B

MA-108 End Sem Examination

24th April 2018, 5:30-7:30 PM

(60 marks)

Instructions :

- 2 marks penalty for wrong roll number, section, tutorial batch or paper code.
- 2 marks penalty for not making index.

1. Let $f(x) = e^{-x}x^2 + e^{-x} - xe^x \sin x + x\cos x - 2\sin x + 3$. Find a constant coefficient linear differential operator of least order which annihilates $f(x)$. (4)

2. Let $L = (D^2+9)^2(D-2)(D^2+2D+2)^2$ be a constant coefficient linear differential operator. Write down a basis for the space of solutions of $Ly = 0$. (4)

3. Solve the IVP $\quad y' = \dfrac{2x+y+1}{x+2y-1}, \quad y(2) = $ [illegible]$.5$ (6)

4. Consider $y' = \tan(x+y)$, $y(x_0) = y_0$. State existence and uniqueness theorem for 1st order ODE. What are the points $(x_0, y_0) \in \mathbb{R}^2$ for which this theorem guarantees that the IVP has a unique solution in some open interval around x_0. (1+3)

5. Consider the ODE $\quad y(x\cos x + 2\sin x) + x(y+1)\sin x \, y' = 0.$ (2+4+2)
   (a) Show that the ODE is not exact.
   (b) Find an integrating factor $\mu(x,y)$.
   (c) Show that after multiplying by $\mu(x,y)$, the ODE becomes exact.

6. Consider the exact ODE $\quad e^x(x^4y^2 + 4x^3y^2 + 1) + (2x^4ye^x + 2y)y' = 0$. Find an implicit solution of it. (6)

7. Solve the IVP $\quad x^2y'' - 3xy' + 3y = x^5, \ y(1) = 0, \ y'(1) = 0.$ (6)

8. Evaluate the integral $\quad \displaystyle\int_0^t x^7 (t-x)^{14}\, dx.$ (4)

9. Solve using Laplace transform $\quad y''' + y = 0, \ y(0) = 1, \ y'(0) = 0, \ y''(0) = 2.$ (6)

10. Find the inverse Laplace transform of $\quad F(s) = \ln\left(\dfrac{s^3-1}{s^3+2s^2}\right)$. (6)

11. Consider the function $\quad f(t) = \begin{cases} 4e^t, & 0 \le t < 3 \\ e^{-t}, & 3 \le t \end{cases}.$ (1+2+3)
    (a) Represent $f(t)$ in terms of unit step functions.
    (b) Find Laplace transform of $f(t)$.
    (c) Find the solution of the IVP $\quad y'' + 2y' + y = f(t), \ y(0) = 0, \ y'(0) = 0,$ in terms of unit step functions.