# COMP 350 Assignment 3

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1.

$$[A \mid b] = \left[\begin{array}{cccc|c} 1 & 2 & 3 & -4 & 6 \\ -2 & 3 & -4 & 5 & -14 \\ 3 & 4 & 5 & -6 & 10 \\ 4 & -5 & -6 & 7 & -4 \end{array}\right] \xrightarrow{r_1 \leftrightarrow r_4} \left[\begin{array}{cccc|c} 4 & -5 & -6 & 7 & -4 \\ -2 & 3 & -4 & 5 & -14 \\ 3 & 4 & 5 & -6 & 10 \\ 1 & 2 & 3 & -4 & 6 \end{array}\right] \xrightarrow[\substack{r_3 \mathrel{+}= -3/4\, r_1 \\ r_4 \mathrel{+}= -1/4\, r_1}]{r_2 \mathrel{+}= 1/2\, r_1} \left[\begin{array}{cccc|c} 4 & -5 & -6 & 7 & -4 \\ 0 & 1/2 & -7 & 17/2 & -16 \\ 0 & 31/4 & 19/2 & -45/4 & 13 \\ 0 & 13/4 & 9/2 & \frac{-23}{4} & 7 \end{array}\right]$$

$$\xrightarrow{r_2 \leftrightarrow r_3} \left[\begin{array}{cccc|c} 4 & -5 & -6 & 7 & -4 \\ 0 & 31/4 & 19/2 & -45/4 & 13 \\ 0 & 1/2 & -7 & 17/2 & -16 \\ 0 & 13/4 & 9/2 & \frac{-23}{4} & 7 \end{array}\right] \longrightarrow \begin{bmatrix} 4 & -5 & -6 & 7 & -4 \\ 0 & 31/4 & 19/2 & \frac{-45}{4} & 13 \\ 0 & 0 & \frac{-236}{31} & \frac{286}{31} & \frac{-522}{31} \\ 0 & 0 & 16/31 & \frac{-38}{31} & 48/31 \end{bmatrix} \longrightarrow \begin{bmatrix} 4 & -5 & -6 & 7 & -4 \\ 0 & 31/4 & 19/2 & -45/4 & 13 \\ 0 & 0 & \frac{-236}{31} & \frac{286}{31} & \frac{-522}{31} \\ 0 & 0 & 0 & \frac{-24}{59} & \frac{24}{59} \end{bmatrix}$$

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} 1 \\ -1 \\ 1 \\ -1 \end{bmatrix}$$

2. The number of flops is

$$\sum_{i=1}^{n}\sum_{j=i}^{n}\sum_{k=j}^{n} 2 = \frac{1}{3}n(n+1)(n+2)$$

.

$$\sum_{i=1}^{n}\sum_{j=i}^{n}\sum_{k=j}^{n} 2$$

$$= \sum_{i=1}^{n}\sum_{j=i}^{n}\sum_{k=1}^{n-j+1} 2$$

$$= \sum_{i=1}^{n} 2\cdot\sum_{j=i}^{n} (n-j+1)$$

$$= \sum_{i=1}^{n} 2\cdot\sum_{j=1}^{n-i+1} (n-(j+i-1)+1)$$

$$= \sum_{i=1}^{n} 2\cdot\sum_{j=1}^{n-i+1} (n-j+i+2)$$

$$= \sum_{i=1}^{n} 2\left(\sum_{j=1}^{n-i+1} (n-i+2) - \sum_{j=1}^{n-i+1} j\right)$$

$$= \sum_{i=1}^{n} 2(n-i+2)(n-i+1) - \frac{2(n-i+1)(n-i+2)}{2}$$

$$= \sum_{i=1}^{n} (n-i+1)(n-i+2)$$

$$= \sum_{i=1}^{n} (n^2-2ni+3n+i^2-3i+2)$$

$$= \sum_{i=1}^{n} (-2ni+i^2-3i) + n(n^2+3n+2)$$

$$= \left(-n\cdot n\cdot(n+1) + \frac{n(n+1)(2n+1)}{6} - \frac{3}{2}n(n+1)\right) + n(n^2+3n+2)$$

$$= n\left(-n^2-n+\frac{2n^2+3n+1}{6} - \frac{9}{6}n - \frac{9}{6}\right) + n(n^2+3n+2)$$

$$= n\left(-\frac{2}{3}n^2 - \frac{2}{3}\cdot 3n - \frac{2}{3}\cdot 2\right) + n(n^2+3n+2)$$

$$= n\left(\frac{-2}{3}\right)(n^2+3n+2) + n(n^2+3n+2)$$

$$= \frac{1}{3}n(n^2+3n+2) = \frac{1}{3}n(n+1)(n+2)$$

3.

a. See q3a_genp.m and q3a_gepp.m for the source code.
   - GENP: $5n + 1n + 3n = \mathbf{9}\boldsymbol{n}$ **flops** and $\mathbf{4} * (\mathbf{2}\boldsymbol{n} + \mathbf{1})$ **memory locations**
   - GEPP: $\mathbf{9}\boldsymbol{n}$ **flops** and $\mathbf{4} * (\mathbf{2}\boldsymbol{n} + \mathbf{1})$ **memory locations**

b. See q3b.m for the source code.

c. See q3c.m for the source code.

```
>> q3b

avg_imp =    3.7894

>> q3b

avg_imp =    2.9249

>> q3b

avg_imp =    4.7632

>> q3c

avg_imp =   6.6232e+11

>> q3c

avg_imp =   6.6881e+11

>> q3c

avg_imp =   6.7002e+11
```