COMP 350 Numerical Computing

Assignment #3: Solving linear systems.

Date given: Wednesday, September 30. Date due: 5pm, Wednesday, October 14, 2015

1. (2 points) Solve the following system using GEPP (Gaussian elimination with partial pivoting):

$$\begin{bmatrix} 1 & 2 & 3 & -4 \\ -2 & 3 & -4 & 5 \\ 3 & 4 & 5 & -6 \\ 4 & -5 & -6 & 7 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} 6 \\ -14 \\ 10 \\ -4 \end{bmatrix}$$

   Show intermediate matrices, vectors and multipliers at each step.

   Note: Do the computations by your hands and don't consider any rounding errors.

2. (3 points) Count the number of flops (floating point operations) in the following pseudocode:

```
for i = 1 : n
    for j = i : n
        for k = j : n
            b_ij = b_ij + a_ik x_kj
        end
    end
end
```

3. (MATLAB programming) Consider the system of $2n+1$ equations:

$$\begin{bmatrix} d_1 & & & & & & a_{2n+1} \\ & d_2 & & & & a_{2n} & \\ & & \ddots & & \iddots & & \\ & & & d_{n+1} & & & \\ & & \iddots & & \ddots & & \\ & a_2 & & & & d_{2n} & \\ a_1 & & & & & & d_{2n+1} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_{n+1} \\ \vdots \\ x_{2n} \\ x_{2n+1} \end{bmatrix} = \begin{bmatrix} b_1 \\ b_2 \\ \vdots \\ b_{n+1} \\ \vdots \\ b_{2n} \\ b_{2n+1} \end{bmatrix}$$

   (a) (8 points) Write two function M-files to solve this system using the GENP and GEPP approaches, respectively. Your programs have to make use of the structure of this system so that they do not do unnecessary computations. How many flops are involved in your programs? How many memory locations are needed?

   (b) (4 points) Write a script M-file to test your two programs on the **same** data: random elements a_i, d_i produced by MATLAB built-in function `randn` (note that a_{n+1} should be equal to d_{n+1}), and b_i defined by $b_i = d_i + a_{2n+2-i}$ for $i \neq n+1$, and $b_{n+1} = d_{n+1}$. Notice that the exact solution $x = [1, 1, \ldots, 1]^T$. Let x_{np} and x_{pp} be the computed solutions by your two programs, respectively. Compute the relative errors $\|x - x_{np}\|_2/\|x\|_2$ and $\|x - x_{pp}\|_2/\|x\|_2$. In your test, you may take $n = 4$.

   (c) (3 points) Now change d_1 in your test example to 10^{-13}, and correspondingly change b_1 so that b_1 still satisfies $b_1 = d_1 + a_{2n+1}$. Again compute the solutions and the corresponding relative errors. Comment on the results.

   Print out the data and computed results.