COMPOSITE MATERIALS PROJECT

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Contents

1 Introduction

The aim of this project is to design a symmetric stacking sequence to fulfill a group of necessities. Design, analyze, and possibly optimize for our stacking sequence are done as a team. Entire calculations and selections of material and ply orientations are shown. Designing the stacking sequences with minimum weight and highest in-plane specific modulus has been our target. As a result of needing complex calculations, we decided to use MATLAB which is a scientific calculation software.

2 Project Definition

In operation as a panel construction on a fighter jet, the square plate with an edge-length of a = 0.4 m. The loads acting on the plate are given.

$$\boldsymbol{N} = \begin{bmatrix} N_x & N_y & N_{xy} \end{bmatrix}^T = \begin{bmatrix} 50 & -50 & 1 \end{bmatrix}^T kN/m \tag{1}$$

$$\boldsymbol{M} = \begin{bmatrix} M_x & M_y & M_{xy} \end{bmatrix}^T = \begin{bmatrix} -2 & 7 & 1 \end{bmatrix}^T N.m/m \tag{2}$$

Designing a symmetric laminate of 6-ply with a 0.25 mm ply thickness and choosing convenient materials and ply orientations is our task. That by, our design can keep the loading statuses defined in Equations 1 and 2. We have the option of selecting various materials for other plies from Table 1. Necessities and limitations are defined below.

- The stresses in individual plies must be lower than the ply strength given in Table 1. The factor of safety (n_s) value equals to 2.
- Magnitudes of global mid-plane strains must be lower than 5*10^-3 m/m. The factor of safety (n_s) value equals to 2.
- Magnitudes of global mid-plane curvatures must be lower than 4 m^-1. The factor of safety (n_s) value equals to 2.

Table 1: Material selection table

Material	E_{11} [GPa]	E_{22} [GPa]	G_{12} [GPa]	ν_{12}	Density [kg/m^3]	σ_{11} [MPa]	σ_{22} [MPa]	τ_{12} [MPa]
Boron/Epoxy	207	19	6.4	0.21	1990	1585	63	131
AS Carbon/Epoxy	128	9	5.7	0.25	1540	1448	62	60
T-300/Epoxy	138	10	6.5	0.21	1550	1448	45	62
HMS Carbon/Epoxy	171	13.8	5.9	0.20	1630	827	86	72
GY-70/Epoxy	262	8.3	4.1	0.25	1690	586	41	97
Kevlar 49/Epoxy	76	5.5	2.1	0.34	1380	1379	28	60
E-Glass/Epoxy	32	4.8	4.8	0.30	1800	1103	97	83

Project steps:

1. Choosing materials and ply orientations for a symmetric stacking sequence for 6 plies.
2. Computing A, B, D matrices and solving the governing equations for mid-plane strains and curvatures.
3. Plotting the changing of global and local strains and stresses along the thickness.
4. Determining in-plane and flexural engineering constants of the laminate.
5. Calculating the mass and specific in-plane axial modulus of the laminate.

3 Calculations

All the calculations were carried out in the MATLAB simulation environment. First, data on Table 1 are defined. Since the aim of the project, manufactured composite is desired to be symmetric. In this context, the plies are grouped as dual; such as 1st and 6th plies are in one group, 2nd -5th, and 3rd -4th. The whole code is written according to this knowledge. These three groups are put into iterations having the interval of [-90,90], with 15° iteration angles for all the 7 materials. Elastic and Shear modulus, Poisson's Ratio of those combinations are calculated. Reducing Compliance Matrix and Reuter Matrix are formed by using zero matrices. Then the matrix elements are filled with related modulus formulas. After that, Minor Poisson's Ratio was calculated. Transfer stiffness matrix, which has sine and cosine values of given iteration angles for combination groups, is created via reduced stiffness matrix.

Another requirements for this calculation are A, B, D stiffness matrices generated. ε_0 is found, and midplane strain is taken from this matrix. For the main aim of the project, the mass of laminate derived from the previous calculations. The density is found.

Through 7 different materials and 13 different angles, 753 571 different laminates that have 6 plies can be created. 330 861 of them provide the conditions specified in the project, but we examined 38 combinations to find the highest Young Modulus and the lowest mass construction.

Firstly, the materials that we chose were GY-70/Epoxy and Kevlar 49/Epoxy. Ply orientations were 0, 90, 0, 0, 90, 0 degrees. Here, 0 degrees is the degree of GY-70/Epoxy and 90 degrees is the degree of Kevlar 49/Epoxy for a symmetric stacking sequence for 6 plies. The reason for our selection is to ensure that the part is of minimum weight and maximum in-plane specific modulus.

Apart from the given parameters, we determined the values of the materials that we selected. Longitudinal Elastic Modulus, Transverse Elastic Modulus, Shear Modulus, Poisson's Ratio, Ultimate Longitudinal Strength, Ultimate Transverse Strength, Ultimate In-plane Shear Strength, and density are entered from Table-1. The necessary formulations were then applied in the program.

$$A_{ij} = \sum_{k=1}^{n} [(\bar{Q}_{ij})]_k (h_k - h_{k-1}), \quad i = 1,2,6; \quad j = 1,2,6,$$

$$B_{ij} = \frac{1}{2} \sum_{k=1}^{n} [(\bar{Q}_{ij})]_k (h_k^2 - h_{k-1}^2), \quad i = 1,2,6; \quad j = 1,2,6,$$

$$D_{ij} = \frac{1}{3} \sum_{k=1}^{n} [(\bar{Q}_{ij})]_k (h_k^3 - h_{k-1}^3), \quad i = 1,2,6; \quad j = 1,2,6.$$

$$\begin{bmatrix} N_x \\ N_y \\ N_{xy} \\ M_x \\ M_y \\ M_{xy} \end{bmatrix} = \begin{bmatrix} A_{11} & A_{12} & A_{16} & B_{11} & B_{12} & B_{16} \\ A_{12} & A_{22} & A_{26} & B_{12} & B_{22} & B_{26} \\ A_{16} & A_{26} & A_{66} & B_{16} & B_{26} & B_{66} \\ B_{11} & B_{12} & B_{16} & D_{11} & D_{12} & D_{16} \\ B_{12} & B_{22} & B_{26} & D_{12} & D_{22} & D_{26} \\ B_{16} & B_{26} & B_{66} & D_{16} & D_{26} & D_{66} \end{bmatrix} \begin{bmatrix} \varepsilon_x^0 \\ \varepsilon_y^0 \\ \gamma_{xy}^0 \\ \kappa_x \\ \kappa_y \\ \kappa_{xy} \end{bmatrix}.$$

Figure 1: A (Extansional), B (Coupling), and D (Bending Stiffness) Formulations and Matrices

2.6529*10^8	3.0220*10^6	0
3.0220*10^6	4.6637*10^7	0
0	0	5.1500*10^6

Table 2: A Matrice

0	-5.6843*10^-14	0
-5.6843*10^-14	2.2737*10^-13	0
0	0	0

Table 3: B Matrice

55.0960	0.5707	0
0.5707	7.3210	0
0	0	1.0073

Table 4: D Matrice

--------------------------A Matrice----------------------|------------------------B Matrice--------------------

2.6529*10^8	3.0220*10^6	0	0	-5.6843*10^-14	0
3.0220*10^6	4.6637*10^7	0	-5.6843*10^-14	2.2737*10^-13	0
0	0	5.1500*10^6	0	0	0
0	-5.6843*10^-14	0	55.0960	0.5707	0
-5.6843*10^-14	2.2737*10^-13	0	0.5707	7.3210	0
0	0	0	0	0	1.0073

--------------------------B Matrice----------------------|------------------------D Matrice--------------------

Table 5: A-B-B-D Matrices

$$\begin{Bmatrix} \varepsilon_x^0 \\ \varepsilon_y^0 \\ \gamma_{xy}^0 \end{Bmatrix} = \begin{Bmatrix} \dfrac{\partial u_0}{\partial x} \\ \dfrac{\partial v_0}{\partial y} \\ \dfrac{\partial u_0}{\partial y} + \dfrac{\partial v_0}{\partial x} \end{Bmatrix}$$

Figure 2: Mid-plane Strains Formulations

2.0083*10^-4
-0.0011
1.9417*10^-4

Table 6: Mid-plane Strains

$$\begin{Bmatrix} \kappa_x \\ \kappa_y \\ \kappa_{xy} \end{Bmatrix} = \begin{Bmatrix} -\dfrac{\partial^2 w_0}{\partial x^2} \\ -\dfrac{\partial^2 w_0}{\partial y^2} \\ -2\dfrac{\partial^2 w_0}{\partial x \partial y} \end{Bmatrix},$$

Figure 3: Mid-plane Curvatures Formulations

-0.0462
0.9598
0.9928

Table 7: Mid-plane Curvatures

Ply Number	1			2			3			4			5			6		
Angle [Degree]	0			90			0			0			90			0		
Location	Top	Middle	Bottom	Top	Middle	Bottom	Top	Middle	Bottom	Top	Middle	Bottom	Top	Middle	Bottom	Top	Middle	Bottom
Sigma_1 [Mpa]	58073973,8	56806011,92	55538050,05	-410377970	-378895624,8	-347413279,6	53002126,29	51734164,42	50466202,54	50466202,54	49198240,66	47930278,79	-221483898,8	-190001553,6	-158519208,4	45394355,03	44126393,16	42858431,28
Sigma_2 [Mpa]	-14521060,81	-13535356,15	-12549651,49	-1391328,036	-1189967,557	-988607,0778	-10578242,17	-9592537,51	-8606832,849	-8606832,849	-7621128,188	-6635423,527	-183165,1615	18195,3176	219555,7967	-4664014,205	-3678309,544	-2692604,883
Tau_12 [MPa]	-2256623,929	-1747833,857	-1239043,785	1239043,785	730253,7123	221463,6399	-221463,6399	287326,4325	796116,5049	796116,5049	1304906,577	1813696,65	-1813696,65	-2322486,722	-2831276,794	2831276,794	3340066,867	3848856,939
Epsilon_1	0,000235512	0,000229732	0,000223952	-0,001565001	-0,001445031	-0,001325062	0,000212392	0,000206612	0,000200832	0,000200832	0,000195052	0,000189272	-0,000845184	-0,000725214	-0,000605245	0,000177711	0,000171931	0,000166151
Epsilon_2	-0,001804939	-0,00168497	-0,001565001	0,000223952	0,000218172	0,000212392	-0,001325062	-0,001205092	-0,001085123	-0,001085123	-0,000965153	-0,000845184	0,000189272	0,000183491	0,000177711	-0,000605245	-0,000485275	-0,000365306
Epsilon_12	-0,000550396	-0,000426301	-0,000302206	0,000302206	0,000178111	5,40E-05	-5,40E-05	7,01E-05	0,000194175	0,000194175	0,00031827	0,000442365	-0,000442365	-0,00056646	-0,000690555	0,000690555	0,00081465	0,000938746

Table 8: Local Stresses and Strain

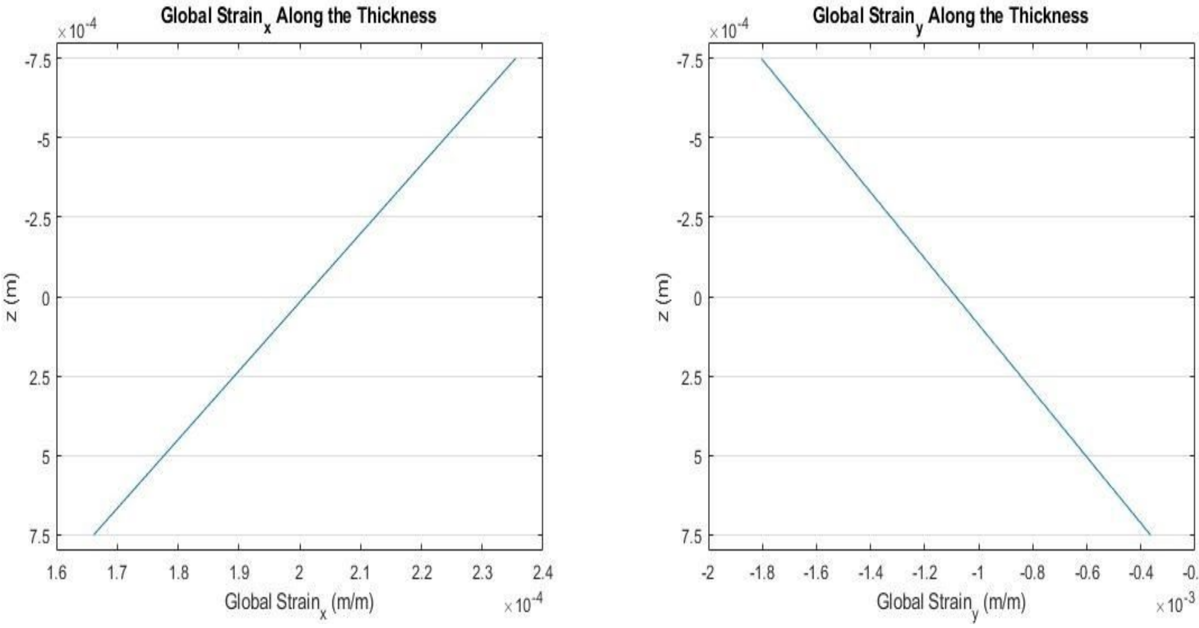

Figure 4: Global Strain_x and Global Strain_y Along The Thickness

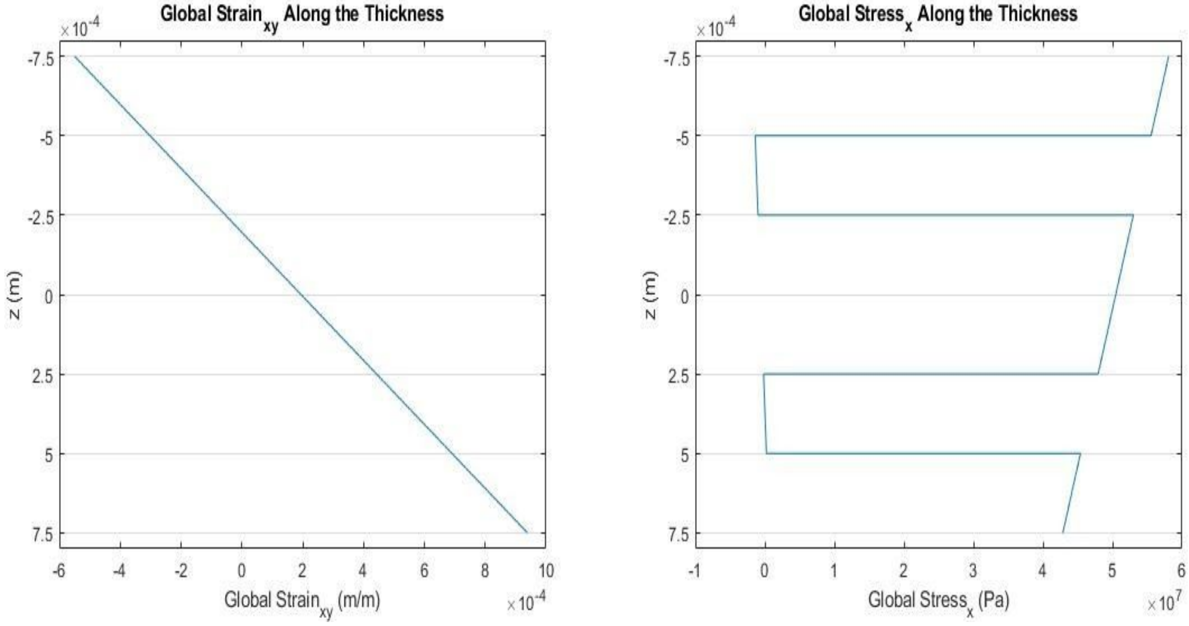

Figure 5: Global Strain_xy and Global Stress_x Along The Thickness

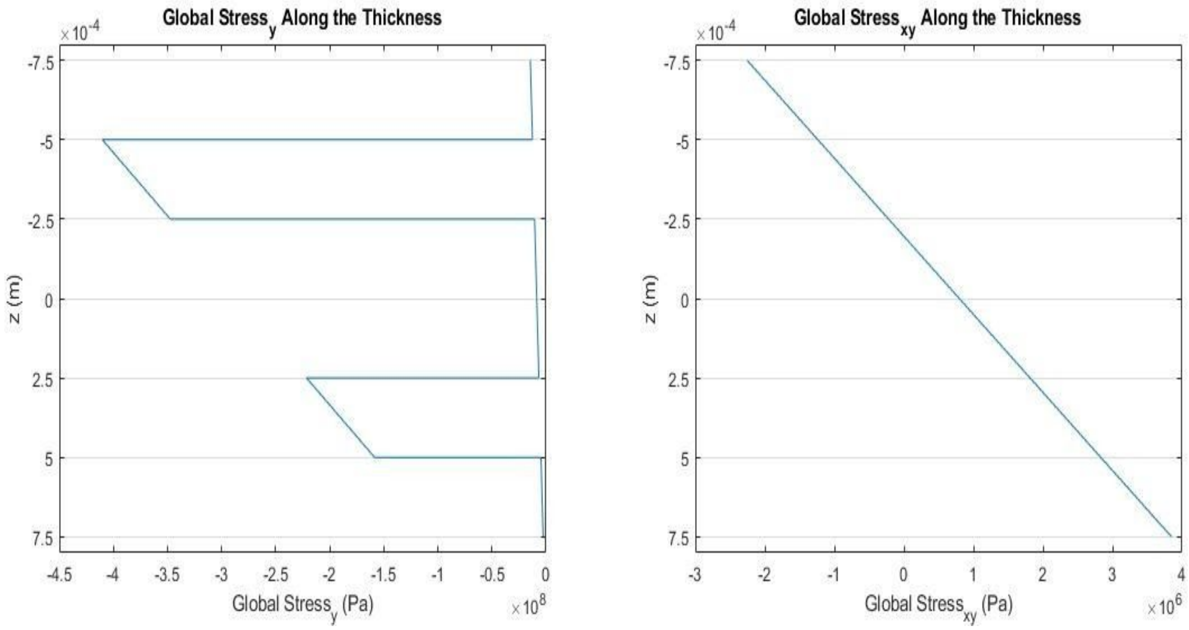

Figure 6: Global Stress_y and Global Stress_xy Along The Thickness

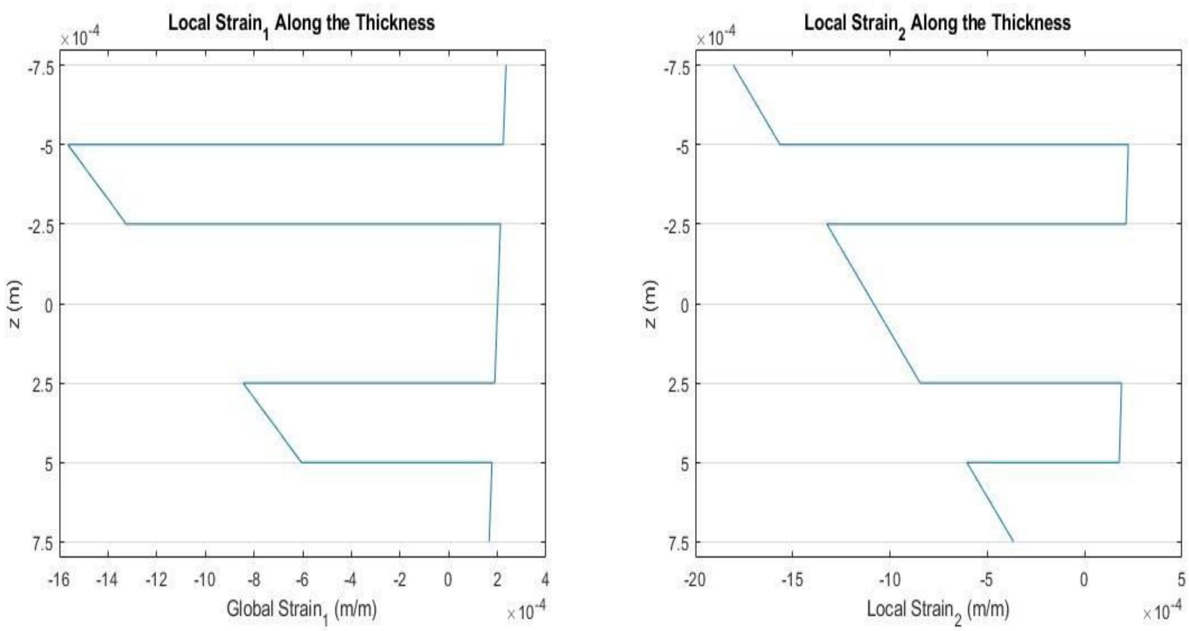

Figure 7: Local Strain_1 and Local Strain_2 Along The Thickness

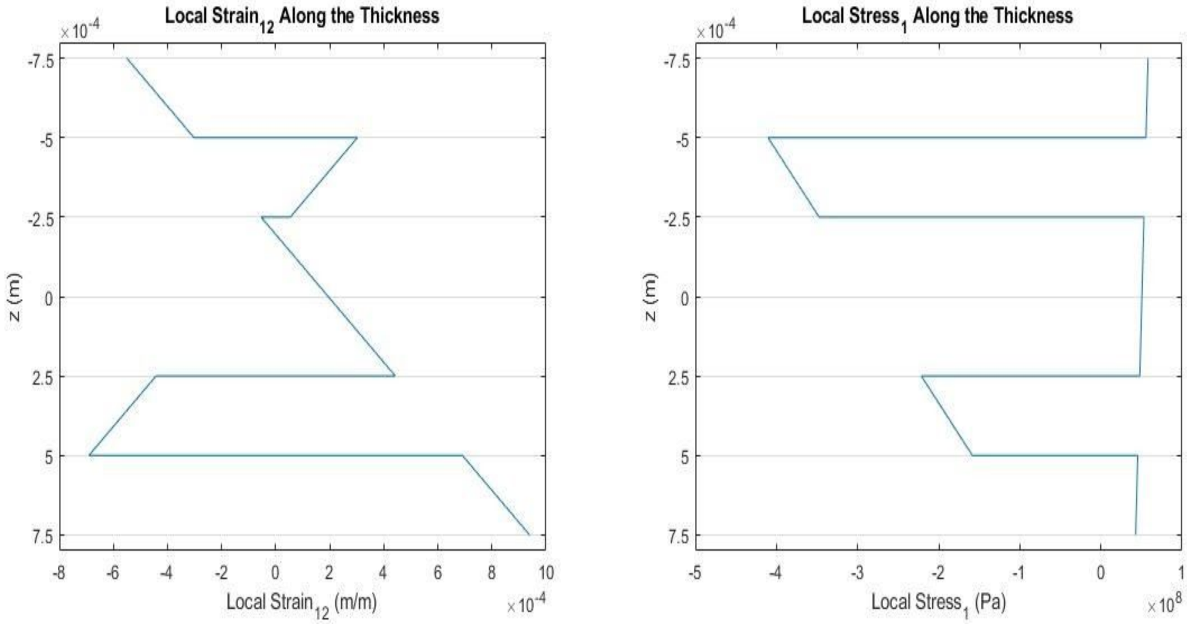

Figure 8: Local Strain_12 and Local Stress_1 Along The Thickness

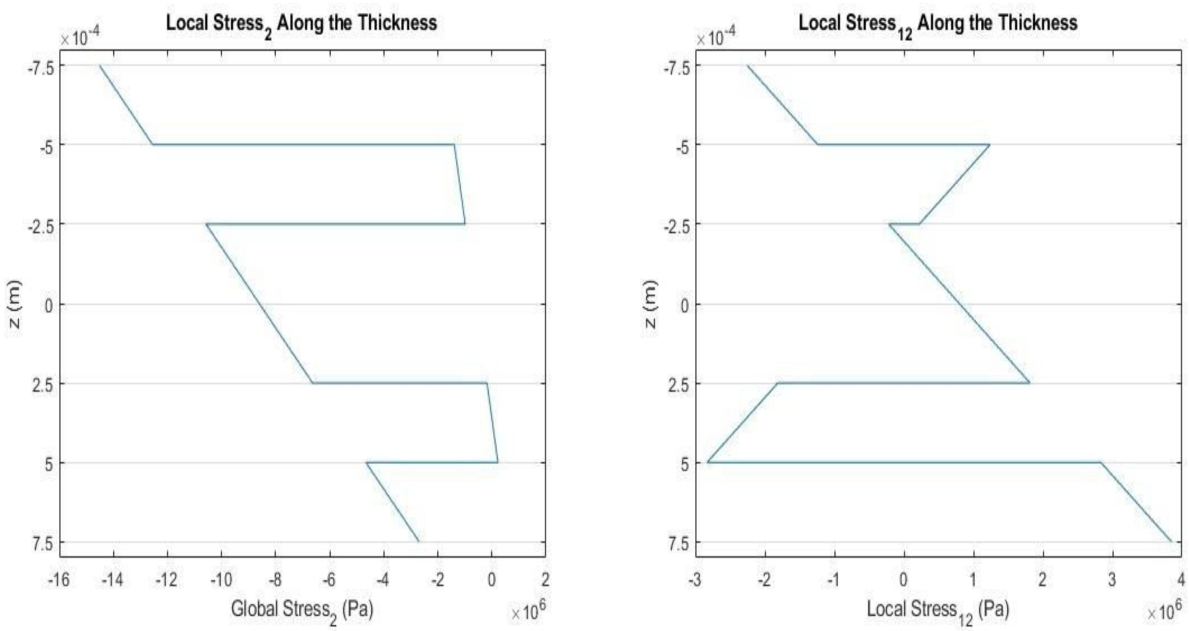

Figure 9: Local Stress_2 and Local Stress_12 Along The Thickness

$$\frac{1}{m_x} = -\frac{1}{\bar{S}_{16}E_1},$$

$$\frac{1}{m_y} = -\frac{1}{\bar{S}_{26}E_1}, \text{ and}$$

$$\nu_{xy} \equiv -\frac{\varepsilon_y}{\varepsilon_x} = -\frac{\bar{S}_{12}}{\bar{S}_{11}}. \quad \frac{1}{m_x} \equiv -\frac{\sigma_x}{\gamma_{xy}E_1} = -\frac{1}{\bar{S}_{16}E_1}. \qquad G_{xy} = \frac{1}{\bar{S}_{66}}.$$

$$\begin{bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \end{bmatrix} = \begin{bmatrix} \frac{1}{E_x} & -\frac{\nu_{xy}}{E_x} & -\frac{m_x}{E_1} \\ -\frac{\nu_{xy}}{E_x} & \frac{1}{E_y} & -\frac{m_y}{E_1} \\ -\frac{m_x}{E_1} & -\frac{m_y}{E_1} & \frac{1}{G_{xy}} \end{bmatrix} \begin{bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{bmatrix}.$$

Figure 10: In-plane and Flexural Engineering Constants of The Laminate Formulations

Longitudinal Modulus [Pa]	Transverse Modulus [Pa]	Shear Modulus [Pa]	Poisson's Ratio (v_xy)	Poisson's Ratio (v_yx)
1.7673*10^11	3.1068*10^10	3.4333*10^9	0.0648	0.0114

Table 9: In-plane Engineering Constants of The Laminate

Longitudinal Modulus [Pa]	Transverse Modulus [Pa]	Shear Modulus [Pa]	Poisson's Ratio (v_xyf)	Poisson's Ratio (v_yxf)
1.9574*10^11	2.6009*10^10	3.5815*10^9	0.0779	0.0104

Table 10: Flexural Engineering Constants of The Laminate

As a result of the calculations, the mass of the part has reached 0.3808 kg.

Volume calculated as 0.00024 m^3. So, density of laminate equals to 1 586 kg/m^3.

Specific Longitudinal Modulus [(Pa*m^3)/kg]	Specific Transverse Modulus [(Pa*m^3)/kg]	Specific Shear Modulus [(Pa*m^3)/kg]
1.1139*10^8	1.9581*10^7	2.1639*10^6

Table 11: Specific In-plane Axial Modulus of The Laminate

References

[1] Autar, K. Kaw, *Mechanics of Composite Materials*, CRC Press, 1997.

[2] Ronald F. Gibson, Principles of Composite Material Mechanics, McGraw-Hill, Inc., 1994.

[3] Cebeci, H., *Composite Materials* Lecture Notes, Istanbul Technical University, 2020.

Appendices

Appendix A: MATLAB Code 1

```
% Composite Materials Project

clc
clear
clear all
format long

% Given Parameters

a = 0.4;                          % An Edge-Length of The Square Plate
[m]
NM = [50e3;-50e3;1e3;-2;7;1]; % Load (N_x;N_y;N_xy;M_x;M_y;M_xy)
[N/m;(N*m)m]
plyno = 6;                        % Ply Number of The Symmetric
Laminate
t = 0.25e-3;                      % Ply Thickness [m]
ns = 2;                           % Factor of Safety

% The stresses in individual plies must be lower than the ply
strength given in table.
% Magnitudes of global mid-plane strains must be lower than 5*10^-3
m/m.
% Magnitudes of global mid-plane curvatures must be lower than 4
m^-1.

% Chooseen Materials and Ply Orientations for A Symmetric Stacking
Sequence for 6 Plies are [0*GY-70Epoxy / 90*KevlarEpoxy /
0*GY-70Epoxy]

% Properties of Choosen Materials

E_1   = [262,76,262,262,76,262]*10^9;      % Longitudinal Elastic
Modulus [Pa]
E_2   = [8.3,5.5,8.3,8.3,5.5,8.3]*10^9;    % Transverse Elastic
Modulus [Pa]
G_12   = [4.1,2.1,4.1,4.1,2.1,4.1]*10^9;  % Shear Modulus [Pa]
v_12   = [0.25,0.34,0.25,0.25,0.34,0.25]; % Major Poisson's Ratio

Strength_Ultimate_11 = [586,1379,586,586,1379,586]*10^6; % Ultimate
Longitudinal Strength [Pa]
```

```
Strength_Ultimate_22 = [41,28,41,41,28,41]*10^6;          % Ultimate
Transverse Strength [Pa]
Strength_Ultimate_12 = [97,60,97,97,60,97]*10^6;          % Ultimate
In-plane Shear Strength [Pa]
Density = [1690,1380,1690,1690,1380,1690];                %
Densities of Materials [kg/m^3]
angle = [0,90,0,0,90,0];                                  % Choosen
Ply Orientations for A Symmetric Stacking Sequence for 6 Plies

% Initial Conditions

Mass = 0;
no_fail = 0;
S = zeros(3,3); % Reducing Compliance Matrix
R = zeros(3,3); % Reuter Matrix
R(1,1) = 1;
R(2,2) = 1;
R(3,3) = 2;
A = zeros(3); % A Matrix
B = zeros(3); % B Matrix
D = zeros(3); % D Matrix

h = plyno*t;             % Total Thickness of Laminate [m]
h_coordinate(1) = -h/2; % [m]
Volume = plyno*t*a^2;   % Total Volume [m^3]

    for n = 1:plyno

        h_coordinate(n+1) = h_coordinate(n)+t; % Locations of the
Ply Surfaces [m]

        % Compliance Matrix

        S(1,1) = 1/E_1(n);
        S(2,2) = 1/E_2(n);
        S(3,3) = 1/G_12(n);
        S(1,2) = -v_12(n)/E_1(n);
        S(2,1) = S(1,2);

        v_21(n) = v_12(n)/E_1(n)*E_2(n); % Minor Poisson's Ratio

        Q = inv(S); % Reduced Stifness Matrix

        c(n) = cosd(angle(n));
        s(n) = sind(angle(n));
        T=[c(n)^2 s(n)^2 2*s(n)*c(n); s(n)^2 c(n)^2 -2*s(n)*c(n);
-s(n)*c(n) s(n)*c(n) c(n)^2-s(n)^2]; % Transfer Matrix
```

```
        Qbar = inv(T)*Q*R*T*inv(R); % Transfer Reduced Stiffnes
Matrix

        A = A+Qbar*(h_coordinate(n+1)-h_coordinate(n));           %
A Matrix
        B = B+1/2*(Qbar*(h_coordinate(n+1)^2-h_coordinate(n)^2)); %
B Matrix
        D = D+1/3*(Qbar*(h_coordinate(n+1)^3-h_coordinate(n)^3)); %
D Matrix

        % ABBD Matrix

        ABBD(1:3,1:3) = A;   % Extansional Stiffness Matrix [Pa*m]
        ABBD(1:3,4:6) = B;   % Coupling Stiffness Matrix [Pa*m^2]
        ABBD(4:6,1:3) = B;
        ABBD(4:6,4:6) = D;   % Bending Stiffness Matrix [Pa*m^3]

        e0 = ABBD\NM;                    %
(Epsilon*0_x;Epsilon*0_x;Gamma*0_xy ; K_x;K_y;K_xy) [m/m;1/m]
        Strain_midplane = e0(1:3,1);     % Midplane Strain [m/m]
        Curvature_midplane = e0(4:6,1); % Midplane Curvatures [1/m]

        Mass = Mass+a^2*t*(Density(n));  % Mass of Laminate [kg]

    end

Density_laminate = Mass/Volume; % Total Density [kg/m^3]

Astar = inv(A); % Extansional Compliance Matrix [1/(Pa*m)]
Dstar = inv(D); % Inverse of the Bending Stiffness Matrix
[1/(Pa*m^3)]

E_x = 1/(h*Astar(1,1));          % Effective In-Plane Longitudinal
Modulus [Pa]
E_y = 1/(h*Astar(2,2));          % Effective In-Plane Tranverse
Modulus [Pa]
G_xy = 1/(h*Astar(3,3));         % Effective In-Plane Shear Modulus
[Pa]
v_xy = -(Astar(1,2)/Astar(1,1)); % Effective In-Plane Poisson's
Ratio (v_xy)
v_yx = -(Astar(1,2)/Astar(2,2)); % Effective In-Plane Poisson's
Ratio (v_yx)

Constant_Inplane = [E_x,E_y,G_xy,v_xy,v_yx];
% In-Plane Engineering Constant of Laminate
```

```
Specific_Modulus_Inplane =
Constant_Inplane(1:3)*(1/Density_laminate); % Specific In-Plane
Modulus [(Pa*kg)/m^3]

E_xf = 12/(h^3*Dstar(1,1));       % Effective Flexural Longitudinal
Modulus [Pa]
E_yf = 12/(h^3*Dstar(2,2));       % Effective Flexural Transverse
Modulus [Pa]
G_xyf = 12/(h^3*Dstar(3,3));      % Effective Flexural Shear
Modulus [Pa]
v_xyf = -(Dstar(1,2)/Dstar(1,1)); % Effective Flexural Poisson's
Ratio (v_xyf)
v_yxf = -(Dstar(1,2)/Dstar(2,2)); % Effective Flexural Poisson's
Ratio (v_yxf)

Constant_Flexural = [E_xf,E_yf,G_xyf,v_xyf,v_yxf]; % Flexural
Engineering Constant of Laminate

    for n = 1:plyno

        z = -plyno*t/2:t/2:plyno*t/2;

        Q = inv(S);
        c(n) = cosd(angle(n));
        s(n) = sind(angle(n));
        T = [c(n)^2 s(n)^2 2*s(n)*c(n); s(n)^2 c(n)^2 -2*s(n)*c(n);
-s(n)*c(n) s(n)*c(n) c(n)^2-s(n)^2]; % Transfer Matrix
        Qbar = inv(T)*Q*R*T*inv(R);

        Strain_global(1:3,3*n-2) = Strain_midplane + z (2*n-1) *
Curvature_midplane; % Global Strain at Top [m/m]
        Strain_global(1:3,3*n-1) = Strain_midplane + z (2*n) *
Curvature_midplane;   % Global Strain at Middle [m/m]
        Strain_global(1:3,3*n) = Strain_midplane + z (2*n+1) *
Curvature_midplane;   % Global Strain at Bottom [m/m]

        SG = Strain_global;
        SG(3,:) = Strain_global(3,:)/2;
        SG1(:,3*n-2:3*n) = T*SG(:,3*n-2:3*n);
        Strain_local = SG1;
        Strain_local (3,:) = 2*SG1(3,:); % Local Strains [m/m]

        Stress_global(:,3*n-2:3*n) = Qbar*Strain_global
(:,3*n-2:3*n); % Global Stresses [Pa]

        Stress_local(:,3*n-2:3*n) = T*Stress_global(:,3*n-2:3*n); %
Local Stresses [Pa]
```

```
    end

Stress_global_x = Stress_global(1,:);  % Global Stresses at X
Direction [N]
Stress_global_y = Stress_global(2,:);  % Global Stresses at Y
Direction [N]
Stress_global_xy = Stress_global(3,:); % Global Stresses at Z
Direction [N]

Strain_global_x = Strain_global(1,:);  % Global Strains at X
Direction [m/m]
Strain_global_y = Strain_global(2,:);  % Global Strains at Y
Direction [m/m]
Strain_global_xy = Strain_global(3,:); % Global Strains at Z
Direction [m/m]

Stress_local_1 = Stress_local(1,:);  % Local Stresses at Direction
1 [N]
Stress_local_2 = Stress_local(2,:);  % Local Stresses at Direction
2 [N]
Stress_local_12 = Stress_local(3,:); % Local Stresses at Direction
3 [N]

Strain_local_1 = Strain_local(1,:);  % Local Strains at Direction 1
[m/m]
Strain_local_2 = Strain_local(2,:);  % Local Strains at Direction 2
[m/m]
Strain_local_12 = Strain_local(3,:); % Local Strains at Direction 3
[m/m]

% Failure Test with Safety Factor = 2

    for n = 1:plyno

        % Tsai-Hill Failure Theory

        S1 = max(abs(Stress_local_1(1,3*n-2:3*n))); % Max Stress at
Direction 1 for Each Individual Ply [Pa]
        X = Strength_Ultimate_11(n)/ns;             % Ultimate
Longitudinal Strength with Factor of Safety [Pa]

        S2 = max(abs(Stress_local_2(1,3*n-2:3*n))); % Max Stress at
Direction 2 for Each Individual Ply [Pa]
        Y = Strength_Ultimate_22(n)/ns;             % Ultimate
Transverse Strength with Factor of Safety [Pa]
```

```
        T12 = max(abs(Stress_local_12(1,3*n-2:3*n))); % Max Stress
at Direction 12 for Each Individual Ply [Pa]
        Z = Strength_Ultimate_12(n)/ns;                % Ultimate
In-plane Shear Strength with Factor of Safety [Pa]

        tsai = (S1/X)^2-((S1*S2)/(X^2))+(S2/Y)^2+(T12/Z)^2; % Must
Be Lower Than 1 According to Tsai-Hill Failure Theory

            if tsai < 1 % Equation of Tsai-Hill Failure Theory

                no_fail = no_fail+1;

                % If value of nonfail is equal to 6, all plies in
laminate pass the stress test.

            end

    end

% When Top, Middle and Bottom Locations of The Surfaces of Plies
are Considered Together

Th =
[z(1),z(2),z(3),z(3),z(4),z(5),z(5),z(6),z(7),z(7),z(8),z(9),z(9),z
(10),z(11),z(11),z(12),z(13)];

    % Plotting

    % Graphics of The Variation of Global Stresses Along The
Thickness

    figure(1);
    plot(Stress_global_x,Th);
    title('Global Stress_x Along The Thickness')
    xlabel('Global Stress_x [Pa]');
    ylabel('z [m]');
    grid on
    yticks (h_coordinate);
    ax = gca;
    ax.XGrid = 'off';
    ax.YDir = 'reverse';

    figure(2);
    plot(Stress_global_y,Th);
    title('Global Stress_y Along The Thickness')
    xlabel('Global Stress_y [Pa]');
    ylabel('z [m]');
```

```
    grid on
    yticks (h_coordinate);
    ax = gca;
    ax.XGrid = 'off';
    ax.YDir = 'reverse';

    figure(3);
    plot(Stress_global_xy,Th);
    title('Global Stress_x_y Along the Thickness')
    xlabel('Global Stress_x_y [Pa]');
    ylabel('z [m]');
    grid on
    yticks (h_coordinate);
    ax = gca;
    ax.XGrid = 'off';
    ax.YDir = 'reverse';

    % Graphics of The Variation of Global Strains Along The
Thickness

    figure(4);
    plot(Strain_global_x,Th);
    title('Global Strain_x Along The Thickness')
    xlabel('Global Strain_x [m/m]');
    ylabel('z [m]');
    grid on
    yticks (h_coordinate);
    ax = gca;
    ax.XGrid = 'off';
    ax.YDir = 'reverse';

    figure(5);
    plot(Strain_global_y,Th);
    title('Global Strain_y Along The Thickness')
    xlabel('Global Strain_y [m/m]');
    ylabel('z [m]');
    grid on
    ax = gca;
    ax.YDir = 'reverse';

    figure(6);
    plot(Strain_global_xy,Th);
    title('Global Strain_x_y Along The Thickness')
    xlabel('Global Strain_x_y [m/m]');
    ylabel('z [m]');
    grid on
    yticks (h_coordinate);
```

```
    ax = gca;
    ax.XGrid = 'off';
    ax.YDir = 'reverse';

    % Graphics of The Variation of Local Stresses Along The
Thickness

    figure(7);
    plot(Stress_local_1,Th);
    title('Local Stress_1 Along The Thickness')
    xlabel('Local Stress_1 [Pa]');
    ylabel('z [m]');
    grid on
    yticks (h_coordinate);
    ax = gca;
    ax.XGrid = 'off';
    ax.YDir = 'reverse';

    figure(8);
    plot(Stress_local_2,Th);
    title('Local Stress_2 Along The Thickness')
    xlabel('Global Stress_2 [Pa]');
    ylabel('z [m]');
    grid on
    yticks (h_coordinate);
    ax = gca;
    ax.XGrid = 'off';
    ax.YDir = 'reverse';

    figure(9);
    plot(Stress_local_12,Th);
    title('Local Stress_1_2 Along The Thickness')
    xlabel('Local Stress_1_2 [Pa]');
    ylabel('z [m]');
    grid on
    yticks (h_coordinate);
    ax=gca;
    ax.XGrid = 'off';
    ax.YDir = 'reverse';

    % Graphics of The Variation of Local Strains Along The
Thickness

    figure(10);
    plot(Strain_local_1,Th);
    title('Local Strain_1 Along The Thickness')
    xlabel('Global Strain_1 [m/m]');
```

```
    ylabel('z [m]');
    grid on
    yticks (h_coordinate);
    ax = gca;
    ax.XGrid = 'off';
    ax.YDir = 'reverse';

    figure(11);
    plot(Strain_local_2,Th);
    title('Local Strain_2 Along The Thickness')
    xlabel('Local Strain_2 [m/m]');
    ylabel('z [m]');
    grid on
    yticks (h_coordinate);
    ax = gca;
    ax.XGrid = 'off';
    ax.YDir = 'reverse';

    figure(12);
    plot(Strain_local_12,Th);
    title('Local Strain_1_2 Along The Thickness')
    xlabel('Local Strain_1_2 [m/m]');
    ylabel('z [m]');
    grid on
    yticks (h_coordinate);
    ax = gca;
    ax.XGrid = 'off';
    ax.YDir ='reverse';
```

Appendix B: MATLAB Code 2

```
% Composite Materials Project

% For Choose Optimized Materials and Ply Orientations for A
Symmetric Stacking Sequence for 6 Plies

clc
clear
clear all
format long

% Properties of Different Materials from Table 1: Material
Selection Table
```

```
% Boron/Epoxy, AS Carbon/Epoxy, T-300/Epoxy, HMS Carbon/Epoxy,
GY-70/Epoxy, Kevlar 49/Epoxy, E-Glass/Epoxy

% All Properties for All Materials from Table 1

E1 = [207,128,138,171,262,76,32]*10^9;             % Longitudinal
Elastic Modulus [Pa]
E2 = [19,9,10,13.8,8.3,5.5,4.8]*10^9;              % Transverse
Elastic Modulus [Pa]
G12 = [6.4,5.7,6.5,5.9,4.1,2.1,4.8]*10^9;          % Shear Modulus
[Ga]
v12 = [0.21,0.25,0.21,0.20,0.25,0.34,0.30];        % Major Poisson's
Ratio
Ro = [1990,1540,1550,1630,1690,1380,1800];         % Density
[kg/m^3]
Str_11 = [1585,1448,1448,827,586,1379,1103]*10^6; % Ultimate
Longitudinal Strength [Pa]
Str_22 = [63,62,45,86,41,28,97]*10^6;              % Ultimate
Transverse Strength [Pa]
Str_12 = [131,60,62,72,97,60,83]*10^6;             % Ultimate
In-Plane Shear Strength [Pa]

b = 0;          % Number of All Combinations
y = 0;          % Number of Stacking Sequence Combinations That
Provide The Desired Conditions
nth = 1;        % Number of Compared Combinations
Ex_spe_max = 0; % Highest Inplane Speci?c Longitudinal Modulus
[(Pa*kg)/m^3]
Mass_Min = 1;   % Minumum Weight [kg]

% To Create Different Symmetric Stacking Sequences with Different
Materials and Angles

for i=1:7                            % There are 7 Different
Materials for 1st and 6th Plies
    for j=1:7                        % There are 7 Different
Materials for 2nd and 5th Plies
        for k=1:7                    % There are 7 Different
Materials for 3rd and 4th Plies
            for ii=-90:15:90         % There are 13 Different Angle
for 1st and 6th Plies
                for jj=-90:15:90     % There are 13 Different Angle
for 2nd and 5th Plies
                    for kk=-90:15:90 % There are 13 Different Angle
for 3rd and 4th Plies

                        angle = [ii,jj,kk,kk,jj,ii];
```

```
                        E_1 =
[E1(i),E1(j),E1(k),E1(k),E1(j),E1(i)];
                        E_2 =
[E2(i),E2(j),E2(k),E2(k),E2(j),E2(i)];
                        v_12 =
[v12(i),v12(j),v12(k),v12(k),v12(j),v12(i)];
                        G_12 =
[G12(i),G12(j),G12(k),G12(k),G12(j),G12(i)];
                        Density =
[Ro(i),Ro(j),Ro(k),Ro(k),Ro(j),Ro(i)];
                        Strength_Ultimate_11 =
[Str_11(i),Str_11(j),Str_11(k),Str_11(k),Str_11(j),Str_11(i)];
                        Strength_Ultimate_22 =
[Str_22(i),Str_22(j),Str_22(k),Str_22(k),Str_22(j),Str_22(i)];
                        Strength_Ultimate_12 =
[Str_12(i),Str_12(j),Str_12(k),Str_12(k),Str_12(j),Str_12(i)];

                        b = b+1;  % All Combinations

                        % Initial Conditions and Given Parameters

                        a = 0.4;                      % An
Edge-Length of The Square Plate [m]
                        NM = [50e3;-50e3;1e3;-2;7;1]; % Load
(N_x;N_y;N_xy;M_x;M_y;M_xy) [N/m;(N*m)m]
                        plyno = 6;                    % Ply Number
of The Symmetric Laminate
                        t = 0.25e-3;                  % Ply
Thickness [m]
                        ns = 2;                       % Factor of
Safety
                        Mass = 0;                     % Mass [kg]
                        no_fail = 0;                  % Number of
Non-Fail Plies
                        S = zeros(3,3);               % Reducing
Compliance Matrix
                        R = zeros(3,3);               % Reuter
Matrix
                        R(1,1) = 1;
                        R(2,2) = 1;
                        R(3,3) = 2;
                        A = zeros(3); % A Matrix
                        B = zeros(3); % B Matrix
                        D = zeros(3); % D Matrix

                        h = plyno*t;             % Total Thickness
of Laminate [m]
```

```
                        h_coordinate(1) = -h/2; % Locations of The
Ply Surfaces [m]
                        Volume = plyno*t*a^2;    % Total Volume
[m^3]

                            for n = 1:plyno % Considering Ply by
Ply

                                h_coordinate(n+1) =
h_coordinate(n)+t; % Locations of The Ply Surfaces [m]

                                % Compliance Matrix

                                S(1,1) = 1/E_1(n);
                                S(2,2) = 1/E_2(n);
                                S(3,3) = 1/G_12(n);
                                S(1,2) = -v_12(n)/E_1(n);
                                S(2,1) = S(1,2);

                                v_21(n) = v_12(n)/E_1(n)*E_2(n); %
Minor Poisson's Ratio

                                Q = inv(S); % Reduced Stifness
Matrix

                                c(n) = cosd(angle(n));
                                s(n) = sind(angle(n));
                                T = [c(n)^2 s(n)^2 2*s(n)*c(n);
s(n)^2 c(n)^2 -2*s(n)*c(n); -s(n)*c(n) s(n)*c(n) c(n)^2-s(n)^2]; %
Transfer Matrix

                                Qbar = inv(T)*Q*R*T*inv(R); %
Transfer Reduced Stiffnes Matrix

                                A =
A+Qbar*(h_coordinate(n+1)-h_coordinate(n));           % A Matrix
                                B =
B+1/2*(Qbar*(h_coordinate(n+1)^2-h_coordinate(n)^2)); % B Matrix
                                D =
D+1/3*(Qbar*(h_coordinate(n+1)^3-h_coordinate(n)^3)); % D Matrix

                                % ABBD Matrix

                                ABBD(1:3,1:3) = A; % Extansional
Stiffness Matrix [Pa*m]
                                ABBD(1:3,4:6) = B; % Coupling
Stiffness Matrix [Pa*m^2]
```

```
                        ABBD(4:6,1:3) = B;
                        ABBD(4:6,4:6) = D; % Bending
Stiffness Matrix [Pa*m^3]

                        e0 = ABBD\NM;                   %
(Epsilon*0_x;Epsilon*0_x;Gamma*0_xy ; K_x;K_y;K_xy) [m/m; 1/m]
                        Strain_midplane = e0(1:3,1);    %
Midplane Strain [m/m]
                        Curvature_midplane = e0(4:6,1); %
Midplane Curvatures [1/m]

                        Mass = Mass+a^2*t*(Density(n)); %
Mass of Laminate [kg]

                end

                Density_laminate = Mass/Volume; % Total
Density [kg/m^3]

                Astar = inv(A); % Extansional Compliance
Matrix [1/(Pa*m)]
                Dstar = inv(D); % Inverse of The Bending
Stiffness Matrix [1/(Pa*m^3)]

                E_x = 1/(h*Astar(1,1));          %
Effective In-Plane Longitudinal Modulus [Pa]
                E_y = 1/(h*Astar(2,2));          %
Effective In-Plane Tranverse Modulus [Pa]
                G_xy = 1/(h*Astar(3,3));         %
Effective In-Plane Shear Modulus [Pa]
                v_xy = -(Astar(1,2)/Astar(1,1)); %
Effective In-Plane Poisson's Ratio (v_xy)
                v_yx = -(Astar(1,2)/Astar(2,2)); %
Effective In-Plane Poisson's Ratio (v_yx)

                Constant_Inplane =
[E_x,E_y,G_xy,v_xy,v_yx];                     % In-Plane
Engineering Constant of Laminate
                Specific_Modulus_Inplane =
Constant_Inplane(1:3)*(1/Density_laminate); % Specific In-Plane
Modulus [(Pa*kg)/m^3]

                E_xf = 12/(h^3*Dstar(1,1));       %
Effective Flexural Longitudinal Modulus [Pa]
                E_yf = 12/(h^3*Dstar(2,2));       %
Effective Flexural Transverse Modulus [Pa]
```

```
                        G_xyf = 12/(h^3*Dstar(3,3));      %
Effective Flexural Shear Modulus [Pa]
                        v_xyf = -(Dstar(1,2)/Dstar(1,1)); %
Effective Flexural Poisson's Ratio (v_xyf)
                        v_yxf = -(Dstar(1,2)/Dstar(2,2)); %
Effective Flexural Poisson's Ratio (v_yxf)

                        Constant_Flexural =
[E_xf,E_yf,G_xyf,v_xyf,v_yxf]; % Flexural Engineering Constant of
Laminate

                            for n = 1:plyno % Considering Ply by
Ply

                                z = -plyno*t/2:t/2:plyno*t/2;

                                Q = inv(S);
                                c(n) = cosd(angle(n));
                                s(n) = sind(angle(n));
                                T = [c(n)^2 s(n)^2 2*s(n)*c(n);
s(n)^2 c(n)^2 -2*s(n)*c(n); -s(n)*c(n) s(n)*c(n) c(n)^2-s(n)^2]; %
Transfer Matrix
                                Qbar = inv(T)*Q*R*T*inv(R);

                                Strain_global(1:3,3*n-2) =
Strain_midplane + z (2*n-1) * Curvature_midplane; % Global Strain
at Top [m/m]
                                Strain_global(1:3,3*n-1) =
Strain_midplane + z (2*n) * Curvature_midplane;   % Global Strain
at Middle [m/m]
                                Strain_global(1:3,3*n) =
Strain_midplane + z (2*n+1) * Curvature_midplane;   % Global Strain
at Bottom [m/m]

                                SG = Strain_global;
                                SG(3,:) = Strain_global(3,:)/2;
                                SG1(:,3*n-2:3*n) = T *
SG(:,3*n-2:3*n) ;
                                Strain_local = SG1;
                                Strain_local (3,:) = 2 * SG1(3,:);
% Local Strains [m/m]

                                Stress_global(:,3*n-2:3*n) = Qbar *
Strain_global (:,3*n-2:3*n); % Global Stresses [Pa]

                                Stress_local(:,3*n-2:3*n) = T *
Stress_global(:,3*n-2:3*n); % Local Stresses [Pa]
```

```
                        end

                    Stress_global_x = Stress_global(1,:);  %
Global Stresses at X Direction [N]
                    Stress_global_y = Stress_global(2,:);  %
Global Stresses at Y Direction [N]
                    Stress_global_xy = Stress_global(3,:); %
Global Stresses at Z Direction [N]

                    Strain_global_x = Strain_global(1,:);  %
Global Strains at X Direction [m/m]
                    Strain_global_y = Strain_global(2,:);  %
Global Strains at Y Direction [m/m]
                    Strain_global_xy = Strain_global(3,:); %
Global Strains at Z Direction [m/m]

                    Stress_local_1 = Stress_local(1,:);  %
Local Stresses at Direction 1 [N]
                    Stress_local_2 = Stress_local(2,:);  %
Local Stresses at Direction 2 [N]
                    Stress_local_12 = Stress_local(3,:); %
Local Stresses at Direction 3 [N]

                    Strain_local_1 = Strain_local(1,:);  %
Local Strains at Direction 1 [m/m]
                    Strain_local_2 = Strain_local(2,:);  %
Local Strains at Direction 2 [m/m]
                    Strain_local_12 = Strain_local(3,:); %
Local Strains at Direction 3 [m/m]

                    % Failure Test with Safety Factor = 2

                        for n = 1:plyno % Considering Ply by
Ply

                            % Tsai-Hill Failure Theory is
Applied

                            S1 =
max(abs(Stress_local_1(1,3*n-2:3*n))); % Max Stress at Direction 1
for Each Individual Ply [Pa]
                            X = Strength_Ultimate_11(n)/ns;
% Ultimate Longitudinal Strength with Factor of Safety [Pa]
```

```
                                S2 =
max(abs(Stress_local_2(1,3*n-2:3*n))); % Max Stress at Direction 2
for Each Individual Ply [Pa]
                                Y = Strength_Ultimate_22(n)/ns;
% Ultimate Transverse Strength with Factor of Safety [Pa]

                                T12 =
max(abs(Stress_local_12(1,3*n-2:3*n))); % Max Stress at Direction
12 for Each Individual Ply [Pa]
                                Z = Strength_Ultimate_12(n)/ns;
% Ultimate In-plane Shear Strength with Factor of Safety [Pa]

                                tsai =
(S1/X)^2-((S1*S2)/(X^2))+(S2/Y)^2+(T12/Z)^2; % Must Be Lower Than 1
According to Tsai-Hill Failure Theory

                                    if tsai < 1 % Equation of
Tsai-Hill Failure Theory

                                        no_fail = no_fail+1;

                                        % If value of nonfail is
equal to 6, all plies in laminate pass the stress test.

                                    end

                            end


                        % The stresses in individual plies must be
lower than the ply strength given in table.
                        % Magnitudes of global mid-plane strains
must be lower than 5*10^-3 m/m.
                        % Magnitudes of global mid-plane curvatures
must be lower than 4 m^-1.

                            if (no_fail) == 6 && (max(abs(
Strain_midplane))<5e-3/ns) && (max(abs(Curvature_midplane))<4/ns)

                                y = y+1; % Number of Stacking
Sequence Combinations That Provide The Desired Conditions

                                Inplane_Mod_spe =
Specific_Modulus_Inplane; % Specific In-Plane Modulus of
Combinations That Provide The Desired Conditions
                                mass = Mass;
% Mass of Stacking Sequence Combinations
```

```
                                % To Compare and Find Highest
Inplane Speci?c Modulus with Minumum Weight

                                    if  Inplane_Mod_spe(1) >=
Ex_spe_max && mass <= Mass_Min

                                        nth = nth+1; % For Keeping
the Properties of Compared Orientations

                                        Ex_spe_max =
Inplane_Mod_spe(1); % Maximum Specific Young's Modulus [Pa]
                                        Mass_Min = mass ;
% Minumum Mass [kg]

                                        % For Creating a Struct
Keeping Essential Properties and Values of Oriantations

                                        Combination(nth).angle =
[ii,jj,kk];                   % Angle Orientations of Laminates
                                        Combination(nth).material =
[i,j,k];                   % Material Orientation of Laminates
                                        Combination(nth).Mass =
mass;                          % Mass of Laminates

Combination(nth).Sp_Modulus_Inplane = Inplane_Mod_spe; % Specific
Inplane Modulus of Laminates
                                        Combination(nth).Strain_Mid
= e0;                      % Mid-plain Strain and Curvatures of
Laminates

Combination(nth).localStress = Stress_local;           % Local
Stresses of Laminates

Combination(nth).StiffnessMatrix = ABBD;               % Stiffness
Matrix of Laminates

                                    end

                            end
                    end
                end
            end
        end
    end
end
```