Asymptotic Behaviour Of Exponential Distribution

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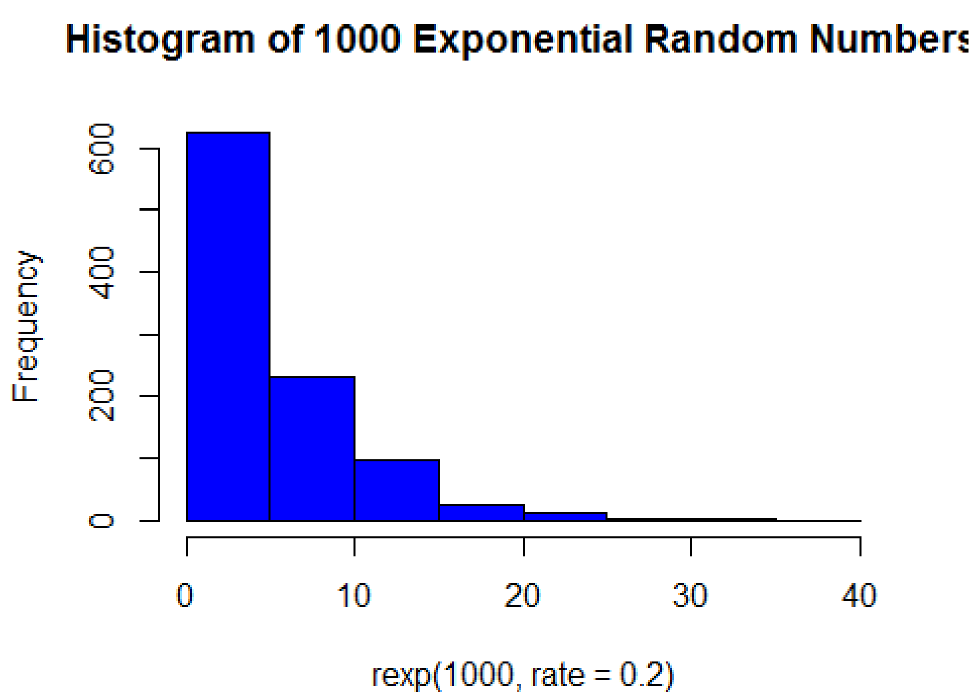
    In this part of the project, we will investigate that the sample mean
and sample variace of the exponential distribution are representatives of
their population (theoriatical) mean and population variance using simulation
study using R. Moreover, it will also investigate that the distbution of the
statistics follows normal distribution as the sample size gets large under
the assumption of Central Limit Theorem using some descriptive as well as
graphical analysis.
 Now since the mean of exponential distribution is 1/lambda and standard
deviation is also 1/lambda and here we have to use **lambda=0.2**, then the
theoriatical mean is 5 and variance for such value of parameter will be 25.
```

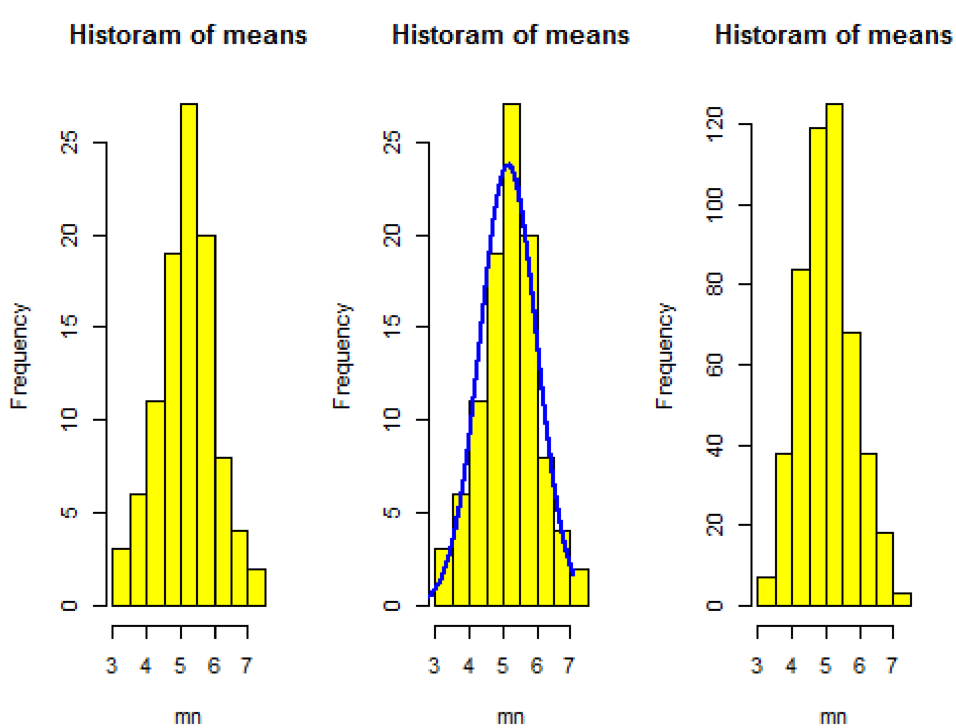
So we will start some simulations to get the required point.

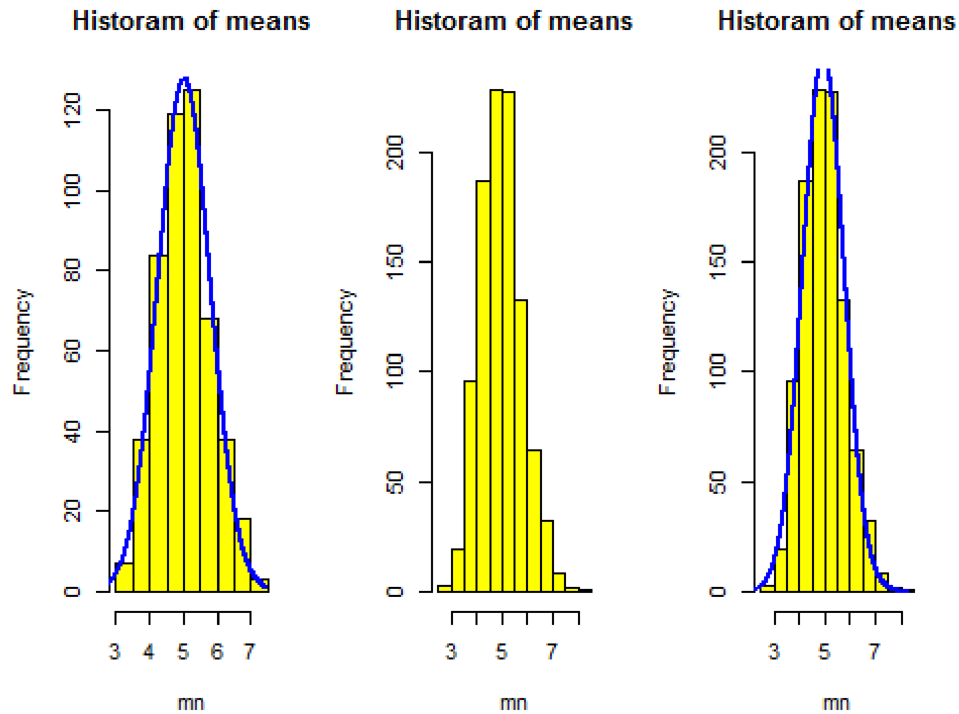
```
hist(rexp(1000, rate=0.2), main= "Histogram of 1000 Exponential Random
Numbers", col="BLUE") # To draw histogram of exponential random numbers
```



As we can see that using the histogram of 1000 exponential random numbers, most of the values are within the group of 0 to 10 which indicates that these values tend to their central point. Now we are going to plot mean of the 40 exponential random numbers to see the behaviour of sample mean as n gets larger using some simulations.

```
par(mfrow=c(1,3)) # splitting graph into three parts
for(j in c(100,500,1000)){
mn<-c()
for(i in 1:j){
     mn[i]<-mean(rexp(40, rate=0.2))
}     # simulated program to evaluate mean of exp random variables
m<-mean(mn); std<-sqrt(var(mn)); # to draw multiple graphs
hist(mn, breaks= 12, col= "yellow", main="Historam of means")
h<-hist(mn, breaks= 12, col= "yellow",main="Historam of means")
xfit<-seq(0,max(mn),length=10000)
yfit<-dnorm(xfit,mean=m,sd=std)
yfit <- yfit*diff(h$mids[1:2])*length(mn)
lines(xfit, yfit, col="blue", lwd=2)}
```





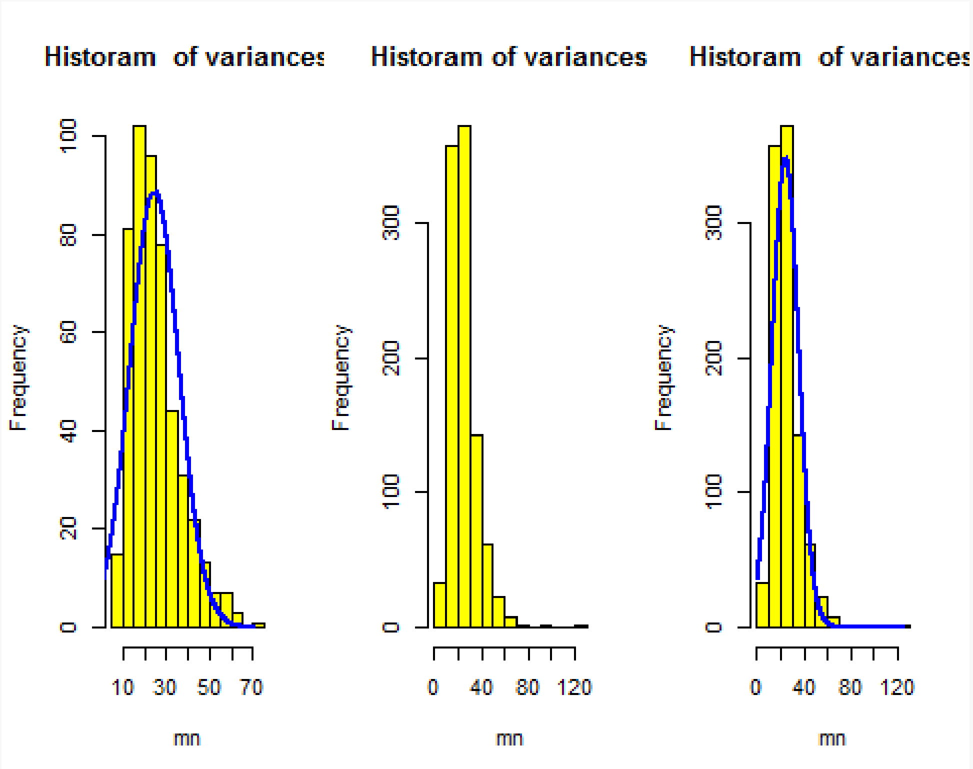

```
par(mfrow=c(1,1))
```

Now we can see from the graph that as the sample size increases the sample mean gets closer to closer their theoretical mean which is 5.

Now again similar method can be adopted for the sample variance, that is,

```
par(mfrow=c(1,3)) # simulated study for variance
for(j in c(100,500,1000)){
mn<-c()
for(i in 1:j){
     mn[i]<-var(rexp(40, rate=0.2))
}
m<-mean(mn); std<-sqrt(var(mn));
hist(mn, breaks= 15, col= "yellow", main="Historam of variances")
h<-hist(mn, breaks= 15, col= "yellow",main="Historam  of variances")
xfit<-seq(0,max(mn),length=10000)
```

```
yfit<-dnorm(xfit,mean=m,sd=std)
yfit <- yfit*diff(h$mids[1:2])*length(mn)
lines(xfit, yfit, col="blue", lwd=2
```

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)}
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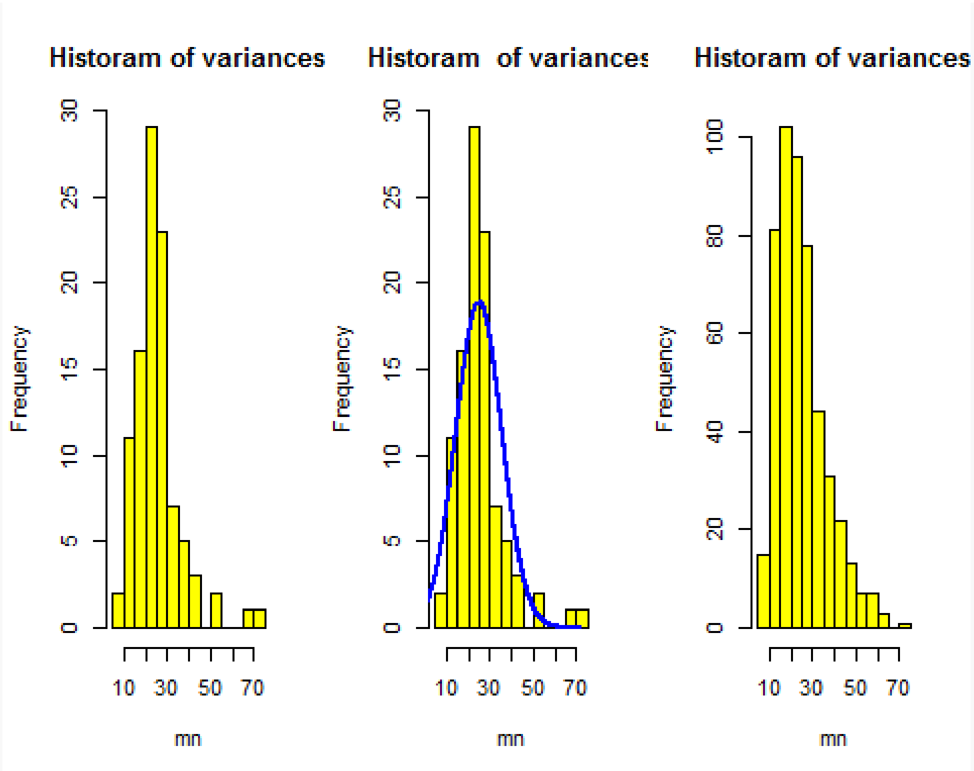


```r
par(mfrow=c(1,1))
```

So we can again see from the graph that as the sample size increases the sample variance gets closer to closer their theoretical variance which is 25. Hence according to central limit theorem, it has been stated that by in exponential distribution, the distribution of mean and variance are approximately normal which is quite obviou from graphs and from descriptive analysis.