

Assignment

August 27, 2024

1. The position vectors of points P and Q are \vec{p} and \vec{q} respectively. The point R divides the line segment PQ in the ratio $3 : 1$ and \vec{S} is the midpoint of the line segment PR . The position vector of \vec{S} is:

- (a) $\frac{\vec{p}+3\vec{q}}{4}$
- (b) $\frac{\vec{p}+3\vec{q}}{8}$
- (c) $\frac{5\vec{p}+3\vec{q}}{4}$
- (d) $\frac{5\vec{p}+3\vec{q}}{8}$

2. For the matrix $A = \begin{bmatrix} 2 & -1 & 1 \\ \lambda & 2 & 0 \\ 1 & -2 & 3 \end{bmatrix}$ to be invertible, the value of λ is:

- (a) 0
- (b) 10
- (c) $\mathbb{R} - \{10\}$
- (d) $\mathbb{R} - \{-10\}$

3. The angle which the line $\frac{x}{1} = \frac{y}{-1} = \frac{z}{2}$ makes with the positive direction of the y -axis is:

- (a) $\frac{5\pi}{6}$
- (b) $\frac{3\pi}{4}$
- (c) $\frac{5\pi}{4}$
- (d) $\frac{7\pi}{4}$

4. The Cartesian equation of the line passing through the point $(1, -3, 2)$ and parallel to the line:

$$\vec{r} = (2 + \lambda)\hat{i} + \lambda\hat{j} + (-1 + \lambda)\hat{k} \quad (1)$$

is:

- (a) $\frac{x-1}{2} = \frac{y+3}{0} = \frac{z-2}{-1}$
- (b) $\frac{x+1}{1} = \frac{y-3}{1} = \frac{z+2}{2}$
- (c) $\frac{x+1}{2} = \frac{y-3}{0} = \frac{z+2}{-1}$
- (d) $\frac{x-1}{1} = \frac{y+3}{1} = \frac{z-2}{2}$

5. If $A = \begin{bmatrix} x & 0 \\ 1 & 1 \end{bmatrix}$ and $B = \begin{bmatrix} 4 & 0 \\ -1 & 1 \end{bmatrix}$ then the value of x for which $A^2 = B$ is:

- (a) -2
 - (b) 2
 - (c) 2 or -2
 - (d) 1
6. Given a curve $y = 7x - x^3$ and x increases at the rate of 2 units per second, the rate at which the slope of the curve is changing when $x = 5$ is:
- (a) -60 units/sec
 - (b) 60 units/sec
 - (c) -70 units/sec
 - (d) -140 units/sec
7. Let $f(x) = \begin{bmatrix} x^2 & \sin x \\ p & -1 \end{bmatrix}$ where p is a constant. The value of p for which $f'(0) = 1$ is:
- (a) R
 - (b) 1
 - (c) 0
 - (d) -1
8. If A and B are events such that $P(A/B) = P(B/A) \neq 0$, then:
- (a) $A \subset B$, but $A \neq B$
 - (b) $A = B$
 - (c) $A \cap B = \emptyset$
 - (d) $P(A) = P(B)$
9. A function $f : \mathbb{R} \rightarrow \mathbb{R}$ defined as $f(x) = x^2 - 4x + 5$ is:
- (a) injective but not surjective
 - (b) surjective but not injective
 - (c) both injective and surjective
 - (d) neither injective nor surjective
10. If A is a square matrix of order 3 such that the value of $|\text{adj}A| = 8$, then the value of $|A^T|$ is:
- (a) $\sqrt{2}$
 - (b) $-\sqrt{2}$
 - (c) 8
 - (d) $2\sqrt{2}$

11. If $\int_{-2}^3 x^2 dx = k \int_0^3 x^2 dx + \int_2^3 x^2 dx$, then the value of k is:
- (a) 2
 - (b) 1
 - (c) 0
 - (d) $\frac{1}{2}$
12. The value of $\int_1^0 \log x dx$ is:
- (a) 0
 - (b) 1
 - (c) e
 - (d) $e \log e$
13. The area bounded by the curve $y = \sqrt{x}$, the y -axis, and between the lines $y = 0$ and $y = 3$ is:
- (a) $2\sqrt{3}$
 - (b) 27
 - (c) 9
 - (d) 3
14. The order of the differential equation:

$$\frac{d^3 y}{dx^3} + x \left(\frac{dy}{dx} \right)^5 = 4 \log \left(\frac{d^4 y}{dx^4} \right)$$

is:

- (a) not defined
 - (b) 3
 - (c) 4
 - (d) 5
15. If the inverse of the matrix $\begin{bmatrix} 7 & -3 & -3 \\ -1 & 1 & 0 \\ -1 & 0 & 1 \end{bmatrix}$ is the matrix $\begin{bmatrix} 1 & 3 & 3 \\ 1 & \lambda & 3 \\ 1 & 3 & 4 \end{bmatrix}$, then the value of λ is:
- (a) -4
 - (b) 1
 - (c) 3
 - (d) 4

16. Find the matrix A^2 , where $A = [a_{ij}]$ is a 2×2 matrix whose elements are given by $a_{ij} = \max(i, j) - \min(i, j)$:
- (a) $\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$
- (b) $\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$
- (c) $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$
- (d) $\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$
17. Derivative of $e^{\sin^2 x}$ with respect to $\cos x$ is:
- (a) $\sin x e^{a \sin^2 x}$
- (b) $\cos x e^{\sin^2 x}$
- (c) $2 \cos x e^{\sin^2 x}$
- (d) $-2 \sin^2 x \cos x e^{\sin^2 x}$
18. The function $f(x) = \frac{x}{2} + \frac{2}{x}$ has a local minimum at x equal to:
- (a) 2
- (b) 1
- (c) 0
- (d) -2

Assertion - Reason Based Questions

Direction: In questions numbers 19 and 20, two statements are given: one labeled Assertion (A) and the other labeled Reason (R). Select the correct answer from the following options:

- (A) Both Assertion (A) and Reason (R) are true and the Reason (R) is the correct explanation of the Assertion (A).
- (B) Both Assertion (A) and Reason (R) are true and Reason (R) is not the correct explanation of the Assertion (A).
- (C) Assertion (A) is true, but Reason (R) is false.
- (D) Assertion (A) is false, but Reason (R) is true.
19. **Assertion (A):** Domain of $y = \cos^{-1}(x)$ is $[-1, 1]$.
Reason (R): The range of the principal value branch of $y = \cos^{-1}(x)$ is $[0, \pi] - \left\{\frac{\pi}{2}\right\}$.