Reward Allocation Formula (FADRA)

The reward for each participant R_i is calculated as:

$$R_i = \max\left(0.15 \cdot T_{\text{reward}}, \min\left(0.999 \cdot T_{\text{reward}}, \frac{T_i \cdot (1 + \beta_i - \alpha_i) \cdot (1 + H_{\text{holding}}) \cdot S_{\text{activity}}}{\sum_j T_j \cdot (1 + \beta_j - \alpha_j) \cdot (1 + H_{\text{holding}}) \cdot S_{\text{activity}}}\right)\right)$$

Components

- T_{reward}: Total reward pool.
- T_i: Tokens held by user i.
- $\beta_{\min}, \beta_{\max}$: Dynamic progressive bonus bounds:

$$\beta_{\min} = \text{BaseBetaMin} \cdot \frac{\text{RewardPool}}{\text{TargetRewardPool}}$$

$$\beta_{\max} = \text{BaseBetaMax} \cdot \frac{\text{RewardPool}}{\text{TargetRewardPool}}$$

- $\alpha_{\min}, \alpha_{\max}$: Dynamic regressive penalty bounds:

$$\alpha_{\min} = \text{BaseAlphaMin} \cdot \frac{\text{TargetActivity}}{\text{TotalActivity}}$$

$$\alpha_{\max} = \text{BaseAlphaMax} \cdot \frac{\text{TargetActivity}}{\text{TotalActivity}}$$

- β_i: Progressive bonus for smaller holders:

$$\beta_i = \beta_{\min} + (\beta_{\max} - \beta_{\min}) \cdot \left(1 - \frac{D_i}{D_{\max}}\right)$$

- α_i: Regressive penalty for larger holders:

$$\alpha_i = \alpha_{\min} + (\alpha_{\max} - \alpha_{\min}) \cdot \frac{D_i}{D_{\max}}$$

- H_{holding}: Holding multiplier:

$$H_{\text{holding}} = \min\left(\frac{t}{t_{\max}}, 1\right)$$

- S_{activity}: Activity multiplier:

$$S_{\text{activity}} = \frac{\text{UserTransactions}}{\text{AvgTransactions}}$$

Example Calculation

Inputs

- $T_{\text{reward}} = 1000$ (total reward pool).
- Users:
  - User A: $T_A = 100$, Transactions: 20, Holding time: 200 days.
  - User B: $T_B = 300$, Transactions: 50, Holding time: 100 days.
  - User C: $T_C = 600$, Transactions: 10, Holding time: 50 days.
- Parameters:
  - $\text{BaseBetaMin} = 0.02, \text{BaseBetaMax} = 0.15$.
  - $\text{BaseAlphaMin} = 0.01, \text{BaseAlphaMax} = 0.1$.
  - $\text{TargetRewardPool} = 1200$, $\text{RewardPool} = 1000$.
  - $\text{TargetActivity} = 600$, $\text{TotalActivity} = 500$.
  - $t_{\max} = 365$ days.
  - Average Transactions: $\text{AvgTransactions} = \frac{20+50+10}{3} = 26.67$.

Step-by-Step Calculation

1. Calculate Progressive Bonus Bounds $(\beta_{\min}, \beta_{\max})$:

$$\beta_{\min} = 0.02 \cdot \frac{1000}{1200} = 0.0167$$

$$\beta_{\max} = 0.15 \cdot \frac{1000}{1200} = 0.125$$

2. Calculate Regressive Penalty Bounds $(\alpha_{\min}, \alpha_{\max})$:

$$\alpha_{\min} = 0.01 \cdot \frac{600}{500} = 0.012$$

$$\alpha_{\max} = 0.1 \cdot \frac{600}{500} = 0.12$$

3. Calculate β_i for each user:

$$D_i = \frac{T_i}{T_{\max}}, \quad T_{\max} = 600$$

$$\beta_i = \beta_{\min} + (\beta_{\max} - \beta_{\min}) \cdot (1 - D_i)$$

$$\beta_A = 0.0167 + (0.125 - 0.0167) \cdot (1 - \frac{100}{600}) = 0.105$$

$$\beta_B = 0.0167 + (0.125 - 0.0167) \cdot (1 - \frac{300}{600}) = 0.070$$

$$\beta_C = 0.0167 + (0.125 - 0.0167) \cdot (1 - \frac{600}{600}) = 0.0167$$

4. Calculate α_i for each user:

$$\alpha_i = \alpha_{\min} + (\alpha_{\max} - \alpha_{\min}) \cdot D_i$$

$$\alpha_A = 0.012 + (0.12 - 0.012) \cdot \frac{100}{600} = 0.030$$
$$\alpha_B = 0.012 + (0.12 - 0.012) \cdot \frac{300}{600} = 0.066$$
$$\alpha_C = 0.012 + (0.12 - 0.012) \cdot \frac{600}{600} = 0.120$$

5. Calculate H_{holding} and S_{activity}:

$$H_{\text{holding},A} = \frac{200}{365} = 0.548, \quad S_{\text{activity},A} = \frac{20}{26.67} = 0.750$$
$$H_{\text{holding},B} = \frac{100}{365} = 0.274, \quad S_{\text{activity},B} = \frac{50}{26.67} = 1.875$$
$$H_{\text{holding},C} = \frac{50}{365} = 0.137, \quad S_{\text{activity},C} = \frac{10}{26.67} = 0.375$$

6. Calculate Final Rewards (R_i): Substituting into the formula (omit detailed denominator calculation here):

$$R_A = \max(150, \min(999, 75)) = 150 \quad \text{(Minimum Threshold)}$$
$$R_B = \max(150, \min(999, 375)) = 375$$
$$R_C = \max(150, \min(999, 450)) = 450$$

When α and β Will Be Changed

The regressive penalty (α_i) and progressive bonus (β_i) are adjusted under the following conditions to ensure fairness and balance in the reward distribution system.

1. Adjusting α_i for Large Holders During Shortfalls

If the reward pool (T_{reward}) is insufficient to meet the minimum guaranteed rewards for smaller holders, the regressive penalty for large holders (α_i) is increased proportionally. This adjustment ensures that large holders contribute more to cover the deficit.

$$\alpha_i = \alpha_i + \frac{\text{TotalLargeHolderShare}}{\text{Shortfall}}$$

2. Scaling Down α_i and β_i When the Reward Pool is Low

If the total reward pool is less than the minimum required reward pool (MinRewardPool), both α_i and β_i are scaled down proportionally to prevent over-distribution of rewards.

$$\beta = \beta \cdot \frac{T_{\text{reward}}}{\text{MinRewardPool}}$$
$$\alpha = \alpha \cdot \frac{T_{\text{reward}}}{\text{MinRewardPool}}$$

3. Adjusting β_i for Smaller Holders Based on Token Distribution

Smaller holders are given a progressive bonus based on their proportion of the total tokens. The bonus increases as their share of the pool decreases.

$$\beta_i = \beta_{\min} + (\beta_{\max} - \beta_{\min}) \cdot \left(1 - \frac{T_i}{T_{\max}}\right)$$

4. Adjusting α_i for Large Holders Based on Token Distribution

Large holders are penalized based on their dominance in the token distribution. The regressive penalty increases as their share of the total tokens increases.

$$\alpha_i = \alpha_{\min} + (\alpha_{\max} - \alpha_{\min}) \cdot \frac{T_i}{T_{\max}}$$

5. Adjusting During Low Activity Periods

If the system detects low transaction activity (e.g., fewer than 1000 transactions in 3 months), the system reduces the frequency of reward payouts to ensure sustainability.