ISTANBUL TECHNICAL UNIVERSITY

Compressible Aerodynamics

UZB 362E

CRN : 30441

Term Project
Design of Rocket Geometry

#Group 5

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Deadline : 23.08.2021

1 Introduction

This paper covers steps of the design of rocket geometry that provides the desired values. Air properties and isentropic relations are used to determine throat section area.

Method of characteristics (MOC) is used for supersonic portion throughout the design process to develop nozzle contour by reducing the length.

The values assigned to group 5 are listed below

Outlet Diameter : $D_e = 2.3\ m$

Nozzle Length : $L = 3.1\ m$

Exit Mach Number : $M_e = 2.4$

2 Introduction to Rocket Nozzle Geometry

A rocket engine uses a nozzle to accelerate hot exhaust to produce thrust as described by Newton's third law of motion. The amount of thrust produced by the engine depends on the mass flow rate through the engine, the exit velocity of the flow, and the pressure at the exit of the engine. The value of these three flow variables are all determined by the rocket nozzle design.

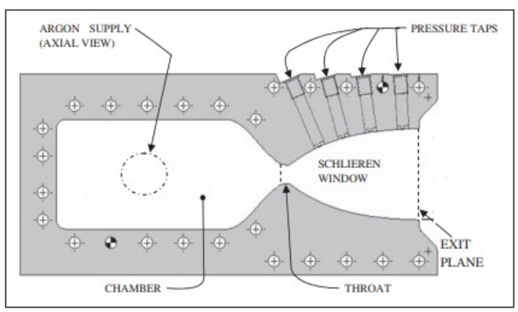

Figure 1: Rocket Nozzle Schematic Geometry

A nozzle is a relatively simple device, just a specially shaped tube through which hot gases flow. Rockets typically use a fixed convergent section followed by a fixed divergent section for the design of the nozzle. This nozzle configuration is called a convergent-divergent, or CD, nozzle. In a CD rocket nozzle, the hot exhaust leaves the combustion chamber and converges down to the minimum area, or throat, of the nozzle. The throat size is chosen to choke the flow and set the mass flow rate through the system. The flow in the throat is sonic which means the Mach number is equal to one in the throat. Downstream of the throat, the geometry diverges and the flow is isentropically expanded to a supersonic Mach number that depends on the area ratio of the exit to the throat. The expansion of a supersonic flow causes the static pressure and temperature to decrease from the throat to the exit, so the amount of the expansion also determines the exit pressure and temperature. The exit temperature determines the exit speed of sound, which determines the exit velocity. The exit velocity, pressure, and mass flow through the nozzle determines the amount of thrust produced by the nozzle.

A number of different proven nozzle configurations are available today. This section describes their geometries and performance. . Nozzles and chambers are usually of circular cross section and have a converging section, a throat at the narrowest location (minimum cross section), and a diverging section.

The converging nozzle section between the chamber and the nozzle throat has never been critical in achieving high performance. The subsonic flow in this section can easily be turned at very low pressure drop and any radius, cone angle, wall contour curve, or nozzle inlet shape is satisfactory. A few small attitude control thrust chambers have had their nozzle at 90 degrees from the combustion chamber axis without any performance loss. The throat contour also is not very critical to performance, and any radius or other curve is usually acceptable. The pressure gradients are high in these two regions and the flow will adhere to the walls. The principal difference in the different nozzle configurations is found in the diverging supersonic-flow section, as described below. The wall surface throughout the nozzle should be smooth and shiny to minimize friction, radiation absorption, and convective heat transfer due to surface roughness. Gaps, holes, sharp edges, or protrusions must be avoided.

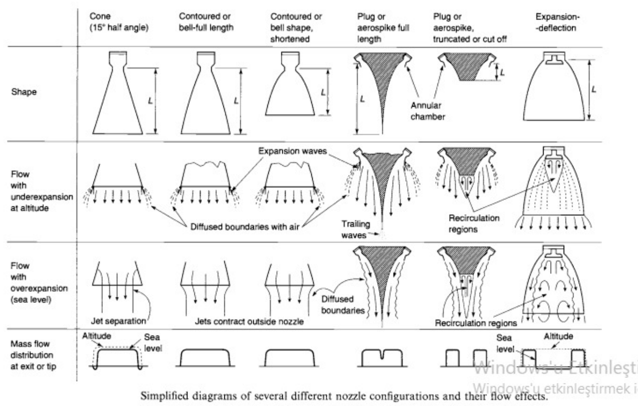

Figure 2: Different Nozzle Configurations

Six different nozzle configurations are shown in Fig.2. The first three sketches show conical and bell-shaped nozzles. The other three have a center body inside the nozzle and have excellent altitude compensation. Although these last three have been ground tested, to date none of them has flown in a space launch vehicle. The lengths of several nozzle types are compared in Fig. 3 . The objectives of a good nozzle configuration are to obtain the highest practical Is, minimize inert nozzle mass, and conserve length.

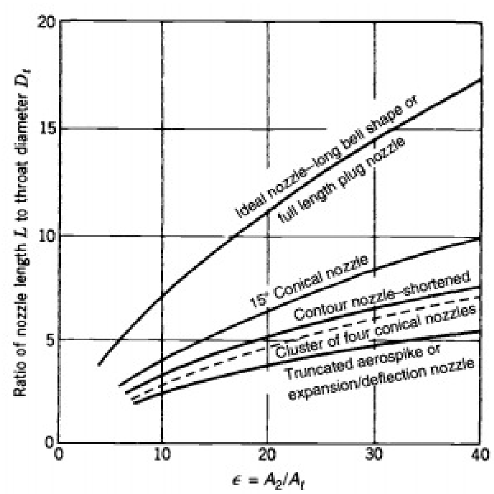

Figure 3: The Lengths of Several Nozzle Types are Compared

3 Determination of Throat Section Area

When determining the throat section area of CD nozzle theoretically, it is known that the mass flow rate is constant in every section of the nozzle, also it is assumed that the flow at throat is in sonic condition. Then continuity equation is written between throat and exit

$$\dot{m} = \rho^* u^* A^* = \rho_e u_e A_e \tag{1}$$

Since $u^* = a^*$, Eq. (1) becomes

$$\frac{A_e}{A^*} = \frac{\rho^*}{\rho_e}\frac{a^*}{u_e} = \frac{\rho^*}{\rho_0}\frac{\rho_0}{\rho_e}\frac{a^*}{u_e} \tag{2}$$

Due to isentropic flow assumption

$$\begin{aligned} \frac{\rho^*}{\rho_0} &= \left(\frac{2}{\gamma+1}\right)^{1/(\gamma-1)} \\ \frac{\rho_0}{\rho_e} &= \left(1 + \frac{\gamma-1}{2}M_e^2\right)^{1/(\gamma-1)} \end{aligned} \tag{3}$$

Also according to definition of M^*

$$\left(\frac{u_e}{a^*}\right)^2 = {M^*}^2 = \frac{[(\gamma+1)/2]\,M_e^2}{1+[(\gamma-1)/2]\,M_e^2} \tag{4}$$

When Eqs. (3), (2) and (4) are combined

$$\left(\frac{A_e}{A^*}\right)^2 = \frac{1}{M_e^2}\left[\frac{2}{\gamma+1}\left(1+\frac{\gamma-1}{2}M_e^2\right)\right]^{(\gamma+1)/(\gamma-1)} \tag{5}$$

When above equation is arranged

$$A^* = A_e M_e \left[\frac{2}{\gamma+1}\left(1+\frac{\gamma-1}{2}M_e^2\right)\right]^{-(\gamma+1)/2(\gamma-1)} \tag{6}$$

Finally, when the given values are substituted

$$A^* = \frac{\pi(2.3)^2}{4}(2.4)\left[\frac{2}{1.4+1}\left(1+\frac{1.4-1}{2}(2.4)^2\right)\right]^{-(1.4+1)/2(1.4-1)}$$

$$A^* = \boxed{1.728915\,m^2}, \qquad D^* = \boxed{1.483686\,m} \tag{7}$$

4 Design of Converging Section

The chamber contraction ratio (chamber cross sectional area to throat area), CR, is an important design parameter and tends to range from about 1.3 to 10 depending on engine thrust level. For example, the high-thrust F-1 engine had CR = 1.3, while SSME had CR = 3. Larger CR values increase chamber diameter and weight and tend to reduce acoustic frequencies.

In this project CR is chosen as 3.

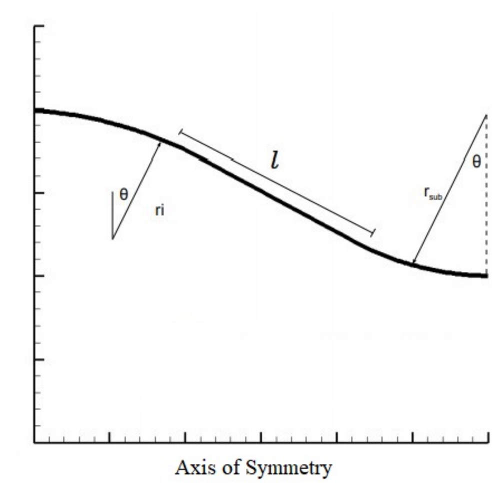

Figure 4: Convergent Section of Countered Nozzle

Due to rule of thumb for sonic part design with smooth transition and without flow separation, radius of curvatures is chosen as $r_i, r_{sub} = 1.5 \times r_{throat}$ and semi-angles of the cones are chosen as $\theta = 30°$.

To summarize design parameters for converging section

$$\begin{aligned}
&CR = 3 \\
&D^*_{calculated} = 0.9622\, m \\
&D_{inlet} = \sqrt{3} \times D^*_{calculated} = 1.66\, m \\
&r_{sub} = r_i = 1.5 \times r_{throat} = 0.7215\, m \\
&\theta = 30°
\end{aligned} \tag{8}$$

5 Design of Diverging Section

To design of diverging section for minimum length nozzle, method of characteristics (MOC) is used step-by-step. Due to flow symmetry with respect to the x-axis design of upper counter of nozzle by MOC is enough.

Steps of Upper Counter Design :

- Prandtl-Meyer (PM) angle at exit (ν_e) is calculated with knowing exit Mach number (M_e).

- Maximum turn angle at wall ($\theta_{w,max}$) is calculated with ν_e.

- MOC is initialized with a proper small non-zero turn angle ($\theta_{initial}$) at throat (point a).

- Turn angle at point a is increased with proper increments.

- Turn and PM angles are calculated point-by-point with proper compatibility constants equations.

- Mach number (M) and Mach angle (μ) of points are calculated.

- Slopes of characteristic lines and coordinates of points are calculated with inlet coordinate and centerline.

MOC is applied for 10, 20 and 50 characteristic lines. Curve fitting with 5 degree polynomial is used to contour nozzle wall and wall is defined as a function of horizontal location.

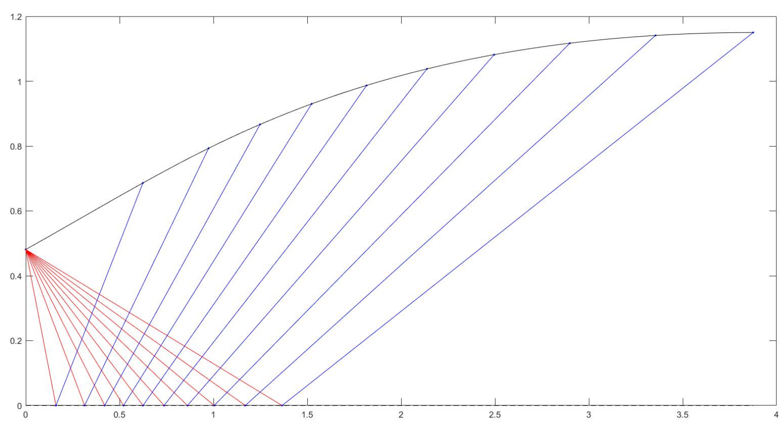

Figure 5: Nozzle Counter with 10 characteristic lines

For 10 characteristic lines diameter of throat is found as $D^*_{calculated} = 0.9621\ m$. However as expected there is a difference between calculated D_t and theoretical $D^*_{theoretic} = 1.4837\ m$. Percent error reaches 35%. When number of characteristic lines are increased $D^*_{calculated}$ decreases and error increases. Also upper wall function of is found as $f(y) = -0.001256x^5 + 0.01437x^4 - 0.05677x^3 + 0.03806x^2 + 0.3246x + 0.481$ with $R^2 = 1$. General formulation and calculation for 65 points with 10 characteristic lines are given below.

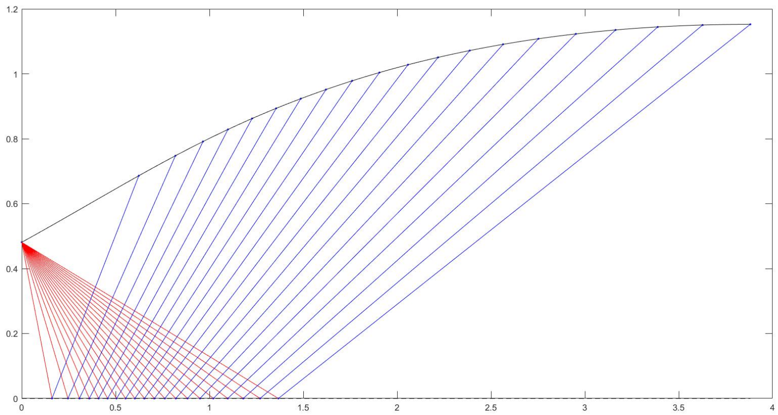

Figure 6: Nozzle Counter with 20 characteristic lines

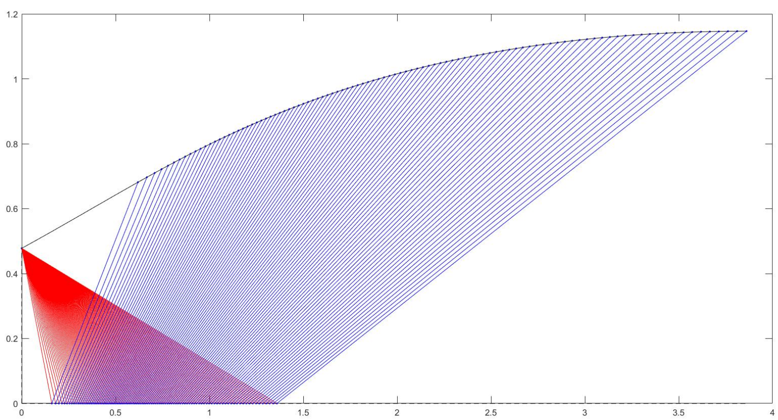

Figure 7: Nozzle Counter with 100 characteristic lines

5.1 Formulation

$$\nu_e = \frac{\gamma+1}{\gamma-1} tan^{-1}\left\{\sqrt{\frac{\gamma-1}{\gamma+1}(M_e^2-1)}\right\} - tan^{-1}\sqrt{M_e^2-1} \tag{9}$$

for $\gamma = 1.4$ and $M_e = 2.4$

$$\nu_e = 36.7465^\circ$$

$$\theta_{w,max} = \frac{\nu_e}{2} = 18.3733^\circ \tag{10}$$

It seems better to choice initial turn angle and increment as $\theta_{initial} = 0.3733^\circ$ and $\Delta\theta = (18.3733^\circ - 0.3733^\circ)/9 = 2^\circ$ for 10 characteristic lines.

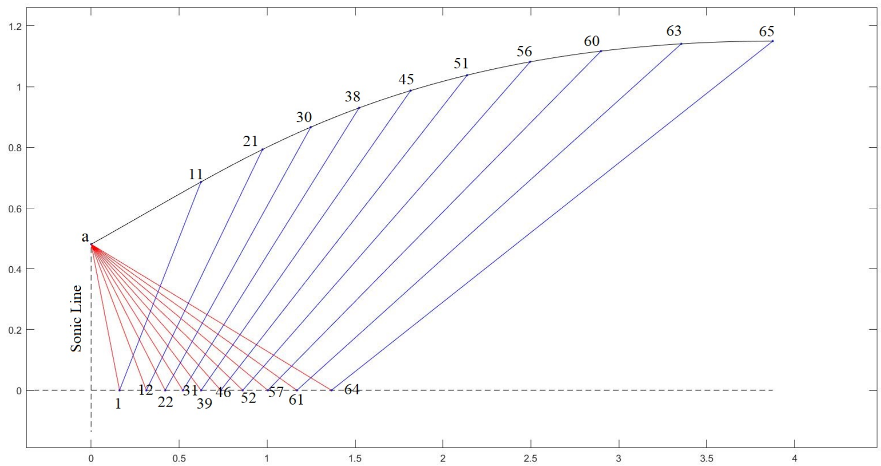

Figure 8: MOC with 10 Characteristic lines

Also to find turn angle, P-M angle, Mach number, Mach angle, average angle, slopes and coordinates between arbitrary points e.g. c and d and intersection of these point, e

$$
\begin{aligned}
&\theta_e = \frac{K_{-(c-e)} + K_{+(d-e)}}{2};\\
&\nu_e = \frac{K_{-(c-e)} - K_{+(d-e)}}{2};\\
&M_e = f(\nu_e);\\
&\mu = f(M_e);\\
&\alpha^-_{c-e} = \frac{\theta_c + \theta_e}{2} - \frac{\mu_c + \mu_e}{2};\\
&\alpha^+_{d-e} = \frac{\theta_d + \theta_e}{2} + \frac{\mu_d + \mu_e}{2};
\end{aligned} \tag{11}
$$

$$
m^-_{c-e} = tan(\alpha^-_{c-e})\,; \qquad\qquad m^+_{d-e} = tan(\alpha^+_{d-e}) \tag{12}
$$

$$
\begin{aligned}
&x_e = \frac{y_c - y_d + m^+_{d-e}x_d - m^-_{c-e}x_e}{m^+_{d-e} - m^-_{c-e}}\\
&y_e = y_d + m^+_{d-e}(x_e - x_d)
\end{aligned} \tag{13}
$$

For example for point 1 :

$$\theta_a = 0.3733°; \quad \nu_a = 0.3733°; \quad \mu_a = 73.7324°$$
$$K_{-_{(a-1)}} = 0.7465°; \quad K_{+_{a'-1}} = -0.7465°$$
$$\theta_1 = 0; \quad \nu_1 = 0.7465°; \quad \mu_1 = 69.6007°$$
$$a^-_{(a-1)} = -71.4799°; \quad a^+_{(a'-1)} = 71.4799°;$$
$$m_{C_{-(a-1)}} = -2.9852; \quad m_{C_{+(a'-1)}} = 2.9852;$$

Continuing with point 2 :

$$\theta_a = 2.3733°; \quad \nu_a = 2.3733°; \quad \mu_a = 60.4387$$
$$K_{-_{(a-2)}} = 4.7465°; \quad K_{+_{1-2}} = -0.7465°$$
$$\theta_2 = 2°; \quad \nu_2 = 2.7465°; \quad \mu_2 = 59.0482°$$
$$a^-_{(a-2)} = -57.5568°; \quad a^+_{(1-2)} = 65.3244°;$$
$$m_{C_{-(a-2)}} = -1.5731; \quad m_{C_{+(1-2)}} = 2.1766,$$

$$\vdots$$

For first wall point 11

$$\theta_a = 18.3733°; \quad \nu_a = 18.3733°; \quad \mu_a = 35.5676°;$$
$$K_{-_{(a-11)}} = 36.74655°; \quad K_{+_{10-11}} = -0.7465°$$
$$\theta_{11} = 18°; \quad \nu_{11} = 18.7465°; \quad \mu_{11} = 35.2664°$$
$$a_{wall_(a-11)} = 18.1854°; \quad a^+_{(10-11)} = 53.2672°;$$
$$m_{wall_(a-11)} = 0.3285; \quad m_{C_{+(10-11)}} = 1.34;$$

$$\vdots$$

$Point\#$	K_-	K_+	θ	ν	M	μ	m_{C_-}	m_{C_+}	m_{wall}	x	y
a	36,7465	-36,7465	18,3733	18,3733	1,7192	35,5676				0	0,4811
1	0,7465	-0,7465	0	0,7465	1,0669	69,6007	-2,9852	2,9852		0,1611	0
2	4,7465	-0,7465	2	2,7465	1,1660	59,0482	-1,5731	2,1766		0,2218	0,1321
3	8,7465	-0,7465	4	4,7465	1,2468	53,3259	-1,1747	1,6767		0,2528	0,1840
4	12,7465	-0,7465	6	6,7465	1,3209	49,2052	-0,9445	1,4975		0,2767	0,2198
5	16,7465	-0,7465	8	8,7465	1,3916	45,9371	-0,7822	1,4056		0,2972	0,2486
6	20,7465	-0,7465	10	10,7465	1,4605	43,2100	-0,6559	1,3551		0,3158	0,2739
7	24,7465	-0,7465	12	12,7465	1,5285	40,8610	-0,5516	1,3288		0,3334	0,2972
8	28,7465	-0,7465	14	14,7465	1,5962	38,7929	-0,4619	1,3187		0,3502	0,3193
9	32,7465	-0,7465	16	16,7465	1,6639	36,9425	-0,3823	1,3207		0,3665	0,3409
10	36,7465	-0,7465	18	18,7465	1,7320	35,2664	-0,3101	1,3321		0,3826	0,3624
11	36,7465	-0,7465	18	18,7465	1,7320	35,2664		1,3400	0,3285	0,6242	0,6861
12	4,7465	-4,7465	0	4,7465	1,2468	53,3259	-1,4381	1,4381		0,3137	0
13	8,7465	-4,7465	2	6,7465	1,3209	49,2052	-1,1210	1,3399		0,3607	0,0631
14	12,7465	-4,7465	4	8,7465	1,3916	45,9371	-0,9186	1,2604		0,3972	0,1090
15	16,7465	-4,7465	6	10,7465	1,4605	43,2100	-0,7694	1,2163		0,4287	0,1474
16	20,7465	-4,7465	8	12,7465	1,5285	40,8610	-0,6503	1,1933		0,4575	0,1817
17	24,7465	-4,7465	10	14,7465	1,5962	38,7929	-0,5504	1,1845		0,4847	0,2139
18	28,7465	-4,7465	12	16,7465	1,6639	36,9425	-0,4635	1,1862		0,5108	0,2449
19	32,7465	-4,7465	14	18,7465	1,7320	35,2664	-0,3860	1,1962		0,5363	0,2754
20	36,7465	-4,7465	16	20,7465	1,8007	33,7332	-0,3153	1,2131		0,5615	0,3060
21	36,7465	-4,7465	16	20,7465	1,8007	33,7332		1,1805	0,3057	0,9742	0,7931
22	8,7465	-8,7465	0	8,7465	1,3916	45,9371	-1,0564	1,0564		0,4204	0
23	12,7465	-8,7465	2	10,7465	1,4605	43,2100	-0,8870	1,0565		0,4659	0,0481
24	16,7465	-8,7465	4	12,7465	1,5285	40,8610	-0,7545	1,0368		0,5057	0,0893
25	20,7465	-8,7465	6	14,7465	1,5962	38,7929	-0,6451	1,0293		0,5424	0,1270
26	24,7465	-8,7465	8	16,7465	1,6639	36,9425	-0,5513	1,0308		0,5772	0,1629
27	28,7465	-8,7465	10	18,7465	1,7320	35,2664	-0,4685	1,0393		0,6109	0,1980
28	32,7465	-8,7465	12	20,7465	1,8007	33,7332	-0,3939	1,0538		0,6441	0,2329
29	36,7465	-8,7465	14	22,7465	1,8704	32,3193	-0,3254	1,0734		0,6771	0,2684
30	36,7465	-8,7465	14	22,7465	1,8704	32,3193		1,0471	0,2679	1,2484	0,8666
31	12,7465	-12,7465	0	12,7465	1,5285	40,8610	-0,8704	0,8704		0,5211	0
32	16,7465	-12,7465	2	14,7465	1,5962	38,7929	-0,7488	0,8949		0,5684	0,0423
33	20,7465	-12,7465	4	16,7465	1,6639	36,9425	-0,6461	0,8962		0,6124	0,0817
34	24,7465	-12,7465	6	18,7465	1,7320	35,2664	-0,5567	0,9037		0,6546	0,1198
35	28,7465	-12,7465	8	20,7465	1,8007	33,7332	-0,4770	0,9163		0,6957	0,1575
36	32,7465	-12,7465	10	22,7465	1,8704	32,3193	-0,4046	0,9334		0,7365	0,1956
37	36,7465	-12,7465	12	24,7465	1,9412	31,0067	-0,3378	0,9544		0,7773	0,2345
38	36,7465	-12,7465	12	24,7465	1,9412	31,0067		0,9327	0,2309	1,5229	0,9300
39	16,7465	-16,7465	0	16,7465	1,6639	36,9425	-0,7499	0,7499		0,6249	0
40	20,7465	-16,7465	2	18,7465	1,7320	35,2664	-0,6520	0,7842		0,6762	0,0402
41	24,7465	-16,7465	4	20,7465	1,8007	33,7332	-0,5658	0,7954		0,7257	0,0796
42	28,7465	-16,7465	6	22,7465	1,8704	32,3193	-0,4883	0,8105		0,7744	0,1191
43	32,7465	-16,7465	8	24,7465	1,9412	31,0067	-0,4176	0,8291		0,8230	0,1594
44	36,7465	-16,7465	10	26,7465	2,0133	29,7812	-0,3520	0,8509		0,8721	0,2012
45	36,7465	-16,7465	10	26,7465	2,0133	29,7812		0,8326	0,1944	1,8158	0,9869
46	20,7465	-20,7465	0	20,7465	1,8007	33,7332	-0,6619	0,6619		0,7369	0
47	24,7465	-20,7465	2	22,7465	1,8704	32,3193	-0,5780	0,7009		0,7940	0,0401
48	28,7465	-20,7465	4	24,7465	1,9412	31,0067	-0,5021	0,7176		0,8508	0,0808
49	32,7465	-20,7465	6	26,7465	2,0133	29,7812	-0,4326	0,7371		0,9078	0,1228
50	36,7465	-20,7465	8	28,7465	2,0869	28,6314	-0,3681	0,7592		0,9656	0,1667
51	36,7465	-20,7465	8	28,7465	2,0869	28,6314		0,7435	0,1584	2,1372	1,0378
52	24,7465	-24,7465	0	24,7465	1,9412	31,0067	-0,5929	0,5929		0,8616	0
53	28,7465	-24,7465	2	26,7465	2,0133	29,7812	-0,5182	0,6345		0,9268	0,0414
54	32,7465	-24,7465	4	28,7465	2,0869	28,6314	-0,4495	0,6545		0,9928	0,0846
55	36,7465	-24,7465	6	30,7465	2,1622	27,5478	-0,3857	0,6768		1,0603	0,1302
56	36,7465	-24,7465	6	30,7465	2,1622	27,5478		0,6631	0,1228	2,4953	1,0818
57	28,7465	-28,7465	0	28,7465	2,0869	28,6314	-0,5363	0,5363		1,0039	0
58	32,7465	-28,7465	2	30,7465	2,1622	27,5478	-0,4682	0,5794		1,0796	0,0439
59	36,7465	-28,7465	4	32,7465	2,2394	26,5230	-0,4047	0,6017		1,1576	0,0908
60	36,7465	-28,7465	4	32,7465	2,2394	26,5230		0,5896	0,0875	2,8982	1,1170
61	32,7465	-32,7465	0	32,7465	2,2394	26,5230	-0,4885	0,4885		1,1695	0
62	36,7465	-32,7465	2	34,7465	2,3186	25,5503	-0,4252	0,5325		1,2590	0,0477
63	36,7465	-32,7465	2	34,7465	2,3186	25,5503		0,5217	0,0524	3,3547	1,1409
64	36,7465	-36,7465	0	36,7465	2,4000	24,6243	-0,4471	0,4471		1,3657	0
65	36,7465	-36,7465	0	36,7465	2,4000	24,6243		0,4583	0,0175	3,8748	1,1500

Table 1: Application of MOC for 10 characteristic lines

6 Drawing

With assigned, chosen and calculated values geometry of rocket nozzle is drawn with CATIA.

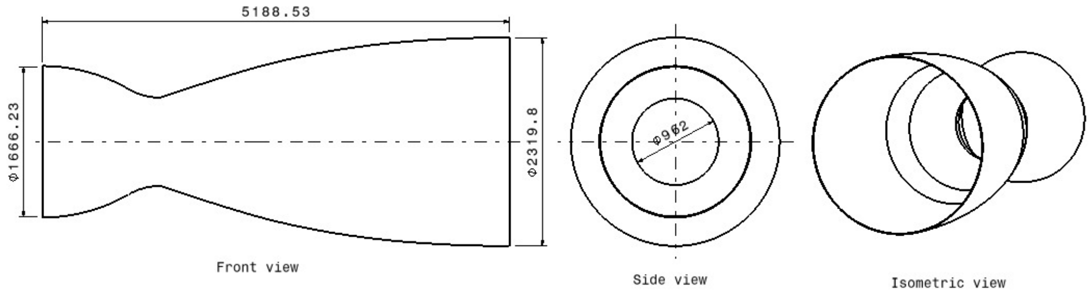

Figure 9: 2D Drawing of Designed Nozzle with Different Perspectives

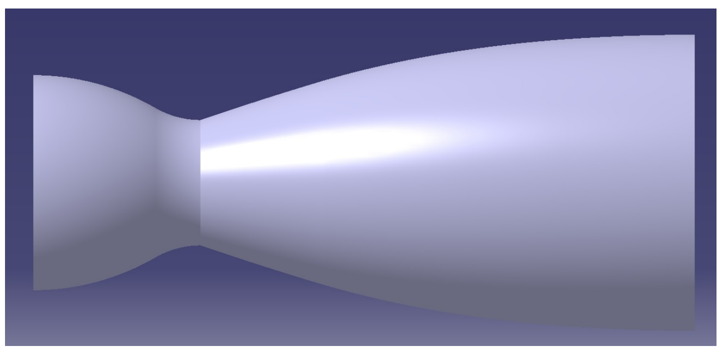

Figure 10: 3D Drawing of Designed Nozzle

References

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7 Appendix

Implemented MatLab Code

```
%% Determination of Throat Section Area
clear; clc;

gama = 1.4;
M = 2.4;
De = 2.3;   %Exit Diameter (m)
Ae = pi*De^2/4; %Exit Area (m^2)

syms At Dt
eq = (Ae / At)^2 == 1/(M^2) * ( 2/(gama+1) * ( 1 + (gama-1)/2 * M^2 ) )...
    ^((gama+1)/(gama-1));

At = double(vpasolve(eq,At));
At = At(2); %Throat Area (m^2)

eq2 = Dt^2*pi/4 == At;
Dt = double(vpasolve(eq2,Dt));
Dt = Dt(2); %Diameter of Throat (m)
save Dt

%% Divergent Section
clear; clc;

gama = 1.4; %Air Property
ncl = 10; %Number of Characteristics Lines

x(1) = 0;
y(1) = 0.4811;      %Change This Value to Achieve Exit Diameter
wx(1) = x(1);
wy(1) = y(1);

%Temporary Variables
l = ncl;
j = 1;

%Exit Conditions
M_exit = 2.4;

PM_exit = sqrt((gama + 1) / (gama-1) ) * atan (sqrt((gama-1)/(gama+1) * ...
    (M_exit^2-1) )) - atan(sqrt(M_exit^2-1));       %Radian
PM_exit = rad2deg(PM_exit);                 %Degree
teta_max = PM_exit / 2;                     %Degree


nnode = (2+(ncl+1)) * ncl/2;    %Number of nodes
node = (0:nnode)';

wall(ncl+1,1) = nnode;


for i = 1:(ncl-1)
```

```
    wall(ncl-i+1,1) = wall(ncl-i+2,1) - (i+1);   %Wall Points
end

teta_int = teta_max - floor(teta_max);  %Initial Turn Angle
inc = (teta_max - teta_int) / (ncl-1);  %Increment of Turn Angle

%Main Calculations
for i = 1 :nnode
    if i < wall(j+1)
        if wall(j) == 0 %Interactions With Point A
            teta(1) = teta_int + (i-1) * inc;
            PM_ang(1) = teta(1);
            Km(1) = teta(1) + PM_ang(1);
            Kp(1) = -Km(1);

            Mach(1,1) = Machfinder(PM_ang(1,1));
            mu(1,1) = asind(1/Mach(1,1));

            teta(i+1,1) = (Km(1) + Kp(i,1))/2;
            PM_ang(i+1,1) = (Km(1) - Kp(i,1))/2;
            Km(i+1,1) = teta(i+1,1) + PM_ang(i+1,1);
            Kp(i+1,1) = teta(i+1,1) - PM_ang(i+1,1);

            Mach(i+1,1) = Machfinder(PM_ang(i+1,1));
            mu(i+1,1) = asind(1/Mach(i+1,1));

            a_minus = (teta(1) + teta(i+1,1)) / 2 - ...
                (mu(1) + mu(i+1,1)) / 2;
            m_C_minus(i+1,1) = tand(a_minus);

            if i == 1   %For Point 1
                y(i+1,1) = 0;
                x(i+1,1) = -y(1) / (m_C_minus(i+1));
                m_C_plus(i+1,1) = -m_C_minus(i+1,1);
                centerlinex(i+1,1) = x(i+1,1);
            else
                a_plus = (teta(i,1) + teta(i+1)) / 2 + (mu(i,1) + ...
                    mu(i+1)) / 2;
                m_C_plus(i+1,1) = tand(a_plus);
                x(i+1,1) = (y(1) - y(i) + m_C_plus(i+1) * x(i) - ...
                    m_C_minus(i+1) * x(1)) / (m_C_plus(i+1) - ...
                    m_C_minus(i+1));
                y(i+1,1) = y(i) + m_C_plus(i+1)* (x(i+1) - x(i));
            end     %Finish of A point Interations

        else %Grid Points Independent of A
            l = ncl - j + 1;
            Km(i+1,1) = Km(i-l);
            Kp(i+1) = - Km(ncl + 2 -l);
            teta(i+1,1) = (Km(i+1,1) + Kp(i+1,1)) / 2;
            PM_ang(i+1,1) = (Km(i+1,1) - Kp(i+1,1)) / 2;

            Mach(i+1,1) = Machfinder(PM_ang(i+1,1));
            mu(i+1,1) = asind(1/Mach(i+1,1));

            %Calculation of Slopes
            a_minus = (teta(i-l,1) + teta(i+1,1)) / 2 - (mu(i-l,1)...
                + mu(i+1,1)) / 2;
            m_C_minus(i+1,1) = tand(a_minus);

            if teta(i+1) == 0  %Centerline
                y(i+1,1) = 0;
                x(i+1,1) = x(i-l) - y(i-l) / (m_C_minus(i+1));
                m_C_plus(i+1,1) = -m_C_minus(i+1,1);
                centerlinex(i+1,1) = x(i+1,1);
```

```
            else
                a_plus = (teta(i+1,1) + teta(i+1,1)) / 2 + (mu(i,1)...
                    + mu(i+1,1)) / 2;
                m_C_plus(i+1,1) = tand(a_plus);
                x(i+1,1) = (y(i-1) - y(i) + m_C_plus(i+1) * x(i) - ...
                    m_C_minus(i+1) * x(i-1)) / (m_C_plus(i+1) - ...
                    m_C_minus(i+1));
                y(i+1,1) = y(i) + m_C_plus(i+1) * (x(i+1) - x(i));
            end
        end

    elseif  i == wall(j+1)    %Wall Points
        Km(i+1) = Km(1);
        Kp(i+1,1) = Kp(i,1);
        teta(i+1,1) = (Km(i+1,1) + Kp(i+1,1)) / 2;
        PM_ang(i+1,1) = (Km(i+1,1) - Kp(i+1,1)) / 2;

        Mach(i+1,1) = Machfinder(PM_ang(i+1,1));
        mu(i+1,1) = asind(1/Mach(i+1,1));
        a_wall = (teta(wall(j)+1) + teta(i)) / 2;
        a_plus = teta(i) + mu(i);

        m_C_wall(i+1,1) = tand(a_wall);
        m_C_plus(i+1,1) = tand(a_plus);
        x(i+1,1) = (y(wall(j)+1) - y(i) + m_C_plus(i+1,1) * x(i) - ...
            m_C_wall(i+1,1) * x(wall(j)+1)) / (m_C_plus(i+1,1) - m_C_wall(i+1)) ;
        y(i+1,1) = y(i) + m_C_plus(i+1,1) * (x(i+1) - x(i));

        wx(j+1,1) = x(i+1,1);
        wy(j+1,1) = y(i+1,1);

        j = j+1;
    end
end

m_C_minus(i+1) = 0;

%Display Outputs
Output.n(1) = "a ";
Output.n(2:nnode+1,1) = (node(2):nnode)';
Output.Km = Km;
Output.Kp = Kp;
Output.teta = teta;
Output.PM_ang = PM_ang;
Output.Mach = Mach;
Output.Machang = mu;
Output.slopem = m_C_minus;
Output.slopep = m_C_plus;
Output.walla = m_C_wall;

Output.x = x;
Output.y = y;

T = struct2table( Output );
T.Properties.VariableNames(1) = {'Point#'};
disp(T)

xi = linspace (min(x),max(x), 100);
yi_sphline = interp1(wx,wy,xi,'spline');
c(1:length(xi)) = 0;
centerlinex = nonzeros(centerlinex);

centerliney = zeros(length(centerlinex),1);

%%%%%%%% Plotting %%%%%%%
```

```
figure(1)
plot(xi,c,'--k'); hold on;
plot(centerlinex,centerliney,'.b'); hold on;
plot(wx,wy,'.b'); hold on;
plot(xi,yi_sphline,'k'); hold on;
for i = 1:length(centerlinex)
plot([wx(1) centerlinex(i)],[wy(1) centerliney(i)], '-r' ); hold on;
end

for i = 1:ncl
    plot( [centerlinex(i) wx(i+1)], [centerliney(i) wy(i+1)], '-b'  ); hold on
end

writetable(struct2table(Output), 'Project.xls')

%Function to Find Mach Number
f = @Machfinder;

function M = Machfinder(PM_ang)
gama = 1.4;
syms M
eq = - deg2rad(PM_ang) + sqrt((gama + 1) / (gama-1) ) * atan (sqrt((gama-1)/...
    (gama+1) * (M^2-1) )) - atan(sqrt(M^2-1)) == 0;
M = abs(double(vpasolve(eq,M)));

end
```