Single Frame Methods and Kalman Filter Based Nanosatellite Attitude Estimation

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I. Abstract

The aim of this paper investigates the proper attitude estimation algorithm for a nano-satellite. Three different methods, TRIAD, q method, TRIAD+EKF and q-method+EKF are chosen for comparison. For that, a Low Earth Orbit (LEO) satellite is examined in terms of attitude dynamics with chosen initial conditions. Sun sensor, magnetometer and nadir sensor are modelled. Then data from dual combinations of these sensors is used by algorithms. Also, all available sensor data used in q method. It is seen that, when sun and nadir sensor information is used in the q method+EKF algorithm, more accurate results are obtained than the others. Also, as expected, Davenport's q method gives attitude with higher accuracy when all sensor information is used.

II. Introduction and Objectives

Nano-satellites are small spacecraft with a mass of 1 to 10 kg and over 1800 nano-satellites have been launched by 2022, mostly in 5 years due to their low cost [1],[2]. Each nano-satellite contains various subsystems to fulfill its mission. One of the essential subsystems is attitude determination and control. But implementing a sensitive attitude determination system on a nano-satellite becomes a challenge due to their limited resources.

The attitude of a nano-satellite can be predicted by determination algorithms that use combination of data from onboard sensors. Sensors are divided into two groups, reference and inertial reference sensors [3]. Reference sensors provide information by measuring direction to a known celestial body. Magnetometers, sun sensors and horizon sensors are the best known reference sensors due to their cost, size and availability. Inertial sensor consists of gyroscope, accelerometer and compass, they measure angular rate in body reference frame.

Attitude determination methods can be examined in two main categories, depending on whether they take into account the dynamic behavior of the spacecraft. The first of these are called single-frame methods and are based on two or more vector observations provided by simultaneous measurements, such as Tri-Axial Attitude Determination (TRIAD) and q method. The second is filtering and uses a combination of the satellite's dynamic model and sensor measurements, by taking into account the sensor error information from the past measurements.

There are many studies in the literature focusing on single frame attitude determination algorithms [4], [5], [6].

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Guler, Conguroglu and Hajiyev (2017) implemented 4 different single-frame methods for attitude estimation using magnetometer and sun sensors data from TIMED satellite [7]. Results of this paper shows that, most accurate estimations are obtained with SVD method and TRIAD gives least accurate prediction when sensor data is available and measured vectors are not parallel. Also they found that while TRIAD and FOAM could not make any predictions in the range where the measured vectors become parallel to each other SVD and q method gave errors of up to 200 percent with higher oscillations in the same range.

Also, many paper about attitude filtering algorithms, mostly Extended Kalman Filter (EKF) can be found. EKF offers more accurate results than single frame methods. In [8] Schmidt et al. (2016), focused on attitude determination of UWE-2 satellite. An EKF algorithm using sun sensor and magnetometer is applied to satellite. Initial attitude estimation for EKF algorithms is obtained from TRIAD algorithm. It was seen that, mean error in roll angle exceed $\pm 20°$ while more accurate estimation is obtained in yaw angle with mean error of near $\pm 11°$.

Ghuffar (2010) investigated several attitude estimation methods for a chosen cubesat [9]. After he modeled Earth's magnetic field, sun vector, and orbital perturbation; he applied attitude estimation algorithms at Matlab by using sun sensors, magnetometer and 3-axis gyros' data. In his study he used Enhanced TRIAD, q method and Kalman filter based algorithms and then for error analysis he compared the results with STK (System Tool Kit). It is seen that TRIAD and q methods give similar results, both give relatively high accuracy in the noise-free condition, but the error increases catastrophically, even with a small noise at sensors. However, the Kalman filter gives most accurate results even with poor initial state estimation and is more robust to sensor bias and noise than others.

In another paper, Finance et al. (2021) compared TRIAD algorithm and Multiplicative Extended Kalman Filter (MEKF) based on real data of UVSQ-SAT [10].They observed that these two methods gives similar results at sunlight, uncertainty for both approximately $\pm 3°$. However, at eclipse higher attitude accuracy is achieved by MEKF with $\pm 10°$ uncertainty while TRIAD gives $\pm 14°$. They also claimed that more frequent measurements would increase the efficiency of the MEKF.

Although advantages of filtering, these algorithms have a higher computer load. Fortunately, for nano-satellites where computational effort is an important trade-off parameter, it can be integrated with single frame algorithms to make it more effective. The paper [11], presents an example of TRIAD integrated UKF algorithm. Coarse attitude information is obtained by TRIAD using sun sensor and magnetometer measurements. Then to improve attitude estimation, UKF is applied by taking into account error in TRIAD algorithm. With TRIAD+UKF algorithm in this study, all Euler angles are obtained with accuracy of less than 0.25 degree at daytime. And even during eclipse, the proposed algorithm gives an error of just over 5 degrees. It is also stated in this paper that the computational load has decreased by more than 30%. Another paper, Gokcay and Hajiyev (2021), covers the study of Kalman filter and TRIAD integration. It was learned from this study that the root-mean-square-error (RMSE) in the roll angle found when TRIAD method is used alone is reduced by over 27% when it is used with EKF. Reduction in error is over 44% when TRIAD algorithm is used with UKF [12]. The paper [13] examines the SVD integration with EKF. At this study 2 EKF algorithm are proposed. Both EKF algorithms use coarse attitude information and covariance from SVD. In the second algorithm, an additional adaptive scaling factor is added to noise covariance of the filter. Then the behaviors of each SVD aided algorithms at eclipse and process and measurement noise increment are examined. At eclipse time, SVD algorithm alone is insufficient. The results obtained in this study also show that SVD+EKF algorithms give more robust results, while SVD oscillates in cases where measurement noise is increased. But when the process noise is increased, first SVD+EKF gives catastrophic errors 50 times larger than the nominal case in roll angle. The second algorithm get more robust results against sensor bias.

At this study, single frame and Kalman filtering based attitude algorithms are examined. For this purpose firstly satellite's non-linear dynamics are simulated. Secondly, 3 conventional attitude sensors which are magnetometer, sun and nadir sensors are modelled assuming each sensor is calibrated. Then, TRIAD, Davenport's q method and EKF algorithms are written via Matlab. Instead of designing the traditional EKF, TRIAD and q-method algorithms are combined with the EKF algorithm separately and single frame methods are used as linearized measurement model of EKF. In these non-traditional EKF algorithms, covariances of TRIAD and q-method are used in EKF to detect errors in measurements. To test the accuracy of the algorithms, different dual sensor combinations are used in each algorithms. In addition, all sensor information is used in the q method to investigate the relationship between accuracy and the number of sensors.

III. Methods & Resources

In this study, different attitude determination methods are examined. For this purpose, firstly, the attitude dynamics of a hypothetical satellite are simulated by assigning appropriate initial conditions of quaternion, angular velocity and orbital parameters. Then orientation of satellite is found with information of attitude dynamics. After that, mathematical and measurements models of magnetometer, sun and horizon sensors are simulated. Proper noises for each sensor are included at measurement models. Obtained sensor information are used as input of attitude determination algorithms. Determination algorithms are written with help of proper equations. Thus, attitude of the chosen satellite is estimated by 4 different algorithms and results are compared.

The study summarized above was carried out in a computer environment without the need for an experimental setup. As computing device personal computer was used and all calculations are done via Matlab R2021a software. Licence for MatLab 2021a is provided by Istanbul Technical University. Matlab is a programming language and computing environment. It is developed by MathWorks company. The extensive libraries and products it offers for different purposes have led to its use in a wide range of engineering and science fields. Interaction between Matlab users is also allowed through the community section on MathWorks' website. It is widely preferred by engineering students for educational purposes.

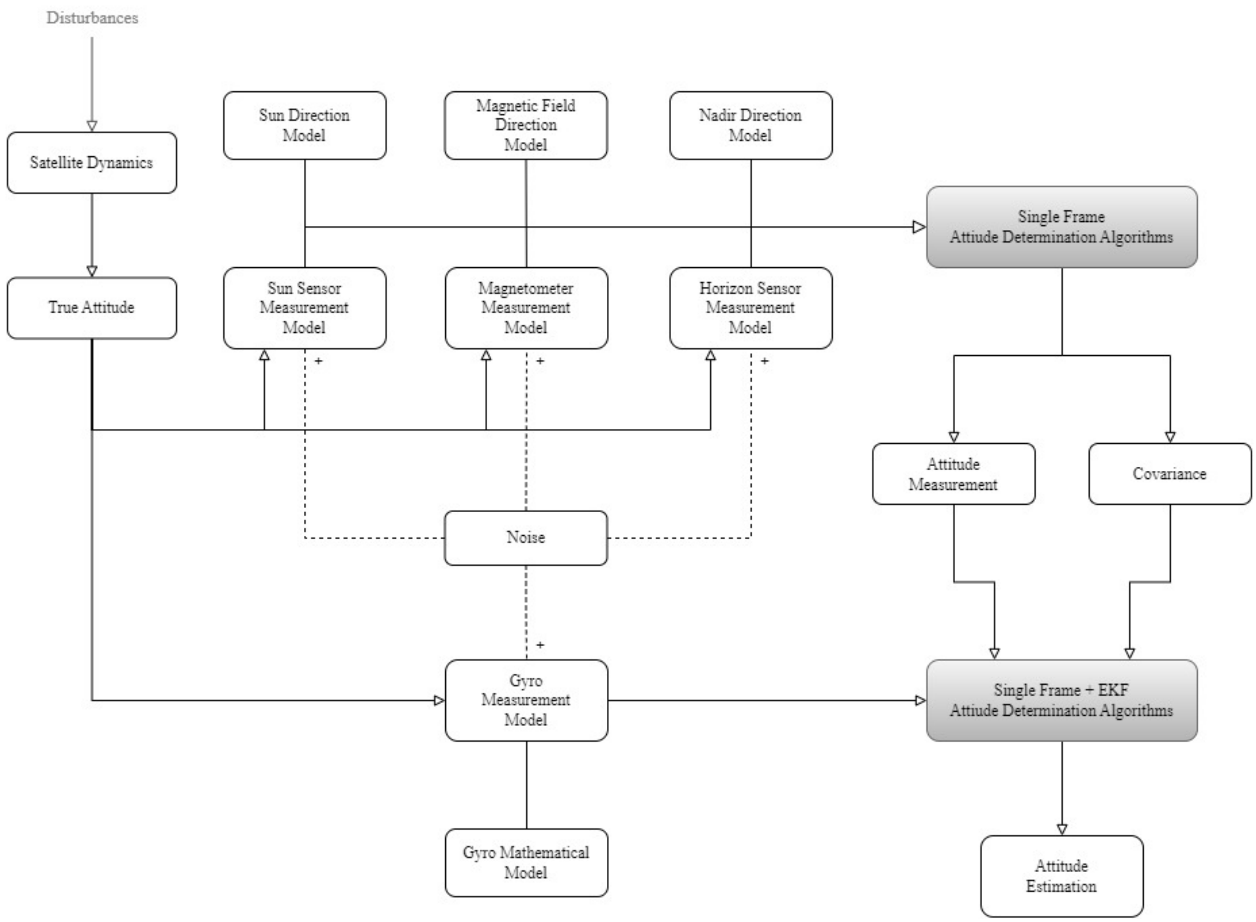

Fig. 1 Scheme of Methodology

Matlab is chosen in this study because of its simplicity, familiarity, and advantages for algebraic and numerical computing capability. The algorithms are written without using the built-in functions found in the Matlab Aerospace Toolbox library. Built-in functions are used only for algebraic operations and linearization process for EKF algorithm.

Each determination algorithms are written as MatLab function. Functions of q-method and TRIAD use measurements vector and variances of sensors as input and they give measured attitude in terms of quaternion and covariance of algorithm as output. So, EKF function uses these covariances and measured attitude as input and then it gives estimated states as output. To test computer load of these functions elapsed time from stopwatch of MatLab are computed. The elapsed time of any algorithm does not exceed 2.5 seconds. Algorithm for q-method has highest elapsed time because it contains eigenvalue computation and arrangement to rid of singularity. Also, transformations between Euler angles and quaternions are one of the time consuming processes. All calculations for one dual combination of sensors are done around 6 seconds. And, all calculations can be easily performed on any computer compatible with MatLab. Algorithms could be written differently to optimize elapsed time, but computer load is not main issue of this study.

The overall methodology of graduation project is shown at Fig 1. So far, the motion of the satellite along one full orbit was calculated by iterative approach with initial values of quaternion and angular rates. Orbital characteristic of the satellite was defined with assigned classical Keplerian orbital elements. A proper distribution for LEO was added. Then, with proper formulation as shown at Eq. (2) to Eq. (7), orientation of the satellite through one full orbit was found in terms of quaternion. For better visualization, obtained quaternion were converted to Euler angles.

Then by using proper mathematical models, Earth's magnetic field vector, sun heading vector and nadir vector in orbit reference frame were simulated. As mathematical models, Earth's Dipole Model, Sun Model and Earth's Oblateness Model were used. Also, measurements of magnetometer, sun sensor, horizon sensor and gyros were simulated with proper equations and addition of Gaussian white noise and ignoring bias terms. Formulation of these measurement and mathematical models are described in Sec. IV.A. On the other hand, attitude determination algorithms were written as functions by following the formulation given at Sec. IV.B to Sec. IV.D. Vector information were used as input of these algorithms and estimated attitude by all algorithms were obtained. Results from each methods were compared with initially found orientation of satellite and fault based on methods were investigated. Detailed process of project is shown in Appendix B. Also, all stages of this study are carried out within the framework of ethical and academic rules as stated in Appendix C.

IV. Attitude Determination Methods

The orientation of a spacecraft in space can be defined with an 3×3 orthogonal matrix. Each element of this matrix specifying the cosine of the angle between the body axis and reference axis, so in literature it is named as direction cosine matrix often, and it can be written as [14] [15]

$$A\boldsymbol{r} = \boldsymbol{b} \tag{1}$$

where A is 3×3 attitude matrix, $\boldsymbol{r}$ is 3×1 position matrix in reference frame and $\boldsymbol{b}$ is 3×1 position matrix in body frame. Basically, goal of attitude determination methods are obtained in this matrix. For this purpose, data from sensors are used. A sensor measures reference vectors in spacecraft frame and also with modelling, resultant vectors in inertial frame are calculated. Reference vectors represent unit vectors in a known direction. Sun heading vector, nadir vector and vector of Earth's magnetic field mostly used as reference. After sensors measures vectors, attitude can be estimated with algorithms. Multiple sensors are integrated into the satellite to improve accuracy, although some advanced algorithms can also work with a single type of sensor. Simpler algorithms, such as TRIAD, require more than one sensor information.

Attitude is usually parameterized with quaternion and Euler angles. Since calculations with quaternion are faster and singularities that may occur in the Euler angle formulation can be avoided, quaternion based parameterization are used in this study. Euler's angles are used only for visualization. Quaternion can be written as

$$\boldsymbol{q} = \boldsymbol{i}q_1 + \boldsymbol{j}q_2 + \boldsymbol{k}q_3 + q_4 \tag{2}$$

where $\boldsymbol{q}$ is unit quaternion, q_4 is scalar part of quaternion, $\{q_1,\ q_2,\ q_3\}$ are pats of Euler vector of quaternion. The kinematics equation of motion of the satellite using quaternion can be written as follows [14]

$$\boldsymbol{q}(t + \Delta t) = \left[\frac{1}{2}\Omega\Delta t\right]\boldsymbol{q}(t) \tag{3}$$

$$\Omega = \begin{bmatrix} 0 & \omega_z & -\omega_y & \omega_x \\ -\omega_z & 0 & \omega_x & \omega_y \\ \omega_y & -\omega_x & 0 & \omega_z \\ -\omega_x & -\omega_y & -\omega_z & 0 \end{bmatrix} \tag{4}$$

where t is time, Δt is sampling time, and $\{\omega_x,\ \omega_y,\ \omega_z\}$ are components of angular velocity in the body frame with respect to the reference frame. To estimate angular velocity, the dynamic equation of the satellite based on Euler's equation is written as

$$J\frac{d\boldsymbol{\omega}}{dt} = \boldsymbol{N} - \boldsymbol{\omega} \times (J\boldsymbol{\omega}) \tag{5}$$

From Eq. (5) components of angular velocity are

$$\omega_x(t + \Delta t) = \omega_x(t) + \frac{\Delta t}{J_x}\left(J_y - J_z\right)\omega_z(t)\omega_y(t) + \frac{\Delta t}{J_x}N_x \tag{6a}$$

$$\omega_y(t + \Delta t) = \omega_y(t) + \frac{\Delta t}{J_y}\left(J_z - J_x\right)\omega_x(t)\omega_z(t) + \frac{\Delta t}{J_y}N_y \tag{6b}$$

$$\omega_z(t + \Delta t) = \omega_z(t) + \frac{\Delta t}{J_z}\left(J_x - J_y\right)\omega_x(t)\omega_y(t) + \frac{\Delta t}{J_z}N_z \tag{6c}$$

where $\{J_x,\ J_y,\ J_z\}$ are components of moments of inertia and $\{N_x,\ N_y,\ N_z\}$ are components of N_T, the disturbance torque acting on the satellite. From reference to body frame attitude matrix, A, in terms of quaternion

$$A = \begin{bmatrix} q_1^2 - q_2^2 - q_3^2 + q_4^2 & 2\left(q_1q_2 + q_3q_4\right) & 2\left(q_1q_3 - q_2q_4\right) \\ 2\left(q_1q_2 - q_3q_4\right) & -q_1^2 + q_2^2 - q_3^2 + q_4^2 & 2\left(q_2q_3 + q_1q_4\right) \\ 2\left(q_1q_3 + q_2q_4\right) & 2\left(q_2q_3 - q_1q_4\right) & -q_1^2 - q_2^2 + q_3^2 + q_4^2 \end{bmatrix} \tag{7}$$

Also transformation to Euler angle can be made with [16],

$$\phi = atan2(A(1,2), A(1,1)) \tag{8a}$$

$$\theta = sin^{-1}(-A(3,1)) \tag{8b}$$

$$\psi = atan2(A(2,3), A(3,3)) \tag{8c}$$

where ϕ, θ and ψ are roll, pitch and yaw angles respectively. And $A(m,n)$ describes the element in the m. row and n. column of attitude matrix.

A. Mathematical & Measurements Models

This part of the article includes the mathematical and measurement models used in this graduation project. At least 2 measured vectors are required to determine the attitude of satellite. So, mathematical models for magnetometer, sun sensor, horizon sensor and gyro are described briefly.

1. Modeling of Earth's Magnetic Field Vector

Earth's dipole model can be used for magnetometer as mathematical model. Magnetic field tensor vector in orbit reference frame can be obtained with this model. Formulation is given at Eq. (9) from Ref. [17].

$$\boldsymbol{H_o} = \begin{bmatrix} \boldsymbol{H_{ox}} & \boldsymbol{H_{oy}} & \boldsymbol{H_{oz}} \end{bmatrix}^T \tag{9a}$$

$$\boldsymbol{H_{ox}}(t) = \frac{M_e}{r_0^3}\{cos(\omega_0 t)\left[cos(\varepsilon_{geo})sin(i) - sin(\varepsilon_{geo})cos(i)cos(\omega_e t)\right] - sin(\omega_0 t)sin(\varepsilon_{geo})sin(\omega_e t)\} \tag{9b}$$

$$\boldsymbol{H_{oy}}(t) = -\frac{M_e}{r_0^3}\left[cos(\varepsilon_{geo})cos(i) + sin(\varepsilon_{geo})sin(i)cos(\omega_e t)\right] \tag{9c}$$

$$\boldsymbol{H_{oz}}(t) = \frac{M_e}{r_0^3}\{sin(\omega_0 t)\left[cos(\varepsilon_{geo})sin(i) - sin(\varepsilon_{geo})cos(i)cos(\omega_e t)\right] - 2sin(\omega_0 t)sin(\varepsilon_{geo})sin(\omega_e t)\} \tag{9d}$$

$$\boldsymbol{\hat{H}_o}(t) = \boldsymbol{H_o}(t)/\|\boldsymbol{H_o}(t)\| \tag{9e}$$

where $\{\boldsymbol{H_{ox}}, \boldsymbol{H_{oy}}, \boldsymbol{H_{oz}}\}$ are components of $\boldsymbol{H_o}$, magnetic field vector in orbit frame with unit of $Tesla$, $\boldsymbol{\hat{H}_o}$ is magnetic field tensor vector in orbit reference, M_e is the magnetic dipole moment of Earth $(Wb.m)$, r_0 is distance between the center of mass of satellite and Earth (m), ω_0 is angular velocity of orbit with respect to the inertial frame (rad/s), ω_e is spin rate of Earth (rad/s), ε_{geo} is inclination of Geomagnetic axis relative to Earth's daily rotating axis (deg) and i is orbit inclination (deg). In addition, measurement model can be represented with Eq. (10)

$$\boldsymbol{H_b}(t) = A(t)\boldsymbol{H_o}(t) + \eta_m(t) + \boldsymbol{b}_m(t) \tag{10a}$$

$$\boldsymbol{\hat{H}_b}(t) = \boldsymbol{H_b}(t)/\|\boldsymbol{H_b}(t)\| \tag{10b}$$

where $\boldsymbol{\hat{H}_b}$ unit vector of measured magnetic field vector in body frame, $\boldsymbol{H_b}$, with unit of $Tesla$. $\boldsymbol{b}_m$ is $3x1$ unknown bias term and η_m is $3x1$ zero mean Gaussian white noise of magnetometer measurement with the characteristic of,

$$E[\eta_{mk}\,\eta_{mj}^T] = \mathbf{1}\sigma_m^2\delta_{kj} \tag{11}$$

where $\mathbf{1}$ is $3x3$ identity matrix, σ_m is the standard deviation of each magnetometer error and δ_{kj} is the Kronecker symbol.

2. Modeling of Sun Direction Vector

Unit reference sun vector in Earth Centered Inertial (ECI) Frame can be obtained with defined mathematical model in Eq. (12) from Ref. [17].

$$\boldsymbol{\hat{S}_{ECI}}(t) = [\cos\lambda_e \quad \sin\lambda_e\cos\varepsilon_{ln} \quad \sin\lambda_e\sin\varepsilon_{ln}]^T \tag{12}$$

where $\boldsymbol{\hat{S}_{ECI}}$ is unit sun vector in the ECI frame, λ_e is ecliptic longitude of the sun (deg) and ε_{ln} is linear model of the ecliptic of the sun in unit of deg. Then, $\boldsymbol{\hat{S}_o}$ unit sun vector in orbit frame can be obtained via transformation matrix with Eq. (13).

$$\boldsymbol{\hat{S}_o}(t) = T_{ECI}^{Orbit}(t)\boldsymbol{\hat{S}_{ECI}}(t) \tag{13}$$

And finally, sun direction in body frame can be found with Eq. (14).

$$\boldsymbol{\hat{S}_b}(t) = A\boldsymbol{S_o}(t) + \eta_s(t) + b_s(t) \tag{14a}$$

$$\boldsymbol{\hat{S}_b}(t) = \boldsymbol{\hat{S}_b}(t)/\|\boldsymbol{\hat{S}_b}(t)\| \tag{14b}$$

where $\boldsymbol{\hat{S}_b}$ tensor sun vector in body frame. $\boldsymbol{b}_s$ is $3x1$ unknown bias term of sun sensor and η_s is $3x1$ zero mean Gaussian white noise of sun sensor measurement with the characteristic of,

$$E[\eta_{sk}\,\eta_{sj}^T] = \mathbf{1}\sigma_s^2\delta_{kj} \tag{15}$$

where σ_s is the standard deviation of each sun sensor error.

3. Modeling of Earth Direction Vector

Unit reference horizon vector in orbit and body frames, $\hat{\boldsymbol{N}}_{\boldsymbol{o}}$ and $\hat{\boldsymbol{N}}_{\boldsymbol{b}}$, can be derived with Earth's oblateness and measurement model as shown Eq. (16) and Eq. (17) from [14] and [18]

$$\boldsymbol{N}_{\boldsymbol{o}}(t) = [\cos\Lambda\cos\Psi \quad \cos\Lambda\sin\Psi \quad \sin\Lambda]^T \tag{16a}$$

$$\hat{\boldsymbol{N}}_{\boldsymbol{o}}(t) = \boldsymbol{N}_{\boldsymbol{o}}(t)/\|\boldsymbol{N}_{\boldsymbol{o}}(t)\| \tag{16b}$$

where Λ is geocentric latitude and Ψ is the azimuth of satellite.

$$\boldsymbol{N}_{\boldsymbol{b}} = A\boldsymbol{N}_{\boldsymbol{o}} + \eta_n(t) + b_n(t) \tag{17a}$$

$$\hat{\boldsymbol{N}}_{\boldsymbol{b}} = \boldsymbol{N}_{\boldsymbol{b}}/\|\boldsymbol{N}_{\boldsymbol{b}}\| \tag{17b}$$

where $\hat{\boldsymbol{N}}_{\boldsymbol{b}}$ is tensor Horizon vector in body reference frame. $\boldsymbol{b}_n$ is $3x1$ unknown bias term of nadir sensor and η_n is $3x1$ zero mean Gaussian white noise of nadir sensor measurement with the characteristic of,

$$E[\eta_{nk}\,\eta_{nj}^T] = \mathbf{1}\sigma_n^2\delta_{kj} \tag{18}$$

where σ_n is the standard deviation of each nadir sensor error.

4. Modelling Gyros

Mathematical model of gyros model can be given with [17],

$$\hat{\omega}_b(t) = \omega_b(t) + \eta_g(t) + \boldsymbol{b}_{\boldsymbol{g}}(\boldsymbol{t}) \tag{19}$$

where $\boldsymbol{\omega}_{\boldsymbol{b}}$ is angular velocity, which can be found by Eq. (6). $\boldsymbol{b}_{\boldsymbol{g}}$ is the gyro bias vector and η_g is zero mean Gaussian white noise with,

$$E[\eta_{gk}\,\eta_{gj}^T] = \mathbf{1}\sigma_g^2\delta_{kj} \tag{20}$$

where σ_g is the standard deviation of each gyros error.

B. TRIAD Method

TRIAD method, introduced by Harold Black, based on two vector observation and algebra. Considering two linearly independent unit vectors both known in spacecraft body and inertial reference frame, the algorithm can be defined as follows [16],

$$A\boldsymbol{r}_i = \boldsymbol{b}_i, \qquad for \quad i = 1, 2 \tag{21}$$

Equation (21) is true only in the absence of measurement errors. Another orthogonal right-handed frame, Triad frame, can be constructed by considering two linearly independent unit vectors derived from sensor measurements and mathematical model. Triad frame in both the spacecraft body and the reference frame can be created as 1) the first component of triad is considered equal to the more accurate vector observation, 2) the second component is set as orthogonal to both reference directions, and 3) the third is chosen to complete orthogonal right-handed frame. After that attitude matrix can be obtained by simple transformation of triads. Algorithm by assuming first measurement is more accurate than second can be written as,

$$\boldsymbol{v}_1 = \boldsymbol{r}_1, \qquad \boldsymbol{v}_2 = \boldsymbol{r}_\times = \frac{\boldsymbol{r}_1 \times \boldsymbol{r}_2}{|\boldsymbol{r}_1 \times \boldsymbol{r}_2|}, \qquad \boldsymbol{v}_3 = \boldsymbol{r}_1 \times \boldsymbol{r}_\times \tag{22a}$$

$$\boldsymbol{w}_1 = \boldsymbol{b}_1, \qquad \boldsymbol{w}_2 = \boldsymbol{b}_\times = \frac{\boldsymbol{b}_1 \times \boldsymbol{b}_2}{|\boldsymbol{b}_1 \times \boldsymbol{b}_2|}, \qquad \boldsymbol{w}_3 = \boldsymbol{b}_1 \times \boldsymbol{b}_\times \tag{22b}$$

where $\{\boldsymbol{v}_1, \boldsymbol{v}_2, \boldsymbol{v}_3\}$ are a triad of vectors in reference body frame and $\{\boldsymbol{w}_1, \boldsymbol{w}_2, \boldsymbol{w}_3\}$ are in spacecraft body frame. Then attitude matrix can be written in terms of triads

$$\hat{A}_{TRIAD} = [\boldsymbol{w}_1 \; \boldsymbol{w}_2 \; \boldsymbol{w}_3][\boldsymbol{v}_1 \; \boldsymbol{v}_2 \; \boldsymbol{v}_3]^T = \sum_{i=1}^{3} \boldsymbol{w}_i \boldsymbol{v}_i^T \tag{23}$$

$$\hat{A}_{TRIAD} = \boldsymbol{b}_1 \boldsymbol{r}_1^T + (\boldsymbol{b}_1 \times \boldsymbol{b}_\times)(\boldsymbol{r}_1 \times \boldsymbol{r}_\times)^T + \boldsymbol{b}_\times \times \boldsymbol{r}_\times^T \tag{24}$$

Covariance of TRIAD algorithm in terms of Euler angles which is given at [19] is

$$P_{\theta\theta}^{TRIAD} = \sigma_1^2 1_{3x3} + \frac{1}{|b_1 \times b_2|} \left[(\sigma_2^2 - \sigma_1^2) b_1 b_1^T + \sigma_1^2 (b_1 . b_2)(b_1 b_2^T + b_2 b_1^T) \right] \tag{25}$$

where

$$\sigma_1^2 = \sigma_{b_1}^2 + \sigma_{r_1}^2 \qquad\qquad \sigma_2^2 = \sigma_{b_2}^2 + \sigma_{r_2}^2 \tag{26}$$

As seen Eq. 22, TRIAD algorithm is inefficient when the angle between reference or observed vectors approaches 0° or 180°. Also, although it is simple and fast, it is difficult to implement with position sensors such as star trackers, it cannot be used in cases where there is no sensor data, such as eclipse, or where measurement vectors are parallel to each other. Therefore TRIAD algorithm is not generally used as main attitude determination algorithm nowadays.

C. Davenport's q method

The Davenport's q method first used on HEAO-1, an X-ray astronomy satellite of NASA [20]. Algorithm based on solution of the well-known Wahba's problem with a unit quaternion. The Wahba's problem is an optimization problem based on least-square in attitude determination with vector. To find the optimal attitude matrix, the loss function, which is the sum of the squared errors for each vector measurements, difference between the measured and transformed vectors from reference frame, must be minimized. Wahba problem is given with [21],

$$\text{Minimize} \qquad L(A) = \frac{1}{2}\sum_{i=1}^{N} a_i |\boldsymbol{b}_i - A\boldsymbol{r}_i|^2 \tag{27}$$

such that

$$A^T A = 1$$

where $L(A)$ is the cost, or loss, function and a_i are non-negative weight of i^{th} measurement. Often the weights are chosen as $a_i = 1/\sigma_i^2$. Quaternion is represented at Eq. (2). Davenport's q-method can be derived as [5]. After some algebraic manipulation, Eq. (27) can be written in very convenient form as,

$$L(A) = \lambda_0 - tr(A\,B^T) \tag{28}$$

where λ_0 is sum of weights and B is 3×3 attitude profile matrix.

$$\lambda_0 = \sum_{i=1}^{N} a_i \tag{29}$$

$$B = \sum_{i=1}^{N} a_i \boldsymbol{b}_i \boldsymbol{r}_i^T \tag{30}$$

To minimize loss function, $tr(A\,B^T)$ must be maximized. If quaternion is used, Eq. (28) can be written again as

$$tr(A\,B^T) = q^T K q \tag{31}$$

$$L(A(q) = \lambda_0 - q^T K(B) q \tag{32}$$

$$K(B) = \begin{bmatrix} B + B^T - tr(B) I_{3\times3} & \boldsymbol{z} \\ \boldsymbol{z}^T & tr(B) \end{bmatrix} \tag{33}$$

$$z = \begin{bmatrix} B_{23} - B_{32} \\ B_{31} - B_{13} \\ B_{12} - B_{21} \end{bmatrix} = \sum_{i=1}^{N} a_i (\boldsymbol{b}_i \times \boldsymbol{r}_i) \tag{34}$$

where $K(B)$ is 4×4 a symmetric trace-less matrix. q-method asserts that eigenvector of K matrix gives optimal quaternion, so it gives best estimate of orientation matrix.

$$K q_{opt.} = \lambda_{max} q_{opt.} \tag{35}$$

$$L(\hat{A}(q_{opt.})) = \lambda_0 - \lambda_{max} \tag{36}$$

In the last step, it is assumed that the covariance of the q method is equal to the covariance of the QUEST method, so it can be written in terms of quaternions as follows [19],

$$P_{qq}^{q-Method} = [\tilde{q}_{opt}] \begin{bmatrix} P_{QQ}^{q-Method} & \mathbf{0} \\ \mathbf{0}^T & \mathbf{0} \end{bmatrix} [\tilde{q}_{opt}]^T \tag{37}$$

where $P_{QQ}^{q-Method}$ is covariance matrix in body system given with :

$$P_{QQ}^{q-Method} = \frac{1}{4} \sigma_{tot}^2 \left[\mathbf{1} - \sum_{i=1}^{n} a_i \boldsymbol{b}_i \boldsymbol{b}_i^T \right]^{-1} \tag{38}$$

and $[\tilde{q}_{opt}]$ is a $4x4$ matrix contains components of q_{opt} found with q method.

$$[\tilde{q}_{opt}] = \begin{bmatrix} q_{opt,4} & -q_{opt,3} & q_{opt,2} & q_{opt,1} \\ q_{opt,3} & q_{opt,4} & -q_{opt,1} & q_{opt,2} \\ -q_{opt,2} & q_{opt,1} & q_{opt,4} & q_{opt,3} \\ -q_{opt,1} & -q_{opt,2} & -q_{opt,3} & q_{opt,4} \end{bmatrix} \tag{39}$$

It must be noted that if the two largest eigenvalues of $K(B)$ matrix are equal q-method can not give an accurate estimate, so it is insufficient [16].

D. Extended Kalman Filter

Kalman filter is a type of state observer that has a predictor-corrector structure. It combines the measurement and prediction to find optimal estimate of attitude and "filter" term stands for filtering noisy process [21]. Kalman filter provides estimation for position, velocity, angle, and angular rate. It also provides uncertainty information by computing covariance directly.

To use Kalman filtering in satellite attitude determination, firstly dynamics of SC must be linearized. With this linearization process, Kalman filter is called as expanded Kalman filter (EKF). EKF can be used by unit quaternion parameterization. Formulation of traditional Kalman filtering algorithms can be written briefly as follows [17] [22] [23]. The process and measurement at time k can be modelled with,

$$\hat{\boldsymbol{x}}(k+1) = \boldsymbol{f}\,[\hat{\boldsymbol{x}}(k),\ k] + \boldsymbol{w}(k) \tag{40a}$$

$$\boldsymbol{z(k)} = \boldsymbol{H}\hat{\boldsymbol{x}}(k) + \boldsymbol{v(k)} \tag{40b}$$

where $\boldsymbol{x}$ is $n \times 1$ state vector contains quaternion and angular rates, $\boldsymbol{x} = [\boldsymbol{q} \quad \omega]^T$ and superscript hat presents prediction. $\boldsymbol{f}[.]$ is the non-linear $n \times n$ transition matrix. $\boldsymbol{w}$ is $n \times 1$ random Gaussian noise vector, $\boldsymbol{z}$ is $m \times 1$ measurement vector. $\boldsymbol{H}$ is m x n is noiseless output matrix and v_k is m x 1 a vector based on measurement error.

The algorithm starts with prediction of $\hat{\boldsymbol{x}}(k+1)$

$$\hat{\boldsymbol{x}}(k+1/k) = \boldsymbol{f}\,[\hat{\boldsymbol{x}}(k),\ k] \tag{41}$$

Priori covariance of estimation, $\boldsymbol{P}(k+1/k)$, can be written as

$$\boldsymbol{P}(k+1/k) = \boldsymbol{F}(k)\boldsymbol{P}(k/k)\boldsymbol{F}^T + \boldsymbol{Q}(k) \tag{42}$$

where $\boldsymbol{F}$ is linearized transition matrix around current state, $\boldsymbol{F}(k) = \frac{\partial \boldsymbol{f}[\hat{x}(k),k]}{\partial \hat{x}(k)}$. $\boldsymbol{Q}$ is process noise covariance matrix. By comparing priori state estimation and measurement information, gain of filter, $\boldsymbol{K}$ can be defined as,

$$\boldsymbol{K}(k+1) = \boldsymbol{P}(k+1/k)\boldsymbol{H}^T \times \left[\boldsymbol{H}\boldsymbol{P}(k+1/k)\boldsymbol{H}^T + \boldsymbol{R}(k)\right]^{-1} \tag{43}$$

where $\boldsymbol{R}$ is measurement noise covariance matrix. After covariance and gain are calculated, prediction can be corrected:

$$\hat{\boldsymbol{x}}(k+1) = \hat{\boldsymbol{x}}(k+1/k) + \boldsymbol{K}(k+1) \times [z(k) - \boldsymbol{H}\hat{\boldsymbol{x}}(k+1/k)] \tag{44}$$

Also priori covariance of algorithm can be updated as,

$$\boldsymbol{P}(k+1/k+1) = [\mathbf{1}_{7x7} - \boldsymbol{K}(k+1)\boldsymbol{H}]\,\boldsymbol{P}(k+1/k) \tag{45}$$

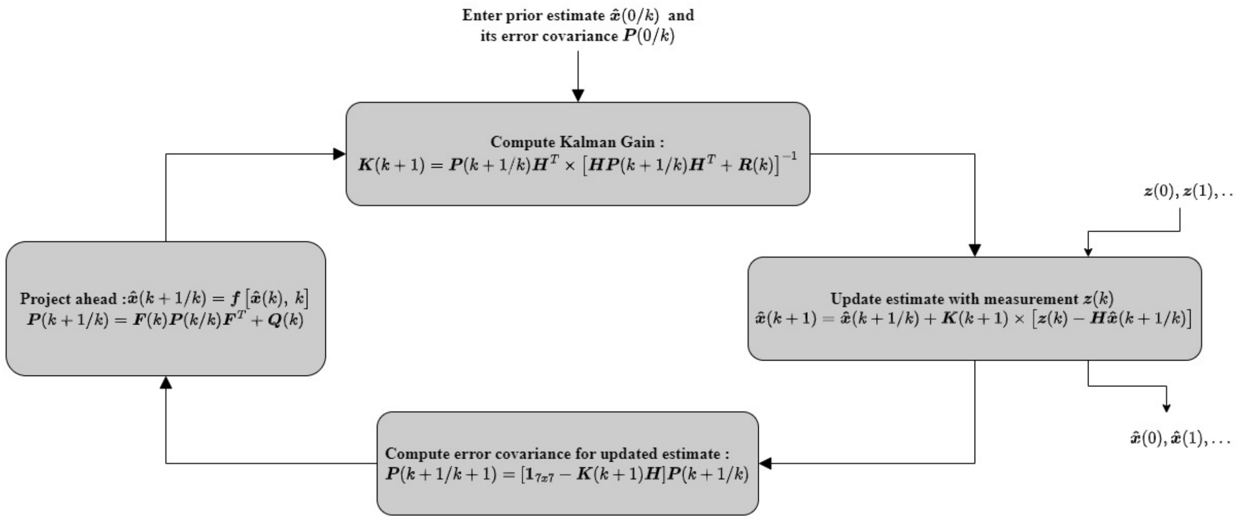

Fig. 2 Expanded Kalman Filter Loop

Overall algorithm for EKF is summarized in Fig. 2. Traditional EKF needs an initial estimation of attitude. If the initial guess is not accurate enough, the algorithm is more likely to diverge. Also, it uses a linearization procedure around current state for non-linear measurement model. Instead of linearization, the measurement can be linearized directly using the coarse attitude information obtained from a simple attitude determination algorithm and EKF algorithm can be initialized properly. This approach reduces the computer load and enables faster convergence.

1. TRIAD-Aided EKF

TRIAD algorithm can be used as measurement model of EKF. So, EKF uses measured quaternions obtained by TRIAD method and it combines them with angular velocity measurement from gyros. So, Eq. (40b) becomes,

$$z(k) = [\hat{\boldsymbol{q}}_{TRIAD}(k) \quad \hat{\omega}_b]^T + \boldsymbol{v}(k) \tag{46}$$

It should be noted that, measurement covariance matrix now depends on covariance of TRIAD algorithm and covariance of gyro and it is written as,

$$\boldsymbol{R}(k) = \begin{bmatrix} P_{qq}^{TRIAD} & 0 \\ 0 & P_{gyros} \end{bmatrix} \tag{47}$$

where P_{qq}^{TRIAD} is $4x4$ covariance matrix of TRIAD in terms of quaternion. It can be found by using Eq. (25) and simple transformation from Euler angles to quaternions at each step. P_{gyros} is $3x3$ covariance matrix of gyros. Other steps of TRIAD+EKF algorithm are same with traditional EKF presented from Eq. (41) to Eq. (45) and at Fig. 2.

2. q Method-Aided EKF

Like the TRIAD method, the q method can also be used as measurement model of EKF. So, Eq. (40b) becomes,

$$z(k) = \begin{bmatrix} \hat{\boldsymbol{q}}_{q-Method}(k) & \hat{\boldsymbol{\omega}}_b \end{bmatrix}^T + \boldsymbol{v}(k) \tag{48}$$

Also, measurement covariance matrix of EKF is updated as,

$$\boldsymbol{R}(k) = \begin{bmatrix} P_{qq}^{q-Method} & 0 \\ 0 & P_{gyros} \end{bmatrix} \tag{49}$$

Other steps of q-Method+EKF algorithm are same with traditional EKF presented from Eq. (41) to Eq. (45) and at Fig. 2.

V. Result and Conclusion

Simulations are conducted for a hypothetical nano-satellite. Attitude of satellite at a circular orbit with altitude of 400 km are obtained in Matlab environment by using Eq. (3) to Eq. (7). Calculations initialized with q_0 = [0.005 0.015 0.01 0.9998] and $\omega_0 = \{0.001\ 0.0015\ 0.001\}$ rad/s and data are collected during one period of nano-satellite with sample time of 0.1 s. The disturbance torque acting on the satellite is assumed as $N_T = 3.6 \times 10^{-10}$ $N.m$. The moments of inertia of the satellite are taken as $J_x = 2.1 \times 10^{-3}$ m^4 $J_y = 2 \times 10^{-3}$ m^4 and $J_z = 1.9 \times 10^{-3}$ m^4. With these parameters, calculated attitude is presented in Fig. 4 and Fig. 5 in terms of quaternion and Euler's angles, respectively. Also, change of angular velocities is given at Appendix A at Fig. 3.

Mathematical models described in Sec.IV.A are applied with the following parameters. The orbit of the satellite is assumed as circular orbit with period of 92.5604 min and angular velocity of orbit ω_{orbit} is calculated as 0.0011 rad/s. Inclination, (i), taken as 97.65°. The argument of perigee (ω) is taken as 207.4°. Calculations for λ_e, ε_{ln}, argument of latitude (u) and Λ are initialized from periapsis at the date 01.01.2022, 00:00. For calculations of measurement models, noise depending on the characteristics of the sensors are included but bias terms are neglected. The standard deviation of magnetometer, nadir sensor, sun sensor and gyros are taken as $\sigma_m = .008$, $\sigma_n = .006$, $\sigma_s = .002$ and $\sigma_g = .006$ respectively. Data from sensor modelling are given at Sec. V.B.

TRIAD, Davenport's q method, TRIAD+EKF and q-Method+EKF algorithms are written as Matlab functions. To compare accuracy of sensor data and algorithms, 3 dual combinations of sensors are used as input of these algorithms. RMSE values of each dual sensor combinations and algorithms are shown at Table 1 to Table 3. Also, Euler angles representation of estimated attitudes are given at Sec. V.C with Figs. 12- 14. In addition, RMSE results are given in Table 4 to analyze the effect of the number and combinations of sensors on the accuracy of Davenport's q method. In

Table 4, Case 1 represents that only magnetometer and nadir sensor data are used, while Case 2 represents that only magnetometer and sun sensor data are used. Only sun and nadir sensor data are used for Case 3 and all available sensor data are used for Case 4.

RMS Error	ϕ (°)	θ (°)	ψ (°)
TRIAD	9.3288	0.8950	3.9870
q method	9.3317	0.8865	3.9812
TRIAD+EKF	7.1811	0.6958	1.2085
q method+EKF	8.8994	0.6287	0.6017

Table 1 RMSE Values - Only Magnetometer and Nadir Sensor

RMS Error	ϕ (°)	θ (°)	ψ (°)
TRIAD	10.8627	0.3311	0.5235
q method	11.1764	0.3478	0.5406
TRIAD+EKF	7.4718	0.1756	0.2622
q method+EKF	5.9120	0.1428	0.2123

Table 2 RMSE Values - Only Magnetometer and Sun Sensor

RMS Error	ϕ (°)	θ (°)	ψ (°)
TRIAD	9.6311	0.2539	0.4198
q method	9.5088	0.2700	0.4260
TRIAD+EKF	6.2870	0.1373	0.2152
q method+EKF	5.9092	0.1214	0.1783

Table 3 RMSE Values - Only Nadir Sensor and Sun Sensor

To analyze effect of sensor accuracy on determination algorithms, RMSE values of single frame methods with different sensor combinations is investigated. The highest error in each algorithm in both yaw and pitch angles is observed when using nadir sensor and magnetometer together. But, contrary to expectations, errors exceeding 10 degrees in the roll angle are observed when the sun and magnetometer are used. This may be because the single frame algorithms are too simple to capture the sudden high change in the true roll angle of the satellite. However, when the roll angle estimates are ignored, it can be said that the combination of sensors with better accuracy increases the accuracy of the attiude by %70 to %80.

EKF highly improves accuracy of single frame algorithms. When a single frame is integrated with EKF, up to 47%, 60% and 80% reduction in roll, pitch and yaw angles respectively are observed. Considering only the improvements in RMSE values, the best results were obtained when integrated with the EKF and q method. When considering only TRIAD+EKF algorithm, the results are between near 22% and 70% more accurate than those obtained with only TRIAD algorithm. On the other hand, the accuracy of q method has been improved from 28% to 85% when it is used with EKF.

According to Table 4 accuracy generally increases when all available data is used, around 100% to 140% lower RMSE values than nadir-magnetometer case are obtained. This results are more accurate than q method+EKF algorithm for nadir sensor and magnetometer combinatiion. However, generally the q method+EKF is more advantageous than using the q method with all available sensor data.

RMS Error	ϕ (°)	θ (°)	ψ (°)
Case 1	9.3317	0.8865	3.9812
Case 2	11.1764	0.3478	0.5406
Case 3	9.5088	0.2700	0.4260
Case 4	8.6167	0.2353	0.3909

Table 4 RMSE Values - q Method and Sensor Combination

As a conclusion, with growing nano-satellite industry, studies on the development of attitude determination systems for LEO nano-satellites have increased. Especially in this field, Kalman filter based algorithms gained importance. In many studies in the literature, attitude estimation methods were investigated with different configurations. According to these studies, Kalman filtering based algorithms give results with higher accuracy than TRIAD and q method. However, considering the computation time, TRIAD method can be preferred due to its simplicity. To decrease the computing time of Kalman filtering algorithm, these algorithms can be combined with single frame methods.

In this graduation study, firstly, the previous studies were examined, and the methodology was determined. Secondly, attitude dynamic were simulated with MatLab and true theoretical orientation of satellite with time was found. Then, necessary unit vectors were obtained from sensor modelling. Attitude of a nano-satellite is estimated by TRIAD, Davenport's q method, TRIAD+EKF and q method+EKF algorithms. At TRIAD+EKF and q method+EKF algorithm, single frame methods are used as linear measurement model in EKF algorithm, so measurement covariance of EKF is taken as covariance of these algorithms. As expected, the error in the algorithms decreases as the sensor accuracy increases. In general, all algorithms provide good accuracy in yaw and pitch angles with RMSE values of under 4°. However, in roll RMSE value exceeds 10° for each single frame methods when only magnetometer and sun sensor are used. This may have been caused by arbitrary selected initial conditions for the satellite. When considering only single frame methods, error due to sensors can be improved by using Davenport's q method with all available sensor data. Also attitude can be estimated with higher accuracy by EKF and single frame method integration and according to obtained results, as the error rate in the single frame method used as the measurement model in the algorithm increases, the EKF generally gives better estimation about attitude.

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Appendix

A. Simulation of Attitude Dynamics of LEO Nano-Satellite

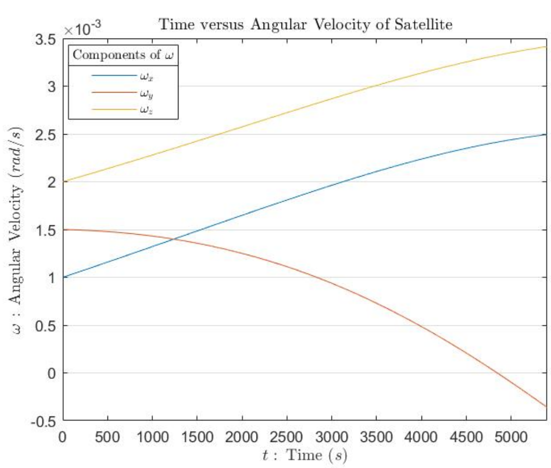

Fig. 3 Components of Angular Velocity versus Time

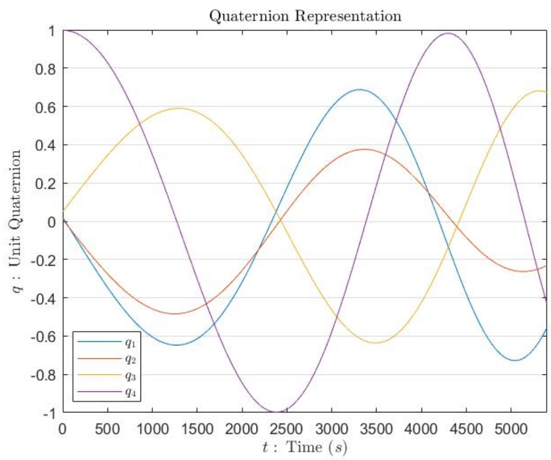

Fig. 4 Components of Quaternion versus Time

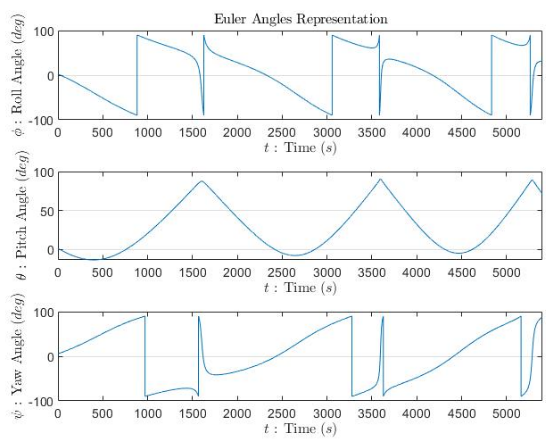

Fig. 5 Euler Angles versus Time

B. Sensor Data

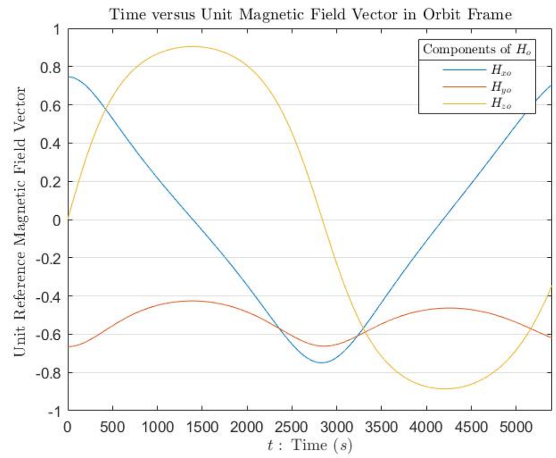

Fig. 6 Unit Magnetic Field Vector in Orbit Frame

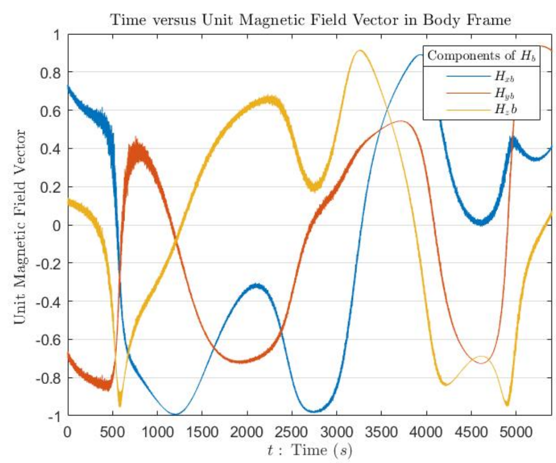

Fig. 7 Unit Magnetic Field Vector in Body Frame

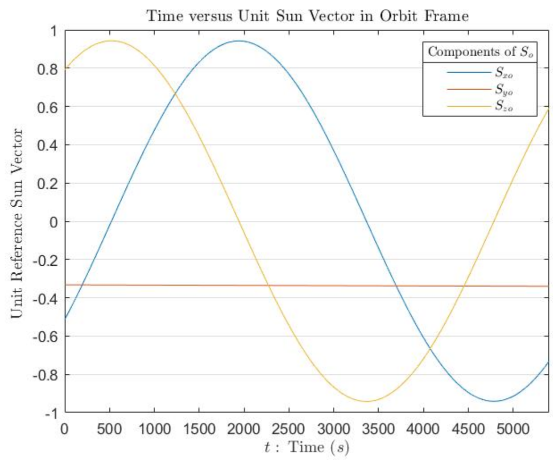

Fig. 8 Unit Sun Vector in Orbit Frame

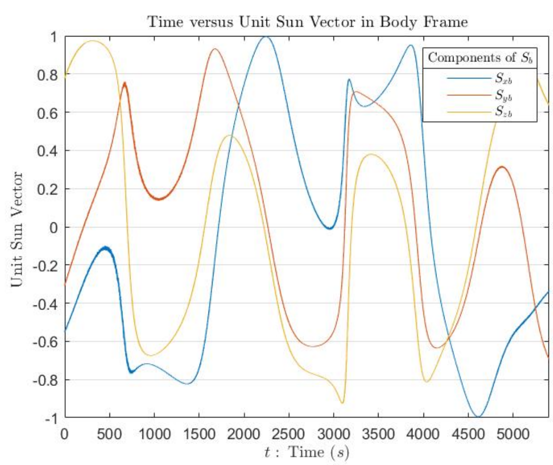

Fig. 9 Unit Sun Vector in Body Frame

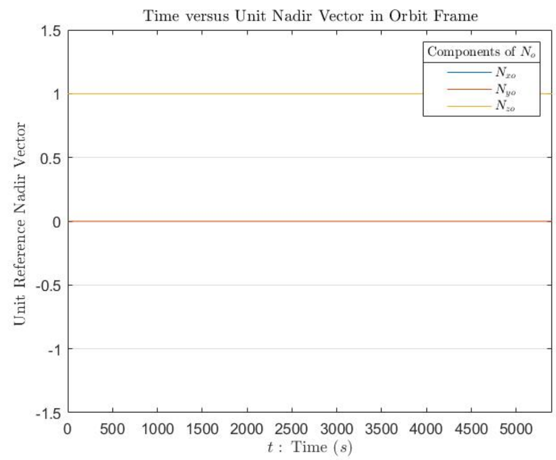

Fig. 10 Unit Nadir Vector in Orbit Frame

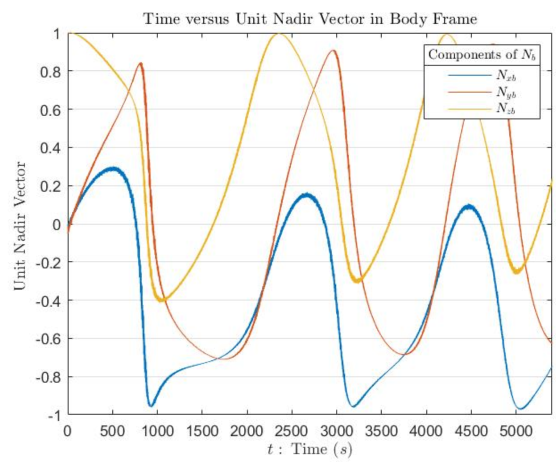

Fig. 11 Unit Nadir Vector in Body Frame

C. Attitude Determination Algorithms

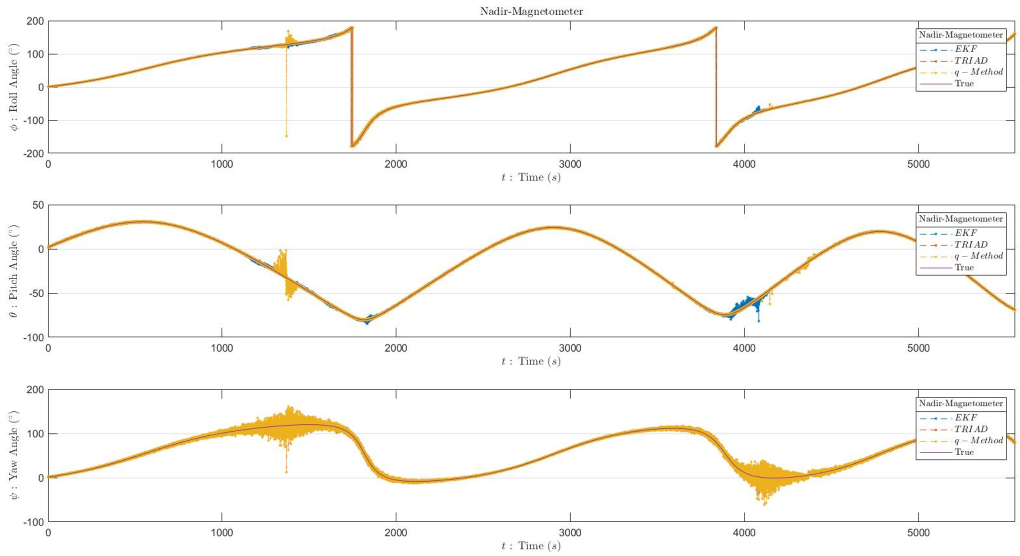

Fig. 12 Estimated Attitudes Whit Magnetometer and Nadir Sensor Data

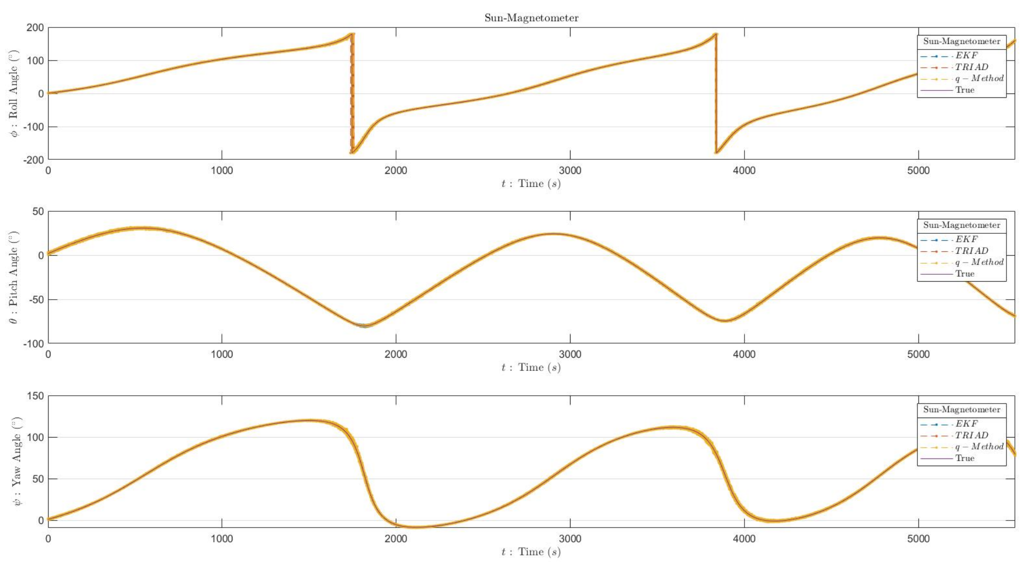

Fig. 13 Estimated Attitudes Whit Magnetometer and Sun Sensor Data

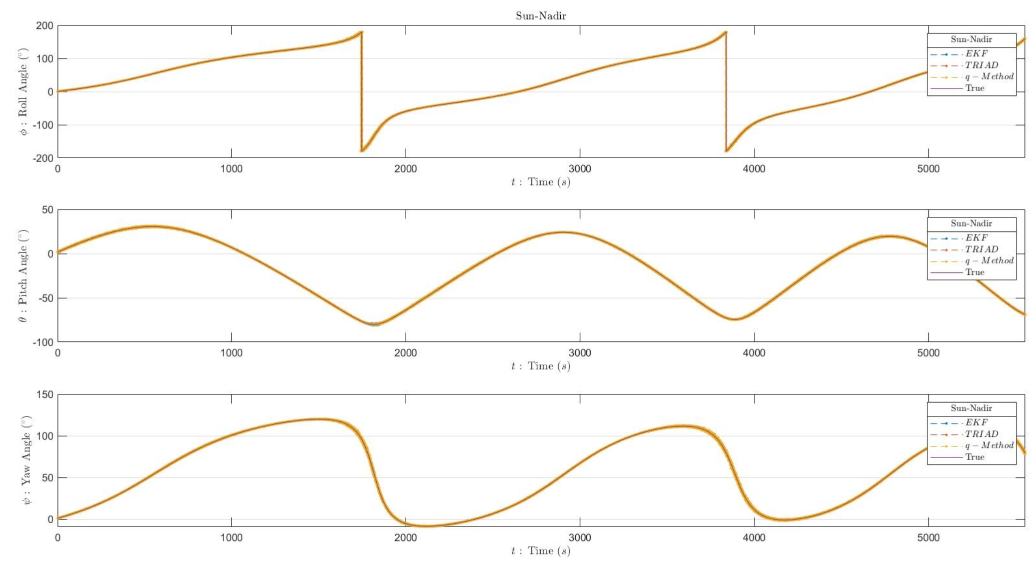

Fig. 14 Estimated Attitudes Whit Sun and Nadir Sensor Data

D. Work Schedule

Single Frame Methods and Kalman Filter Based Nanosatellite Attitude Estimation - Neslihan Gülsoy - 12.06.2022																												
Work Schedule																												
Course :	**UCK/UZB 4901(E)**														**UCK/UZB 4902(E)**													
Week :	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**	**13**	**14**	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**	**13**	**14**
Performed Studies:																												
Choosing the topic																												
Literature review																												
Examination of similar theses																												
Modelling of attitude dynamics and orientation																												
Modelling of sensor measurements																												
Application of determination methods																												
Application of TRIAD method																												
Application of q method																												
Application of TRIAD+EKF method																												
Obtaining results																												
Representation and analysis of results																												
Comparing of results and error analysis																												
Preparing the presentation																												
Preparation of a paper from the study																												

Table 5 Work Schedule

E. Statement of Ethics

I herewith declare that this graduation project is entirely and solely my own work. This paper has been prepared by adhering to ethical conduct and academic rules. I further declare that, all materials and results in this paper have been cited and referenced in full accordance with these rules and conduct.

Neslihan Gülsoy