

# Learning Rules With Numerical and Categorical Attributes from Linked Data Sources

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# Overview

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- 3 Inductive Logic Programming
- 4 Learning Rules With Numerical and Categorical Attributes
- 5 Experiments

# Semantic Web

## Semantic Web

*“provides a common framework that allows data to be shared and reused across application, enterprise, and community boundaries”*

# Semantic Web

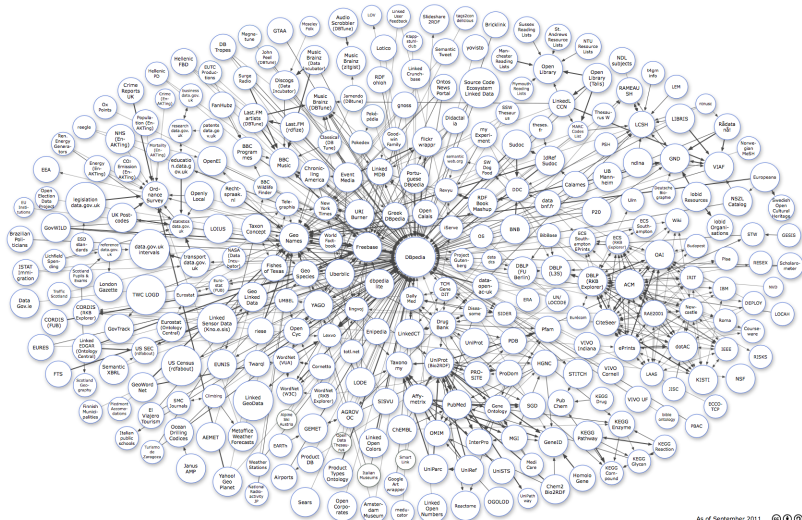
## Semantic Web

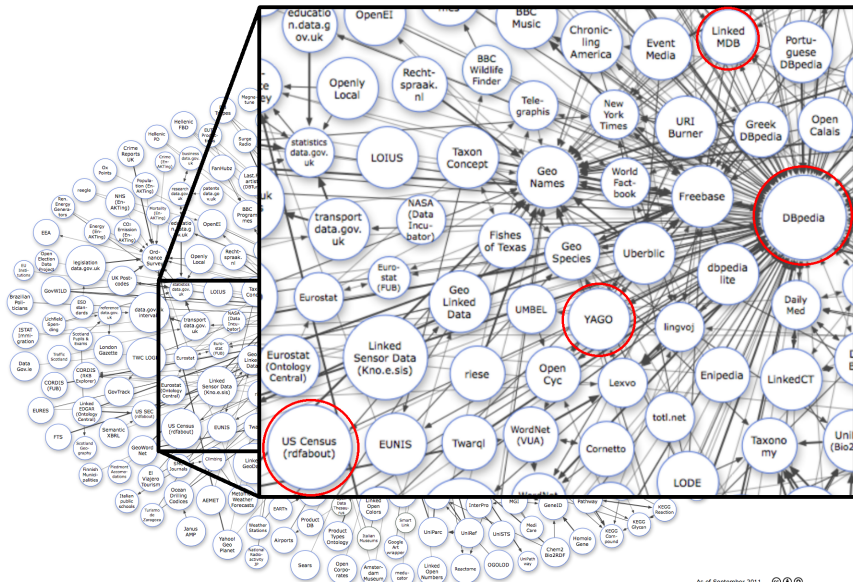
*“provides a common framework that allows data to be shared and reused across application, enterprise, and community boundaries”*

## Linked Data

*“collection of interrelated datasets on the Web”*

*“recommended best practises for exposing, sharing, and connecting pieces of data, information and knowledge on the Semantic Web”*





# Motivation

Learn Datalog rules from data:

$$\underbrace{livesIn(X, Y)}_{head} \text{ :- } \underbrace{isMarriedTo(X, Z), livesIn(Z, Y)}_{body}$$

Support and confidence thresholds

- ▶ Support:  $supp(head \text{ :- } body) = supp(head \wedge body)$
- ▶ Confidence:  $conf(head \text{ :- } body) = \frac{supp(head \wedge body)}{supp(body)}$

# Rules with constants

Refining rules with constants is relevant

$$\textit{speaks}(X, Z) \text{ :- } \textit{livesIn}(X, W)$$

Searching constants for  $Z$  and  $W$  we can learn:

$$\textit{speaks}(X, \textit{english}) \text{ :- } \textit{livesIn}(X, \textit{australia})$$
$$\textit{speaks}(X, \textit{spanish}) \text{ :- } \textit{livesIn}(X, \textit{argentina})$$
$$\textit{speaks}(X, \textit{portuguese}) \text{ :- } \textit{livesIn}(X, \textit{brasil})$$

What about numerical constants?

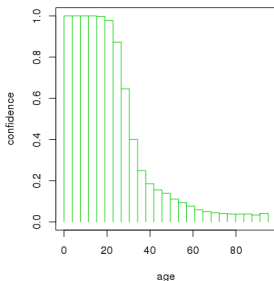
$$\textit{speaks}(X, \textit{english}) \text{ :- } \textit{hasIncome}(X, \$3.71\textit{Billion})$$
$$\textit{speaks}(X, \textit{portuguese}) \text{ :- } \textit{livesIn}(X, W), \textit{hasPopulation}(W, 193946886)$$



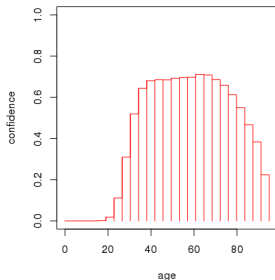
# Refining rules with numerical intervals

$\text{maritalStatus}(X, \text{single}) \text{ :- } \text{age}(X, Y) \text{ [conf=0.40]}$   
 $\text{maritalStatus}(X, \text{married}) \text{ :- } \text{age}(X, Y) \text{ [conf=0.46]}$   
 $\text{maritalStatus}(X, \text{widowed}) \text{ :- } \text{age}(X, Y) \text{ [conf=0.06]}$

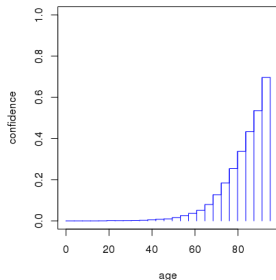
Single



Married



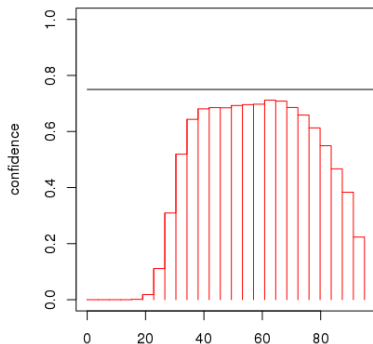
Widowed



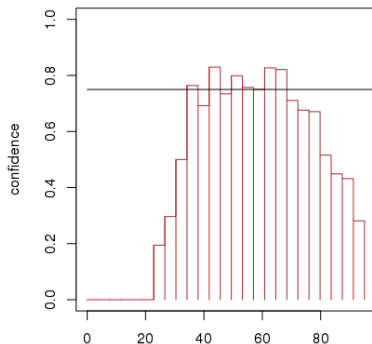
# Combine with categorical constants

For  $\text{maritalStatus}(X, \text{married})$ ,  $\text{minConf} = 0.75$  is not satisfied.  
Refine by State?

USA

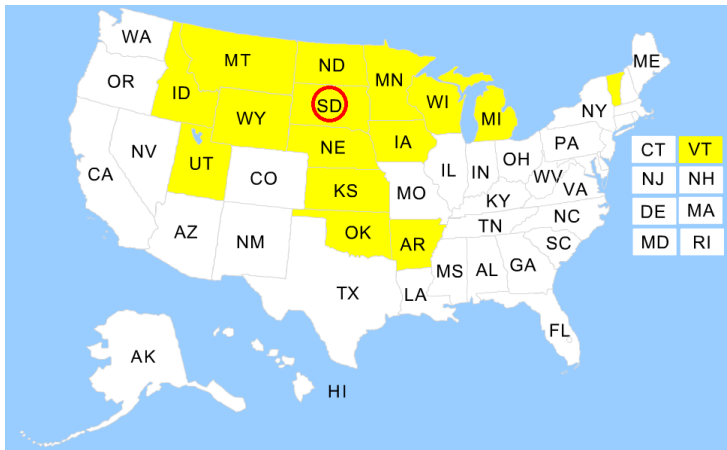


South Dakota



# Combine with categorical constants

We can find intervals for  $Y$  that satisfy  $\text{minConf} = 0.75$  for  $\text{maritalStatus}(X, \text{widowed}) \text{ :- } \text{livesIn}(X, \text{sd}), \text{age}(X, Y)$  and...



# Base-rule and Refined-rule

- **Base-rule:** Numerical argument with no constant

$r_1 : \text{maritalStatus}(X, \text{single}) :- \text{livesIn}(X, sd), \text{age}(X, Y)$   
 $[\text{conf}=0.49, \text{supp}=2368]$

- **Refined-rule:** Base-rule with restricted numerical variable

$r_2 : \text{maritalStatus}(X, \text{single}) :- \text{livesIn}(X, sd), \text{age}(X, Y), Y \in [33, 67]$   
 $[\text{conf}=0.77, \text{supp}=1092]$

We are interested in refinements that bring a significant confidence gain:

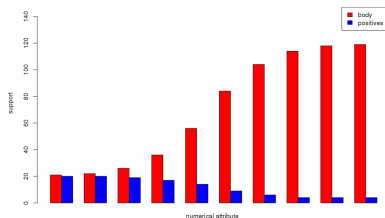
$$\text{gain}_{r_{ref}, r_{base}} = \frac{\text{conf}(r_{ref})}{\text{conf}(r_{base})} \quad (1)$$

For our example:  $\text{gain}_{r_2, r_1} = \frac{0.77}{0.49} = 1.57$

What base-rules have refined-rules with significant confidence gain?

- ▶ Satisfy support threshold
- ▶ Do not necessarily satisfy confidence threshold
- ▶ Divergent body and positives (body $\wedge$ head) probability distributions

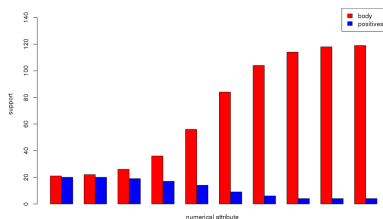
## Frequency histograms



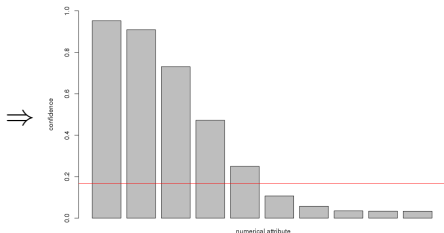
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Frequency histograms



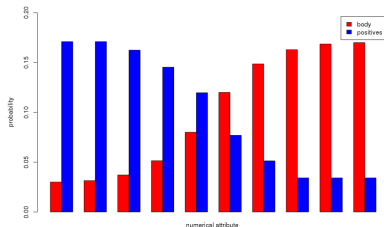
Confidence distribution



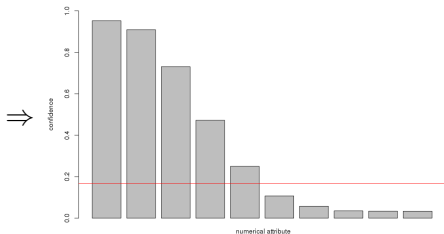
What base-rules have refined-rules with significant confidence gain?

- ▶ Satisfy support threshold
- ▶ Do not necessarily satisfy confidence threshold
- ▶ Divergent body and positives (body $\wedge$ head) probability distributions

Body and Positives distributions



Confidence distribution



# Motivation

## Problem?

- ▶ Search space grows exponentially with the number of predicates and constants
- ▶ Querying support and confidence distributions is very expensive

## Idea:

- ▶ Analyze combinations of numerical and categorical properties
- ▶ Measure their level of interestingness
- ▶ Extend top-down ILP to detect and suggest interesting combinations



# Logic Programming Concepts

- ▶ Literal: predicate symbol with bracketed n-tuple, e.g:  
 $L = \text{livesIn}(X, Y)$
- ▶ Clause: a disjunction of literals (negated or not), e.g:  
 $c = (L_1 \vee L_2 \vee \dots \vee \neg L_{m-1} \vee \neg L_m)$
- ▶ Safe Datalog Rule: every variable in the head appear in a non-negated literal in the body, negated literal variables in the body should appear in some positive literal in the body, e.g.:  
 $\text{speaks}(X, Y) \text{ :- } \text{wasBornIn}(X, Z), \text{hasOfficialLanguage}(Z, Y)$
- ▶ Hypothesis: a set of clauses  $\mathcal{H}$ 
  - ▶ Completeness:  $\mathcal{H}$  covers all positive examples
  - ▶ Consistency:  $\mathcal{H}$  covers no negative examples

# Inductive Logic Programming (ILP)

Inductive Logic Programming: Finds a hypothesis  $\mathcal{H}$  that covers all positive, and no negative examples

*positiveExamples + negativeExamples + backgroundKnowledge  $\rightarrow$  hypothesis*

Training Examples	Background Knowledge
daughter(mary,ann) +	parent(ann,mary)
daughter(eve,tom) +	parent(ann, tom)
daughter(tom,ann) -	parent(tom,eve)
daughter(eve,ann) -	parent(tom,ian)
	female(ann)
	female(mary)
	female(eve)

$\mathcal{H} = \text{daughter}(X, Y) \text{ :- } \text{female}(X), \text{parent}(Y, X)$

# Inductive Logic Programming (ILP)

## Approaches

- ▶ Bottom-up: Start with least general  $\mathcal{H}$  then perform generalizations
- ▶ **Top-down**: Start with most general  $\mathcal{H}$  then perform specializations
  - ▶ Specialization loop: adds literals to a clause and ensures consistency
  - ▶ Covering loop: adds clauses to the hypothesis and ensures completeness
  - ▶ Apriori-style pruning

What about large, noisy, and incomplete LOD datasets such as YAGO and DBpedia?:

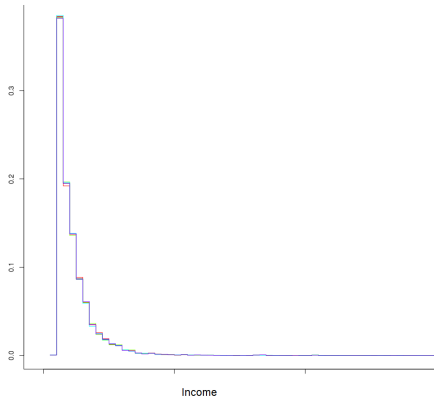
- ▶ Sample data to reduce size
- ▶ Restrict the number of literals in a clause
- ▶ Tolerate a certain level of inconsistency and incompleteness

$$\text{Expected Accuracy: } A(c) = P(e \in \mathcal{E}^+ | c) = \frac{n^+(c)}{n^+(c) + n^-(c)}$$

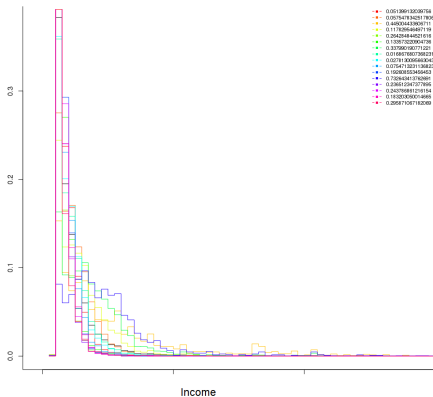
# Correlation between Literals

Let's say we want to refine a clause with  $hasIncome(X, Y)$  with an interval for  $Y$ . Refine by  $quarterOfBirth$  or  $hasEducation$ ?

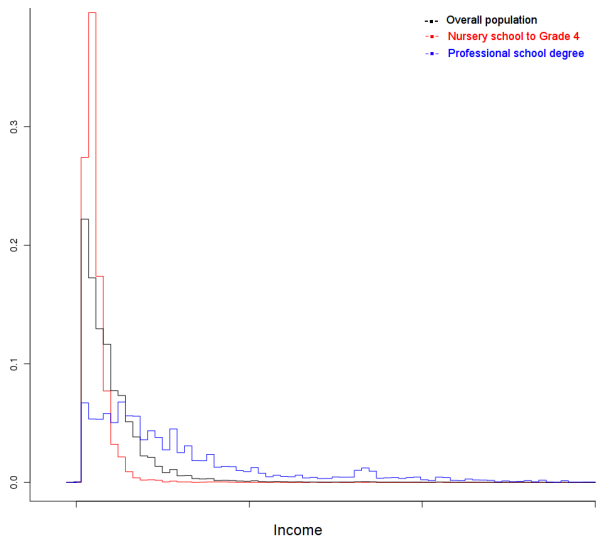
$hasIncome(X, Y), quarterOfBirth(X, Z)$



$hasIncome(X, Y), hasEducation(X, Z)$



# Correlation between Literals



## Educational Levels

Nursery school to grade 4  
 Grade 5 or grade 6  
 Grade 7 or grade 8  
 Grade 9  
 Grade 10  
 Grade 11  
 Grade 12 no diploma  
 High school graduate  
 Some college (< 1 year)  
 Some college ( $\geq$  1 year)  
 Associate's degree  
 Bachelor's degree  
 Master's degree  
 Professional school degree  
 Doctorate degree

# Interestingness Measure

How to measure the interestingness of adding a literal  $l$  to a clause  $c$ ?

- ▶ Extract the frequency histograms of  $\{c\}$  and  $\{c \wedge l\}$  over a numerical attribute  $Y$
- ▶ Normalize the histograms to obtain their probability distributions, and measure their divergence (e.g., with Kullback-Leibler)

But, divergence alone isn't a good idea because:

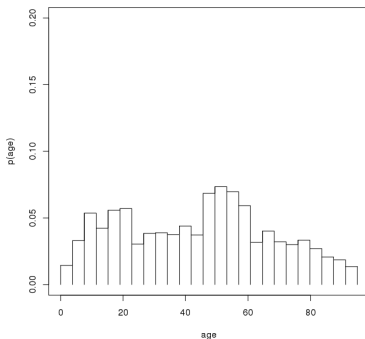
- ▶ Lower support histograms are more likely to have a divergent distribution (*sampling error*)
- ▶ Rules with high support are still interesting

Then combine both measures:  $\text{divergence} * \text{support}$

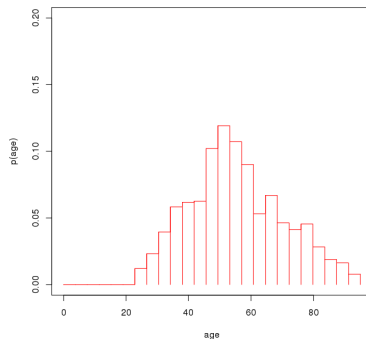
# Interestingness Measure

$$\underbrace{\text{maritalStatus}(X, \text{married})}_{l=\text{head}} \text{ :- } \underbrace{\text{livesIn}(X, \text{sd}), \text{age}(X, Y)}_{c=\text{body}}$$

$\text{age}(X, Y), \text{livesIn}(X, \text{sd})$



$\text{age}(X, Y), \text{livesIn}(X, \text{sd}), \text{maritalStatus}(X, \text{married})$



# Correlation Lattice

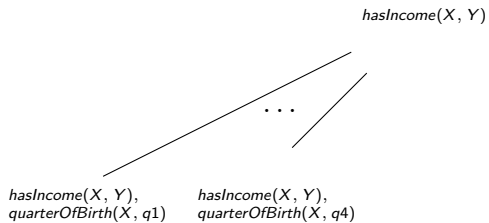
- ▶ Build a lattice similar to an *itemset lattice*
- ▶ Numerical property  $r(X, Y)$  as root
- ▶ The “items” are literals that can be joined with the root's non-numerical variable  $X$
- ▶ Root's numerical attribute  $Y$  is discretized in  $k$  buckets  $\{b_1, \dots, b_k\}$
- ▶ Each node  $x$  has a frequency histogram  $h(x) = \langle h_1(x), \dots, h_k(x) \rangle$  from its clause support distribution  
 where  $h_i(x) = \text{supp}(x | Y \in b_i)$  and  $|h(x)|_1 = \text{supp}(x)$



# Correlation Lattice

*hasIncome*( $X, Y$ )

# Correlation Lattice



# Correlation Lattice

*hasIncome*(*X*, *Y*)

...

...

*hasIncome*(*X*, *Y*),  
*quarterOfBirth*(*X*, *q1*)

*hasIncome*(*X*, *Y*),  
*quarterOfBirth*(*X*, *q4*)

*hasIncome*(*X*, *Y*),  
*hasEducation*(*X*, *nursery*)

*hasIncome*(*X*, *Y*),  
*hasEducation*(*X*, *phd*)

# Correlation Lattice

*hasIncome*(*X*, *Y*)

...

...

...

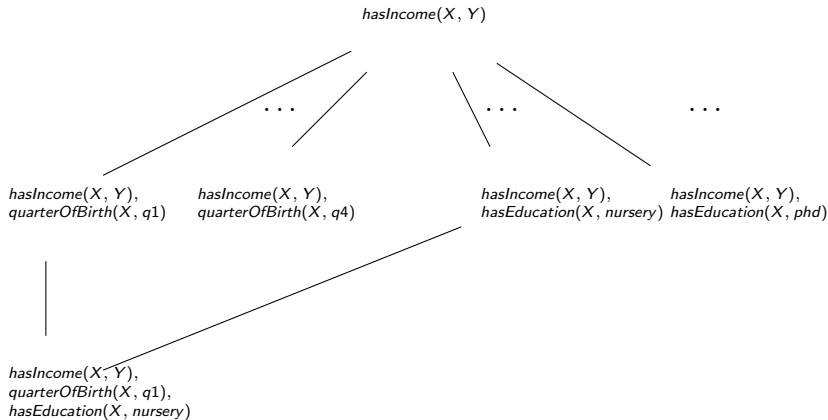
*hasIncome*(*X*, *Y*),  
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*hasIncome*(*X*, *Y*),  
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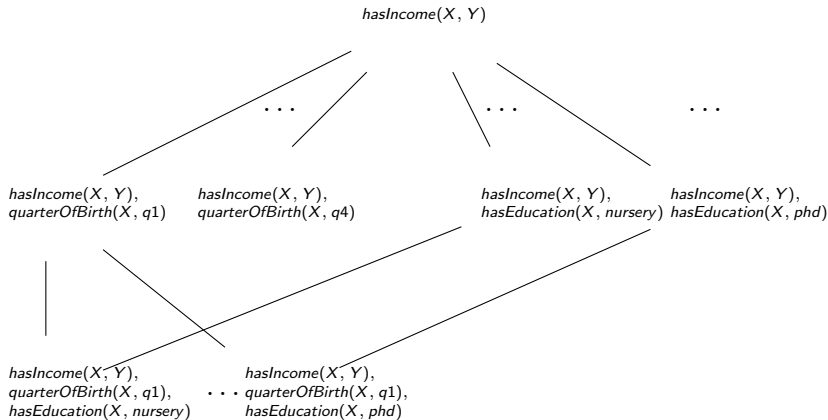
*hasIncome*(*X*, *Y*),  
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*hasEducation*(*X*, *phd*)

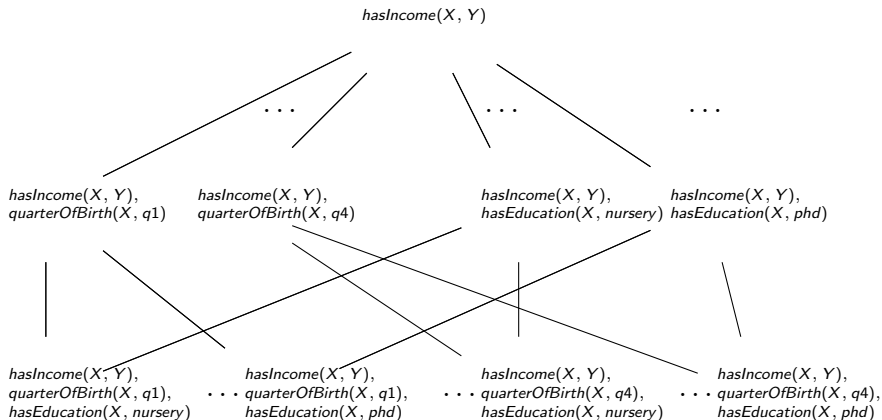
# Correlation Lattice



# Correlation Lattice



# Correlation Lattice



# Correlation Lattice

- ▶ Number of nodes in a lattice with  $\ell$  levels  $n$  properties and  $m$  constants per property:

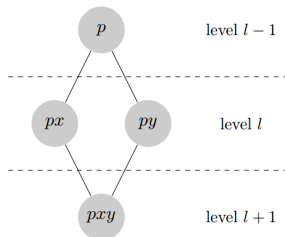
$$\sum_{i=1}^{\ell} \binom{nm}{i} \quad (2)$$

- ▶ Too expensive, we need to reduce size
  - ▶ Prune by support (safe)
  - ▶ Restrict  $\ell$  to the maximum clause size allowed in the core-ILP
  - ▶ Restrict the literals added to the lattice in order to reduce  $n$  and  $m$
  - ▶ Prune by interestingness or independence (heuristics)



# Independence checks

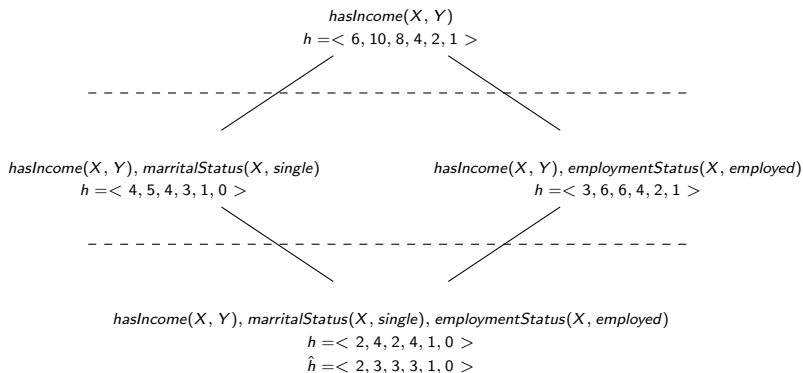
- Checks if a pair of nodes joining nodes are independent given their common parent



(where  $p$  is a clause,  $x$  and  $y$  are literals, s.t.  $x \neq y$  and  $x, y \notin p$ )

- Estimate  $\hat{h}(pxy)$  assuming independence of  $x$  and  $y$  given  $p$
- Query actual  $h(pxy)$  and perform a Pearson's chi-squared test
  - $H_0 = x$  and  $y$  are independent given  $p$
  - $H_1 = x$  and  $y$  are dependent given  $p$

# Independence checks



$$\chi^2 = \sum_{i=1}^k \frac{(h_i - \hat{h}_i)^2}{\hat{h}_i} = 1 \quad \Rightarrow \quad p\text{-value} = 0.96$$

# Independence checks

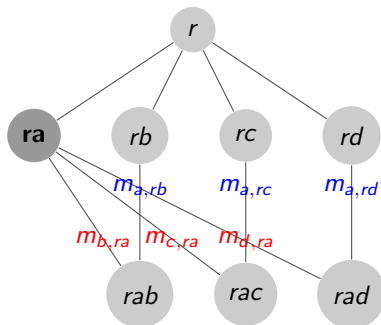
- ▶ If there's not enough evidence of dependence, we assume independence, then:  

$$x \text{ :- } p, y \equiv x \text{ :- } p$$

$$y \text{ :- } p, x \equiv y \text{ :- } p$$
- ▶ The lower the p-value (greater  $\chi^2$ ), the greater the evidence that  $x$  and  $y$  are dependent given  $p$ , therefore the more interesting it is to join the nodes  $py$  and  $px$
- ▶ As heuristics, we can set a maximum *p-value* threshold to prune independent nodes

# Refinement Suggestions

In the ILP refinement loop, the clauses have a fixed head while the body is refined. Assuming we have  $a$  as head literal,  $r$  as root and  $b, c, d$  as possible new literals:

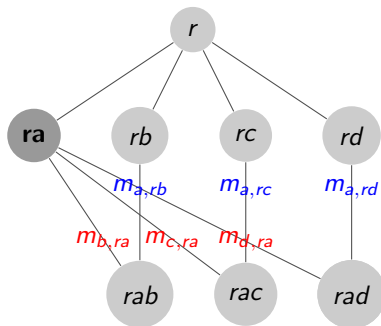


$$a \mid \begin{array}{l} b [m_{a,rb}] \\ c [m_{a,rc}] \\ d [m_{a,rd}] \end{array}$$

What literal is more interesting to add to the clause  $a:-r$ ?

# Refinement Suggestions

In the ILP refinement loop, the clauses have a fixed head while the body is refined. Assuming we have  $a$  as head literal,  $r$  as root and  $b, c, d$  as possible new literals:



$$a \mid \begin{array}{l} b [m_{a,rb}] \\ c [m_{a,rc}] \\ d [m_{a,rd}] \end{array}$$

What literal is more interesting to add to the clause  $a:-r$ ?

$$\operatorname{argmax}_{i \in \{b, c, d\}} m_{a,ri}$$

# Search in the Lattice

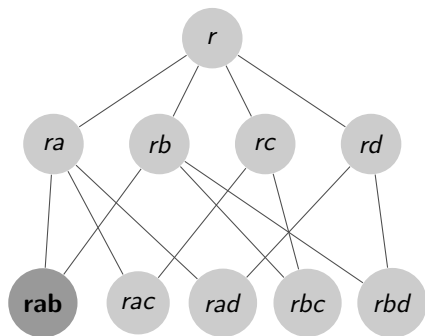
What has to be done?

- ▶ Search the node with body literals
- ▶ For each child of such node check head literal can be further added, if so collect the new literal and the interestingness value of adding the head
- ▶ Sort the possible new literals by interestingness

Alternative?

- ▶ Create mapping in every node with the possible head literals as key and sorted literals to be added to body as value
- ▶ Only add entry if head and new literal not independent given body

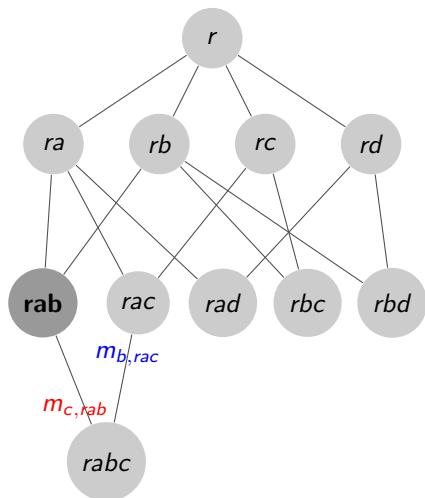
# Refinement Suggestions



Suggestions Map

|

# Refinement Suggestions

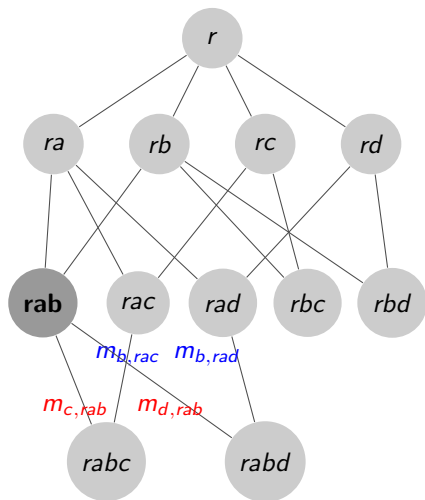


Suggestions Map

$b$	$c$	$[m_{b,rac}]$
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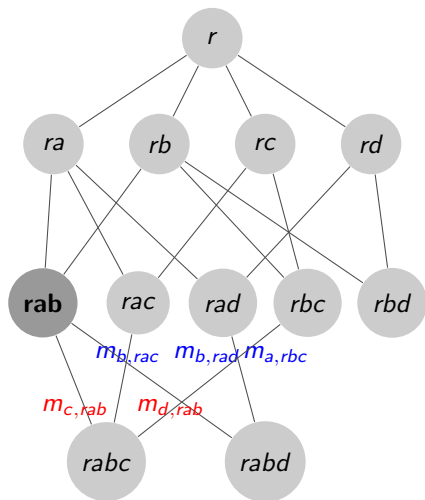
# Refinement Suggestions



Suggestions Map

$$b \left| \begin{array}{l} c [m_{b,rac}] \\ d [m_{b,rad}] \end{array} \right.$$

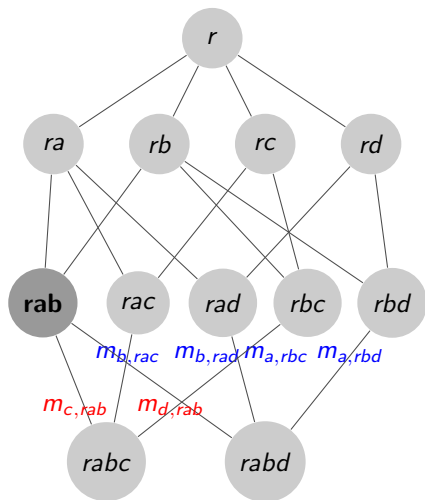
# Refinement Suggestions



Suggestions Map

$b$	$c [m_{b,rac}]$
	$d [m_{b,rad}]$
<hr/>	
$a$	$c [m_{a,rbc}]$

# Refinement Suggestions



Suggestions Map

$b$	$c [m_{b,rac}]$ $d [m_{b,rad}]$
$a$	$c [m_{a,rbc}]$ $d [m_{a,rbd}]$

# Incorporating the Lattice in the Core-ILP

In the refinement step, we detect clauses with body containing a lattice root

- ▶ If clause satisfies support threshold and does not satisfy confidence threshold
- ▶ Then search in lattice for body literals and head
- ▶ Check the interestingness of adding the head to the body and analyze whether to search for numerical intervals
- ▶ Query the lattice for suggestions of interesting literals to be added to the clause

# Experiments

## Overall Settings:

- ▶ We compare 4 interestingness measures:
  1. ● *supp*: Support Only
  2. ■ *kl supp*: KL-divergence\*Support
  3. ● *kldiv*: KL-divergence Only
  4. ★ *jssupp*: JS-divergence\*Support
- ▶ Thresholds:
  - ▶  $minConf = 0.75$
  - ▶  $minSupp = 25$
  - ▶  $minGain = 1.25$

## 1<sup>st</sup> Experiment: evaluation of the Correlation Lattice

- ▶ All data joined by person only (anonymized)
- ▶ All properties categorical (categories as literals)
- ▶ Create a lattice for *hasIncome* property

## 2<sup>nd</sup> Experiment: evaluation of the ILP extension

- ▶ All data joined by person only (anonymized)
- ▶ All properties categorical (categories as literals)

# 1<sup>st</sup> Experiment

Figure: Build time per lattice size

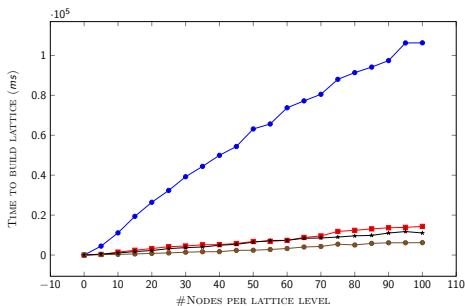
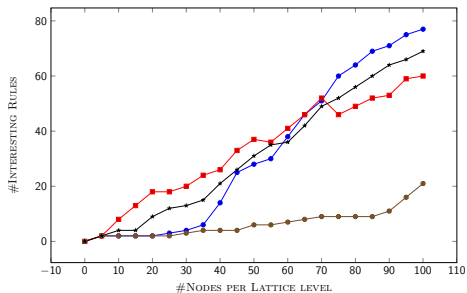


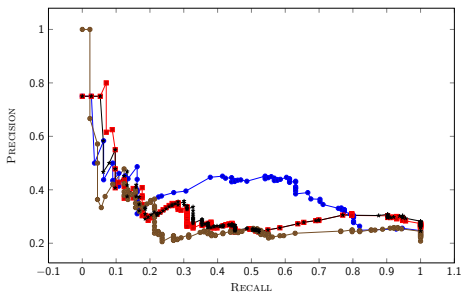
Figure: Interesting rules per lattice size



Legend: [● *supp* ■ *klsupp* ● *kldiv* ★ *jssupp*]

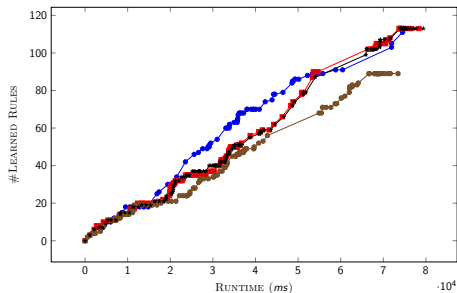
## 2<sup>nd</sup> Experiment

Figure: Precision-Recall graph from interestingness predictions (rules with *runtime* attribute)



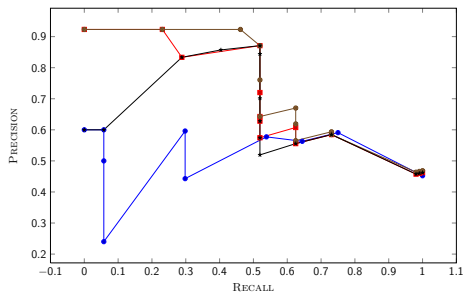
Legend: [● *supp* ■ *klsupp* ● *kldiv* ☆ *jssupp*]

Figure: Interesting rules per runtime (rules with attribute *runtime*)

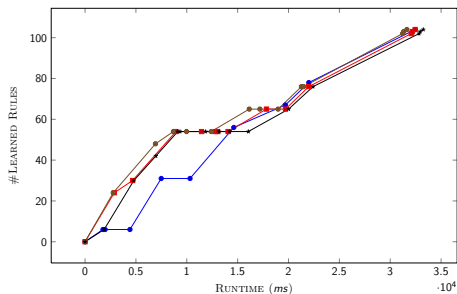


# 1<sup>st</sup> Experiment

**Figure:** Precision-Recall graph from interestingness predictions (rules with *budget* attribute)



**Figure:** Interesting rules per runtime (rules with *budget* attribute)





# Thank you