PRE-SESSION MATH EXAM

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1.1. $\frac{x^{n+2}}{x^{n-2}} = x^{n+2-n+2} = x^4$

1.2. $x^{-1} \cdot 8 = 2$

$\frac{8}{x} = 2$

$x = 4$

1.3. $a = 5$

$b = 10$

$(a^b)^0 = 1$

1.4. $\frac{\sqrt{4x}}{\sqrt{x}} = \frac{2\sqrt{x}}{\sqrt{x}} = 2$

1.5. $x^2 + (x+1)^2 = (x+2)^2$

$x^2 + (x^2+2x+1) = x^2+4x+4$

$2x^2+2x+1 = x^2+4x+4$

$x^2 - 2x - 3 = 0$

$x_{1,2} = \frac{2 \pm \sqrt{4+12}}{2} = \frac{2 \pm 4}{2} = \begin{cases} x_1 = 3 \\ x_2 = -1 \end{cases}$

1.6. $2^x > 1024$

$\ln 2^x > \ln 1024$

$x \ln 2 > \ln 1024$

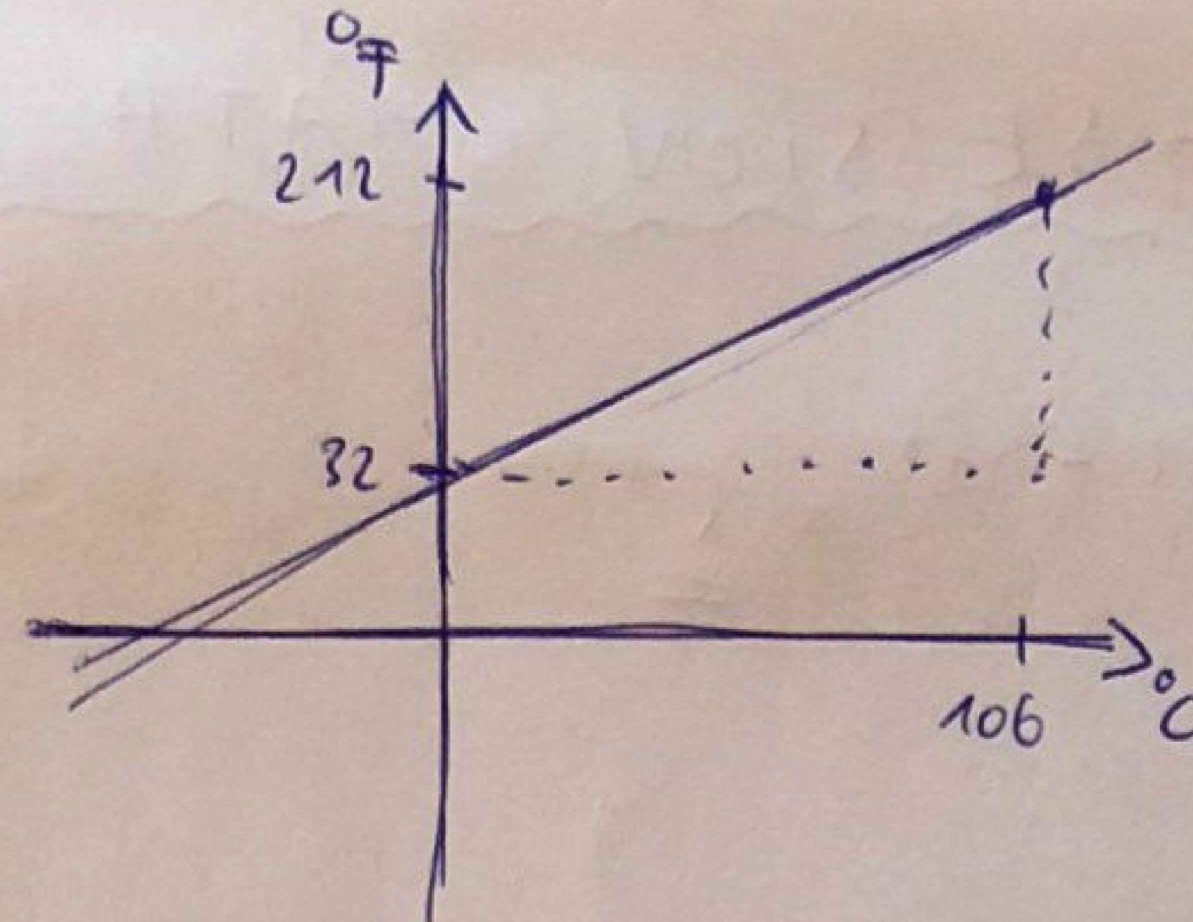
$x > \frac{\ln 1024}{\ln 2}$

$x > \frac{\ln 2^{10}}{\ln 2}$

$x > \frac{10 \ln 2}{\ln 2}$

$x > 10$

2.1.



$0°C = 32F$

$100C = 212F$

slope: $\frac{212-32}{100-0} = \underline{1.8} \rightarrow \underline{F = 1.8C + 32}$

$C = 1.8C + 32$

$-32 = 0.8C$

$\underline{-40 = C} \rightarrow 1.8 \cdot (-40) + 32 = \underline{\underline{-40}}$

2.2. $f(x) = 5x + 4$

$y = f(3) = 5 \cdot 3 + 4 = \underline{\underline{19}}$

2.3. $x^2 - 4x + 3 = 0$

$(x-1)(x-3) = 0$

$$\boxed{\begin{array}{l} x_1 = 1 \\ x_2 = 3 \end{array}}$$

2.4. In case of annual compounding, effective interest rate:

$10 \cdot (1.02)^{90} = \underline{\underline{59.4313}}$

In case of continuous compounding:

$10 \cdot e^{0.02 \cdot 90} = 10 \cdot e^{1.8} = \underline{\underline{60.4965}}$

2.5. $e^{\ln 5} = 5$ (by definition $a^{\log_a b} = b$)

3.1. $\sum_{i=1}^{\infty} \frac{12}{6^i}$

summation formula for an infinite geometric series:

$a + ar + ar^2 \ldots = \frac{a}{1-r}$, if $|r| < 1$

$\downarrow$

$a = 12$
$r = \frac{1}{6}$ $\rightarrow \sum_{i=1}^{\infty} \frac{12}{6^i} = \frac{12}{5/6} - 12 = \underline{\underline{2.4}}$

3.2. $\lim_{x \to 1} \frac{6^{1-x}}{x} = \underline{\underline{1}}$

$6^{1-x} \rightarrow 1$

$x \rightarrow 1$

3.3. $f(x) = x^5 - 8$ at $x = -3$

$f'(x) = 5 \cdot x^4 \rightarrow 5(-3)^4 = \underline{\underline{405}}$

3.4. $\frac{d}{dx} \frac{x^3 + 2x - 1}{x - 2} = \frac{(3x^2 + 2)(x-2) - (x^3 + 2x - 1) \cdot 1}{(x-2)^2} =$

$= \frac{3x^2 \cdot x - 6x^2 + 2x - 4 - x^3 - 2x + 1}{(x-2)^2} = \underline{\underline{\frac{2x^3 - 6x^2 - 3}{(x-2)^2}}}$

3.5. $\frac{d^2}{dx^2} 4x^4 + 4x^2 = (16x^3 + 8x)' = \underline{\underline{48x^2 + 8}}$

3.6. $\frac{d}{dx} \frac{\ln x}{e^x} = \frac{1/x \cdot e^x - e^x \cdot \ln x}{e^{2x}} = \frac{e^x(1/x - \ln x)}{e^{2x}} = \underline{\underline{\frac{1/x - \ln x}{e^x}}}$

3.7. $f(x) = 3x^2 - 5x + 2$

- $\underline{f(x) = 0}$

$3x^2 - 5x + 2 = 0$

$$x_{1,2} = \frac{5 \pm \sqrt{25 - 4 \cdot 3 \cdot 2}}{6} = \frac{5 \pm 1}{6} = \begin{cases} \underline{x_1 = 1} \\ \underline{x_2 = \frac{2}{3}} \end{cases}$$

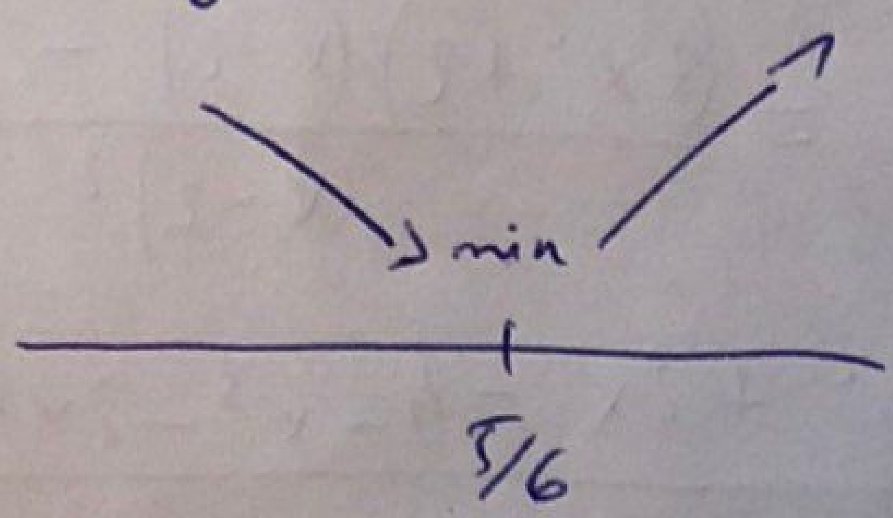
- $\underline{f'(x) = 0}$

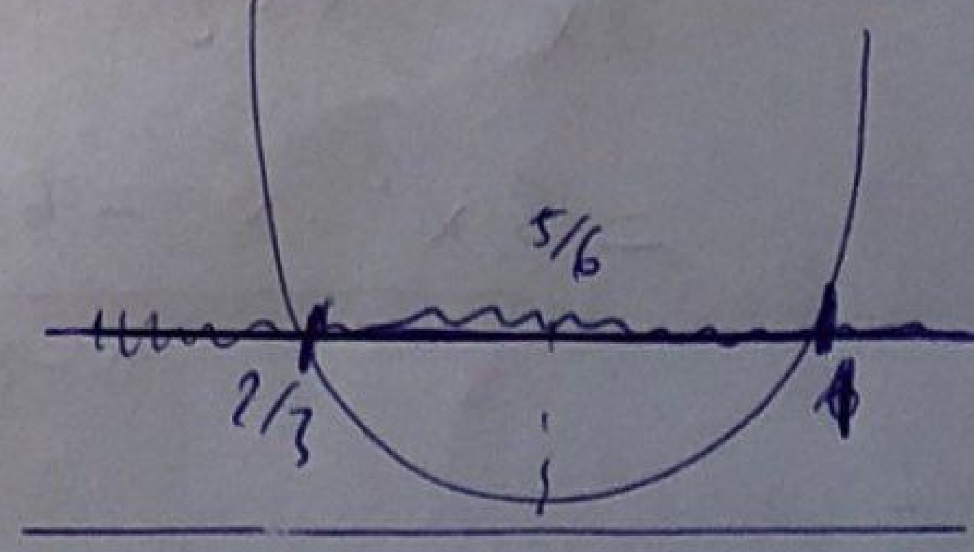
$f'(x) = 6x - 5 = 0$

$\underline{x = \frac{5}{6}}$ → the function has a minimum/maximum point at $x = \frac{5}{6}$

- check $x > \frac{5}{6}$ and $x < \frac{5}{6}$

$$\left.\begin{aligned} f'(1) &= 6 \cdot 1 - 5 = 1 > 0 \\ f'(0) &= 6 \cdot 0 - 5 = -5 < 0 \end{aligned}\right\} \rightarrow \underline{x = \tfrac{5}{6}} \text{ is a local minimum}$$





- check the second derivative

$f''(x) = (6x - 5)' = 6 > 0$ → convex ∪

→ ∅ inflection point, no convexity changes

x	$x < 2/3$	$2/3$	$2/3 < x < 5/6$	$5/6$	$5/6 < x < 1$	1	$1 < x$
$f(x)$	+	0	−	−	−	0	+
$f'(x)$	−	−	−	0	+	+	+
shape	↘	↘	↘	local minimum	↗	↗	↗
$f''(x)$	+	+	+	+	+	+	+
convexity	convex ∪	∪	∪	∪	∪	∪	∪

3.8. $f(x,y) = x^2+y^3$

$f(2,3) = 2^2+3^2 = 4+27 = \underline{\underline{31}}$

3.9. $f(x,y) = \ln(x-y)$

$x-y>0$

$\underline{\underline{x>y}}$

3.10. $\frac{\partial x + xy^3}{\partial x} = \underline{\underline{5x^4+y^3}}$

3.11. $f(x,y) = x^2y^2+10$

$$\left.\begin{aligned} f'_x &= 2x\cdot y^2 = 0 \\ f'_y &= 2y\cdot x^2 = 0 \end{aligned}\right\} \rightarrow \underline{\text{local minimum for every } x=0 \ \& \ y=0}$$

3.12. $f(x) = x^2y^2 \rightarrow \max$ constraint: $x+y=10$

$\downarrow$

$x+y-10=0$

$$\boxed{\mathcal{L} = x^2y^2 - \lambda(x+y-10)}$$

$$\left.\begin{aligned} \frac{\partial \mathcal{L}}{\partial x} &= 2xy^2 - \lambda = 0 \\ \frac{\partial \mathcal{L}}{\partial y} &= 2yx^2 - \lambda = 0 \end{aligned}\right\} \rightarrow 2xy^2 = 2yx^2$$

$xy^2 = yx^2$

$\boxed{x=y}$

$\frac{\partial \mathcal{L}}{\partial \lambda} = x+y-10 \longrightarrow x+x-10=0$

$2x=10$

$\boxed{\begin{aligned} x&=5 \\ y&=5 \end{aligned}}$

4.1.

$$A = \begin{bmatrix} 2 & 6 \\ 5 & 1 \\ 1 & 9 \end{bmatrix} \qquad B = \begin{bmatrix} 1 & 1 & 7 \\ 2 & 8 & 2 \end{bmatrix}$$

$$A \cdot B = \begin{bmatrix} 2 \cdot 1 + 6 \cdot 2 & 2 \cdot 1 + 6 \cdot 8 & 2 \cdot 7 + 6 \cdot 2 \\ 5 \cdot 1 + 1 \cdot 2 & 5 \cdot 1 + 1 \cdot 8 & 5 \cdot 7 + 1 \cdot 2 \\ 1 \cdot 1 + 9 \cdot 2 & 1 \cdot 1 + 9 \cdot 8 & 1 \cdot 7 + 9 \cdot 2 \end{bmatrix}$$

$$A \cdot B = \begin{bmatrix} 14 & 50 & 26 \\ 7 & 13 & 37 \\ 19 & 73 & 25 \end{bmatrix}$$

4.2.

$$A = \begin{bmatrix} 2 & 2 \\ 4 & 6 \\ 1 & 3 \end{bmatrix} \qquad B = \begin{bmatrix} 1 & 9 & 1 \\ 2 & 1 & 2 \end{bmatrix}$$

$$B \cdot A = \begin{bmatrix} 1 \cdot 2 + 9 \cdot 4 + 1 \cdot 1 & 1 \cdot 2 + 9 \cdot 6 + 1 \cdot 3 \\ 2 \cdot 2 + 1 \cdot 4 + 2 \cdot 1 & 2 \cdot 2 + 1 \cdot 6 + 2 \cdot 3 \end{bmatrix}$$

$$B \cdot A = \begin{bmatrix} 39 & 59 \\ 10 & 16 \end{bmatrix}$$

4.3.

$$A = \begin{bmatrix} 7.1 & 9.1 & 4.7 \\ 2 & 7.8 & 1.1 \\ 4 & 4.44 & 0 \end{bmatrix} \rightarrow A^T = \begin{bmatrix} 7.1 & 2 & 4 \\ 9.1 & 7.8 & 4.44 \\ 4.7 & 1.1 & 0 \end{bmatrix}$$

4.4

$$A = \begin{bmatrix} 1 & 9 \\ 2 & 8 \end{bmatrix}$$

$$\det(A) = 1 \cdot 8 - 9 \cdot 2 = \underline{\underline{-10}}$$

5.1.

$$\Omega = \{11, 12, 13, 14, 15, 16,$$
$$21, 22, 23, 24, 25, 26,$$
$$31, 32, 33, 34, 35, 36,$$
$$41, 42, 43, 44, 45, 46,$$
$$51, 52, 53, 54, 55, 56,$$
$$61, 62, 63, 64, 65, 66\}$$

5.2.

1% use drug → 99% accuracy

99% Ø use drug → 99.5 accuracy

method 1

let's say sample = 10.000

		Predicted		
		T	F	Σ
Actual	T	99	1	100
	F	49.5	9850.5	9900
				10.000

$$\frac{99 + 49.5}{10.000} = 0.01485 = 1.485\%$$

method 2

A : positive test

B_1 : drug use

B_2 : non drug user

$$P(A) = \sum_{i=1}^{2} P(B_i) \cdot P(A \mid B_i) =$$

$$= 0.01 \cdot 0.99 + 0.99 \cdot 0.005 = 0.01485 = \underline{\underline{1.485\%}}$$

5.3.

$$P(B_1|A) = \frac{P(A|B_1) \cdot P(B_1)}{\sum_{i=1}^{2} P(B_i) \cdot P(A|B_i)} =$$

$$= \frac{0.01 \cdot \cancel{0.99}}{0.01 \cdot \cancel{0.99} + \cancel{0.99} \cdot 0.005} = \frac{0.01}{0.015} = \frac{2}{3} = \underline{\underline{66.\dot{6}\,\%}}$$