Assignment #5
Numerical Computing (COMP 350)

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1.
- Vandermonde form (using `vander_coef.m`):

$$p(x) = 2 - x + 2x^2 - x^3$$

- Lagrange form (by hand):

$$\begin{aligned}
p(x) &= (x-1)(x-2)(x-3)(x-4)\left(\frac{\frac{2}{(1-2)(1-3)(1-4)}}{(x-1)} + \frac{\frac{0}{(2-1)(2-3)(2-4)}}{(x-2)} + \frac{\frac{-10}{(3-1)(3-2)(3-4)}}{(x-3)} + \frac{\frac{-34}{(4-1)(4-2)(4-3)}}{(x-4)}\right) \\
&= (x-1)(x-2)(x-3)(x-4)\left(\frac{\frac{2}{(-1)(-2)(-3)}}{(x-1)} + \frac{\frac{-10}{(2)(1)(-1)}}{(x-3)} + \frac{\frac{-34}{(3)(2)(1)}}{(x-4)}\right) \\
&= (x-1)(x-2)(x-3)(x-4)\left(\frac{2/-6}{(x-1)} + \frac{-10/-2}{(x-3)} + \frac{-34/6}{(x-4)}\right) \\
&= \frac{-1}{3}\cdot(x-2)(x-3)(x-4) + 5\cdot(x-1)(x-2)(x-4) + \frac{-17}{3}\cdot(x-1)(x-2)(x-3) \\
&= \frac{-1}{3}\cdot(x^3-9x^2+26x-24) + 5\cdot(x^3-7x^2+14x-8) + \frac{-17}{3}\cdot(x^3-6x^2+11x-6) \\
p(x) &= -x^3 + 2x^2 - x + 2
\end{aligned}$$

- Newton form (by hand using `newton_coef.m` for a_1, a_2, a_3, a_4):

$$\begin{aligned}
p_0(x) &= 2 \\
p_1(x) &= 2 - 2(x-1) \\
p_2(x) &= 2 - 2(x-1) - 4(x-1)(x-2) \\
p_n(x) = p_3(x) &= 2 - 2(x-1) - 4(x-1)(x-2) - (x-1)(x-2)(x-3) \\
p_n(x) &= -x^3 + 2x^2 - x + 2
\end{aligned}$$

See `vander_coef.m`, `vander_pval.m` and `gepp.m` for the code used in (a).

See `newton_coef.m` and `newton_pval.m` (retrieved from Professor Chang's website) for the code used in (c).

See `q1.m` for the code using all these functions.

2.

$$\begin{aligned}
p(x) &= 0.0385 + 0.1323x + 0.6211x^2 + 1.8681x^3 - 8.2312x^4 + 13.1349x^5 - 13.1349x^6 \\
g(x) &= 0.7192 - 2.3862x^2 + 1.7195x^4
\end{aligned}$$

See Table 1 for the values at the 13 equally spaced points.

See Figure 1 for the graph of $f(x)$, $p(x)$, $S(x)$ and $g(x)$.

See `newton_coef.m` and `newton_pval.m` for the code used to calculate $p(x)$.

See `splinecubic_coef.m` and `splinecubic_pval.m` for the code used to calculate $S(x)$.

See `leastsquares_coef.m` and `leastsquares_pval.m` for the code used to calculate $g(x)$.

See `q2.m` for the code generating the functions, tables and graph. .

x	$f(x)$	$f(x)-p(x)$	$f(x)-S(x)$	$f(x)-g(x)$
-1	0.0384615	0	0	-0.0140397
-0.833333	0.0544629	-0.553416	-0.0186871	0.163105
-0.666667	0.0825688	$-1.38778e-17$	$-1.38778e-17$	0.0842385
-0.5	0.137931	0.229347	0.0539594	-0.0921964
-0.333333	0.264706	0	$5.55112e-17$	-0.210596
-0.166667	0.590164	-0.181723	-0.129022	-0.0640851
0	1	0	$2.22045e-16$	0.280795
0.166667	0.590164	-0.181723	-0.128559	-0.0640851
0.333333	0.264706	$-2.22045e-16$	$-1.66533e-16$	-0.210596
0.5	0.137931	0.229347	0.0525706	-0.0921964
0.666667	0.0825688	$-2.91434e-16$	$1.38778e-17$	0.0842385
0.833333	0.0544629	-0.553416	-0.0135945	0.163105
1	0.0384615	$2.498e-15$	$-6.93889e-18$	-0.0140397

Table 1: $f(x)$, $f(x)-p(x)$, $f(x)-S(x)$, and $f(x)-g(x)$ at 13 equally spaced points on the interval $[-1,1]$.

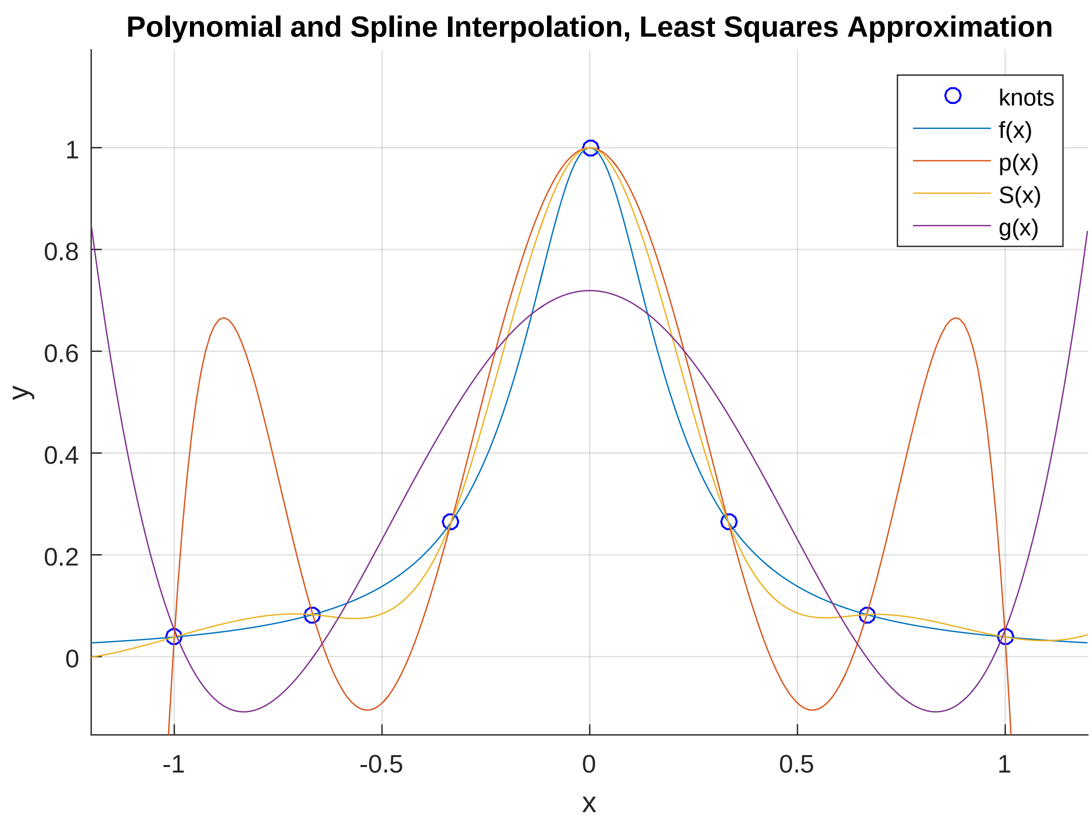

Figure 1: $p(x)$ is the interpolating polynomial of degree 6 for $f(x)$ by the Newton approach. $S(x)$ is the natural cubic spline function interpolating $f(x)$ and $g(x) = a + bx^2 + cx^4$ approximates $f(x)$ by the least squares.