Self-organization and Selection
in the Emergence of Vocabulary

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Abstract

Human language may have started from a consistent set of mappings between meanings and signals. These mappings, referred to as the early vocabulary, are considered to be the results of conventions established among the agents of a population. In this study, we report simulation models for investigating how such conventions can be reached. We propose that convention is essentially the product of self-organization of the population through interactions among the agents; and that cultural selection is another mechanism that speeds up the establishment of convention. Whereas earlier studies emphasized either one or the other of these two mechanisms, our focus is to integrate them into one hybrid model. The combination of these two complementary mechanisms, i.e. self-organization and cultural selection, provides a plausible explanation for cultural evolution which progresses with high transmission rate. Furthermore, we observe that as the vocabulary tends to convergence there is a uniform tendency to exhibit a sharp phase transition.

Summary

Language is one of the defining characteristics specific to humans. It is well known that language evolves continuously at high rate. However, the answers to why and how human language emerged and changes are still being pursued. In this study, we speculate that human language may have started from a consistent set of mappings between meanings and signals. These mappings are considered to be the results of conventions established among the individuals of a population. We use simulation models to investigate how such conventions can be reached.

Keywords

1 Introduction

Language is generally considered to be one of the most important characteristics that differentiates humans from other species. The question as to how fully-fledged human language came into being has been pursued for centuries; various theories have been suggested and many controversies exist [1]. Recently there has been growing interest in the study of language emergence and change within the framework of complex adaptive systems [2]. In this study, we focus on the emergence of vocabulary within a population of early humans using simulation models, as is often adopted in the study of complex adaptive systems.

Most would agree that the first step in human language evolution was communication systems using a number of holistic signals, like those found in primates and other animals such as bees and birds [3]. These signals are not equivalent to the "names" or "words" as we use these terms today. Each signal may refer instead to a complex mix of meanings. The signal for "danger", for example, may imply a cry of fear, a warning – "run, there is danger coming", as well as a general reference for the dangerous predator. Later, such a signal would have been narrowed down to name a single object or class of objects, and used in a non-situation specific fashion [4]. The realization of signals as symbols representing objects or events is referred as the "naming insight" [5]. In the language development of a normal child, the naming insight comes so naturally that most parents do not notice the exact moment of this event without special attention. On the contrary, chimpanzees need intensive teaching to learn to name [6].

How the naming insight occurred in the phylogeny of the hominid line is still a mystery. However, we may assume that there was a stage during which early humans became equipped with the naming insight. What interests us in this study is that once the naming insight was achieved in the agents, each of whom might have his or her own way to name in an arbitrary manner, how did the actual naming of objects or events become consistent across the entire population? Some twenty centuries ago, two great philosophers, continents apart, arrived at a similar observation that names are formed by convention [7]. Xunzi in China taught that "names have no intrinsic appropriateness" and "names have no intrinsic reality" —the appropriateness and reality of names are both given by convention. At about the same time in Greece, Plato wrote that "any name which you give is the right one, and if you change that and give another, the new name is as correct as the old."

In this study, we use simulation models and mathematical analysis to explore the process of how convention is achieved to form a coherent naming system—which we may refer to as a *vocabulary*. In our models, at first, a number of agents in a population each have their own agent-specific way for naming a set of objects. Upon interacting with each other, some are led to modify their naming systems. Agents may not be concerned with and may not care about the general communication performance at the population level, but instead concentrate only on their *own* communication performance with other agents. Without any explicit or implicit design, however, a consistent common vocabulary develops as an emergent property of the population. Our proposed model of vocabulary emergence shares the same spirit as the "invisible hand" mechanism suggested by Keller [8], which has been echoed by Kirby: "the local, individual actions of many speakers, hearers, and acquirers of language across time and space conspire to produce non-local, universal patterns of variation" [9]. In fact such phenomena have been widely studied in many complex systems in various disciplines and are described as "self-organization" [10]. A system is defined as self-organizing "if it acquires a functional, spatial, or temporal structure without specific interference from the outside" [11]. In recent studies of language emergence, there have been several reports explicitly adopting this framework, which use computer simulation models to study the self-organization mechanism of the emergence of vocabulary [12] and sound systems [13].

From the evolution point of view, self-organization plays a role in the synchronic and horizontal interaction in a population. However, self-organization has not addressed the evolution process in the diachronic aspect, i.e. from generation to generation. On the other hand, natural selection has been generally considered as the major mechanism for evolution in terms of gene transmission across generations, i.e. vertical transmission. In the study of language evolution there are two views related to natural selection. In one view, some consider language as the product of a biological organ, the Language Acquisition Device (LAD); they think it is the LAD that must have evolved through natural selection [14]. Alternatively, Dawkins introduced the term "meme" to refer to a new type of replicating unit in cultural evolution as a counterpart of the gene in biological evolution [15]. In this sense, a language is an elaborate complex of memes, which evolve based on linguistic or cultural selection[1]. It is the languages *themselves* that adapt for their own "survival" in the transmission from speaker to speaker. It is impossible to evaluate which language, e.g. English, Chinese or any other, is better, in terms of its overall fitness by taking into account learnability, expressiveness, complexity and so on. However, we argue that when considering some specific aspects, for example the possibility of ambiguity in terms of the number of homophones[2], different language subsystems indeed can be compared with respect to their fitness. Subsystems with higher fitness will have a higher probability to diffuse more into the next generation. In this study we assume that vocabulary can be transmitted from generation to generation as a result of children's learning from parents and that the communicative consistency of the vocabulary determines the possibility in which it is transmitted to the next generation. In this way, we study the effect of cultural selection in this vertical transmission process and examine how selection couples with self-organization in catalyzing the formation of a consistent vocabulary in the population.

In the study of modeling vocabulary emergence, there have been a few related studies. Among them, Steels [12], as mentioned above, focuses on the self-organization mechanism, while Hurford [16] and Nowak & Krakauer [17] mainly emphasize the selection process. In this study, we propose that both mechanisms are indispensable for structure emergence in evolution. Using the "tinkerer" metaphor introduced by Jacob [18][3], we highlight the combined function of these two "tinkerers" in language evolution.

The rest of the paper is organized as follows. First, we report an imitation model in Section 2, which simulates how a common vocabulary is formed by agents imitating each other either merely randomly or by following the majority. We use Markov chain theory to analyze the model. A detailed proof of the convergence is given in the appendix. In Section 3 we present an interaction model which uses a probabilistic representation of vocabulary. Different parameters in the model are investigated by simulation and a few interesting observations are reported. Section 4 introduces a hybrid model which combines the self-organization and cultural selection mechanisms. Conclusions and discussion are given in the last section.

2 The imitation model

The strong ability of imitation in human, even from early infancy, has been extensively documented in the studies reported by many investigators, e.g. Meltzoff [19]. While other

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[1]In this study, however, we still use the term "natural selection" in a broad sense to refer to the selection mechanism in language evolution.

[2]Two or more words are homophones if they are pronounced the same but differ in meaning, such as the words 'too' and 'two'.

[3]Jacob states that "natural selection ... works like a tinkerer, who does not know exactly what he is going to produce, but uses whatever he finds around him, whether it be pieces of string, fragments of wood, or old cardboards, ... to produce some kind of workable object."

social animals, particularly the primates, also imitate, it appears that the tendency is by far the strongest and most general in our species. We assume that imitation may serve as the most explanatory mechanism for the formation of a common vocabulary. Before establishing a consistent way of naming things, early humans very likely made use of their propensity for imitation; the younger ones imitating their elders, the followers imitating the leaders or, just by chance, their neighbors. Such imitation between agents can be seen as self-organization in the population.

In this study, we set up a model to simulate the process of word imitation by agents in a population. Assume that in a population of P_s agents there are M classes of objects[4] that the agents must name in their daily communication, and U different utterances[5] that the agents can make. Each agent's vocabulary consists of a set of associations, or one-to-one mappings, between meanings and utterances. Each agent can create and change his or her own vocabulary by imitation, similar to the model proposed by Steels [12]. We assume that, at first, each agent already has his or her own specific meaning-utterance mappings. For example, the M-U mappings of two agents (A and B) might be as shown in Table 1.

Table 1 about here

Let us assume that when they interact with each other, the two agents only communicate a single meaning. When they are not using the same word to represent that meaning, an imitation event is likely to occur; for example A may imitate B when they communicate meaning m_1 by replacing his own u_2 with B's u_5, or vice versa. We assume that each agent is equipped with an imitation strategy. For simplicity, we assume that such imitation events involve no errors. There are many possible imitation strategies that can be conceived. In this study we report two strategies that may be the most realistic[6]:

- Strategy 1: imitating by random direction—either A imitates B, or vice versa, with equal probability.

- Strategy 2: imitating by following the majority—A imitates B if the utterance B uses is shared by more agents than the utterance A uses.

Before studying how imitation affects agents' vocabulary in a population, we need to design some criteria to evaluate how well the vocabularies of the members of a population work for conveying meanings. First, we need to consider the population consistency, denoted by C, i.e. how many consistent meaning-utterance mappings there are among the agents. Second, we consider the effect of homophones, which are words that have the same form but different meanings. The existence of homophones is likely to cause confusion. Therefore, assuming that the fewer the homophones, the better the vocabulary, we select a second criterion to be the distinctiveness of the vocabulary, denoted by D. C and D are calculated as follows. For a population with P_s agents,

1. The overall *consistency* for all meanings is

$$\mathbf{C} = \frac{1}{M} \sum_{i=1}^{M} \mathbf{C_i}$$

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[4] A class of objects containing a single object is permissible.

[5] In this report "utterance", "signal" and "word" are used interchangeably for the sake of convenience, particularly when discussing other studies, though we acknowledge that there are important differences among them.

[6] We have considered other strategies such as following the authority and homophone avoidance. The detailed simulation results are reported in an earlier study by Wang & Ke [20].

where M is the number of meanings, and $\mathbf{C_i}$ is the consistency for meaning i, which is defined as the proportion of matched pairs (i.e. the number of pairs using the same utterance for meaning i) among all possible pairs in the population,

$$\mathbf{C_i} = \frac{\sum_j \binom{S_{ij}}{2}}{\binom{P_s}{2}}$$

where S_{ij} is the number of agents that use utterance u_j for meaning m_i. The following holds,

$$\sum_j S_{ij} = P_s, \quad \text{for any } i$$

2. The overall *distinctiveness* for all agents' vocabulary is

$$\mathbf{D} = \frac{1}{P_s} \sum_{I=1}^{P_s} \mathbf{D}_I$$

where $\mathbf{D}_I$ is a measure of the degree of homophony in agent I's vocabulary defined by

$$\mathbf{D}_I = \frac{\sum_j \xi_j}{N}$$

where N is the number of distinctive utterances that the I-th agent uses for the M meanings, and ξ_j is the probability that the I-th agent correctly interprets u_j, which is inversely proportional to the number of meanings that u_j represents.

Both C and D are positive values but no larger than 1. When $C = 1$, all agents have the same vocabulary, while $D = 1$ means that none of the agents have any homophones in their vocabulary.

We carried out a number of simulations for a fix number of meanings ($M = 10$) and different population sizes, P_s, and different numbers of signals, U, to study the change of C and D. Table 2 shows the average C and D from 100 runs for different combinations of P_s and U. In each run 20000 interactions were carried out. From the simulation results, we find that in most cases the whole population can reach consistency, i.e. $C = 1$. However, homophones seem to be unavoidable, i.e. $D < 1$, even when the number of signals (U) outweighs the number of meanings (M) by five times; this is consistent with the common observation that large numbers of homophones exist in most natural languages. The converged vocabulary with $C = 1$ and $D < 1$ corresponds to the lexicon in sub-optimal state which has been discussed in Nowak et al. [21]. In their model, the lexicon could reach a coherent but unstable state where one signal can refer to more than one meaning, and small amounts of error will destroy the coherence.

When P_s increases, the population reaches consistency in its vocabulary more slowly, as expected, using either Strategy 1 or Strategy 2. However, in Strategy 2, i.e. following the majority, when U is larger, it becomes more difficult for C to reach 1 when the population size is small, for instance $P_s = 10$. This occurs when several competing mappings happen to co-exist, which map to the same meaning and each of which is used by an equal number of agents, in other words, when there is exactly the same number of agents using each of the competing mappings.

We study the consistency in these two strategies and pursue a formal mathematical proof under a simplified condition ($U = 2$ and $M = 1$) using Markov chain theory; the proof is given in the Appendix. We observe from the proof that consistency can be reached with Strategy 1 and, in most cases, with Strategy 2 except when there are competing mappings as described above.

Table 2 about here

Our model is conceptually close to the model reported by Steels [12], in which a group of agents share a coherent vocabulary by changing their own private vocabulary by imitating others. Steels's model incorporates probabilities for the mappings whereby imitation depends on communicative success. Similarly, in our model, strategy 2, i.e. "following the majority", there is also an implicit fitness measure involved. A uni-directional selection mechanism drives the system towards the emergence of consistency; this is implemented by assuming that the agents have a complete sampling of the whole population and therefore know of the majority tendency in the population, though this may not be realistic especially when the population size is large. On the other hand, for the proposed strategy 1 in our model, the population can converge by purely random imitation through local interactions without any explicit measure of fitness or knowledge of the majority in the population. The coherent state of a convergent vocabulary achieved by random imitation may not be optimal due to there being a large number of homophones, i.e. D is much smaller than 1 under some conditions, such as shown in Table 2. In this model we have not considered the learners' effect on the development vocabulary. Nowak et al. [24] study three different types of learners' strategies, and find that random learning strategies are more sensitive to noise than parental or role model learning.

3   The interaction model

3.1   Probabilistic representation of vocabulary

The U-M mapping in the previous section assumes that each meaning is represented uniquely by a single utterance at any one time. However, one may speculate that words are represented in the mind in a probabilistic manner [16]. At some stage, a particular meaning may be represented by several forms/utterances with certain probabilities. And some utterances may possibly be associated with several meanings. Consequently we hypothesize a process in which there is competition among several meanings or utterances. When one wants to express a meaning, several possible utterances which are associated with the meaning will rival each other. The speaker will produce the utterance which "beats" other utterances in the production competition. Similarly, when the listener hears an utterance, he/she will interpret the utterance as the meaning which beats the other meanings associated with that utterance. These two processes are possibly separate, which implies that there are two different sets of mappings respectively for speaking and listening. Here in this study we adopt such a representation of two matrices, one for speaking and one for listening, corresponding to the active and passive matrices in Nowak & Krakauer [17] and the transmission and reception matrices in Hurford [16].

We set the initial speaking and listening mappings for each agent randomly. Table 3 shows an example of the initial speaking and listening mappings of one agent(S) with $M = U = 3$.

Table 3 about here

In the speaking matrix (P) the rows sum up to 1, i.e. $\sum_{j=1}^{U} p_{ij} = 1$, while the columns of the listening matrix (Q) sum up to 1, i.e. $\sum_{i=1}^{M} q_{ij} = 1$. This assumes that each agent can always select an utterance to convey a meaning (speaking) and can always infer a meaning from an utterance (listening). However, they might not have formed a fixed way to express the meanings. Then how did early humans organize their internal mappings? This should be considered differently from ontogenetic vocabulary development because children construct their mental lexicon by learning from relatively consistent mappings. Although the linguistic environment that a child is exposed to today is also constantly changing, composed of various idolects with a certain diversity[7], the degrees of diversity in the two processes, i.e. the phylogenetic and ontogenetic development of vocabulary, are considered to be at different levels.

Hurford [16] first applied simulation studies to this question. He hypothesizes three idealized learning strategies, Imitator, Calculator and Saussurean. Hurford's simulations show that the Saussurean strategy, which restricts the learner to identical speaking and listening matrices, is more successful than the other two strategies in evolutionary terms. Hurford speculates that the Saussurean strategy would have been selected against other conceivable strategies due to its clear advantage. We would infer from this speculation that we should always have identical speaking and listening vocabularies if the Saussurean strategy is an inherited learning ability. However, we are skeptical that this is true. As we observe from their vocabulary development, children often go through a stage of overextension, i.e. they use one word to refer to several meanings or objects which have similar physical or functional features. For example, "ball" may refer to a ball, a balloon, an observatory dome etc. [23]. But they can understand when adults talk with them using adults' vocabulary. We believe this is a reflection of non-identical speaking and listening vocabulary in children. We speculate that when there is no established common vocabulary and everyone has his own way to name things at the early stage of language development, such an inconsistency would have to exist as well. In fact we can still observe the non-identical phenomena in modern society. Very often we can understand a word though we never use it when we speak. It is generally thought that people typically have a different, usually larger, listening vocabulary than speaking vocabulary. Therefore, we hypothesize that the speaking matrix is not necessarily the same as the listening matrix.

Oliphant and Batali [24] propose an "obverter" model, where agents adopt a strategy very similar to Hurford's calculator strategy. In their model, new learners are continuously inserted into a randomly initiated population. The learner is able to observe the speaking and listening behaviors of other agents and accordingly constructs his/her speaking and listening matrices by following the majority. It is shown that a coordinated system can be achieved if the learner have a *complete* observation of the whole population. The convergence slows down and deteriorates much if learners have only a limited number of observations, which is more plausible, however, in real situations. Furthermore, an important factor in the convergence of the obverter model is that, although the initial population starts with random probabilistic matrices, the learner's speaking and listening matrices are in binary representation, and once formed they will not change any more. These assumptions make the convergence not surprising at all, much as we have shown in the simulation of Strategy 2 of the imitation model above.

Nowak and Krakauer [17] report a similar simulation of the emergence of signal-meaning associations in terms of two probabilistic matrices. Their model, similar to that of Hurford [16], takes selection as the basic principle to guide the evolution of signal-meaning associa-

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[7]Ross [22] gives a good example demonstrating this diversity in a linguistic community, by studying English speakers' judgements on grammaticality of a number of sentences.

tions in a population of agents from a random initial condition to sub-optimum states. In Nowak and Krakauer's model, each agent has a measure of the fitness of its signal-meaning associations with regard to its ability to communicate successfully and produces offspring proportional to its fitness. Children learn the associations by sampling their parents' speech. Their simulation shows that the population can converge to a set of consistent mappings. However, the optimal state, i.e. unique one-to-one associations of signal and meaning, is not guaranteed; sometimes the population settles at a sub-optimal state whereby, for example, two different signals are associated with one object, or one signal is associated with two different objects. Furthermore the resultant speaking and listening matrices in their model are not guaranteed to be "compatible"; by compatible we mean that the two matrices are identical or the speaking matrix is a subset of the listening matrix when $U > M$. Nowak et al. [21] considered this a paradoxical result in their model.

In this study, we report two models which adopt the same probabilistic representation of vocabulary as Hurford [16] and Nowak & Krakauer [17]. The interaction model is introduced in this section and the hybrid model in Section 4.

Our proposal differs from previous studies in that agents construct their vocabulary by continually interacting with other agents. All agents modify their probabilistic speaking and listening matrices independently according to the success or failure of each interaction. After a number of interactions, the agents in the population come up with a set of consistent mappings. The population behaves like a self-organizing system where order emerges from the interactions between agents within the system. The following describes the details of our interaction model and its simulation results.

3.2 Interactions between agents

In an interaction event in the simulation, two agents are randomly selected, one as a speaker (S), and one as a listener (L), each agent having a speaking matrix and a listening matrix, both of size $M \times U$. Consider, for instance, two agents, agent S starting with a set of mappings as presented in Table 3 given earlier and agent L with mappings shown in Table 4 below ($M = U = 3$).

Table 4 about here

Suppose that in an interaction event speaker S decides to convey meaning m_2, say, which is randomly selected. Because the mappings are interpreted as probabilities that associate a meaning with an utterance, rather than directly choosing the utterance with the biggest probability for that meaning, S will choose an utterance by sampling. Similarly, the listener receives the utterance sent by the speaker and will interpret the utterance by choosing one meaning from those which have associations with that utterance by sampling. In the simulation, the roulette wheel sampling method is used in both the speaking and listening process [25].

3.3 The adjustment after one interaction

If the meaning interpreted by the listener is the same as the meaning intended by the speaker, an interaction is seen as successful. Consequently the associations between the intended meaning (m) and the utterance (u) are strengthened by an adjustment variable Δ, both in the speaker's speaking matrix and the listener's listening matrix, i.e. p_{mu}^S and q_{mu}^L. Accordingly, the mappings of p_{mi}^S ($i \neq u$) in the speaker's speaking matrix and q_{ju}^L ($j \neq m$) in the listener's listening matrix are decreased by re-normalization, in order to maintain the con-

straints $\sum_{j=1}^{U} p_{ij} = 1$, and $\sum_{i=1}^{M} q_{ij} = 1$. This way of adjusting the mapping is based on the assumption that once the speaker decides to use one particular utterance to express a certain meaning in that particular interaction, he will be less likely to use other utterances for that meaning. This is just one of many possible strategies that can be conceived. Similarly, once the listener realizes that an utterance refers to one particular meaning, he will be less likely to associate other utterances with that meaning.

The interaction fails if the meaning interpreted by the listener is not the same as the meaning intended by the speaker. The speaker is assumed to modify his current mappings for that meaning as he notices that his intention was not understood by the listener. At the same time, the listener also reduces the mapping between the utterance and the meaning he interprets wrongly, while increasing the mappings associated with other meanings. The above situations are hypothesized based on the assumption that the interactions between the agents are intentional, which implies that both the speaker and the listener are consciously taking part in the communication interaction. Furthermore, the agents are both able to judge whether the communication is successful with the help of the information from the environment in which the interaction takes place. Upon assuming these, we are consciously avoiding the danger of attributing "mind-reading" to the agents, which means that the agents do not have access to each other's internal states.

The adjustment variable (Δ) measures by how much a mapping is reinforced when an interaction is successful and by how much it is weakened upon failure. This variable plays an important role in the interaction model. It is easy to understand that Δ should be neither too large nor too small, otherwise it will result in oscillations when adjusting the agents' mappings.

Furthermore, we propose a multiple-listener hypothesis, which finds psychological precursors in chimpanzees [26]. We speculate that groups of early humans were small [27] and often gathered together. In an interaction, when one speaker speaks, it is very likely that more than one listener would be involved in the communication interaction. Here, we assume that there is only one listener that the speaker intends to talk to but that any other listener who interprets the speaker's meaning correctly will also correspondingly reinforce their mapping as we assume there is some direct reward or benefit from such a successful communication. However, when the communication fails, only the intended listener and the speaker adjust their mappings. This is based on the assumption that the side-listeners would not be affected when failing to interpret the speaker's utterance as the communication is not intended to them, though this may not necessarily be true.

3.4 Evaluation of the communication system

Before starting the simulation, we design four measures for evaluating the communication system: the similarity of the mapping matrices (SI), the individual convergence rate (IC), the population convergence rate (PC), and the convergence time (CT).

1. $SI(I, J)$ is a measure of the similarity between the vocabularies of two agents, I and J, and is defined in terms of the sum of the differences between corresponding elements in both their speaking matrices and their listening matrices, i.e. for two agents I and J,

$$SI(I, J) = 1 - \frac{1}{(U-1)M + (M-1)U} \sum_{i=1}^{U} \sum_{j=1}^{M} (|p_{ij}^{(I)} - p_{ij}^{(J)}| + |q_{ij}^{(I)} - q_{ij}^{(J)}|) \quad (1)$$

The population similarity SI is defined as the average similarity between all possible pairs of agents. When SI reaches 1, all agents have identical speaking and listening matrices.

2. $IC(I)$ is a measure of the degree of consistency of an individual's speaking and listening mappings, and is defined as the proportion of elements in each of the two matrices that are smaller than a certain threshold (δ, set as 0.05 here),

$$IC(I) = \frac{1}{2}\left(\frac{1}{M(U-1)}\sum_{i=1}^{M}\sum_{j=1}^{U}\Theta(p_{ij}^{(I)}) + \frac{1}{U(M-1)}\sum_{i=1}^{M}\sum_{j=1}^{U}\Theta(q_{ij}^{(I)})\right) \tag{2}$$

where $\Theta(k) = 1$ if $k < \delta$, else $\Theta(k) = 0$.

The purpose of this definition is to determine when a vocabulary has converged to a state in which each meaning is associated with a single dominant utterance in the speaking matrix and each utterance is associated with a unique meaning in the listening matrix. Table 5 gives an example of $IC(I) = 1$. Although this is a *convergent* state, it is not a *stable* state because the speaking matrix is not compatible with the listening matrix, implying that the agent cannot successfully interact with him/herself except about m_3.

Table 5 about here

3. PC is an index of the population convergence, which is the summation of communicative consistency between all possible pairs in the population. For a pair of agents, I and J, the communicative consistency $PC(I, J)$ is calculated by the following formula:

$$PC(I, J) = \frac{1}{2UM}\sum_{i=1}^{U}\sum_{j=1}^{M}(p_{ij}^{(I)}q_{ij}^{(J)} + p_{ij}^{(J)}q_{ij}^{(I)}) \tag{3}$$

When $PC = 1$, it means that any pair of agents in the population can successfully interact with each other for all meanings. Simulation shows that PC will equal 1 only when $SI = 1$ and $IC = 1$, which means that the system defined by the interaction model has reached a stable state. Table 6 shows an example of a stable state for the system when $M = U = 3$; there are five other stable states for such a system, which can be easily derived by finding one-to-one mappings for a 3×3 matrix. If the matrix is $N \times N$, then the number of possible matrices is simply $N!$. In the simulation, the agents do not stop interacting until the system has reached a stable state or they have completed a given number of interactions.

Table 6 about here

4. Convergence time, CT, is the number of interactions taken for PC to reach a certain threshold (set as 0.99 here) above which the population is considered to have converged.

3.5 Simulation results

In the simulation, several parameters are investigated: the population size (P_s), the number of available utterances that each agent can produce or perceive (U), the number of meanings that need to be conveyed (M), and the adjustment variable (Δ).

- The convergence trend

  When M and U are not too big, we can easily observe the convergent trend. Figure 1 shows an example of a population with 10 agents, initialized randomly, with a vocabulary size of $M = U = 3$. Figure 2 shows an example of one run with $M = U = 5$. It is

interesting to observe that there is always a long period of oscillation before the population starts to converge. The convergence is not gradual but rather is quite abrupt. This is reminiscent of the "phase transition" which has been discussed as a common pattern of emergence in physical, biological and social systems [10]. The global structure abruptly emerges, which seems to be reached by chance. As it is hard to rigorously analyze the condition of convergence for the current model, we can only demonstrate the phase transition by simulation, while we have undertaken no systematic analysis to determine when the transition point will occur.

Figure 1 about here

Figure 2 about here

- **The effect of adjusting variable Δ**

  As we have discussed above, the adjustment variable Δ is crucial; it should be neither too large nor too small. The following Table 7 shows the convergence time for different values of Δ. The table shows the percentage of runs for which convergence occurs, and the average, minimum, maximum and variance of convergence time taken from 500 runs. It can be seen that there is an optimal Δ for each vocabulary size. For $M = U = 3$ the optimal Δ must lie in the interval $[0.1, 0.3]$, while for $M = U = 4$ or 5, the optimal value must lie in the interval $[0.05, 0.2]$.

  Table 7 about here

- **The effect of population size**

  Figure 3 shows the convergence percentage and the average convergence time for different population sizes, P_s, taken from 500 runs. For most population sizes, the convergence time increases as the population size increases, at a rate faster than linear. However, it is interesting to note that when the population size is small, here smaller than 5, the convergence is slow. There seems to be an optimal population size which converges the fastest—P_s=10 for $M = U = 3$ and Δ=0.3.

  The optimal population size observed above is in fact an artefact due to the constraint that agents are not allowed to interact with themselves. When we remove this constraint, allowing agents to interact with themselves and update their vocabulary accordingly, it is found that the convergence becomes much faster, and there is no optimal population size (Figure 3); the smaller the population size, the easier it is for the population to achieve a consistent vocabulary. Such an effect of "self-talking" is not surprising, because not only is an agent's listening vocabulary formed by observing other agents' speaking behavior, the agent's speaking vocabulary, too, is indirectly influenced by others' speaking behavior by interacting with him/herself. This can be considered as an indirect imitation, which is very plausible in real situations as we have discussed in Section 2. The observation of optimal population size under the condition that "self-talking" is prohibited is unexpected. We could a preliminary explanation which is that in a very small population the majority hardly takes effect accumulatively, and agents are very sensitive to the small adjustment after each interaction with any other agent and thus keep oscillating. When self-talk allowed, the indirect imitation helps to suppress the oscillation to a large extent.

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Figure 3 about here

- The outcome when the speaking and listening matrices do not have the same size, i.e. $M \neq U$

  As mentioned earlier, the speaking and listening matrices are developed independently. It is therefore possible that the agents end up with different mappings for the two matrices. Nevertheless, under the current configuration with $M = U$, when the population converges the two matrices are always the same. Only when $M \neq U$ does the population actually converge with different speaking and listening matrices. It would appear to be a significant emergent property of the system that the speaking matrix is always a subset of the listening matrix when $U > M$, an example of which is shown below. The simulation results show that such system indeed emerges, which corresponds to the observation of non-identical speaking and listening vocabulary in reality as we have discussed in Section 3.1.

  Table 8 about here

The above four sections report various factors determining the convergence of a consistent vocabulary. We can see that the self-organization is sensitive to the variables such as the adjustment variable and the population size. However, the attainment of the stable state is not guaranteed, at least within the maximum permitted number of interactions, 1000,000 for simulations reported here.

4   The hybrid model

The imitation and interaction models in the previous sections have only treated the horizontal interactions in a fixed population in one generation. In this section, we will build upon the interaction model and add to it vertical transmission from generation to generation. As mentioned in the Introduction, the mechanism of cultural selection is often used to study vertical transmission. No matter by gene transmission or meme transmission, the replicators which have higher fitness to the environment will have higher probability to be transmitted to the next generation. In this study, we assume that the communicative consistency of the vocabulary determines the possibility that it is transmitted to the next generation. In other words, the consistency of the agent's vocabulary is regarded as its fitness and is related to the successfulness of it being learned by the next generation. The fitness of agent I's vocabulary $F(I)$ is defined here as the average consistency when the agent communicates with all agents including himself/herself,

$$F(I) = \frac{1}{P_s} \sum_{J=1}^{P_s} PC(I, J)$$

where $PC(i, j)$ is defined in Section 3.4.

We implement vertical transmission as follows. Assume that each child samples "exhaustively" their parent's speaking and listening matrices to form his own initial vocabulary, with the result that the child has nearly the same speaking and listening matrices as that of the parent. In addition, to reflect the errors which often occur during learning, we also allow occasional small mutations in the matrices. Such implementation is different from Nowak & Krakauer [17] in that their model does not assume an exhaustive sampling but instead a limited sampling, allowing unique mappings between objects and sounds to be achieved more quickly. This is the reason that the population can converge very quickly in their simulation.

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In our hybrid model, after learning from the parents and building their own initial mappings, the agents of a new generation go through a number of interactions as described in the interaction model. For the sake of simplicity, we set the number of interactions to 1000 regardless of the population size, though it may be true that a larger population should provide more opportunities for each agent to interact with more different agents. Table 9 shows the convergence time for various vocabulary sizes. Figure 4 demonstrates the convergence trends of one run of simulation when $P_s = 10$ and $M = U = 5$.

Table 9 about here

Figure 4 about here

In the interaction model, when the population size becomes large or the vocabulary size increases, it is hard for the population to converge to a consistent vocabulary, even when the maximum number of interactions is set as large as 1,000,000. However, the life span of a human being is limited and the time for an individual to learn is always finite. Therefore, interaction among a single generation of a population cannot guarantee the formation of a consistent vocabulary. If the population cannot develop a consistent vocabulary for one generation, and the next generation starts from scratch again, then it would be hard for a common vocabulary to emerge. On the contrary, if the partially developed vocabulary in the previous generation can be transmitted and developed cumulatively, the chance for the emergence of a consistent vocabulary will be much higher. From the simulation in this section, we can observe the effect of cultural selection: a consistent vocabulary emerges much more quickly and with higher probability in the hybrid model than in the interaction model.

To make an approximate comparison of the convergence time between these two models, we calculate the equivalent convergence time for the hybrid model as the number of generations multiplied by the number of interactions per generation, and compare this figure with the total number of interactions in the interaction model. From Tables 7 and 9, we see that under the same condition, for example, $M = U = 5$ and $\Delta = 0.1$, the hybrid model is about 10 times faster, in terms of average convergence time, than the interaction model. Furthermore, the hybrid model has a higher convergence rate (100%) than interaction model (90%).

5 Conclusions and discussions

In this study, we have used simulation and mathematical models to explore the emergence of vocabulary. We propose that a common vocabulary, the first ordered system in human language phylogeny, would have formed as the result of conventions established in the population through local interactions. Our approach of studying the emergence of patterns from the interactions among the agents falls into the general setting of agent-based modeling, which has been widely used in the study of various complex systems with emergent properties. There are only a small number of rules describing each agent's behavior. However, the order emerges from the locally coupled interactions [28].

The reported imitation and interaction models show that interaction between agents in a population can cause them to arrive at a coherent vocabulary. We speculate that, especially in the early stages of language emergence when there was no established communication system, synchronic interactions among agents were possibly the only way for early humans to form conventions and to attain mutual understanding. The imitation and interaction models have allowed us to study how the convention may be achieved under various conditions. The simulations demonstrate that without external design or driving force, coherence among

the entire population can be reached only through self-organization by each agent. Each agent concentrates on adjusting him/herself to be better understood by others and to better understand others for his/her own benefit. Nevertheless, this individual focus can lead to global consistency.

The interaction hypothesis, however, only works with small populations and small vocabulary size. We speculate that cultural selection plays a role in speeding up and passing on the progress of interaction between agents through generations. Those vocabularies which have a higher fitness through self-organization will have a higher probability to be transmitted to the next generation. Such an external selection mechanism accumulates the benefits obtained by each agent and spreads them through the population, and therefore speeds up the emergence of a consistent communication system.

Language emergence and evolution have been widely studied under the framework of cultural evolution [29]; cultural transmission evolves much faster than biological transmission. We propose that it is the combined effect of horizontal self-organization plus cultural selection in vertical transmission that results in high speed evolution. We believe the hybrid model in this study has demonstrated this effect.

We observe that as the vocabulary tends to convergence there is a tendency to exhibit a sharp phase transition. The global structure abruptly emerges after a long period of oscillation. It seems that there is some threshold after which the system converges quickly. However, it is not yet clear how to obtain or predict this threshold.

In the demographic study of early humans, it has been pointed out that for hunter-gatherers there exists an optimal size of population for the formation of small local groups of 10-50 persons [30]. Interestingly, our simulation also shows that there is an optimal population size for the emergence of early vocabulary. If the population size is too small, a shared vocabulary does not easily emerge; meanwhile, as the population size is increased, the convergence time increases faster than linearly. Therefore, we speculate that it is still very likely that language emerged in groups having an optimal size, neither too big nor too small. We note that this optimum size occurs only under the constraint that agents do not interact with themselves, though the plausibility of this constraint is still under question. The observed population size (i.e. 10-15) is smaller than that deemed likely among modern human groups. Further investigation in this aspect would be worthwhile.

Other work can be done along the line of the models we have presented here as well. In the current model, agents do not make use of their previous experience when modifying their vocabulary. Agents update their vocabulary matrix with a fixed value, only according to the successfulness of the current interaction. However, by common sense, experience is an important factor which could affect agents' behavior. Therefore we could further investigate the effect of experience. We assume that agents have a memory of their own several previous interactions and modify the matrices according to the current and previous interaction results. Preliminary simulation results show that convergence can thus be obtained in larger vocabulary such as $M = 10$, $U = 10$ and population sizes up to 100. Detailed results will be given in successive reports.

One may also incorporate one or a group of model agents who already have consistent mappings in the population and see how this agent or group affects the population's convergence. The current model of generation transmission is rather simple as each child learns a parent's vocabulary by sampling with some small learning errors. Komarova & Nowak [31] recently report analytic models and simulations for incomplete and incorrect vocabulary learning. Further systematic study of the effect of learning errors could be a useful direction of exploration.

Acknowledgement

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Appendix: A Markov chain proof of convergence

Definition 1 *A **Markov chain** is defined as a sequence $X_0, X_1, \ldots$ of discrete random variables with the property that the conditional distribution of X_{n+1} given $X_0, X_1, \ldots, X_n$ depends only on the value of X_n but not further on $X_0, X_1, \ldots, X_{n-1}$; i.e. for any set of values $h, j, \ldots, k$ belonging to the discrete state space,*

$$Pr(X_{n+1} = k | X_0 = h, \ldots, X_n = j) = Pr(X_{n+1} = k | X_n = j).$$

Definition 2 *Given a Markov chain M, a **closed set of states** is any proper subset of states in M such that there is no arc from any of the states in C to any state not in C. Particularly, a closed set with only one element (state) is called an **absorbing state**.*

Property 1 *For transition matrices corresponding to finite Markov chains, the multiplicity of the eigenvalue $\lambda = 1$ is equal to the number of closed sets.*

Property 2 *Let P be an $M \times M$ transition matrix, $\mathbf{x_i}$ and $\mathbf{y_i}$ are the linearly independent left eigenvectors and right eigenvectors corresponding to λ_i, i.e. $\mathbf{x_i}^T P = \lambda_i \mathbf{x_i}^T$ and $P\mathbf{y_i} = \lambda_i \mathbf{y_i}$. P^k can be represented as*

$$P^k = \sum_{i=0}^{M} \lambda_i^k \mathbf{y_i} \mathbf{x_i^T}$$

Property 3 *As k tends to infinity,*

$$P_\infty = \lim_{k \to \infty} \sum_{i:\lambda_i=1} \mathbf{y_i} \mathbf{x_i}^T$$

A Markov chain for the imitation model

We begin this mathematical investigation by considering a simplified model with one meaning ($M = 1$), two available utterances ($U = 2$, i.e. u_1 and u_2), and P_s number of agents in the population. Writing the number of agents who use utterance u_1 as k ($k = 0, 1, \ldots, P_s$), after each imitation interaction k can change in one of only three possible ways: keep constant, decrease by 1, or increase by 1. In other words, k_{n+1} depends only on k_n, the situation at the previous time instant. Therefore, by Definition 1, this process can be viewed as a Markov chain.

Strategy 1

The Markov chain for the state relationships in one imitation interaction for Strategy 1 can be represented as the following Figure 5:

Figure 5 about here

The transition matrix $\mathbf{P}$ (with size $(P_s + 1) \times (P_s + 1)$) for the Markov chain is

$$\mathbf{P} = \begin{bmatrix} 1 & 0 & 0 & 0 \ldots & & & 0 \\ c_1 & d_1 & c_1 & 0 \ldots & & & 0 \\ 0 & c_2 & d_2 & c_2 \ldots & & & 0 \\ \vdots & \vdots & \vdots & & & & \vdots \\ & & & \ldots c_j & d_j & c_j \ldots & \\ \vdots & \vdots & \vdots & & & & \vdots \\ & & & & c_1 & d_1 & c_1 \\ 0 & 0 & 0 & 0 \ldots & & & 1 \end{bmatrix}$$

where

$$c_j = \frac{j(P_s - j)}{P_s(P_s - 1)}$$

and

$$d_j = \frac{j(j-1) + (P_s - j)(P_s - j - 1)}{P_s(P_s - 1)}.$$

According to the definition of closed set of states in a Markov chain, we can easily see that there are only two closed states in the above Markov chain shown in Figure 5. Therefore the multiplicity of $\lambda = 1$ is 2, i.e. there are two eigenvalues equal to 1. Other eigenvalues are all smaller than 1.

Taking $\mathbf{x_i}^T \mathbf{P} = \lambda \mathbf{x_i}^T$, $\mathbf{x_i}^T$ can be seen as an input to the Markov chain. Setting $\lambda = 1$ means that the output will be the same as the input. It is easy to calculate that $\mathbf{x_1} = [1\ 0\ 0\ 0 \ldots 0]^T$ and $\mathbf{x_2} = [0\ 0\ 0\ 0 \ldots 1]^T$. To calculate $\mathbf{y_1}$ and $\mathbf{y_2}$, we need to solve a series of equations.

From

$$\mathbf{P}[y_1\ y_2\ \ldots y_{P_s+1}]^T = [y_1\ y_2\ \ldots y_{P_s+1}]^T$$

we have

$$c_1 y_1 + d_1 y_2 + c_1 y_3 = y_2$$
$$c_2 y_2 + d_2 y_3 + c_3 y_4 = y_3$$
$$\ldots$$
$$c_1 y_{P_s-1} + d_1 y_{P_s} + c_1 y_{P_s+1} = y_{P_s}$$

Thus we obtain $\mathbf{y_1} = [1\ \frac{P_s-1}{P_s} \ldots \frac{1}{P_s}\ 0]^T$. Similarly, we can obtain $\mathbf{y_2} = [0\ \frac{1}{P_s} \ldots \frac{p-1}{P_s}\ 1]^T$. Therefore,

$$\mathbf{P}^\infty = \mathbf{y_1}\mathbf{x_1}^T + \mathbf{y_2}\mathbf{x_2}^T$$
$$= \begin{bmatrix} 1 & 0\ 0\ 0 \ldots & 0 \\ \frac{P_s-1}{P_s} & 0\ 0\ 0 \ldots & \frac{1}{P_s} \\ \vdots & \vdots\ \vdots & \vdots \\ \frac{1}{P_s} & 0\ 0\ 0 \ldots & \frac{P_s-1}{P_s} \\ 0 & 0\ 0\ 0 \ldots & 1 \end{bmatrix}$$

which indicates that the population will always converge to a state in which either all agents use u_1 or all agents use u_2. Different initial conditions cause the Markov chain to converge to either of the stable states with different probabilities.

Consider a more complicated model with $M = 1, U > 2$. We use a mathematical induction method to prove the emergence of consistency by the following steps:

1. We have proved convergence for $M = 1, U = 2$.

2. Assume that we can prove convergence $M = 1, U = i$. That is, after a sufficient number of interactions, the agents will all use the same utterance.

3. When $U = i + 1$, the set of all utterances $\{u_1, u_2, ..., u_i, u_{i+1}\}$ can be divided into two subsets $Set1 = \{u_1, u_2, ..., u_i\}$ and $Set2 = \{u_{i+1}\}$. Similarly we can construct a Markov chain which contains $P_s + 1$ states, $k = 0, 1, 2, ..., P_s$. A state denotes the number of agents that use the utterance in $Set1$. As the number of interactions tends to infinity,

the system will converge to one of only two possible states: $k = 0$ or $k = P_s$. That is to say, finally there are only two possible situations for the population, either all agents use u_{i+1} or all agents use one of the utterances in $Set1$. As we have the assumption in step [2], the population is convergent with $U = i$ number of available utterances. Thus, when $U = i + 1$, the system can reach convergence also.

Strategy 2

Consider again the simplest case of $M = 1$, $U = 2$. The transition matrix for the Markov chain is as follows: if P_s is an even number,

$$
\mathbf{P} =
\begin{bmatrix}
1 & 0 & 0 & 0 \dots & & & & & & & 0 \\
e_1 & f_1 & 0 & 0 \dots & & & & & & & 0 \\
0 & e_2 & f_2 & 0 \dots & & & & & & & 0 \\
\vdots & \vdots & \vdots & & & & & & & & \vdots \\
& & & \dots & e_{\frac{P_s}{2}-1} & f_{\frac{P_s}{2}-1} & 0 \dots & & & & \\
& & & \dots & 0 & 1 & 0 & & & & \\
& & & \dots & 0 & f_{\frac{P_s}{2}+1} & e_{\frac{P_s}{2}+1} \dots & & & & \\
\vdots & \vdots & \vdots & & & & & & & & \vdots \\
& & & & & & & & 0 & f_1 & e_1 \\
0 & 0 & 0 & 0 \dots & & & & & & & 1
\end{bmatrix}
$$

while if P_s is an odd number,

$$
\mathbf{P} =
\begin{bmatrix}
1 & 0 & 0 & 0 \dots & & & & & & & 0 \\
e_1 & f_1 & 0 & 0 \dots & & & & & & & 0 \\
0 & e_2 & f_2 & 0 \dots & & & & & & & 0 \\
\vdots & \vdots & \vdots & & & & & & & & \vdots \\
& & & \dots & e_{\frac{P_s-1}{2}} & f_{\frac{P_s-1}{2}} & 0 \dots & & & & \\
& & & \dots & 0 & f_{\frac{P_s+1}{2}} & e_{\frac{P_s+1}{2}} \dots & & & & \\
\vdots & \vdots & \vdots & & & & & & & & \vdots \\
& & & & & & & & 0 & f_1 & e_1 \\
0 & 0 & 0 & 0 \dots & & & & & & & 1
\end{bmatrix}
$$

where

$$
e_j = 2 \times \frac{j(P_s - j)}{P_s(P_s - 1)}
$$

and

$$
f_j = \frac{j(j - 1) + (P_s - j)(P_s - j - 1)}{P_s(P_s - 1)}.
$$

The Markov chains have the states shown in the following two graphs:

Figure 6 about here

Figure 7 about here

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It is easy to derive $\mathbf{P}^\infty$ using a method similar to Strategy 1. If P_s is an even number,

$$\mathbf{P}^\infty = \begin{bmatrix} 1\,0\,0\,0\,\ldots & 0 \\ 1\,0\,0\,0\,\ldots & 0 \\ \vdots\;\vdots\;\vdots & \vdots \\ 1\,0\,0\,0\,\ldots & 0 \\ 0\,0\,0\,0\,\ldots & 0 \\ 0\,0\,0\,0\,\ldots & 1 \\ \vdots\;\vdots\;\vdots & \vdots \\ 0\,0\,0\,0\,\ldots & 1 \\ 0\,0\,0\,0\,\ldots & 1 \end{bmatrix}$$

while if P_s is an even number,

$$\mathbf{P}^\infty = \begin{bmatrix} 1\,0\,0\,0\,\ldots & 0 \\ 1\,0\,0\,0\,\ldots & 0 \\ \vdots\;\vdots\;\vdots & \vdots \\ 1\,0\,0\,0\,\ldots & 0 \\ 0\,0\,0\,0\,\ldots & 1 \\ \vdots\;\vdots\;\vdots & \vdots \\ 0\,0\,0\,0\,\ldots & 1 \\ 0\,0\,0\,0\,\ldots & 1 \end{bmatrix}$$

Thus it is proved that the population can reach consistency after long term interaction, except in the situation that there is an equal number of agents using each of the two utterances at the beginning of the interaction when P_s is an even number.

Figure Captions:

Figure 1: The convergent trends from an example simulation of the interaction model (P_s=10, $M = U = 3$, $\Delta = 0.2$). Three measures of the convergence (SI, PC and IC) are shown. A consistent vocabulary emerges after 3553 interactions. An abrupt phase transition can be observed around 3000th interaction.

Figure 2: The convergent trends from an example simulation of the interaction model (P_s=10, $M = U = 5$, $\Delta = 0.1$). A consistent vocabulary emerges after 443781 interactions. An abrupt phase transition can be observed around 44000th interaction.

Figure 3: The relationship between P_s and CT for the interaction model. When agents are not allowed to interact with themselves (i.e. no self-talk), an optimal population size can be observed; when self-talk is allowed, the convergence time increases nonlinearly as the population size increases.

Figure 4: The convergent trends from an example simulation of the hybrid model (P_s=10, $M = U = 5$, $\Delta = 0.1$). Three measures of the convergence (SI, PC and IC) are shown. A consistent vocabulary emerges after 50 generations. 1000 interactions take place per generation.

Figure 5: The Markov chain for a simplified model of imitation strategy 1, i.e. random imitation. $U = 2, M = 1$.

Figure 6: Markov chain for a simplified model of imitation strategy 2, i.e. following the majority. P_s is an even number.

Figure 7: Markov chain for a simplified model of imitation strategy 2, i.e. following the majority. P_s is an odd number.
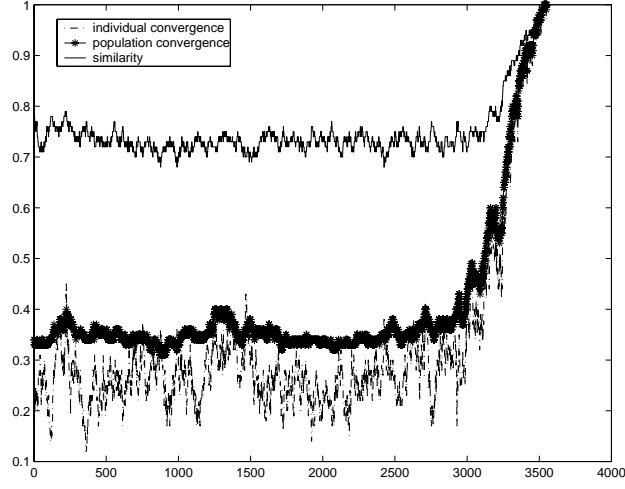
Figures and Tables

Figure 1: The convergent trends from an example simulation of the interaction model (P_s=10, $M = U = 3$, $\Delta = 0.2$). Three measures of the convergence (SI, PC and IC) are shown. A consistent vocabulary emerges after 3553 interactions. An abrupt phase transition can be observed around 3000th interaction.
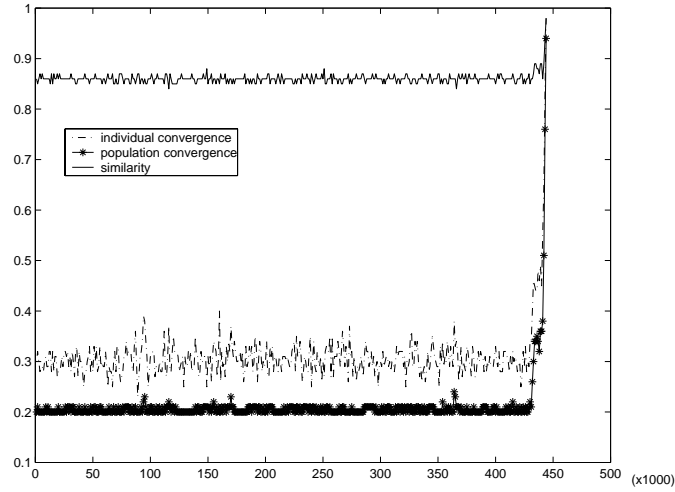

Figure 2: The convergent trends from an example simulation of the interaction model (P_s=10, $M = U = 5$, $\Delta = 0.1$). A consistent vocabulary emerges after 443781 interactions. An abrupt phase transition can be observed around 44000th interaction.
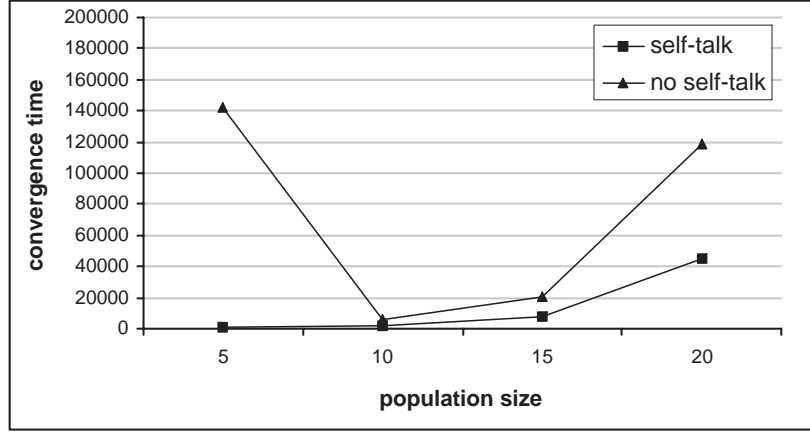
Figure 3: The relationship between P_s and CT for the interaction model. When agents are not allowed to interact with themselves (i.e. no self-talk), an optimal population size can be observed; when self-talk is allowed, the convergence time increases nonlinearly as the population size increases.

Table 1: Two agents, A and B, and their U-M mappings between utterances (U) and meanings (M)

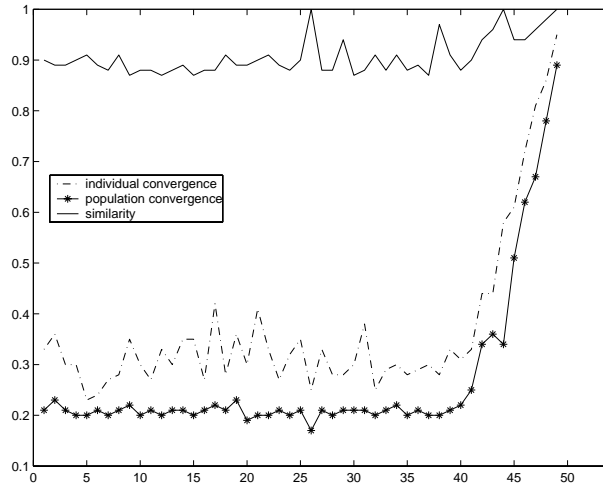|	m_1	m_2	m_3	m_4	$\ldots$
$\Psi(A)$	u_2	u_4	u_1	u_4	$\ldots$
$\Psi(B)$	u_5	u_3	u_1	u_1	$\ldots$

Figure 4: The convergent trends from an example simulation of the hybrid model (P_s=10, $M = U = 5$, $\Delta = 0.1$). Three measures of the convergence (SI, PC and IC) are shown. A consistent vocabulary emerges after 50 generations. 1000 interactions take place per generation.
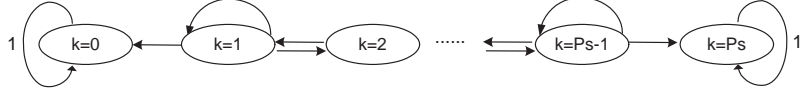
Figure 5: The Markov chain for a simplified model of imitation strategy 1, i.e. random imitation. $U = 2, M = 1$.
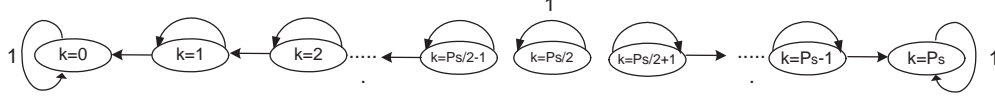

Figure 6: Markov chain for a simplified model of imitation strategy 2, i.e. following the majority. P_s is an even number.
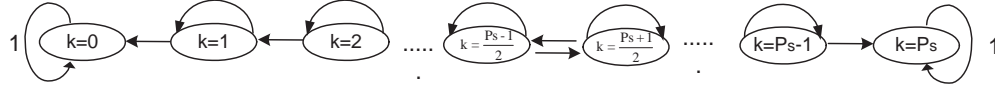

Figure 7: Markov chain for a simplified model of imitation strategy 2, i.e. following the majority. P_s is an odd number.

Table 2: Average consistency C and distinctiveness D of 100 runs for various P_s and U, Strategy 1 and 2 (imitation model)

S1	C			D		
	U=10	U=30	U=50	U=10	U=30	U=50
P_s=10	1.00	1.00	1.00	0.77	0.91	0.94
P_s=30	0.98	0.98	0.98	0.76	0.92	0.95
P_s=50	0.82	0.80	0.80	0.78	0.92	0.95
S2	C			D		
	U=10	U=30	U=50	U=10	U=30	U=50
P_s=10	1.00	0.80	0.63	0.74	0.92	0.95
P_s=30	1.00	1.00	1.00	0.73	0.92	0.95
P_s=50	1.00	1.00	1.00	0.74	0.91	0.94

Table 3: Probabilistic speaking and listening mapping matrices of agent S

p_{ij}	u_1	u_2	u_3
m_1	0.3	0.4	0.3
m_2	0.4	0.55	0.05
m_3	0.7	0.2	0.1

q_{ij}	u_1	u_2	u_3
m_1	0.1	0.3	0.6
m_2	0.5	0.3	0.3
m_3	0.4	0.4	0.1

Table 4: Probabilistic speaking and listening mapping matrices of agent L

p_{ij}	u_1	u_2	u_3
m_1	0.3	0.2	0.5
m_2	0.4	0.3	0.1
m_3	0.3	0.5	0.4

q_{ij}	u_1	u_2	u_3
m_1	0.2	0.1	0.7
m_2	0.6	0.2	0.1
m_3	0.2	0.7	0.2

Table 5: An example of convergent vocabulary with incompatible speaking and listening matrices

p_{ij}	u_1	u_2	u_3
m_1	1	0	0
m_2	0	1	0
m_3	0	0	1

q_{ij}	u_1	u_2	u_3
m_1	0	1	0
m_2	1	0	0
m_3	0	0	1

Table 6: An example of a stable state: all agents have the same speaking and listening matrices.

p_{ij}	u_1	u_2	u_3
m_1	1	0	0
m_2	0	1	0
m_3	0	0	1

q_{ij}	u_1	u_2	u_3
m_1	1	0	0
m_2	0	1	0
m_3	0	0	1

Table 7: The convergent time for different adjustment variable Δ (interaction model)

M	U	Δ	converged %	CT_{avg}	CT_{min}	CT_{max}	CT_{var}
3	3	0.3	100%	5958	878	27401	4288
3	3	0.2	100%	1759	759	4249	592
3	3	0.1	100%	1840	919	4246	514
3	3	0.05	100%	2916	1493	6289	718
4	4	0.3	0%	nc^a	nc	nc	0
4	4	0.2	100%	81294	2823	414821	72865
4	4	0.1	100%	7631	2941	22348	3131
4	4	0.05	99.8%	18072	5056	nc	65457
5	5	0.3	0%	nc	nc	nc	0
5	5	0.2	0%	nc	nc	nc	0
5	5	0.1	90.0%	402948	9016	nc	315519
5	5	0.05	21.4%	893577	35813	nc	234069

^{a}Note: If the population can not converge when the simulation completes the given number of interactions in a run, then this run is considered unconverged. When all 500 runs are unconverged, "nc" is indicated in the table. This applies to the following tables as well.

Table 8: An example of convergent vocabulary when $U > M$: Two types of resultant speaking and listening matrices, speaking being a subset of listening vocabulary (interaction model)

p_{ij}^1	u_1	u_2	u_3	u_4
m_1	1	0	0	0
m_2	0	1	0	0
m_3	0	0	1	0

p_{ij}^2	u_1	u_2	u_3	u_4
m_1	0	0	0	1
m_2	0	1	0	0
m_3	0	0	1	0

q_{ij}^1	u_1	u_2	u_3	u_4
m_1	1	0	0	1
m_2	0	1	0	0
m_3	0	0	1	0

q_{ij}^2	u_1	u_2	u_3	u_4
m_1	1	0	0	1
m_2	0	1	0	0
m_3	0	0	1	0

Table 9: The convergent time for different vocabulary sizes (hybrid model)

M	U	Δ	converged %	CT_{avg}	CT_{min}	CT_{max}	CT_{var}
3	3	0.3	100%	3	1	12	1
4	4	0.2	100%	20	2	129	16
5	5	0.1	100%	37	6	156	26
6	6	0.1	98.2%	552	25	na	486
7	7	0.1	46.6%	3876	69	na	964