

Multi-view triangulation and Non-linear optimization

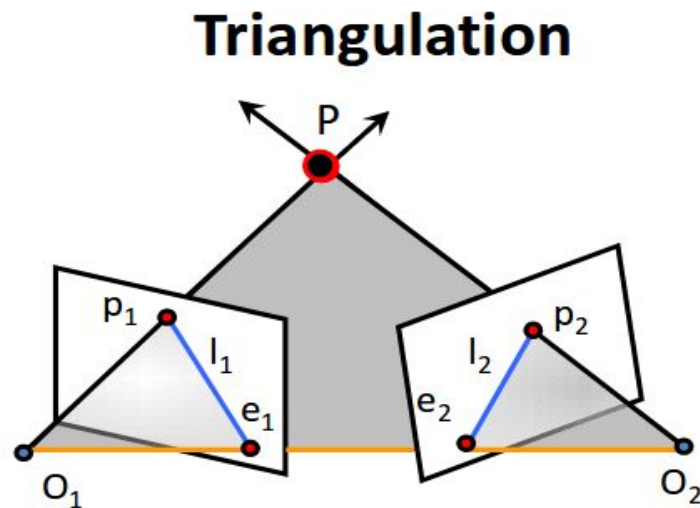
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1. Multi-view triangulation

Multi-view triangulation is a straight forward extension of 2-view triangulation which I have already coded in the previous assignment.

We have used projection matrices of all the 8 views and setup a least square system of the form $Ax = b$ and then solve it using SVD. For example 3D point X_3 must satisfy the following constraints $P_1^*X_3 = x_{13}$, $P_2^*X_3 = x_{23}$, ..., $P_8^*X_3 = x_{83}$, where x_{13} denotes the 2D projection of X_3 in image 1, x_{23} denotes the 2D projection of X_3 in image 2,..., x_{83} denotes the 2D projection of X_3 in image 8.

Triangulation

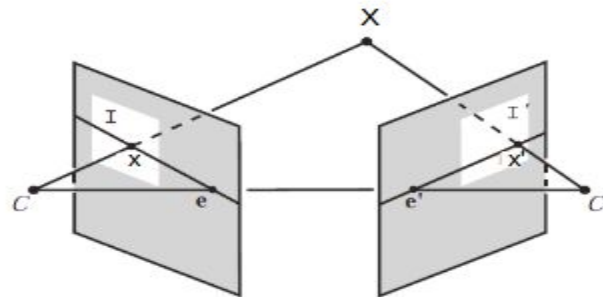


Parameter Solving for All View triangulation

Similar to 2 view triangulation we will make A matrix for 8 views provided and take the SVD to get the homogenous world coordinates(4 x 1 vector). We will divide the fourth coordinate to get our 3d world coordinates.(3 x 1 vector)

Triangulation: Linear Solution

- Generally, rays $C \rightarrow x$ and $C' \rightarrow x'$ will not exactly intersect
- Can solve via SVD, finding a least squares solution to a system of equations



$$\mathbf{x} = \mathbf{P}\mathbf{X}$$

$$\mathbf{x}' = \mathbf{P}'\mathbf{X}$$

$$\mathbf{A}\mathbf{X} = \mathbf{0} \quad \mathbf{A} = \begin{bmatrix} u\mathbf{p}_3^T - \mathbf{p}_1^T \\ v\mathbf{p}_3^T - \mathbf{p}_2^T \\ u'\mathbf{p}_3^{T'} - \mathbf{p}_1^{T'} \\ v'\mathbf{p}_3^{T'} - \mathbf{p}_2^{T'} \end{bmatrix}$$

Triangulation: Linear Solution

Given $\mathbf{P}, \mathbf{P}', \mathbf{x}, \mathbf{x}'$

1. Precondition points and projection matrices
2. Create matrix \mathbf{A}
3. $[\mathbf{U}, \mathbf{S}, \mathbf{V}] = \text{svd}(\mathbf{A})$
4. $\mathbf{X} = \mathbf{V}(:, \text{end})$

$$\mathbf{x} = w \begin{bmatrix} u \\ v \\ 1 \end{bmatrix} \quad \mathbf{x}' = w' \begin{bmatrix} u' \\ v' \\ 1 \end{bmatrix}$$

$$\mathbf{P} = \begin{bmatrix} \mathbf{p}_1^T \\ \mathbf{p}_2^T \\ \mathbf{p}_3^T \end{bmatrix} \quad \mathbf{P}' = \begin{bmatrix} \mathbf{p}_1^{T'} \\ \mathbf{p}_2^{T'} \\ \mathbf{p}_3^{T'} \end{bmatrix}$$

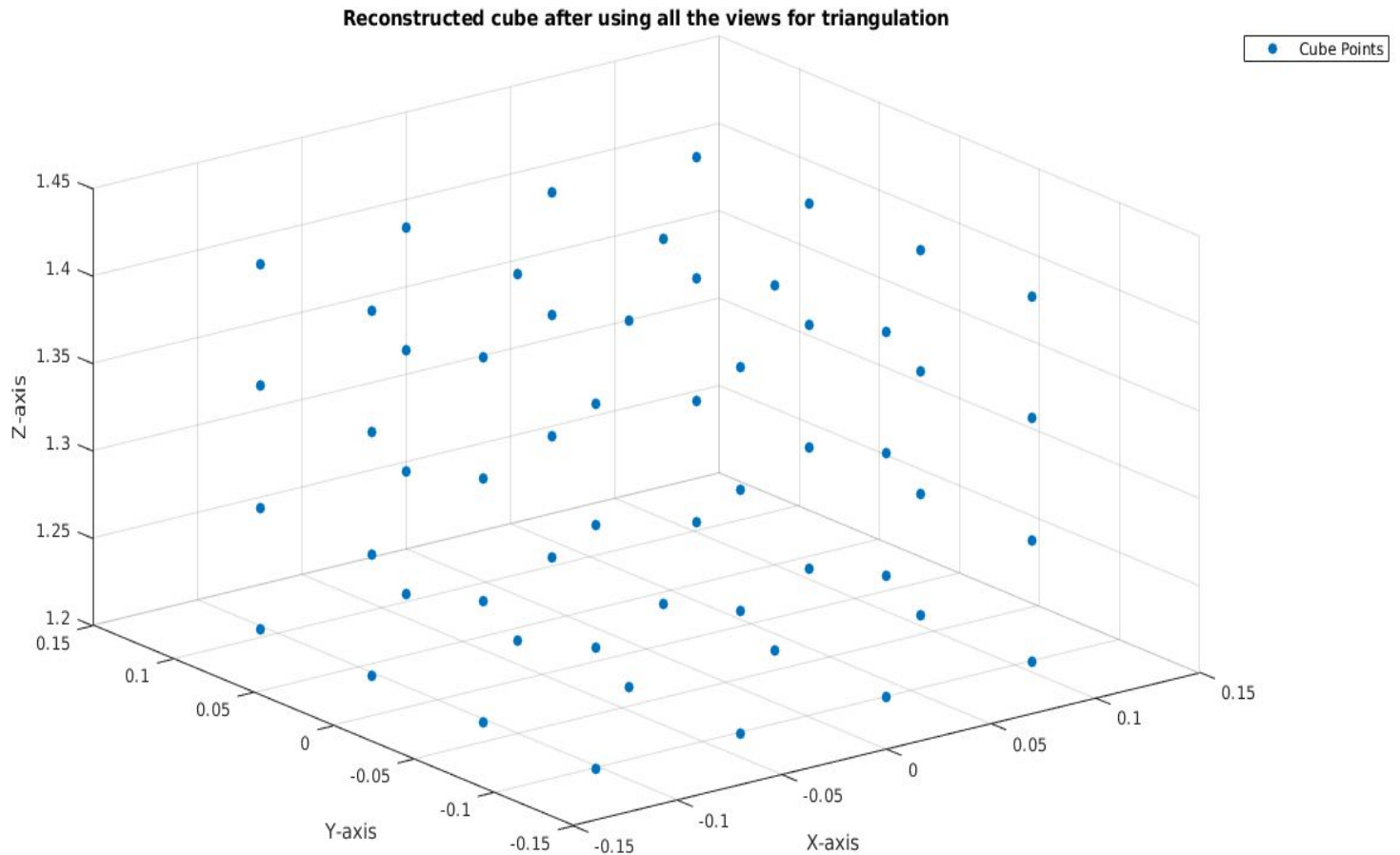
$$[\dots^T \quad \dots^T]$$

ALL View Triangulation Function

```
function [pts3D] = allviewsalgebraicTriangulation(pts2D, ProjMat_1,  
ProjMat_2, ProjMat_3,ProjMat_4,ProjMat_5,ProjMat_6,ProjMat_7,ProjMat_8)  
    %% Calculating the 3D coordinates using Linear Triangulation %%  
    A = [pts2D(1,1)*ProjMat_1(3,:) - ProjMat_1(1,:);  
pts2D(1,2)*ProjMat_1(3,:) - ProjMat_1(2,:);  
    pts2D(2,1)*ProjMat_2(3,:) - ProjMat_2(1,:); pts2D(2,2)*ProjMat_2(3,:) -  
- ProjMat_2(2,:);  
    pts2D(3,1)*ProjMat_3(3,:) - ProjMat_3(1,:); pts2D(3,2)*ProjMat_3(3,:) -  
- ProjMat_3(2,:);  
    pts2D(4,1)*ProjMat_4(3,:) - ProjMat_4(1,:); pts2D(4,2)*ProjMat_4(3,:) -  
- ProjMat_4(2,:);  
    pts2D(5,1)*ProjMat_5(3,:) - ProjMat_5(1,:); pts2D(5,2)*ProjMat_5(3,:) -  
- ProjMat_5(2,:);  
    pts2D(6,1)*ProjMat_6(3,:) - ProjMat_6(1,:); pts2D(6,2)*ProjMat_6(3,:) -  
- ProjMat_6(2,:);  
    pts2D(7,1)*ProjMat_7(3,:) - ProjMat_7(1,:); pts2D(7,2)*ProjMat_7(3,:) -  
- ProjMat_7(2,:);  
    pts2D(8,1)*ProjMat_8(3,:) - ProjMat_8(1,:); pts2D(8,2)*ProjMat_8(3,:) -  
- ProjMat_8(2,:);];  
    [~, ~, V ] = svd(A);  
    pts3D = V(:,end);  
end
```

Given P1,P2,....,P8 , we have solved for 3D location of all points using least squares .

The reconstructed points are as given below :



2. Levenberg-Marquardt (LM) Algorithm for non-linear least square

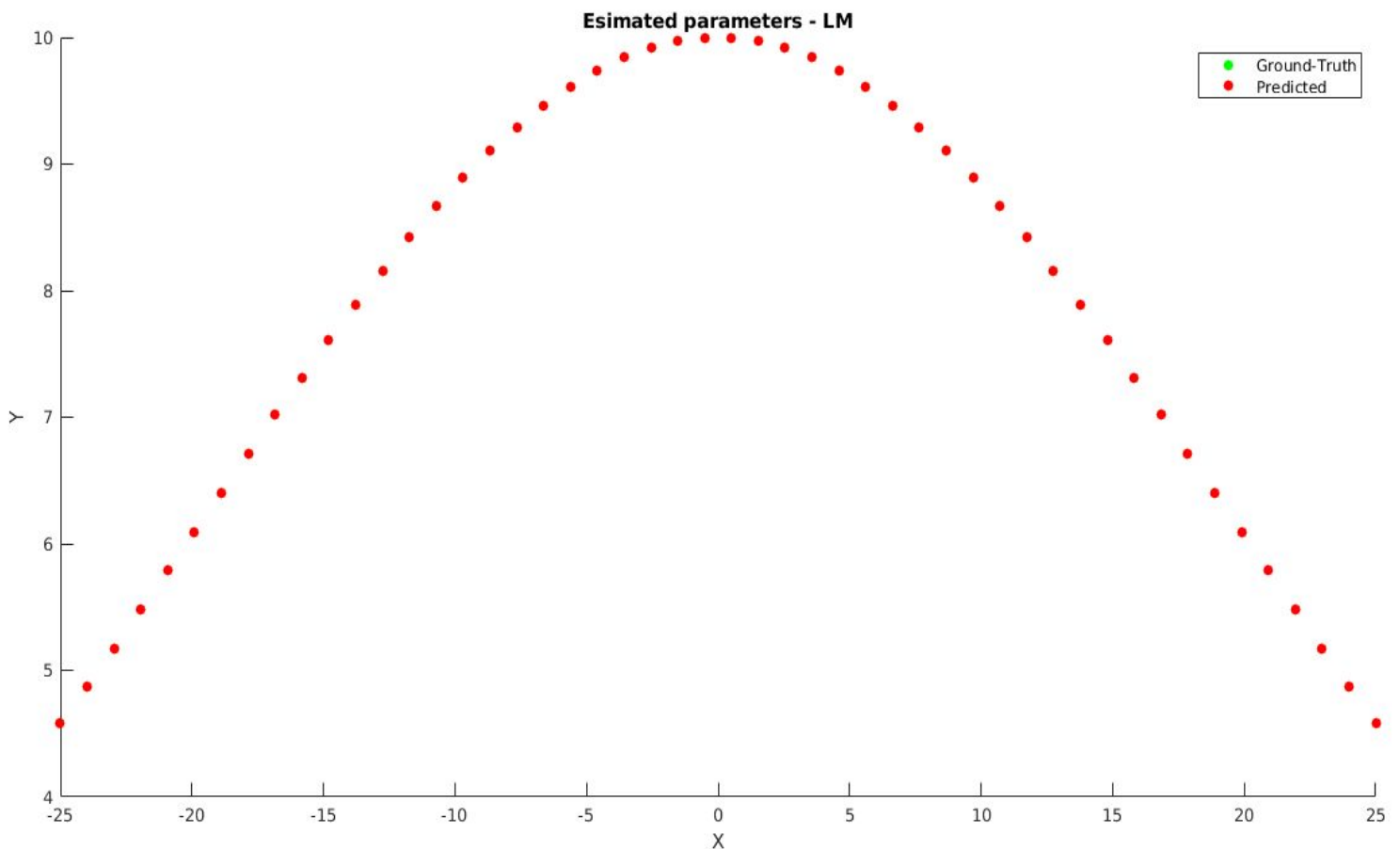
We have implemented ONLY the trust-region strategy of Levenberg-Marquardt (LM) method for non-linear least square problems. The required code for the trust region of LM algorithm has been written in the file '**testLevenbergMarquardt.m**' .

The error or difference in outputs obtained was of the order $8.2510e-16$. The obtained parameters after some 22 iterations (after which the program stops) is same as input parameters i.e

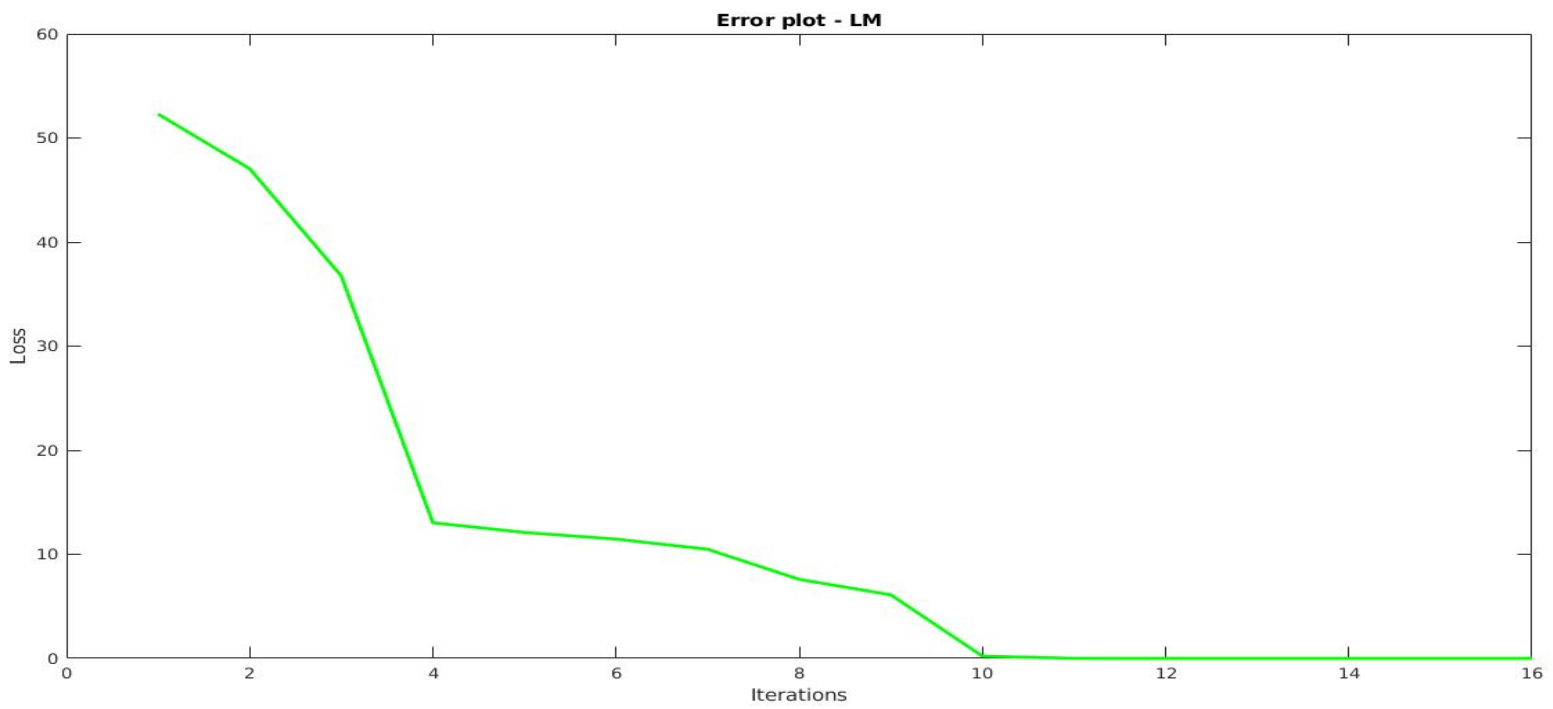
$a = 10$, mean = 0 , variance = 20 .

Levenberg-Marquardt (LM) is quite better than gauss Newton method where difference/error in outputs is about 0.5694.

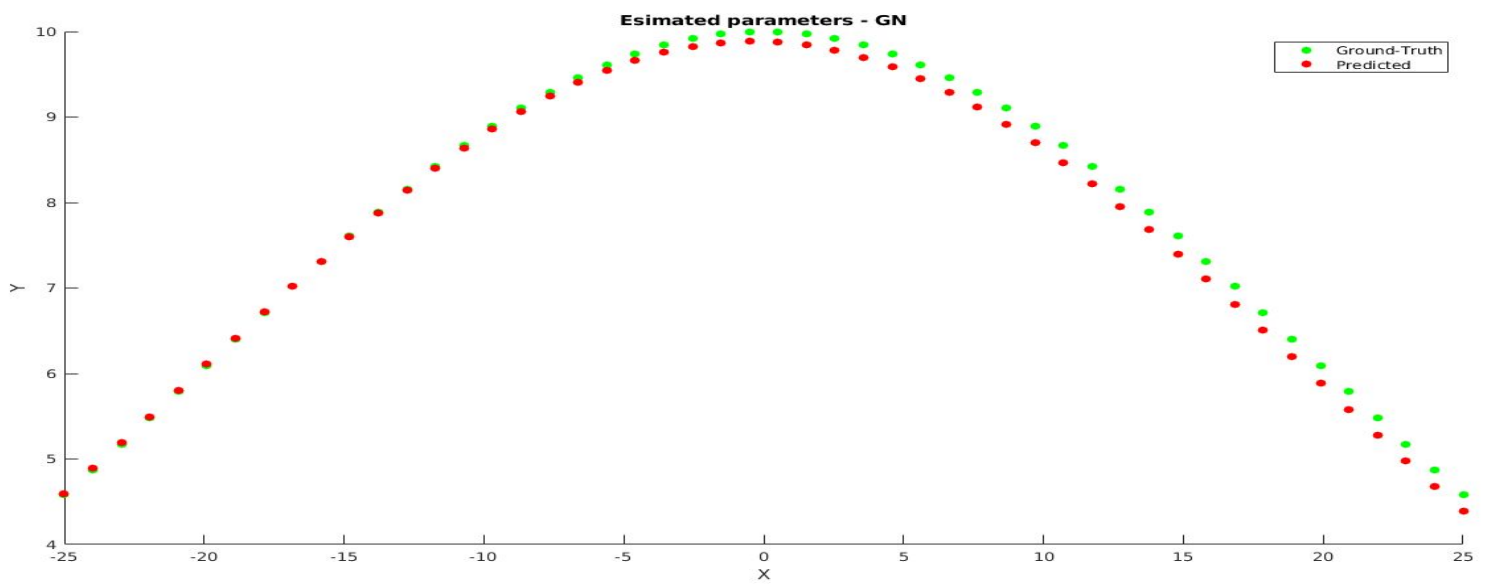
Estimated parameters- LM



Error Plot LM



Estimated Parameters Gaussian Newton



Error Plot Gaussian Newton

