Image Processing with Quantum Computing

Master of Science

in

Big Data Analytics and Information Technology

by

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Declaration

I, Gulshan Savanth Dommu, hereby declare that this advance system project entitled **Image Processing with Quantum Computing** submitted by me under the guidance and supervision of *Dr. Rajkumar Pandiyampalayam Venkatachalam.*

1 IMAGE PROCESSING WITH QUANTUM COMPUTING

1.1 Quantum Addition

Comparing classical vs Quantum

```
from qiskit import QuantumCircuit, assemble, Aer
from qiskit.visualization import plot_histogram
```

```
from qiskit_textbook.widgets import binary_widget
binary_widget(nbits=5)
```

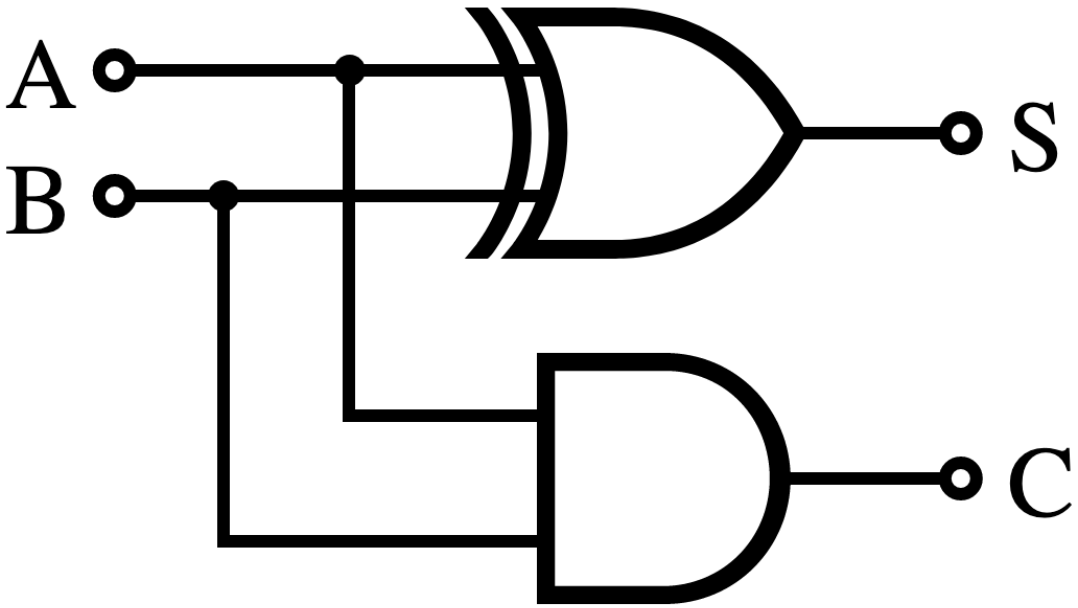
```
VBox(children=(Label(value='Toggle the bits below to change the binary number.'),␣
↪Label(value='Think of a numb...
```

```
HTML(value='<pre>Binary   Decimal\n 00000 = 0</pre>')
```

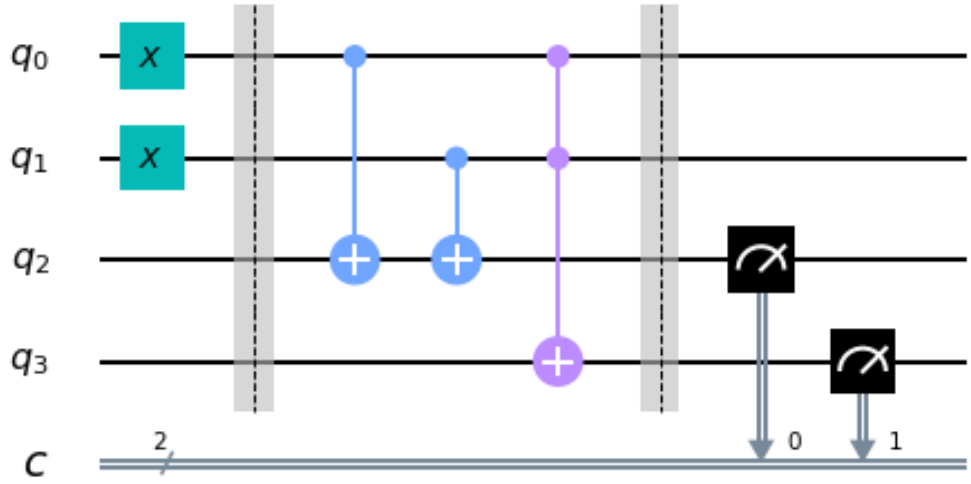
1.2 2. Computation as a diagram

Whether we use qubits or bits, we need to manipulate them to transform the input into the desired output. For the simplest programs with very few bits, it makes sense to represent this process with a diagram called a schematic. They have inputs on the left, outputs on the right, and operations represented by fuzzy symbols in between. These operations are called "gates", mainly for historical reasons.

This is an example of what a typical bit-based computer circuit might look like.



For quantum computers, we use the same basic idea, but with different conventions for representing the symbols used for inputs, outputs, and operations. Here is a quantum circuit representing the same process as above.



A circuit typically needs to do three things. 1. First encode the input, 2. computation 3. extract the output.

In our first quantum circuit, we will focus on the last of these tasks. First, create a circuit with 8 qubits and 8 outputs.

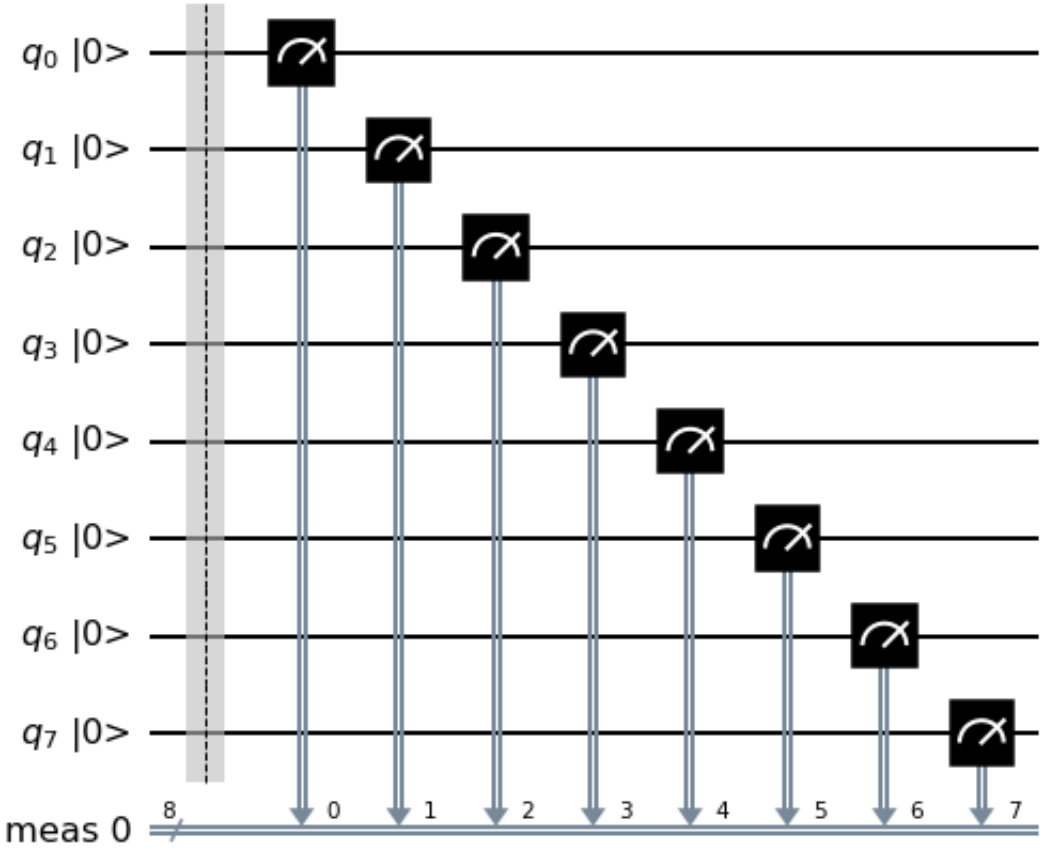
```
qc_output = QuantumCircuit(8)
```

Qc_output is created by Qiskit using QuantumCircuit.
QuantumCircuit takes as an argument the number of qubits in the quantum circuit. The Output extraction in quantum circuits is done with an operation called **measure_all()**. Each measurement instructs a specific qubit to give an output on a specific output bit. The qc_output.measure_all() command adds a measurement to each qubit of the qc_output circuit and also some classical bits to write the output.

```
qc_output.measure_all()
```

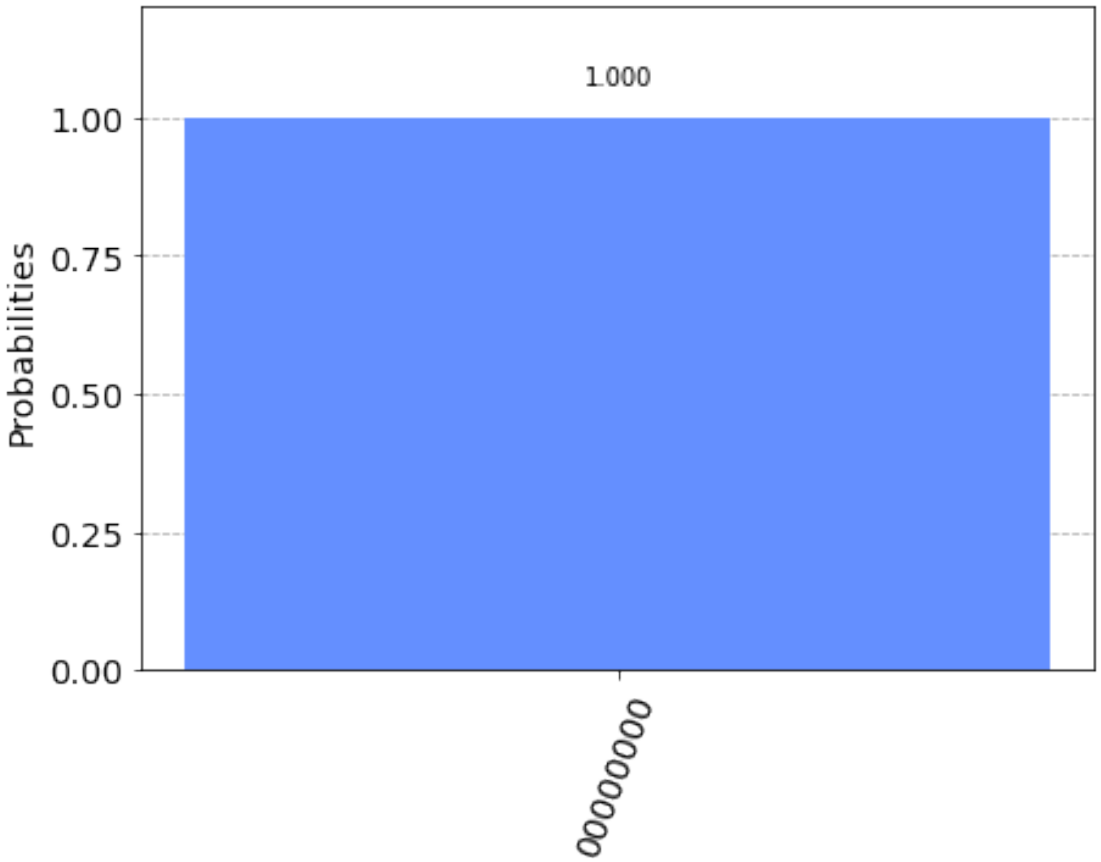
This is our circuit with something added.

```
qc_output.draw(initial_state=True)
```



The qubit is always initialized and the output is 0. Since we didn't do anything with the qubits in the circuit above, this is exactly what we get when we measure them. This can be verified by running the circuit many times and plotting the results in a histogram. Note that the result is always 00000000: 0 from each qubit.

```
sim = Aer.get_backend('aer_simulator')
result = sim.run(qc_output).result()
counts = result.get_counts()
plot_histogram(counts)
```



The reason we do this so often and display the results as a histogram is that quantum computers can have randomness in their results. In this case, the quantum is doing nothing, so we're pretty sure we'll only get a result of "00000000".

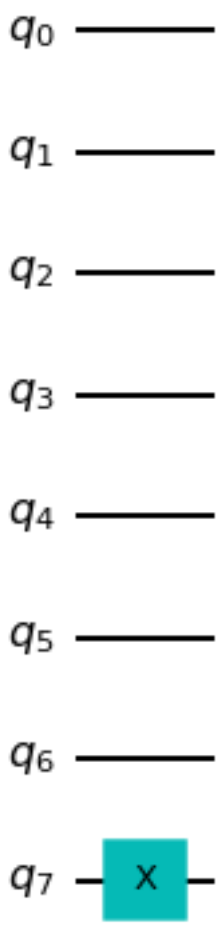
1.3 Creating an Adder Circuit

1.3.1 Encoding an input

Now let's see how to encode another binary string as input. This requires a so-called NOT gate. This is the most basic operation a computer can perform. Simply invert the bit values: 0 becomes 1, 1 becomes 0. For qubits, it's an operation called x that acts as NOT.

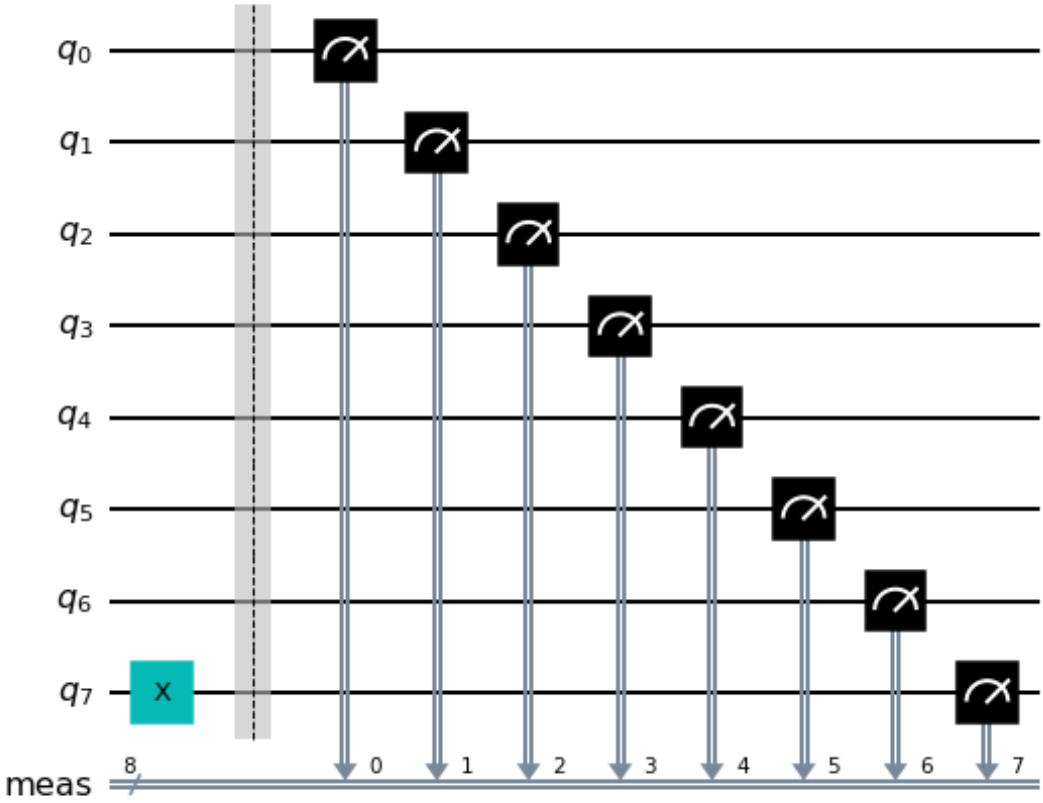
Below, create a new circuit specifically for encoding and call it qc_encode. For now, we're just specifying the number of qubits.

```
qc_encode = QuantumCircuit(8)
qc_encode.x(7)
qc_encode.draw()
```



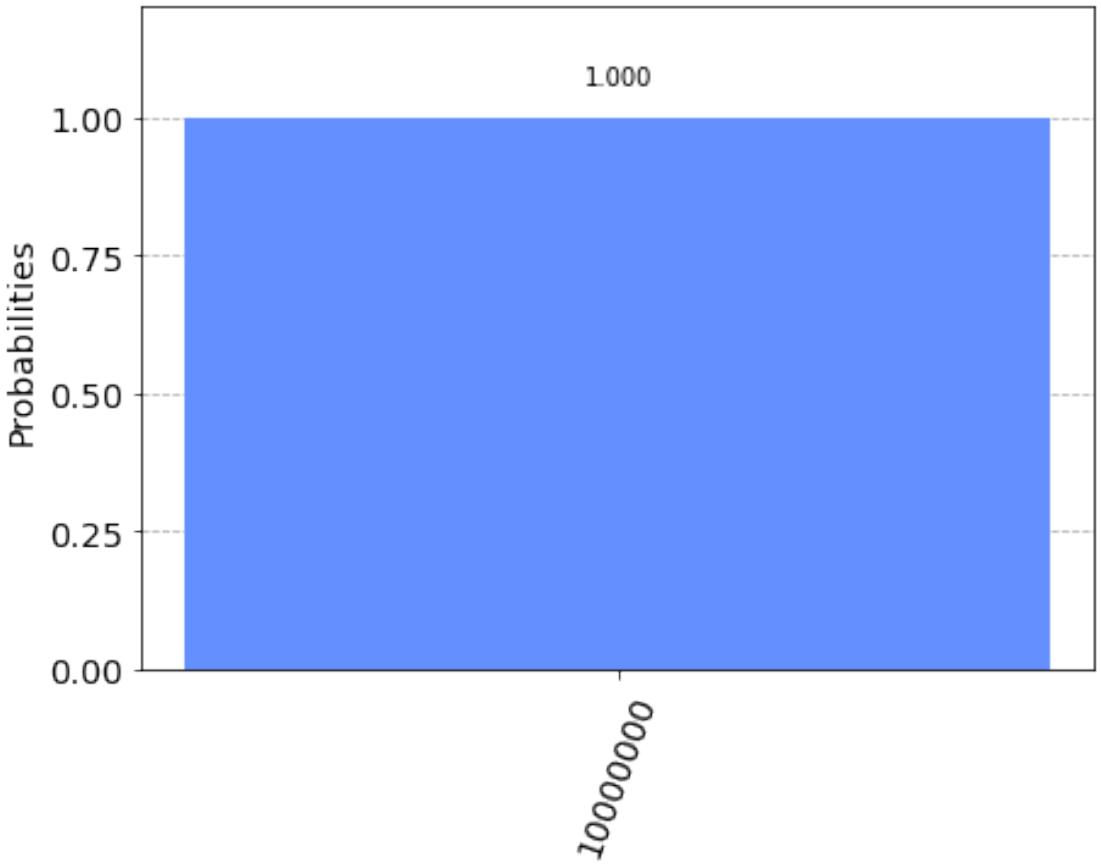
Extraction of the result can be done with the circuit we have before: qc_output.

```
qc_encode.measure_all()
qc_encode.draw()
```



Now you can run the combined circuit and see the results.

```
sim = Aer.get_backend('aer_simulator')
result = sim.run(qc_encode).result()
counts = result.get_counts()
plot_histogram(counts)
```



Now the computer prints the string 10000000 instead.

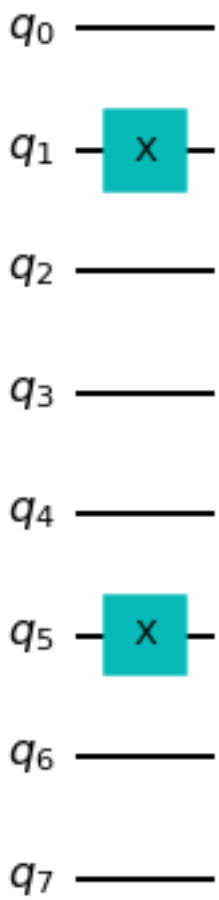
The inverted bit from qubit 7 is at the left end of the string. This is because Qiskit numbers the bits in strings from right to left. Some people prefer to number the bits in reverse, but Qiskit's system certainly has its advantages when using bits to represent numbers. Specifically, this means that qubit 7 tells us how many 27

are in our number. Flipping this bit writes the number 128 to a simple 8-bit computer.

Now write another number for yourself. For example you can make your age. Use your search engine to find out what the number looks like in binary format (ignore if it contains '0b') and add a 0 to the left if it's less than 128 .

```
qc_encode = QuantumCircuit(8)
qc_encode.x(1)
qc_encode.x(5)

qc_encode.draw()
```



Take an input that we have encoded, and turn it into an output

1.3.2 Adding with Qiskit

All that's left so far is what we already know how to do. This is because breaking everything down to 2 bits leaves only 4 possible ones that need to be calculated. Here are the four basic sums (all answers are written in 2 bits for consistency):

0+0 = 00 (0+0=0 in decimal) 0+1 = 01 (0+1=1 in decimal) 1+0 = 01 (1+0=1 in decimal) 1+1=10 (1+1=2 in decimal)

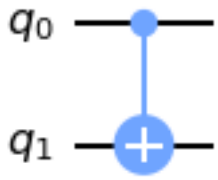
This is called a half adder. If our computer implements this and we can chain many of them, we can add anything.

XOR gate

Input 1	Input 2	XOR Output
0	0	0
0	1	1
1	0	1
1	1	0

In a quantum computer, the work of an XOR gate is done by a controlled NOT gate,CNOT. In Qiskit it's called cx for even shorter. The circuit diagram is drawn as shown below.

```
qc_cnot = QuantumCircuit(2)
qc_cnot.cx(0,1)
qc_cnot.draw()
```



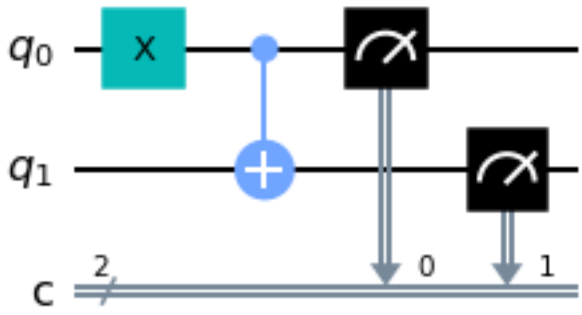
This applies to pairs of qubits. One acts as a control qubit (the one with the little dot). The other acts as a target qubit (with a + in the big circle).

There are several ways to explain the effect of CNOT. One thing we can say is that we look at two input bits to see if they are the same or different. Then overwrite the target qubit with the response. 0 if equal, 1 otherwise. Image

Another way to describe CNOT is that if the control is 1, do NOT to the target, otherwise do nothing. This explanation is just as valid as the previous one (actually, the gates are named).

Try each possible input and try CNOT yourself. For example, here is a circuit that tests CNOT on input 01.

```
qc = QuantumCircuit(2,2)
qc.x(0)
qc.cx(0,1)
qc.measure(0,0)
qc.measure(1,1)
qc.draw()
```



Run this circuit and you will see that the output is 11. This is likely due to one of the following reasons:

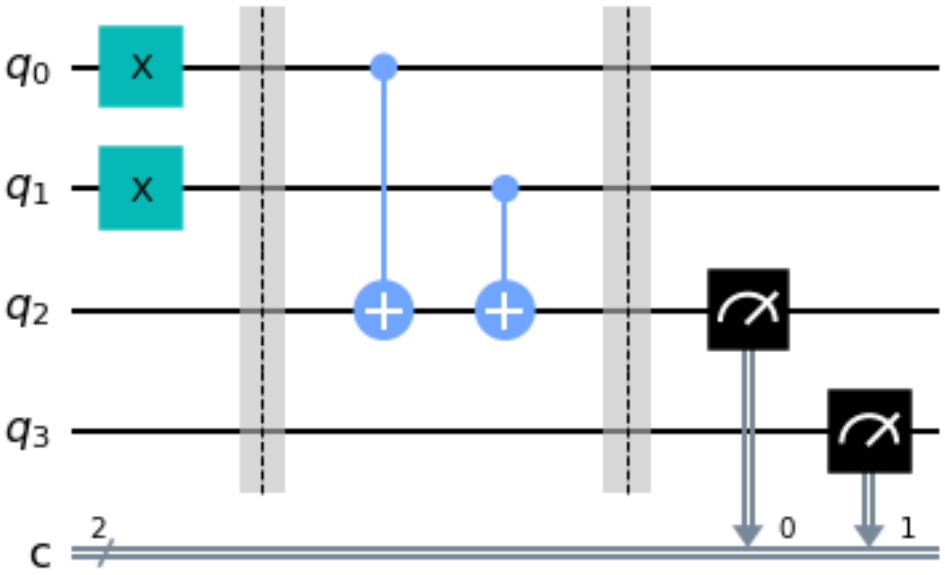
CNOT calculates whether the input values are different and determines that they are different. In other words, we want to output 1. To do this, he overwrites the state of qubit 1 (you know, it's on the left side of the bitstring), transforming 01 into 11. This flips the 0 in qubit 1 to 1 and turns 01 to 11.

Input (q1 q0)	Output (q1 q0)
00	00
01	11
10	10
11	01

In the half adder we don't want to overwrite the input. Instead, we want to write the result to another qubit pair. Two CNOTs are available for this purpose.

```
qc_ha = QuantumCircuit(4,2)
# encode inputs in qubits 0 and 1
qc_ha.x(0) # For a=0, remove this line. For a=1, leave it.
qc_ha.x(1) # For b=0, remove this line. For b=1, leave it.
qc_ha.barrier()
# use cnots to write the XOR of the inputs on qubit 2
qc_ha.cx(0,2)
qc_ha.cx(1,2)
qc_ha.barrier()
# extract outputs
qc_ha.measure(2,0) # extract XOR value
qc_ha.measure(3,1)

qc_ha.draw()
```



Looking again at the four possible sums, we see that there is only one case where this is 1 out of 0: 1+1=10. Occurs only if both bits to add are 1.

To compute this part of the output, just have the computer check that both inputs are 1. If so, and only if so, we need to create a NOT gate on qubit 3. This inverts it to the value 1 that we want in this case and gives us the output we want.

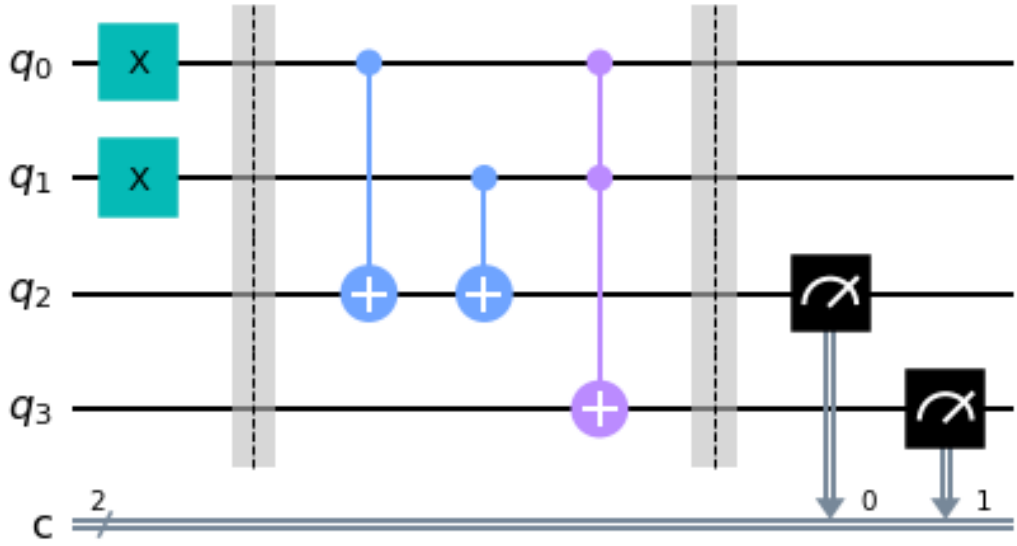
For this we need a new gate. Like CNOT, but controlled by two qubits instead of one. This will NOT execute on the target qubit only if both controls are in the 1 state. This new gate is called Tofori. For those familiar with Boolean logic gates, this is basically an AND gate.

In Qiskit, Toffoli is rendered with the ccx command.

```
qc_ha = QuantumCircuit(4,2)
# encode inputs in qubits 0 and 1
qc_ha.x(0) # For a=0, remove the this line. For a=1, leave it.
qc_ha.x(1) # For b=0, remove the this line. For b=1, leave it.
qc_ha.barrier()
# use cnots to write the XOR of the inputs on qubit 2
qc_ha.cx(0,2)
qc_ha.cx(1,2)
# use ccx to write the AND of the inputs on qubit 3
qc_ha.ccx(0,1,3)
qc_ha.barrier()
# extract outputs
qc_ha.measure(2,0) # extract XOR value
```

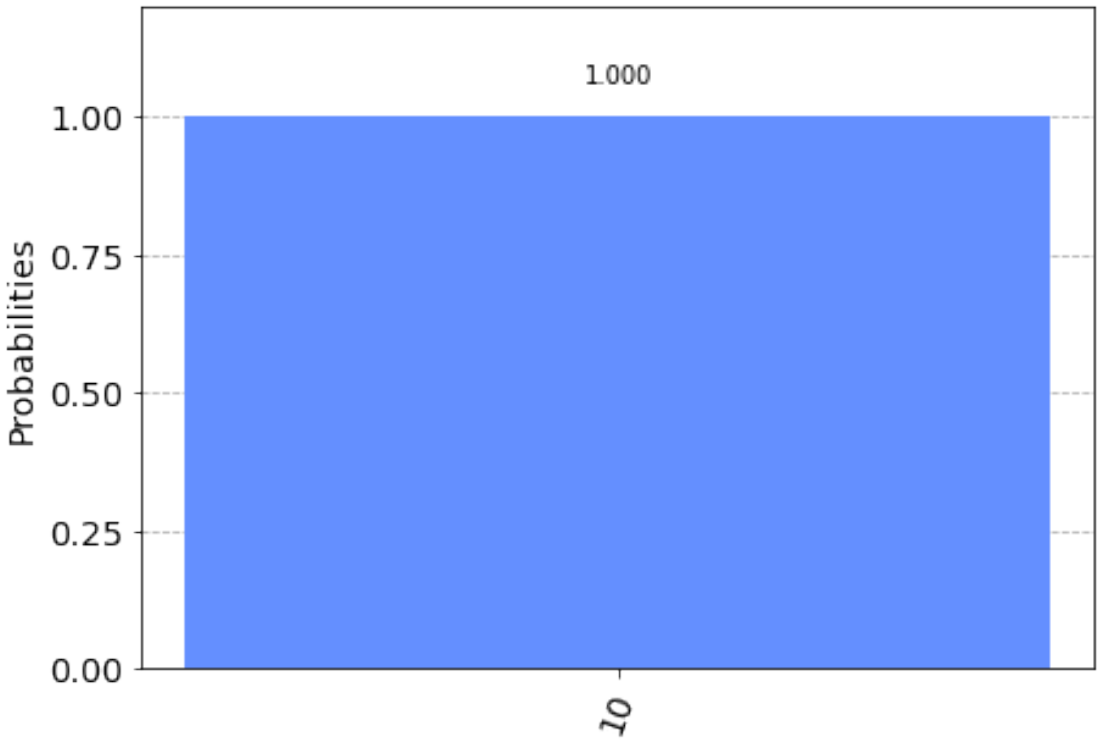
```
qc_ha.measure(3,1) # extract AND value

qc_ha.draw()
```



This example computes 1+1 because both input bits are 1. Let's see what we get.

```
qobj = assemble(qc_ha)
counts = sim.run(qobj).result().get_counts()
plot_histogram(counts)
```



The result is 10, which is the binary representation of the number 2.

Solved 1+1 math problem!

Now try it with three other possible inputs to show that the algorithm also returns correct results for these can do.

Half adder contains everything needed for addition. Using NOT, CNOT, and Toffoli gates, you can write a program to add any number of sets of any size.

These three gates are sufficient for all other operations. In fact, you can do it without CNOT. Also, NOT gates are only needed to generate 1-valued bits. Toffoli gates are essentially atoms of mathematics. This is the simplest element from which other problem-solving techniques can be assembled.

As we will see, quantum computing divides atoms.

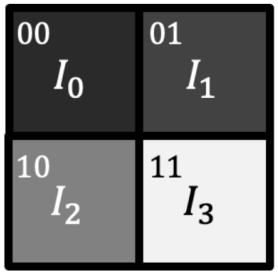
Detecting Image Edges using Quantum Computing

Quantum Probability Image Encoding (QPIE)

The QPIE representation uses the probability amplitudes of quantum states to store pixel values in conventional images. If we have n qubits, we can access at most 2 n states in the superposition. QPIE takes advantage of this fact to design efficient and robust encoding schemes for black-and-white (B&W) or RGB images, exponentially reducing the memory required to store the data. This means that only two qubits are required to store a 4-pixel image. For an 8-pixel image, you need 3 qubits

$$n = \lceil \log_2 N \rceil$$

Converting Classical Image to QPIE



where the vectors are ($I0$, $I1$, $I2$, $I3$) (or ($I00$, $I01$, $I01$, $I11$) in subscript binary representation) and various pixels (00,01,10,11) of color intensity (8-bit B&W color)) as a 2D matrix to replace traditional 2×2

$$I = (I_{yx})_{N_1 \times N_2}$$

An image can be expressed in pixel intensity as follows: is the intensity of the pixel at location (x,y)

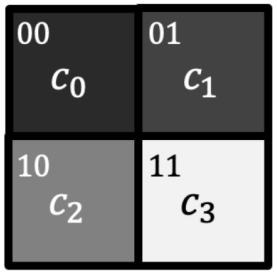
in the 2D image, starting the axes from the upper left corner of the image.

Now we need to express these pixel intensities as probability amplitudes for a particular quantum state. To do this, we need to normalize the pixel intensities so that the sum of the squares of all probability amplitudes is 1. For each Iyx corresponding to each Iyx

Normalization can be done as

$$c_i = \frac{I_{yx}}{\sqrt{\sum I_{yx}^2}}$$

Quantum Image looks like this



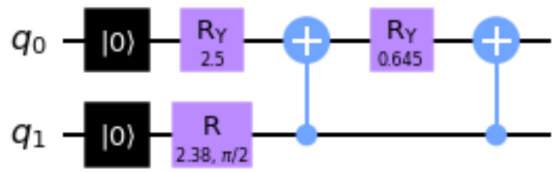
The normalized pixel color value of each pixel Pi is assigned to each quantum state

$|\text{Img}\rangle$ as:-

$$|\text{Img}\rangle = c_0 |00\rangle + c_1 |01\rangle + c_2 |10\rangle + c_3 |11\rangle$$

Such a state can be prepared very efficiently just by using some rotation gates and CNOT gates,

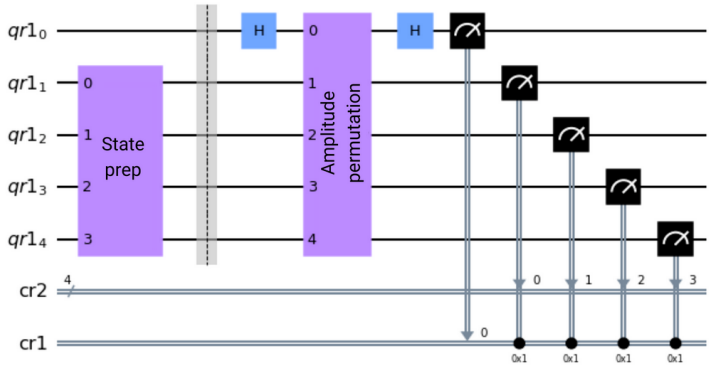
$$|\text{Img}\rangle = \sum_{i=0}^{2^n-1} c_i |i\rangle \tag{1.5}$$



Quantum Circuit

Let us take a sample 4×4 image, flattened and represented as the vector: $(0, 0.9, 0, 0, 0.5, 0.6, 0.3, 0, 0, 0.2, 0.7, 0.8, 0, 0, 1, 0)$,

The QHED quantum circuit for the above image can be generalized as:



The measurements of data qubits ($qr1_1$, $qr1_2$, $qr1_3$, and $qr1_4$) is dependent on the measurement outcome of auxiliary qubit ($qr1_0$) being in the state $|1\rangle$

Implemenation of Quantum Edge Detection

[34]:
```
from qiskit import *
from qiskit import IBMQ
from qiskit.compiler import transpile, assemble
from qiskit.tools.jupyter import *
from qiskit.visualization import *

import numpy as np
```

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ASP_ImageProcessingQua

Code

Python 3 (ipykernel)

Implemenation of Quantum Edge Detection

[34]:
```
from qiskit import *
from qiskit import IBMQ
from qiskit.compiler import transpile, assemble
from qiskit.tools.jupyter import *
from qiskit.visualization import *

import numpy as np
import matplotlib.pyplot as plt
from matplotlib import style
style.use('bmh')
```

[45]:
```
# A 8x8 binary image represented as a numpy array
image = np.array([[0, 0, 0, 0, 0, 0, 0, 0],
                  [0, 1, 1, 1, 1, 1, 0, 0],
                  [0, 1, 1, 1, 1, 1, 1, 0],
                  [0, 1, 1, 1, 1, 1, 1, 0],
                  [0, 1, 1, 1, 1, 1, 1, 0],
                  [0, 0, 0, 1, 1, 1, 1, 0],
                  [0, 0, 0, 1, 1, 1, 1, 0],
                  [0, 0, 0, 0, 0, 0, 0, 0]])

# Function for plotting the image using matplotlib
def plot_image(img, title: str):
    plt.title(title)
    plt.xticks(range(img.shape[0]))
    plt.yticks(range(img.shape[1]))
    plt.imshow(img, extent=[0, img.shape[0], img.shape[1], 0], cmap='viridis')
    plt.show()
```

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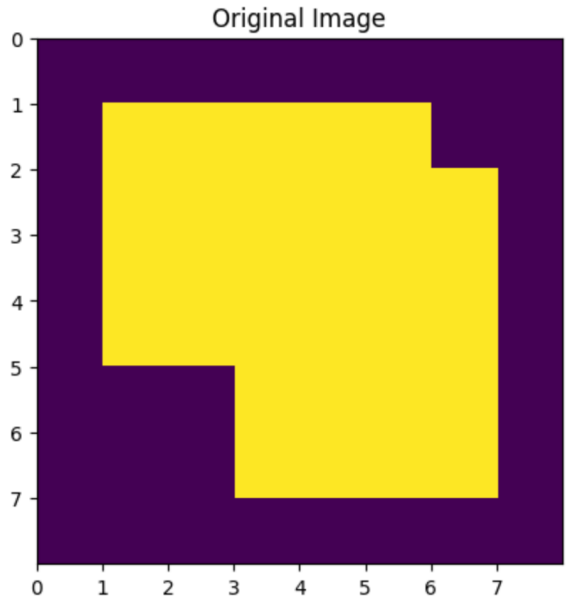
ASP_ImageProcessingQua

Code

Python 3 (ipykernel)



[36]:
```
# Convert the raw pixel values to probability amplitudes
def amplitude_encode(img_data):

    # Calculate the RMS value
    rms = np.sqrt(np.sum(np.sum(img_data**2, axis=1)))
```

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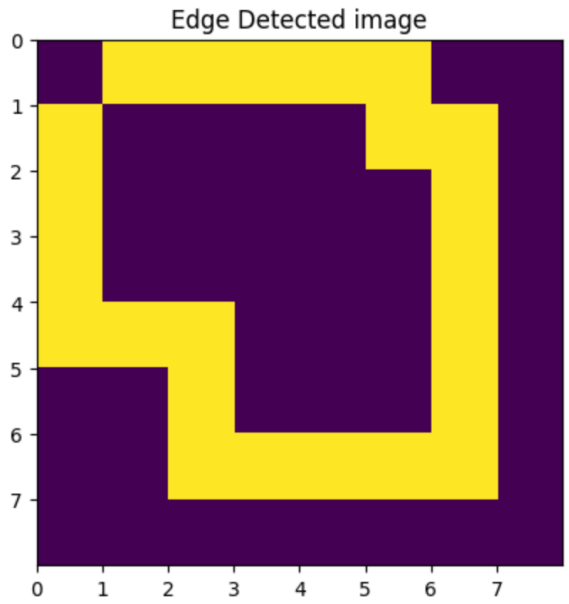
ASP_ImageProcessingQua

Code

Python 3 (ipykernel)



[47]:
```
# Initialize the number of qubits
data_qb = 2
anc_qb = 1
total_qb = data_qb + anc_qb

# Create the circuit for horizontal scan
```

References

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