COMP 350 Numerical Computing
Assignment #6, Numerical Integration

Date given: Monday, Nov 16. Date due: 5:00pm, Monday, Nov 30, 2015.

1. (12 points) The function $\text{erf}(t) = \frac{2}{\sqrt{\pi}} \int_0^t e^{-x^2} dx$ is called an error function in statistics.

   (a) Compute erf(3) by the recursive trapezoid rule. Stop the iteration until the difference between two consecutive computed integrals is smaller than or equal to 10^{-5}.

   (b) Compute erf(3) by the adaptive Simpson's method. Try to avoid redundant function evaluation. Take $\epsilon = 10^{-5}$ and `level_max=30`.

   For both methods, report the number of function evaluations and print the final results and the MATLAB codes as well.

2. (8 points) In class we gave the following Gaussian two-point quadrature rule:

$$\int_{-1}^{1} f(x)dx \approx f\left(-\frac{\sqrt{3}}{3}\right) + f\left(\frac{\sqrt{3}}{3}\right).$$

   Suppose we want to compute $\int_a^b f(x)dx$. We divide the interval $[a, b]$ into n equal subintervals $[x_i, x_{i+1}]$, $i = 0, 1 \ldots, n-1$. For each subinterval we apply the Gaussian two-point quadrature rule, leading to the composite Gaussian two-point quadrature rule.

   (a) Derive this composite rule.

   (b) Suppose the number of subintervals used in Question 1(a) is m. Use the composite rule given in 2(a) to compute erf(3) by taking $n = \lfloor \frac{m+1}{2} \rfloor$. Report the number of function evaluations. Compare the result with that obtained in Question 1(a) (you can regard the result obtained by MATLAB built-in function `erf` as the true result). Print the final results and the MATLAB codes as well.