Assignment #2
Numerical Computing (COMP 350)

Guillaume Labranche (260585371)

due on 30 September 2015

1. $x_{single} = 41$ and $x_{double} = 49$
2. Because this sequence initally rises past N_{max} ($3.402823466 \cdot 10^{38}$) before converging to 0. So at some n, $x_n = \infty$ and since the program depends on x_n to compute x_{n+1}, once it hits such $x_n = \infty$ it will always stay at ∞. The sequence rises to high values before coming back down because 100^n is much higher than $n!$ for at least the first 100 values of n. **Bonus**: See function *bonus()* in file q2.c .
3. (a) At first the sequence converges to 0, but then diverges. This is because for large values of n, 2^{-n} is close to 0 and therefore $\sqrt{1 + 2^n x_n}$ goes to 1 and so $(\sqrt{1 + 2^n x_n} - 1)$ goes to 0. This cancelation error (loss of significance) is then amplified by the term 2^{n+1}. So we end up with lots of errors in the calculation as n gets larger.

n	x_n	$x_n - \ln(x_0 + 1)$
1	0.8284271247461901	0.1352799441862448
2	0.7568284600108842	0.0636812794509389
3	0.7240618613220612	0.0309146807621159
4	0.7083805188386214	0.0152333382786761
5	0.7007087569317336	0.0075615763717883
6	0.6969143073088293	0.0037671267488840
7	0.6950273424387610	0.0018801618788157
8	0.6940864128518361	0.0009392322918909
9	0.6936165847594014	0.0004694041994561
10	0.6933818296999492	0.0002346491400039
11	0.6932644918933768	0.0001173113334315
12	0.6932058329179385	0.0000586523579932
13	0.6931765059118140	0.0000293253518687
14	0.6931618430291042	0.0000146624691589
15	0.6931545117428328	0.0000073311828875
16	0.6931508461384652	0.0000036655785199
17	0.6931490133459732	0.0000018327860279
18	0.6931480969521431	0.0000009163921978
19	0.6931476387558178	0.0000004581958725
20	0.6931474096578540	0.0000002290979088
21	0.6931472951089290	0.0000001145489837
22	0.6931472378346371	0.0000000572746918
23	0.6931472091973774	0.0000000286374321
24	0.6931471948792023	0.0000000143192570
25	0.6931471877214790	0.0000000071615337
26	0.6931471841453458	0.0000000035854005
27	0.6931471823627362	0.0000000018027909
28	0.6931471814750694	0.0000000009151241
29	0.6931471810094081	0.0000000004494628
30	0.6931471808347851	0.0000000002748398
31	0.6931471808347851	0.0000000002748398
32	0.6931471806019545	0.0000000000420092
33	0.6931471806019545	0.0000000000420092
34	0.6931471806019545	0.0000000000420092
35	0.6931471787393093	−0.0000000018206360
36	0.6931471750140190	−0.0000000055459263
37	0.6931471675634384	−0.0000000129965069
38	0.6931471526622772	−0.0000000278976681
39	0.6931471228599548	−0.0000000576999905
40	0.6931470632553101	−0.0000001173046352
41	0.6931469440460205	−0.0000002365139248
42	0.6931467056274414	−0.0000004749325039
43	0.6931467056274414	−0.0000004749325039
44	0.6931457519531250	−0.0000014286068203
45	0.6931457519531250	−0.0000014286068203
46	0.6931457519531250	−0.0000014286068203
47	0.6931457519531250	−0.0000014286068203
48	0.6931457519531250	−0.0000014286068203
49	0.6931152343750000	−0.0000319461849453
50	0.6931152343750000	−0.0000319461849453
51	0.6931152343750000	−0.0000319461849453
52	0.6928710937500000	−0.0002760868099453
53	0.6923828125000000	−0.0007643680599453
54	0.6914062500000000	−0.0017409305599453
55	0.6914062500000000	−0.0017409305599453
56	0.6875000000000000	−0.0056471805599453
57	0.6875000000000000	−0.0056471805599453
58	0.6875000000000000	−0.0056471805599453
59	0.6875000000000000	−0.0056471805599453
60	0.6250000000000000	−0.0681471805599453

Table 1: Results from the initial formula

(b) We can improve the formula by using algebraic manipulation to remove the 2^{n+1} term:

$$x_{n+1} = 2^{n+1}(\sqrt{1+2^{-n}x_n} - 1)$$
$$x_{n+1} = 2^{n+1}(\sqrt{1+2^{-n}x_n} - 1) \cdot \frac{\sqrt{1+2^{-n}x_n}+1}{\sqrt{1+2^{-n}x_n}+1}$$
$$x_{n+1} = \frac{2^{n+1}(\sqrt{1+2^{-n}x_n}^2 - 1^2)}{\sqrt{1+2^{-n}x_n}+1}$$
$$x_{n+1} = \frac{2^{n+1}(2^{-n}x_n)}{\sqrt{1+2^{-n}x_n}+1}$$
$$x_{n+1} = \frac{2x_n}{\sqrt{1+2^{-n}x_n}+1}$$

n	x_n	$x_n - \ln(x_0+1)$	n	x_n	$x_n - \ln(x_0+1)$
1	0.8284271247461901	0.1352799441862448	31	0.6931471806718098	0.0000000001118645
2	0.7568284600108842	0.0636812794509389	32	0.6931471806158777	0.0000000000559324
3	0.7240618613220612	0.0309146807621159	33	0.6931471805879117	0.0000000000279664
4	0.7083805188386214	0.0152333382786761	34	0.6931471805739287	0.0000000000139834
5	0.7007087569317337	0.0075615763717884	35	0.6931471805669371	0.0000000000069919
6	0.6969143073088294	0.0037671267488841	36	0.6931471805634414	0.0000000000034961
7	0.6950273424387612	0.0018801618788159	37	0.6931471805616936	0.0000000000017483
8	0.6940864128518364	0.0009392322918911	38	0.6931471805608197	0.0000000000008744
9	0.6936165847594015	0.0004694041994562	39	0.6931471805603827	0.0000000000004374
10	0.6933818296999493	0.0002346491400040	40	0.6931471805601642	0.0000000000002189
11	0.6932644918933770	0.0001173113334317	41	0.6931471805600550	0.0000000000001097
12	0.6932058329179387	0.0000586523579934	42	0.6931471805600004	0.0000000000000551
13	0.6931765059118140	0.0000293253518687	43	0.6931471805599730	0.0000000000000278
14	0.6931618430291043	0.0000146624691590	44	0.6931471805599594	0.0000000000000141
15	0.6931545117428319	0.0000073311828866	45	0.6931471805599526	0.0000000000000073
16	0.6931508461384656	0.0000036655785203	46	0.6931471805599493	0.0000000000000040
17	0.6931490133459748	0.0000018327860295	47	0.6931471805599476	0.0000000000000023
18	0.6931480969521525	0.0000009163922072	48	0.6931471805599467	0.0000000000000014
19	0.6931476387558471	0.0000004581959018	49	0.6931471805599463	0.0000000000000010
20	0.6931474096578458	0.0000002290979005	50	0.6931471805599461	0.0000000000000008
21	0.6931472951088831	0.0000001145489378	51	0.6931471805599460	0.0000000000000007
22	0.6931472378344111	0.0000000572744658	52	0.6931471805599460	0.0000000000000007
23	0.6931472091971775	0.0000000286372323	53	0.6931471805599460	0.0000000000000007
24	0.6931471948785614	0.0000000143186161	54	0.6931471805599460	0.0000000000000007
25	0.6931471877192534	0.0000000071593081	55	0.6931471805599460	0.0000000000000007
26	0.6931471841395995	0.0000000035796542	56	0.6931471805599460	0.0000000000000007
27	0.6931471823497726	0.0000000017898273	57	0.6931471805599460	0.0000000000000007
28	0.6931471814548591	0.0000000008949138	58	0.6931471805599460	0.0000000000000007
29	0.6931471810074024	0.0000000004474571	59	0.6931471805599460	0.0000000000000007
30	0.6931471807836740	0.0000000002237287	60	0.6931471805599460	0.0000000000000007

Table 2: Results from the improved formula

Note: Please see attached C files q1.c, q2.c, q3.c for the source code.