Assignment

August 27, 2024

1. The position vectors of points P and Q are $\vec{p}$ and $\vec{q}$ respectively. The point R divides the line segment PQ in the ratio $3:1$ and $\vec{S}$ is the midpoint of the line segment PR. The position vector of $\vec{S}$ is:

   (a) $\frac{\vec{p}+3\vec{q}}{4}$

   (b) $\frac{\vec{p}+3\vec{q}}{8}$

   (c) $\frac{5\vec{p}+3\vec{q}}{4}$

   (d) $\frac{5\vec{p}+3\vec{q}}{8}$

2. For the matrix $A = \begin{bmatrix} 2 & -1 & 1 \\ \lambda & 2 & 0 \\ 1 & -2 & 3 \end{bmatrix}$ to be invertible, the value of λ is:

   (a) 0

   (b) 10

   (c) $\mathbb{R} - \{10\}$

   (d) $\mathbb{R} - \{-10\}$

3. The angle which the line $\frac{x}{1} = \frac{y}{-1} = \frac{z}{2}$ makes with the positive direction of the y-axis is:

   (a) $\frac{5\pi}{6}$

   (b) $\frac{3\pi}{4}$

   (c) $\frac{5\pi}{4}$

   (d) $\frac{7\pi}{4}$

4. The Cartesian equation of the line passing through the point $(1, -3, 2)$ and parallel to the line:

$$\vec{r} = (2+\lambda)\hat{i} + \lambda\hat{j} + (-1+\lambda)\hat{k} \tag{1}$$

   is:

   (a) $\frac{x-1}{2} = \frac{y+3}{0} = \frac{z-2}{-1}$

   (b) $\frac{x+1}{1} = \frac{y-3}{1} = \frac{z+2}{2}$

   (c) $\frac{x+1}{2} = \frac{y-3}{0} = \frac{z+2}{-1}$

   (d) $\frac{x-1}{1} = \frac{y+3}{1} = \frac{z-2}{2}$

5. If $A = \begin{bmatrix} x & 0 \\ 1 & 1 \end{bmatrix}$ and $B = \begin{bmatrix} 4 & 0 \\ -1 & 1 \end{bmatrix}$ then the value of x for which $A^2 = B$ is:

(a) -2

(b) 2

(c)

(d) 2 or -2

(e) 1

6. Given a curve $y = 7x - x^3$ and x increases at the rate of 2 units per second, the rate at which the slope of the curve is changing when $x = 5$ is:

   (a) -60 units/sec

   (b) 60 units/sec

   (c) -70 units/sec

   (d) -140 units/sec

7. Let $f(x) = \begin{bmatrix} x^2 & \sin x \\ p & -1 \end{bmatrix}$ where p is a constant. The value of p for which $f'(0) = 1$ is:

   (a) R

   (b) 1

   (c) 0

   (d) -1

8. If A and B are events such that $P(A \mid B) = P(B \mid A) \neq 0$, then:

   (a) $A \subset B$, but $A \neq B$

   (b) $A = B$

   (c) $A \cap B = \emptyset$

   (d) $P(A) = P(B)$

9. A function $f : \mathbb{R} \to \mathbb{R}$ defined as $f(x) = x^2 - 4x + 5$ is:

   (a) injective but not surjective

   (b) surjective but not injective

   (c) both injective and surjective

   (d) neither injective nor surjective

10. If A is a square matrix of order 3 such that the value of $|\text{adj}A| = 8$, then the value of $\mod A^T$ is:

    (a) $\sqrt{2}$

    (b) $-\sqrt{2}$

    (c) 8

(d) $2\sqrt{2}$

11. If $\int_{-2}^{3} x^2 \text{ dx} = \text{k} \int_{0}^{3} x^2 \text{ dx} + \int_{2}^{3} x^2 \text{ dx}$, then the value of k is:

    (a) 2

    (b) 1

    (c) 0

    (d) $\frac{1}{2}$

12. The value of $\int_{1}^{0} \log x \, dx$ is:

    (a) 0

    (b) 1

    (c) e

    (d) $e \log e$

13. The area bounded by the curve $y = \sqrt{x}$, the y-axis, and between the lines $y = 0$ and $y = 3$ is:

    (a) $2\sqrt{3}$

    (b) 27

    (c) 9

    (d) 3

14. The order of the differential equation:

$$\frac{d^3y}{dx^3} + x\left(\frac{dy}{dx}\right)^5 = 4\log\left(\frac{d^4y}{dx^4}\right)$$

    is:

    (a) not defined

    (b) 3

    (c) 4

    (d) 5

15. If the inverse of the matrix $\begin{bmatrix} 7 & -3 & -3 \\ -1 & 1 & 0 \\ -1 & 0 & 1 \end{bmatrix}$ is the matrix $\begin{bmatrix} 1 & 3 & 3 \\ 1 & \lambda & 3 \\ 1 & 3 & 4 \end{bmatrix}$, then the value of λ is:

    (a) -4

    (b) 1

    (c) 3

    (d) 4

16. Find the matrix A^2, where $A = \left[a_{ij}\right]$ is a 2×2 matrix whose elements are given by $a_{ij} = \max(i, j) - \min(i, j)$:

    (a) $\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$

    (b) $\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$

    (c) $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

    (d) $\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$

17. Derivative of $e^{\sin^2 x}$ with respect to $\cos x$ is:

    (a) $\sin x e^{a \sin^2 x}$

    (b) $\cos x e^{\sin^2 x}$

    (c) $2\cos x e^{\sin^2 x}$

    (d) $-2 \sin^2 x \cos x e^{\sin^2 x}$

18. The function $f(x) = \frac{x}{2} + \frac{2}{x}$ has a local minimum at x equal to:

    (a) 2

    (b) 1

    (c) 0

    (d) -2

Assertion - Reason Based Questions

Direction: In questions numbers 19 and 20, two statements are given: one labeled Assertion (A) and the other labeled Reason (R). Select the correct answer from the following options:

(A) Both Assertion (A) and Reason (R) are true and the Reason (R) is the correct explanation of the Assertion (A).

(B) Both Assertion (A) and Reason (R) are true and Reason (R) is not the correct explanation of the Assertion (A).

(C) Assertion (A) is true, but Reason (R) is false.

(D) Assertion (A) is false, but Reason (R) is true.

19. **Assertion (A):** Domain of $y = \cos^{-1}(x)$ is $[-1, 1]$.
Reason (R): The range of the principal value branch of $y = \cos^{-1}(x)$ is $[0, \pi] - \left\{\frac{\pi}{2}\right\}$.