Introduction

WQD7011 Numerical Optimization

Numerical Optimization

- Objectives of the course:
    1. Explain the key principles and values pertinent to numerical optimization and linear algebra.
    2. Apply and implement numerical solution methods.
    3. Interpret the numerical solutions with respect to their accuracy and suitability.
- Topics including:
    - Unconstrained Optimization (Line Search and Trust Region)
    - Constrained Optimization (Linear and Quadratic Programming)
- Tools: Octave and Python

Form of Conduct

- Friday, 6pm to 9pm (MM4)
- Lecture + Tutorial + Lab
- All the materials will be uploaded to Spectrum
- All the submission will be via Spectrum.
- Announcement and notices are made via Spectrum.
- Additional exercises will be given every week (independent learning).
- Applied + Problem solving
- Class Representative – point of contact
- Telegram (username: ongsimying)

References

1. Numerical Optimization, Jorge Nocedal and Stephen Wright, second edition, Springer-Verlag, 2006.
2. Numerical Methods and Optimization: An Introduction Sergiy Butenko, Panos M. Pardalos, Chapman and Hall/CRC 2014

Assessments

- Continuous Assessments (Week 1 to Week 14):
  - Quizzes (10 marks)
  - Mid Term Examination (20 marks)
  - Group Assignment (30 marks)
    - Part 1 (10 marks)
    - Part 2 (20 marks)
- Final Examination (40 marks)

Lecturer

- S. ONG
- Email: simying.ong@um.edu.my
- Telegram: ongsimying
- Room No.: A-03-2D
- Consultation

Your Turn for Introduction

- Enter the following link: https://padlet.com/ongsimying/WQD7011Intro

- Then, post a **note** which contains:
  - Your selfie
  - Your name (nickname to be called in the class)
  - Your background (Bachelor Degree)
  - Other information

Introduction to Optimization

- Optimization is:
  - Important tool in decision science and analysis if physical systems.
  - First, identify some objective (e.g., profit, time, potential energy) – also called as variables or unknown
  - Then, identify a quantitative measure of the performance of the system.
  - Goal: find the values of variables that optimize the objective.

Modelling

- Process of identifying objective, variables, and constraints for a given problem.
- What happened if
  - The model is too simplistic?
  - The model is too complex?

Optimization Algorithm

- After model formulation, optimization algorithm is applied to find its solution, usually WITH COMPUTER.
- NO universal algorithm for all solutions
- Collection of algorithm – tailored to a particular type of optimization problem
- Who choose the algorithm?
  - User

Successful Application?

- Recognize whether solution + algorithm has succeeded in its task of finding a solution.
- Optimality Conditions
  - Checking that the current set of variables is indeed the solution of the problem
  - If NOT satisfied, useful information can be extracted to improve the estimation.
- Sensitivity Analysis
  - Reveals the sensitivity of the solution to changes in the model and data
- **Any changes to the model? Process repeated.**

Mathematical Formulation

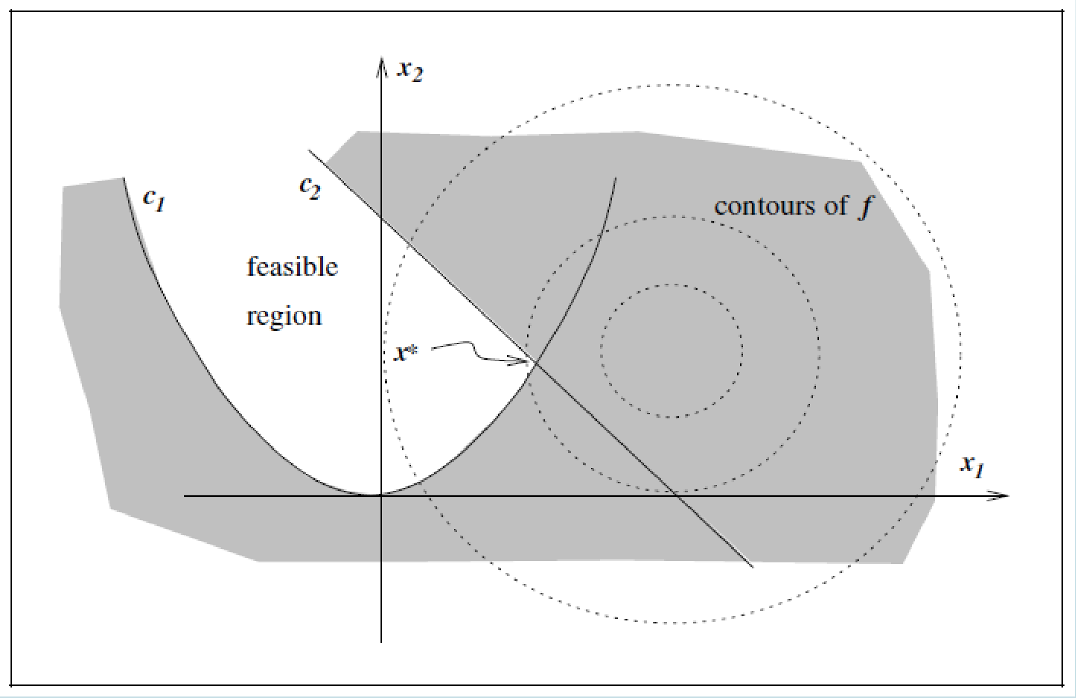
- Optimization is the minimization or maximization of a function subject to constraints on its variables.
- Formal notation:
  - x is the vector of variables, also called as unknowns or parameter.
  - f is the objective function, a (scalar) function of x that we want to maximize or minimize.
  - c_i are constraint functions, which are scalar functions of x that define certain equations and inequalities that the unknown vector x must satisfy

Mathematical Formulation

- Optimization problem can be written as follows:
- $\min_{x \in R^n} f(x)$ subject to $\begin{matrix} c_i(x) = 0, i \in \varepsilon \\ c_i(x) \geq 0. i \in \tau \end{matrix}$
- where ε and τ are sets of indices for equality and inequality constraints, respectively.

Mathematical Formulation (Example)

- Consider the problem:
- $\min_{x \in R^n} (x_1 - 2)^2 + (x_2 - 1)^2$ subject to $\begin{matrix} x_1^2 - x_2 \leq 0 \\ x_1 + x_2 \leq 2 \end{matrix}$



Example: Transportation Problem

We begin with a much simplified example of a problem that might arise in manufacturing and transportation. A chemical company has 2 factories F_1 and F_2 and a dozen retail outlets $R_1, R_2, \ldots, R_{12}$. Each factory F_i can produce a_i tons of a certain chemical product each week; a_i is called the *capacity* of the plant. Each retail outlet R_j has a known weekly *demand* of b_j tons of the product. The cost of shipping one ton of the product from factory F_i to retail outlet R_j is c_{ij}.

The problem is to determine how much of the product to ship from each factory to each outlet so as to satisfy all the requirements and minimize cost. The variables of the problem are x_{ij}, $i = 1, 2$, $j = 1, \ldots, 12$, where x_{ij} is the number of tons of the product shipped from factory F_i to retail outlet R_j; see Figure 1.2. We can write the problem as

$$\min \sum_{ij} c_{ij} x_{ij} \tag{1.3a}$$

$$\text{subject to} \sum_{j=1}^{12} x_{ij} \le a_i, \quad i = 1, 2, \tag{1.3b}$$

$$\sum_{i=1}^{2} x_{ij} \ge b_j, \quad j = 1, \ldots, 12, \tag{1.3c}$$

$$x_{ij} \ge 0, \quad i = 1, 2, \quad j = 1, \ldots, 12. \tag{1.3d}$$

Continuous VS Discrete Optimization

- Discrete optimization: takes in integer and binary variables, or specific sets of objects
  - Including: Integer programming
- Continuous optimization: uses continuous variables
  - Feasible set is usually uncountably infinite, as when the components of x are allowed to be real numbers.
- Which ONE is easier to solve? And WHY?

Stochastic and Deterministic Optimization

- Stochastic Optimization
  - Model cannot be fully specified because it depends on quantities that are unknown at the time of formulation.
  - Example: future interest rates, future demands for a product.
  - Use quantifications (based on users knowledge) of the uncertainty to produce solutions that optimize the expected performance of the model.
- Deterministic Optimization
  - Model is completely known

Optimization Algorithms

- Iterative
- Begin with initial guess of the variable x and generate a sequence of improved estimates (called "iterates") until they terminate, hopefully at solution.
- Strategies on moving are varies.
- Unconstrained optimization (unbound values): Line Search and Trust Region

Optimization Algorithms

- Good algorithm should posses these properties:
  - Robustness: should perform well on a wide variety of problems in their class, for all reasonable values of the starting points.
  - Efficiency: Should not require excessive computer time or storage.
  - Accuracy: Should be able to identify a solution with precision, without being overly sensitive to errors in the data or to the arithmetic rounding errors when implemented in computer.
- These goals are conflicting sometimes. HOW?

REVISION QUESTIONS

GROUP DISCUSSION

- Group yourself in a group of 3-5 persons.
- Identify the daily-life problem that can be solved by using optimization.
- Present it to the class:
  - Problem
  - Objective
  - Variables / parameter involved
  - Constraints