CFD Analysis of NACA 1408 at Varying Angles of Attack

Neslihan GULSOY*
Department of Aeronautical/Astronautical Engineering, Istanbul Technical University

I. Nomenclature

α	=	angle of attack (deg)
LE	=	leading edge
TE	=	trailing edge
c	=	chord length (m)
w	=	induced velocity (m/s)
γ	=	strength of vortex sheet
Γ	=	total circulation
ξ	=	distance from trailing edge (m)
A_0, A_n	=	coefficients of Fourier series
a_0	=	lift curve slope
$\alpha_{L=0})$	=	zero lift angle of attack (deg)
Re	=	Reynold number
c_d	=	drag coefficient
L'	=	lift force per unit span (N/m)
c_l	=	lift coefficient per unit span
$c_{m,c/4}$	=	moment coefficient per unit span around aerodynamic center
ρ_∞	=	density (kg/m^3)
m	=	mass (kg)
V_∞	=	free-stream velocity (m/s)
c	=	chord length (m)

II. Introduction

Thin airfoil theory was developed by Max Munk [1] for inviscid incompressible flow around an 2 dimensional airfoil with relatively small angle of attack by neglecting thickness effect. This theory has an important place in the aerodynamics application because it is a practical approach. This theory models a vortex sheet in a potential flow and it takes into account the Kutta condition at trailing edge to calculate lift and moment [2].

This paper is prepared as part of CA course and it contains a study on the classical thin airfoil theory for a cambered airfoil, NACA 1408. Shape of airfoil is generated by an online tool. Mean chamber line is obtained by curve fitting methods, then fitting curve is used as input for thin airfoil algorithm. After that necessary variables lift and moment coefficients are computed analytically. All calculation is conducted via MatLab. After that, obtained results are compared with experimental data to determine accuracy of theory. This study showed that, the theory gives similar results with experimental data at relatively small angles of attack. However, the error rate increases with angle of attack, and the theory is insufficient for angles exceeding the critical angle of attack.

*Undergraduate Student at ITU, Aerospace Engineering, gulsoy16@itu.edu.tr

III. Classical Thin Airfoil Theory Derivation

Thin airfoil theory is based on the assumption that shape of an airfoil with small thickness distribution can be replaced by its mean chamber line. This theory is derived for the condition that freestream is a potential flow and it makes use of the Kutta's Theorem . Theorem computes lift and moment acting on the airfoil by modelling strength of vortex sheet γ that ensures that Kutta's condition at trailing edge and the chamber line is a streamline of flow.

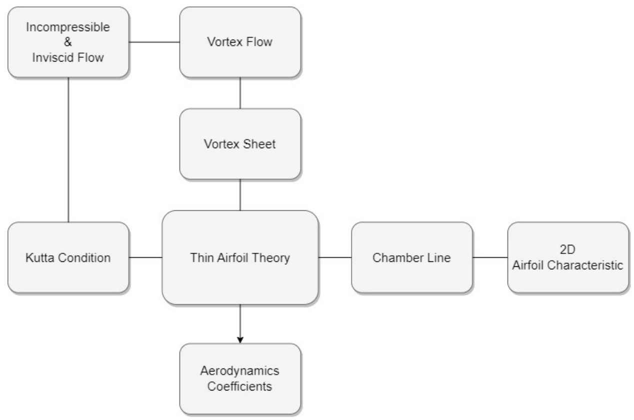


Fig. 1 Thin Airfoil Theory Scheme

For a thin airfoil whit a very small thickness compared to its chord ($t \ll c$) it can be assumed that the distribution of the circulation, vortex sheet, is placed along the chamber line. But, if the chamber of the airfoil is small then the circulation is distributed continuously along the chord line as shown in Fig. 2. Vortex strength per unit length is can be shown by $\gamma = \gamma(x)$ due to the small chamber assumption. Noted that the Kutta condition must be satisfied at the trailing edge $\gamma(c) = 0$.

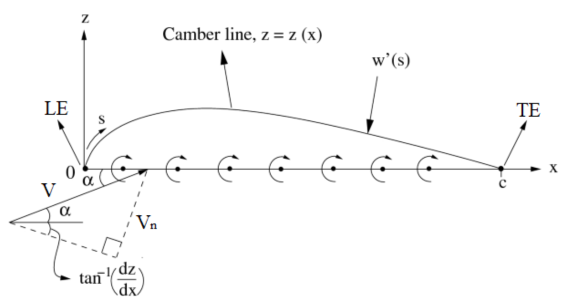


Fig. 2 Distribution of vortex on the chord line [3]

$w'(s)$ is the component of velocity normal to the camber line induced by the vortex sheet, $w(x)$ is the component of velocity normal to the chord line induced by the vortex sheet, and dz/dx is the slope of the chamber line.

It is known that the velocity normal to streamline will be zero. Then for chamber line to be streamline of flow below statement can be written.

$$V_{\infty,n} + w'(s) = 0 \tag{1}$$

From Fig. 2 it is seen that,

$$V_{\infty,n} = V_\infty sin\left[\alpha + tan^{-1}\left(-\frac{dz}{dx}\right)\right] \tag{2}$$

For at small α, $tan^{-1}(-dz/dx) \approx dz/dx$. So left term of Eq. 1 becomes

$$V_{\infty,n} = V_\infty(a - dz/dx) \tag{3}$$

To expand left term of Eq. 1 it is noted that for a small chamber $w'(s) \approx w(x)$. The tangential component of elemental velocity induced by the vortex is given as,

$$dv_\theta = dw(x) = -\frac{d\Gamma}{2\pi r} \tag{4}$$

At a distance ξ, circulation is $d\Gamma = \gamma(\xi)d\xi$. Also, it is noted that $r = x - \xi$. Then Eq. 4 can be rewritten as,

$$dw = -\frac{\gamma(\xi)d\xi}{2\pi(x-\xi)} \tag{5}$$

So velocity at x that is induced by all the elemental vortex sheets along the chord line is,

$$w(x) = -\frac{1}{2\pi}\int_0^c \frac{\gamma(\xi)d\xi}{(x-\xi)} \tag{6}$$

Then by substituting Eq. 3 and Eq. 6 to Eq. 1 below equation can be obtained.

$$\frac{1}{2\pi}\int_0^c \frac{\gamma(\xi)d\xi}{(x-\xi)} = V_\infty(a - dz/dx) \tag{7}$$

The polar coordinate system is more convenient than Cartesian system. Coordinate transformation is introduced as $\xi = c/2(1 - cos\theta)$, $d\xi = c/2sin\theta d\theta$ and $x = c/2(1 - cos\theta_0)$. θ is the general angle while θ_0 represents a fixed value of angle. So Eq. 7 is rewritten as,

$$\frac{1}{2\pi}\int_0^\pi \frac{\gamma(\xi)sin\theta d\theta}{(cos\theta - cos\theta_0)} = V_\infty(a - dz/dx) \tag{8}$$

Eq. 8 is fundamental governing equation of Thin Airfoil Theory. Aim of this paper is solving this equation for a chambered airfoil, NACA 1408. Therefore, next section focus on solution for chambered airfoil.

A. Thin Chambered Airfoil

For a thin chambered airfoil to find the distribution of the circulation along the chord line is more complex than symmetric airfoil with $dz/dx = 0$ solution. Though strength of vortex sheet can be expanded in form of Fourier series as,

$$\gamma(\theta) = 2V_\infty\left(A_0\frac{cos\theta + 1}{sin\theta} + \sum_{n=1}^{\infty} A_n sinn\theta\right) \tag{9}$$

After insertion Eq. 9 to Eq. 8,

$$\frac{1}{\pi}\int_0^\pi \frac{A_0 cos\theta + 1}{cos\theta - cos\theta_0}d\theta + \frac{1}{\pi}\sum_{n=1}^{\infty}\int_0^\pi \frac{A_n sinn\theta sin\theta d\theta}{cos\theta - cos\theta_0} = \alpha - \frac{dz}{dx} \tag{10}$$

By using Glauert's integrals [4] and the help of trigonometric series, coefficients of Fourier series are found as,

$$A_0 = \alpha - \frac{1}{\pi}\int_0^\pi \frac{dz}{dx}d\theta_0 \tag{11}$$

$$A_n = \frac{2}{\pi}\int_0^\pi \frac{dz}{dx}cosn\theta_0 d\theta_0 \tag{12}$$

Total circulation on airfoil given by $\Gamma = c/2\int_0^\pi \gamma(\theta)sin\theta d\theta$ also found as,

$$\Gamma = cV_\infty\left(\pi A_0 + \frac{\pi}{2}A_1\right) \tag{13}$$

Lift per span of airfoil can be calculated from Kutta-Joukowski theorem that is,

$$L' = \rho_\infty V_\infty \Gamma \tag{14a}$$

$$= \rho_\infty V_\infty^2 c\left(\pi A_0 + \frac{\pi}{2}A_1\right) \tag{14b}$$

With lift force, an important unitless aerodynamic variable, lift coefficient per unit span can be computed as,

$$c_l = \frac{L'}{1/2\rho_\infty V_\infty^2 c(1)} \tag{15a}$$

$$= \rho_\infty V_\infty^2 c\left(\pi A_0 + \frac{\pi}{2}A_1\right) \tag{15b}$$

$$= 2\pi\left[\alpha + \frac{1}{\pi}\int_0^\pi \frac{dz}{dx}\left(cos\theta_0 - 1\right)d\theta_0\right] \tag{15c}$$

The lift curve slope defines the relation between the angle of attack and the lift coefficient $c_l = a_0(\alpha - \alpha_{L=0})$. For a thin airfoil it will be $a_0 = 2\pi$. So zero lift angle of attack can be written by given statements as,

$$\alpha_{L=0} = -\frac{1}{\pi}\int_0^{\pi}\frac{dz}{dx}(cos\theta_0 - 1)d\theta_0 \tag{16}$$

Other important aerodynamic coefficient is moment coefficient per unit span and value of it around aerodynamic center is computed as,

$$c_{m,c/4} = \frac{\pi}{4}(A_2 - A_1) \tag{17}$$

By the derivation above theory asserts that : 1) lift slope for a cambered airfoil is 2π, 2) lift coefficient is function of angle of attack and mean chamber line, 3) moment coefficient around aerodynamics center depend only on the shape of airfoil.

IV. Numerical Application of Theory

Derived formula mentioned in Sec. III are applied to a conventional airfoil, NACA 1408. Firstly shape of airfoil is obtained by an online airfoil generator at Ref. [5] with 200 station. Shape data is transferred to MatLab environment for calculation. To obtained chamber line, it is assumed that y-axis values of both upper and bottom profile placed at same x-axis station. Then mean chamber stations are obtained by mean value of y-axis values. Then for analytical calculation, chamber line is found by fourth degree curve fitting of mean chamber. So, an robust curve is obtained with $R^2 = 0.9996$ as shown at Fig 3.

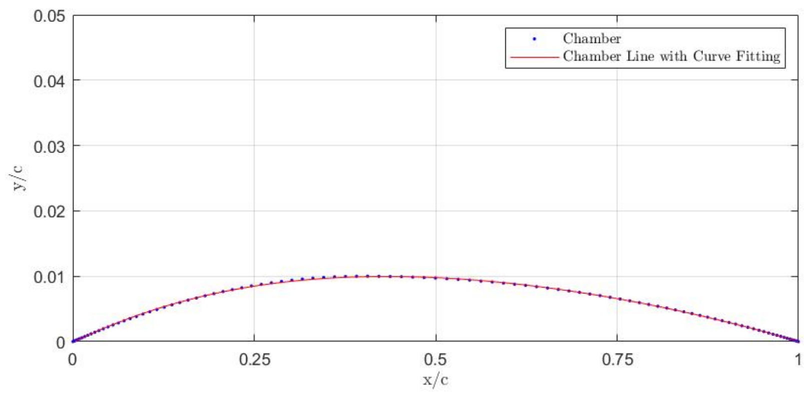


Fig. 3 Chamber Line as 4^{th} Polynomial Curve

Mean chamber line and differential it in terms of chord length of are given with,

$$\frac{y_c}{dx} = -0.0003\left(\frac{x}{c}\right)^4 + 0.0009\left(\frac{x}{c}\right)^3 - 0.0044\left(\frac{x}{c}\right)^2 - 0.0018\left(\frac{x}{c}\right) + 0.0098$$

$$\frac{dy_c}{dx} = -0.0010\left(\frac{x}{c}\right)^3 + 0.0028\left(\frac{x}{c}\right)^2 - 0.0088\left(\frac{x}{c}\right) - 0.0018$$

Then by coordinate transformation given with $x = (c/2)(1 - cos\theta)$, slope of chamber line is obtained in terms of θ as,

$$\frac{dz}{dx} = 1.3\times10^{-4}\cos\theta^3 + 3.0284\times10^{-4}\cos\theta^2 + 0.0034\cos\theta - 0.0056$$

Also, necessary Fourier coefficients are calculated with 'Symbolic Math Toolbox' as,

$$A_{0@\alpha=0^\circ} = 0.0055, \quad A_1 = 0.0035, \quad A_2 = 1.5142\times10^{-4}, \quad A_3 = 3.25\times10^{-5}, \quad A_{4,\ldots,n} = 0$$

Then, strength of vortex sheet per unit length along the camber line at zero angle of attack is found as,

$$\gamma(\theta) = 2V_\infty\left(\frac{\cos\theta + 1}{\sin\theta}(\alpha + 0.0055) + 3.25\times10^{-5}\sin 3\theta + 1.5142\times10^{-4}\sin 2\theta + 0.0035\sin\theta\right)$$

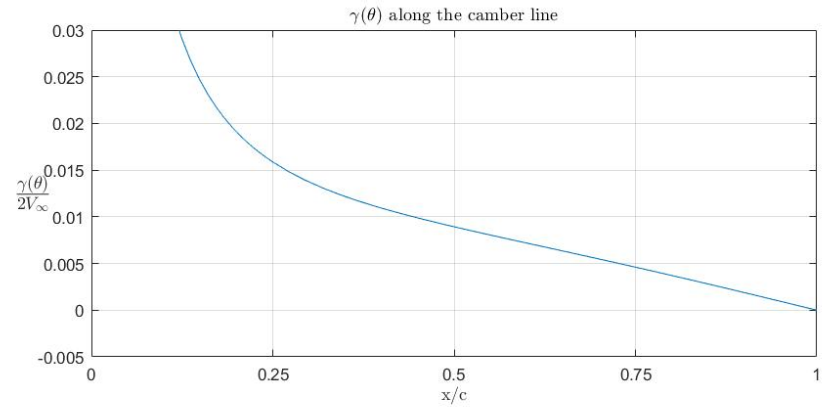
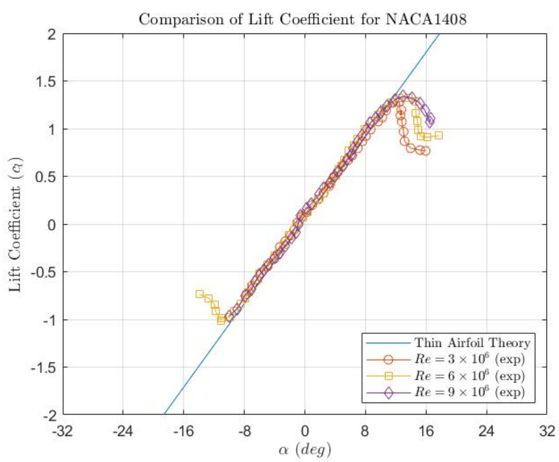
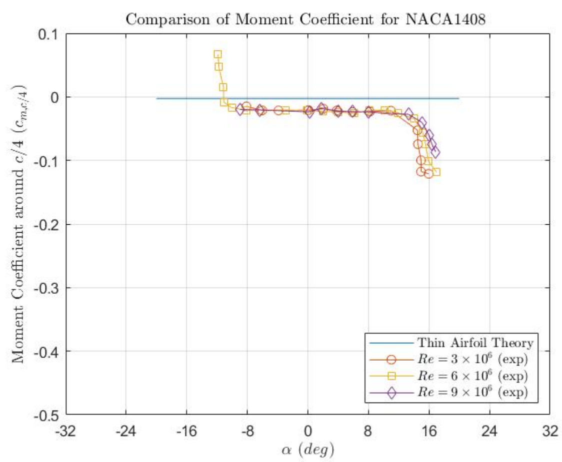


Fig. 4 $\gamma(\theta)$ Along the Chamber Line



(a) Lift Coefficient Comparison



(b) Moment Coefficient Comparison

Fig. 5 Comparison of Aerodynamic Characteristic of NACA 1408

Under zero angle of attack $\gamma(\theta)$ is given at Fig. 4. Analytical c_l under varying angle of attack is given at Fig 5a.

Also other important variables zero lift angle of attack, $\alpha_{L=0}$ and pitching moment coefficient around aerodynamic center $c_{m,c/4}$ are computed as,

$$\alpha_{L=0} = -0.4138°, \qquad c_{m,c/4} = -0.0026$$

V. Comparison with Experimental Data& Conclusion

To investigate the limit and accuracy of thin airfoil theory, experimental data for NACA 1408 and analytical results are compared in Fig. 5. Experimental data are obtained from Ref. [6] by a digitizer program Ref. [7].

As seen in Fig. 5a and Table 1 the Thin Airfoil Theory gives results with error of $\approx 10\%$ at relatively small angles of attack, i.e. in the range of $\alpha \pm 6°$. Although this error rate is not at the desired level, it is acceptable considering the simplicity of the theory. However, as the angle of attack increases, the error rate continues to increase and approaches after a certain angle range. Limits of this range is called as critical angle of attack and it is approximately $\alpha = -15°$ and $\alpha = 16°$ for NACA 1408. When this limits are exceeded, the airplane is in a stall condition and theory is no longer usable.

On the other hand when Fig. 5b is examined it is seen that, $c_{m,c/4}$ computed with thin airfoil theory is constant while experimental $c_{m,c/4}$ varies with angle of attack. Error varies with angle of attack and error in $c_{m,c/4}$ is about 50 times larger than error in c_l in range of $\alpha \pm 6°$. This situation is due to the inference that $c_{m,c/4}$ calculated with theory only depend on mean chamber line of airfoil.

α (deg)	**Lift Coefficient c_l**				**Moment Coefficient $c_{m,c/4}$**			
	Thin Airfoil Theory	Experimental Data			Thin Airfoil Theory	Experimental Data		
		Re = $3x10^6$	Re = $6x10^6$	Re = $9x10^6$		Re = $3x10^6$	Re = $6x10^6$	Re = $9x10^6$
-16	-1.7092	NaN				NaN		
-8	-0.8319	≈-0.7				≈ -0.015		
-4	-0.3933	≈-0.34				≈-0.021		
0	0.0454	≈ 0.05			-0.0026	≈ -0.022		
4	0.4840	≈ 0.54				≈ -0.023		
8	0.9227	≈ 0.95				≈ -0.024		
16	1.8000	≈ 0.8	≈ 0.9	≈ 1.2		≈ -0.12	≈ -0.1	≈-0..06

Table 1 Table

As a conclusion, the thin airfoil theory, which was derived nearly 3 decades ago, is still used frequently today to compute lift and moment acting on aircraft. The accuracy of the theory depends on the accuracy of potential flow assumption and ratio of airfoil thickness to it's chord length. To test accuracy and limits of thin airfoil theory, it is applied for a chambered airfoil NACA 1408.

With all the above results, the following conclusions can be drawn, thin airfoil theory lack the ability to model stall conditions. Error when stall occurs, reaches to ∞. Because the thin airfoil theory is based on potential flow assumption, as known viscous effect increases with angle of attack and theory makes no inferences about drag. Although, theory is useful and it can be preferred for small angle of attack due to it's simplicity.

References

[1] Munk, M. M., "NACA-TR-191 : Elements of the Wing Section Theory and of the Wing Theory," , 1925. ID : 19930091257.

[2] Ali, N., Komol, M. M., and Saki, M., "Study on Thin Airfoil Theory Performance Test of Elliptical Wing as Compared to Model Mosquito Wing and NACA 64A012 Mod Airfoil," *European Journal of Engineering Research and Science*, Vol. 3, 2018, p. 48. https://doi.org/10.24018/ejers.2018.3.4.665.

[3] Kaushik, M., *Thin Airfoil Theory*, Springer Singapore, Singapore, 2019, pp. 127–144. https://doi.org/10.1007/978-981-13-1678-4_5, URL https://doi.org/10.1007/978-981-13-1678-4_5.

[4] Gradinaru, and Stelian, "PAIRS OF GLAUERT INTEGRALS," , n.d. URL https://citeseerx.ist.psu.edu/document?repid=rep1&type=pdf&doi=43923df9b700146d4c1a3cdbee995ec76927b030.

[5] "NACA 4 digit airfoil generator (NACA 2412 AIRFOIL)," , n.d. URL http://airfoiltools.com/airfoil/naca4digit.

[6] Abbott, I. H., Doenhoff, A. E. V., and Stivers, L. S., "Report No. 824 - Summary of Airfoil Data," , n.d. URL https://engineering.purdue.edu/~aerodyn/AAE514/Spring%202011/naca-report-824.pdf.

[7] "WebPlotDigitizer - Copyright 2010-2022 Ankit Rohatgi," , n.d. URL https://apps.automeris.io/wpd/.