette., 501*, 460-467.

The codes generating these results can be found on:
https://github.com/gullirg/TuringPatterns/tree/6-optimise-code

Special thanks to Dr Attila Attila Csikasz-Nagy, Gregory Szep, Shaheim Ogbomo-Harmitt, Phoenix Kayler, Emeline Pineau.

[1] Department of Physics, King's College London, Strand Building, Strand Campus, Strand, London, WC2R 2LS

guglielmo.reggio@kcl.ac.uk