$(y|\beta) \sim N(X\beta, \Sigma)$ $\quad \Sigma = \sigma^2 I$, diag$(\sigma_1^2, \dots, \sigma_N^2)$ $\quad \beta \sim N(m, V)$

$$\begin{pmatrix} y \\ \beta \end{pmatrix} \sim N\left(\begin{matrix} Xm \\ m \end{matrix} , \begin{bmatrix} \Sigma + XVX^T & XV \\ VX & V \end{bmatrix} \right)$$

$(\beta | y) \sim N(\hat{\beta}, \hat{C})$

$\hat{C} = V - VX^T(XVX^T + \Sigma)^{-1}XV$ ★

$\hat{\beta} = m + VX^T[XVX^T + \Sigma]^{-1}(y - Xm)$

Woodbury Matrix Identity

★ $(A + LTR)^{-1} = A^{-1} - A^{-1}L(T^{-1} + RA^{-1}L)^{-1}RA^{-1}$

$\hat{C} = V - VX^T\underbrace{(XVX^T + \Sigma)^{-1}}_{n\times n \text{ matrix}}XV = \underbrace{(V^{-1} + X^T\Sigma^{-1}X)^{-1}}_{p \times p \text{ matrix}}$ $\quad$ easier to invert a pxp matrix

$\hat{\beta} = m + VX^T[XVX^T + \Sigma]^{-1}(y - Xm) = \hat{C}[\underbrace{V^{-1}m}_{\text{prior mean}} + \underbrace{X^T\Sigma^{-1}y}_{\text{data}}]$

$\hat{C}[\frac{1}{\tau^2} I m + \frac{1}{\sigma^2} X^T y] \quad V = \tau^2 I$

$\hat{C} = (\frac{1}{\tau^2} I + X^T \frac{1}{\sigma^2} I X)^{-1} \quad \Sigma = \sigma^2 I$

$E(\beta | y) = (\frac{1}{\tau^2} I + \frac{1}{\sigma^2} X^T X)^{-1} (\frac{1}{\tau^2} m + \frac{1}{\sigma^2} X^T y)$

$= \frac{1}{\sigma^2} (\frac{1}{\tau^2} I + \frac{1}{\sigma^2} X^T X)^{-1} (\frac{\sigma^2}{\tau^2} m + X^T y)$

$= (\underbrace{\frac{\sigma^2}{\tau^2}}_{\to \lambda} I + X^T X)^{-1} (\frac{\sigma^2}{\tau^2} m + X^T y)$

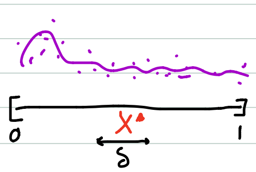
Ridge Regression $= (\lambda I + X^T X)^{-1} (\lambda m + X^T y) \quad \lambda = \frac{\sigma^2}{\tau^2} = \frac{1/\sigma^2}{1/\tau^2}$ $\quad$ signal-to-noise ratio

Estimation /

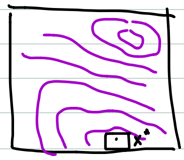
Tychonov reglarization

Folk Theorem for Multivariate Stats

- To maintain given MSE, n must grow exponentially w/ dimension



$X = [0, 1]$ $\quad$ What # of points fall w/i $\delta/2$ of x^*? $\quad n\delta$



$X = [0,1] \times [0,1]$

$f(x_1, x_2)$ $\quad$ $x^* = (x_1^*, x_2^*)$ $\quad$ w/in $\frac{\delta}{2}$ of x^* $\quad$ $n\delta^2$

$f(x^*)$ $\quad$ $n\delta^3$

$n\delta^{10}$

$n\delta^{1000}$

$MSE \approx \left[\frac{c}{n^{4/(4+p)}}\right]$ $\quad$ p = dimension

$n \propto \left[\frac{c}{MSE}\right]^{p/4}$

data needs to grow exponentially