Camera Calibration Report

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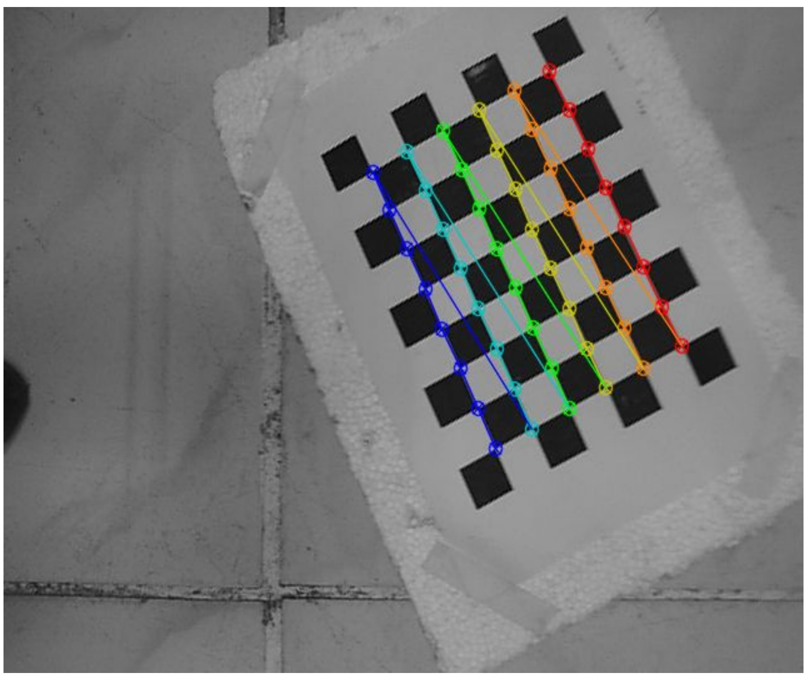
The Matlab code in the folder, finds out the intrinsic parameters of a camera using [Zhang's method].

Code Explanation

In the code 3 images Img1, Img2, Img3 were read(minimum number of images required to find out intrinsic parameters). After it since we knew cube size ,i.e, 2.4 cm and number of cubes, so ground truth was generated by considering first point as origin and then moving 2.4 cm in X and Y direction. Since all points are on same plane, so Z coordinate will be equal and were assumed to be equal to zero.



M matrix was generated by the given formula -

$$
\begin{aligned}
\boldsymbol{p} &= (p_k) = \mathrm{vec}(\mathsf{P}^\mathsf{T}) \\
\boldsymbol{a}_{x_i}^\mathsf{T} &= (-\mathbf{X}_i^\mathsf{T}, \mathbf{0}^\mathsf{T}, x_i\mathbf{X}_i^\mathsf{T}) \\
&= (-X_i, -Y_i, -Z_i, -1, 0, 0, 0, 0, x_iX_i, x_iY_i, x_iZ_i, x_i) \\
\boldsymbol{a}_{y_i}^\mathsf{T} &= (\mathbf{0}^\mathsf{T}, -\mathbf{X}_i^\mathsf{T}, y_i\mathbf{X}_i^\mathsf{T}) \\
&= (0, 0, 0, 0, -X_i, -Y_i, -Z_i, -1, y_iX_i, y_iY_i, y_iZ_i, y_i)
\end{aligned}
$$

V matrix was calculated was applying SVD on M matrix -

- Singular value decomposition (SVD)

$$\underset{2I\times 12}{\mathsf{M}} = \underset{2I\times 12}{U}\ \underset{12\times 12}{\mathsf{S}}\ \underset{12\times 12}{V^\mathsf{T}} = \sum_{i=1}^{12} s_i \boldsymbol{u}_i \boldsymbol{v}_i^\mathsf{T}$$

with properties $U^\mathsf{T} U = I_{12}$, $V^\mathsf{T} V = I_{12}$ and $s_1 \geq s_2 \geq ... \geq s_{12}$

Last row of V was chosen, matrix which was corresponding lowest eigenvalue of S matrix.

$$\Omega = \boldsymbol{p}^\top \left(\sum_{i=1}^{12} s_i^2 \boldsymbol{v}_i \boldsymbol{v}_i^\top \right) \boldsymbol{p}$$

- Due to orthogonality of V

$$\boldsymbol{v}_i \boldsymbol{v}_j^\top = \begin{cases} 0 & \text{if } i \neq j \\ 1 & \text{if } i = j \end{cases}$$

- If we choose $\boldsymbol{p} = \boldsymbol{v}_i$

$$\Omega = \boldsymbol{v}_i^\top \, (s_i^2 \boldsymbol{v}_i \boldsymbol{v}_i^\top) \boldsymbol{v}_i = s_i^2 \, \boldsymbol{v}_i^\top \boldsymbol{v}_i \boldsymbol{v}_i^\top \boldsymbol{v}_i = s_i^2$$

- Choosing $\boldsymbol{p} = \boldsymbol{v}_{12}$ (the singular vector belonging to the smallest singular value s_{12}) minimizes Ω

Similar process was applied on all three images.

Than V matrix was calculated by the below formula -

- The matrix V is given as

$$V = \begin{pmatrix} \boldsymbol{v}_{12}^T \\ \boldsymbol{v}_{11}^T - \boldsymbol{v}_{22}^T \end{pmatrix} \quad \text{with} \quad \boldsymbol{v}_{ij} = \begin{bmatrix} h_{1i}h_{1j} \\ h_{1i}h_{2j} + h_{2i}h_{1j} \\ h_{3i}h_{1j} + h_{1i}h_{3j} \\ h_{2i}h_{2j} \\ h_{3i}h_{2j} + h_{2i}h_{3j} \\ h_{3i}h_{3j} \end{bmatrix}$$

elements of H

- For one image, we obtain

$$\begin{pmatrix} \boldsymbol{v}_{12}^T \\ \boldsymbol{v}_{11}^T - \boldsymbol{v}_{22}^T \end{pmatrix} \boldsymbol{b} = 0$$

B matrix was obtained by applying SVD on the V matrix,

Then Cholesky decomposition was used on B matrix to obtain the desired K matrix.

$$\mathsf{B} = \begin{pmatrix} b_{11} & b_{12} & b_{13} \\ b_{12} & b_{22} & b_{23} \\ b_{13} & b_{23} & b_{33} \end{pmatrix} \qquad \mathrm{chol}(\mathsf{B}) = \mathsf{A}\mathsf{A}^T$$

$$\mathsf{A} = \mathsf{K}^{-T}$$