Theoretical Cosmology
Part IV: Dark Energy

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ICG PhD Lectures, November 2021

1. Introduction to Big-Bang cosmology	15 November
2. Hot thermal Universe	19 November
3. Inflation	22 November
4. *Dark energy*	*26 November*

Plan for today

1. *Observational evidence for acceleration*
2. *Cosmological constant problem*
3. *Alternative explanations*
4. *Dynamical DE, Modified gravity – challenges*
5. *Growth history and observational tests*

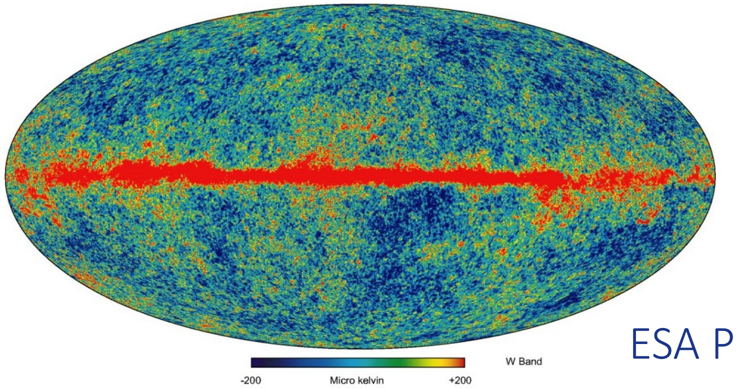
References

- Kazuya Koyama's 2016 lecture notes (especially the observation section)
- *Dark Energy*, David Weinberg and Martin White, in *Review of Particle Physics*.
- *Everything You Always Wanted To Know About The Cosmological Constant Problem (But Were Afraid To Ask)*, Jérôme Martin, arXiv:1205.3365
- Several research papers, references included in slides.

Basic Assumptions

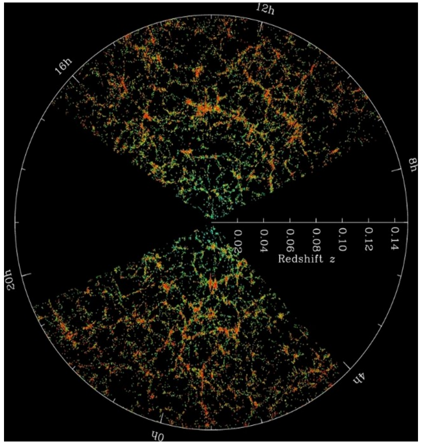
1. Cosmological Principle

- *Isotropy*: CMB photons with temperature fluctuations $\frac{\Delta T}{T} \sim 10^{-5}$



ESA Planck

- *Homogeneity*: Galaxy distribution



SDSSIII

FLRW metric:

$$ds^2 = -dt^2 + a(t)^2 ds_3^2$$

Constant curvature space
(open, closed, flat)

Basic Assumptions

2. General Relativity (GR)

- Equations of motion

$$G_{\mu\nu} \equiv \underbrace{R_{\mu\nu} - \frac{1}{2}\, g_{\mu\nu} R}_{geometry} = \underbrace{8\,\pi\, G_N\, T_{\mu\nu}}_{matter}$$

- Matter

$$T_{\mu\nu} = (\rho + P) u_\mu u_\nu + P g_{\mu\nu} \;, \qquad u_\mu = (1,\, 0,\, 0,\, 0)$$

- Contracted Bianchi identity

$$\nabla^\mu G_{\mu\nu} = 0 \quad \Rightarrow \quad \nabla^\mu T_{\mu\nu} = 0$$

Basic Assumptions

3. Matter contents of the Universe

- All matter components (baryons+CDM, radiation, ...) independently satisfy their individual conservation equation. No interaction between different components

$$\dot{\rho}_i + 3\,H\,(1+w_i)\rho_i = 0\,, \qquad w_i = \frac{P_i}{\rho_i} \quad \Longrightarrow \quad \rho_i \propto a^{-3(1+w_i)}$$

- Matter evolution is determined by the equation of state

$$w_r = \frac{1}{3}\,, \qquad w_m = 0\,, \qquad w_{DE} = ?$$

- Dimensionless density parameters

$$\Omega_i = \frac{8\,\pi\,G_N\,\rho_i}{3\,H^2}\,, \qquad \Omega_K = -\frac{K}{a^2\,H^2} \quad \Longrightarrow \quad \sum_i \Omega_i + \Omega_K = 1$$

What do we measure?

- Assumption (1) implies

$$ds_3^2 = d\chi^2 + f_K(\chi)^2(d\theta^2 + \sin\theta\, d\phi^2)\,, \qquad f_K(\chi) \equiv \frac{1}{\sqrt{-K}}\sinh(\sqrt{-K}\,\chi)$$

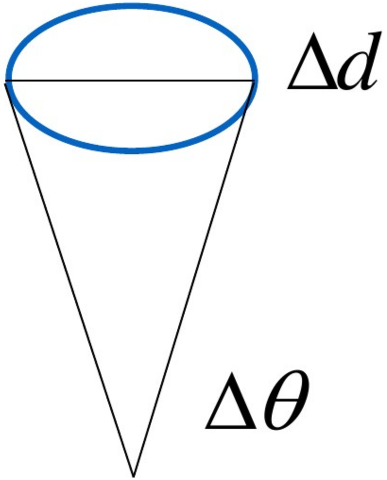
- We detect light

$$ds^2 = -dt^2 + a^2(t)ds_3^2 = 0$$

- Light emitted at time t and comoving distance χ, seen today at $(t_0, 0)$

$$\chi = \int_t^{t_0} \frac{dt'}{a(t')} = \frac{1}{a_0}\int_0^z \frac{dz}{H}$$

Redshift:
$z \equiv \frac{a_0}{a} - 1$



- Luminosity distance and angular distance

$$d_L = f_K(\chi)(1+z)\,, \qquad d_A = \frac{d_L}{(1+z)^2}$$

Cosmological model

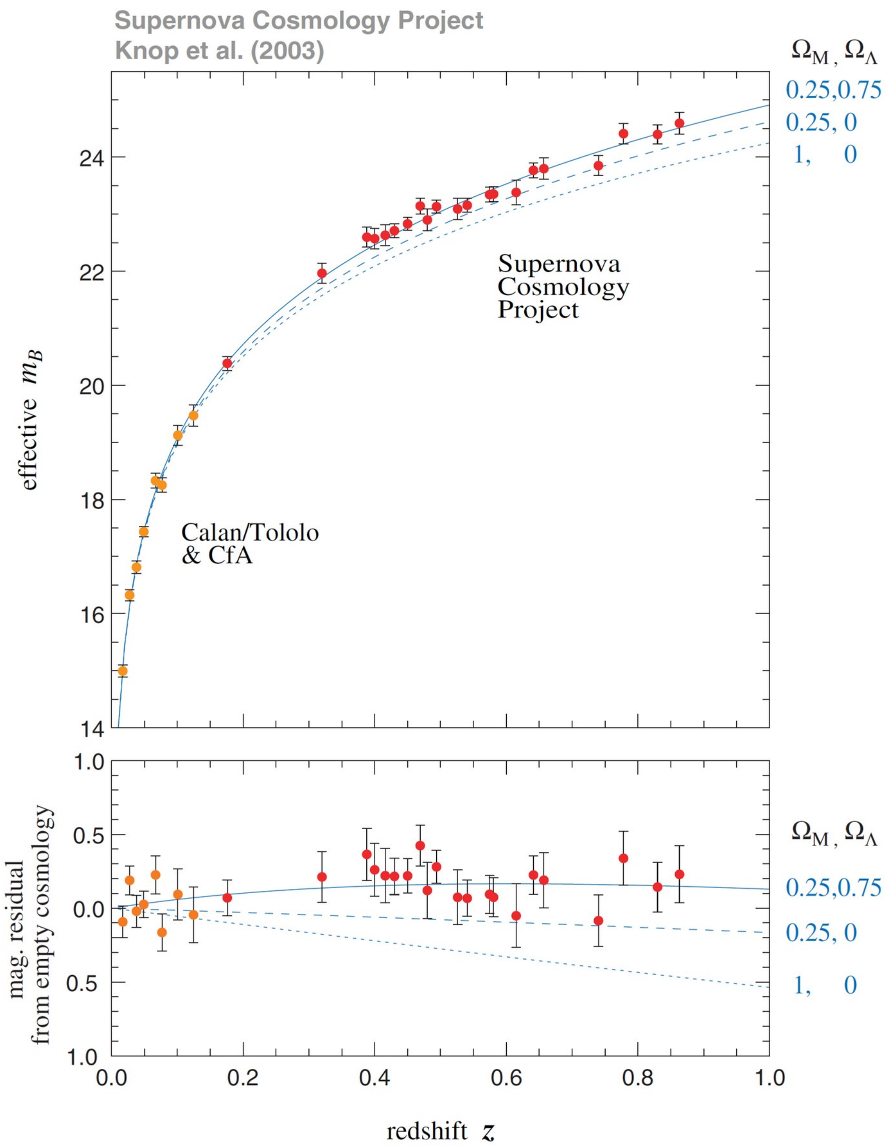
- Model based on assumptions (2) and (3)

$$\frac{H(z)^2}{H_0^2} = \Omega_{m,0}(1+z)^3 + \Omega_{r,0}(1+z)^4 + \Omega_{K,0}(1+z)^2 + \Omega_{DE,0}\,(1+z)^{3(1+w_{DE})}$$

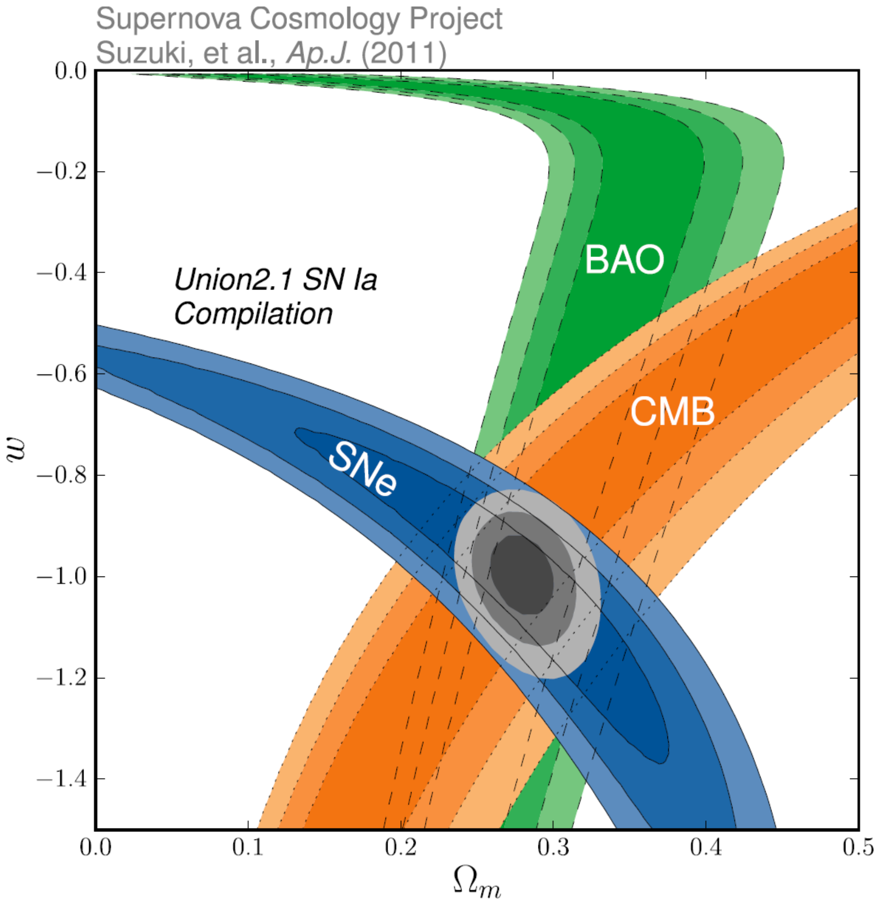
- ΛCDM model

$$w_{DE} = -1\,, \quad \Omega_{DE,0} = \Omega_\Lambda\,, \quad (\Omega_{r,0} = 5\times 10^{-5})$$

- Distance measurements
  - Supernovae d_L
  - CMB d_A
  - BAO d_A



Concordance



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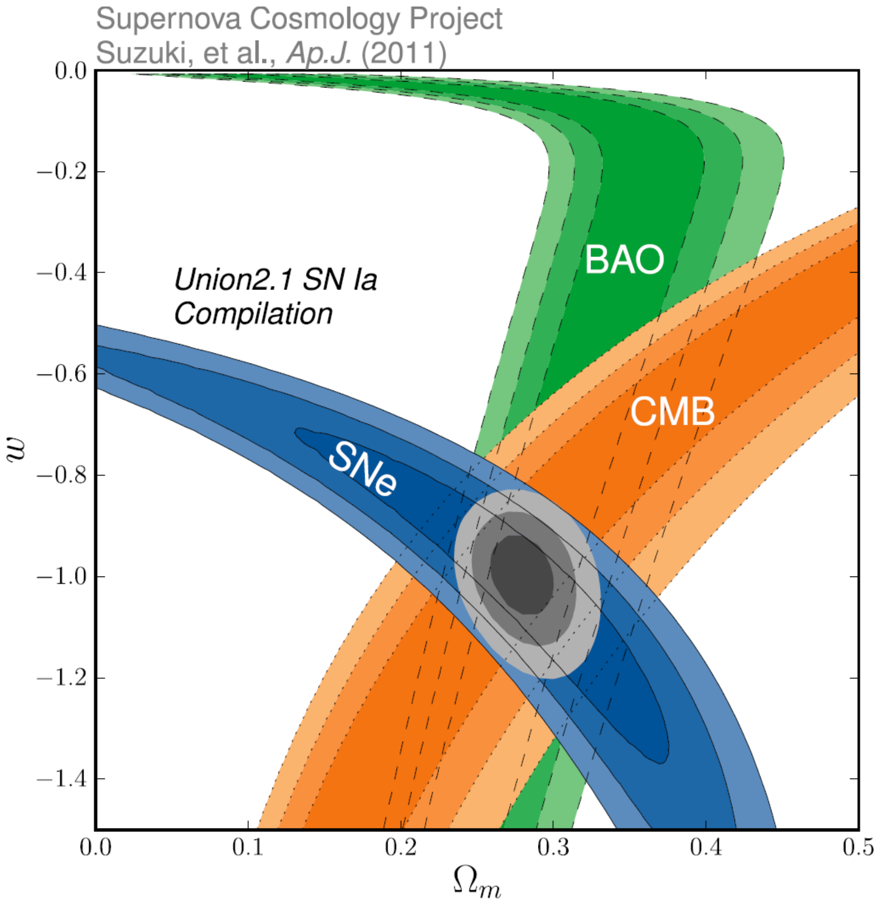

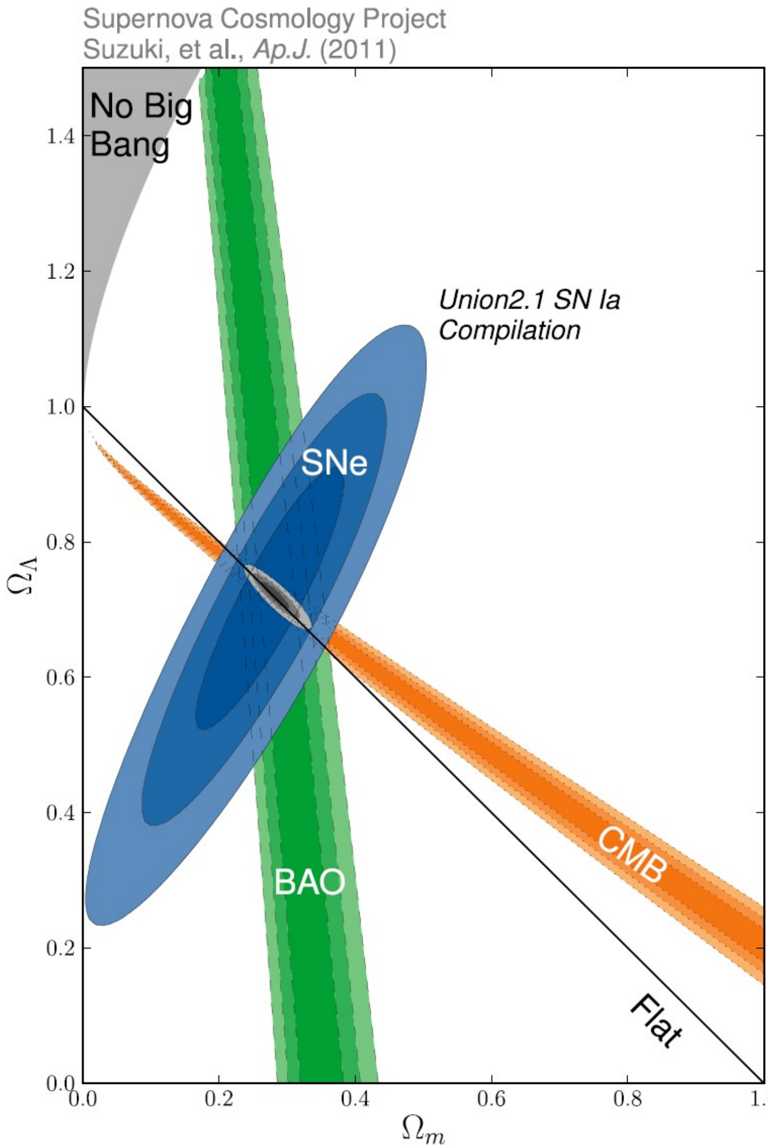

Adding cosmological constant to theory

- Action

$$S = \frac{1}{16\,\pi\,G}\int d^4x\,\sqrt{-g}(R - 2\,\Lambda)$$

- Einstein's equations

$$G_{\mu\nu} = 8\,\pi\,G\,T_{\mu\nu} - \Lambda\,g_{\mu\nu}$$

$$\begin{aligned} H^2 &= \frac{8\,\pi\,G_N}{3}\,\rho - \frac{K}{a^2} + \frac{\Lambda}{3} \\ \frac{\ddot{a}}{a} &= -\,\frac{4\,\pi\,G_N}{3}\,(\rho + 3\,P) + \frac{\Lambda}{3} \end{aligned}$$

$$\left(\rho_\Lambda = \frac{\Lambda}{8\,\pi\,G_N} = -P_\Lambda\right)$$

- Cosmological constant is unaffected by the expansion. When it dominates (which is now) the universe accelerates $\ddot{a} > 0$.

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- ✓ ΛCDM works well to explain observations
- ✓ c.c. can be implemented into General Relativity with minimal effort

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- Let's get some numbers. We are in natural units, so using

$$G_N = \frac{1}{8\,\pi\,M_{Pl}^2} \sim 10^{-38}\mathrm{GeV}^{-2} \quad \text{and} \quad H_0 \sim 10^{-42}\mathrm{GeV}$$

we get

$$\rho_\Lambda = \frac{3\,H_0^2}{8\,\pi\,G_N} \sim 10^{-47}\mathrm{GeV}^4 \sim (10^{-3}\mathrm{eV})^4$$

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- c.c. looks like vacuum energy from QFT. Let's compare.

How big is vacuum energy?

- Quantum fields have zero-point energy $E_0 = g_i \frac{\omega}{2} = \frac{g_i}{2}\sqrt{p^2 + m_i^2}$

# of degrees of freedom
$g > 0 \quad \text{(boson)}$
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- Quartic divergence! *Naïve approach:* new physics at $p_{max} = M_{Pl}$ provides a natural cutoff?

$$\rho_{vac} \sim M_{Pl}^4 = 10^{121} \rho_\Lambda^{obs}$$

$\boxed{\textit{Disagreement at 121 orders!}}$

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- We can do better than this. The cutoff is *not physical.* Renomalisation can tame the divergence.

Vacuum energy is big, but *how* big?

- Imposing Lorentz invariance of the vacuum state, the 1-loop renormalised vacuum energy is

$$\rho_{vac}^{ren} = \sum_i g_i \, \frac{m_i^4}{64\,\pi^2} \, \log\left(\frac{m_i^2}{\mu^2}\right)$$

Koksma, Prokopec 1105.6296

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- Contributions:
  - Top quark $m_t \sim 173\,\mathrm{GeV}\,,\ g_t = -12$ ⇒ $\Delta\rho_{vac} \sim -2\,\times 10^{9}\mathrm{GeV}^4 = -10^{56}\,\rho_\Lambda^{obs}$

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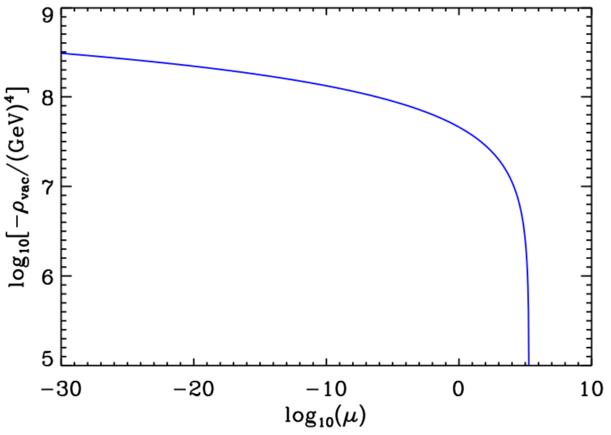
- Including all contributions

$$\rho_{total} = \rho_{bare} + \sum_i g_i \frac{m_i^4}{64\,\pi^2} \log\frac{m_i^2}{\mu^2} + \rho_{EW} + \rho_{QCD} + \ldots$$

Martin 1205.3365

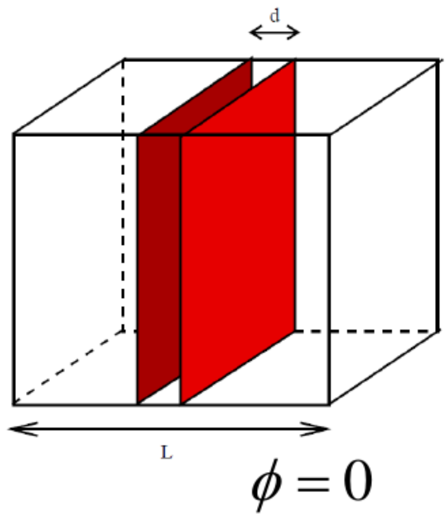


Is vacuum energy real?

- Example: Casimir effect.
  Boundary in $\hat{x}$ direction: momentum discrete

$$\phi(\vec{x}) = \phi(\vec{x} + L\,\vec{n})\,, \quad \vec{p} = \left(\frac{n\,\pi}{d}, p_y, p_z\right)\,, \quad n = 1, 2, 3\ldots$$

- Zero-point energy $\frac{\omega}{2} = \frac{1}{2}\sqrt{\left(\frac{n\,\pi}{d}\right)^2 + p_y^2 + p_z^2}$



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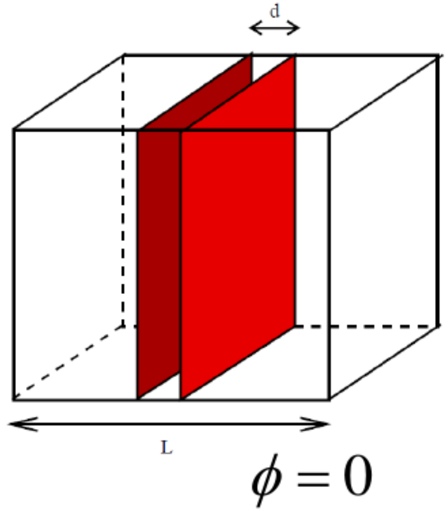

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- Total zero-point energy per unit area

$$\frac{E(d)}{A} = \sum_{n=1}^{\infty} \int \frac{dp_y dp_z}{(2\,\pi)^2} \left[\frac{1}{2}\sqrt{\left(\frac{n\,\pi}{d}\right)^2 + p_y^2 + p_z^2}\right]$$

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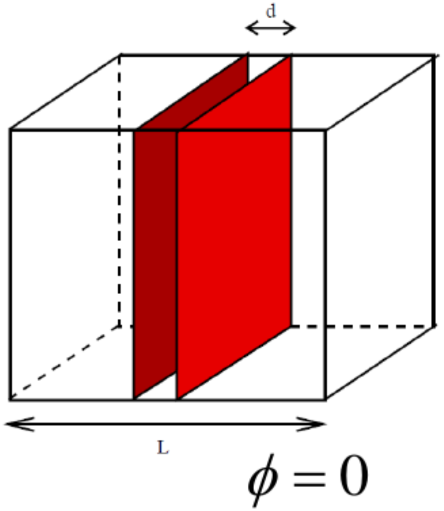

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Regulator:
$F_{reg}(a) = \mathrm{e}^{-a\,\omega}$

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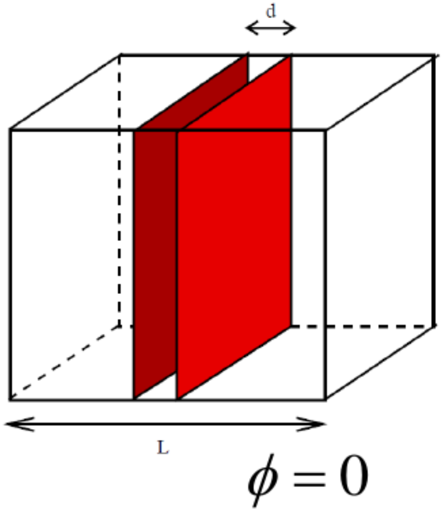

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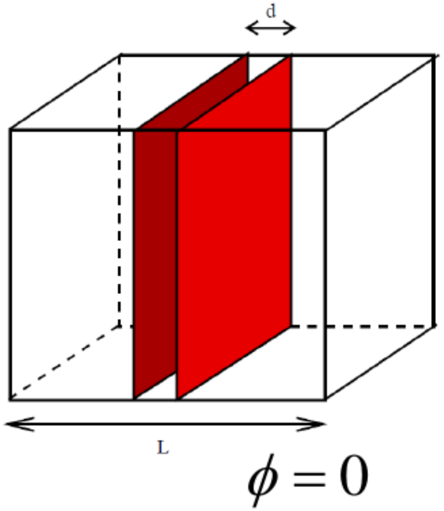

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Regulator:
$F_{reg}(a) = \mathrm{e}^{-a\,\omega}$

- Total energy $E_{tot}(d)$ diverges as $a \to 0$.
- Force between the two plates is finite

$$F = -\frac{1}{A}\frac{\partial E_{tot}(d)}{\partial d} = -\frac{\pi^2}{240\,d^4}$$

Confirmed by measurements

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- What's wrong with fine-tuning? We have renormalised vacuum energy. Fix bare c.c. such that:

$$\rho_{bare} = \frac{\Lambda_{bare}}{8\,\pi\,G_N} = \rho_{obs} - \rho_{vac} \quad \Rightarrow \quad S = \frac{1}{16\,\pi\,G} \int d^4x\,\sqrt{-g}\,(R - 2\,\Lambda_{bare})$$

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Old c.c. problem: *Why doesn't the vacuum energy gravitate? Where did it go?*

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- Vacuum energy very sensitive to high energy physics! At each loop order, the tuning is spoiled and have to re-tune.

$\Rightarrow$ Unstable under radiative corrections   see e.g. Padilla 1502.05296

Old cosmological constant problem

- In QFT (flat spacetime), we don't deal with zero-point energy: Casimir force is determined by the change in energy, not its value.
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- Something is missing. New symmetry? UV modification of gravity?

New c.c. problem / coincidence problem

Let's assume somehow the UV physics resolves the old c.c. problem.

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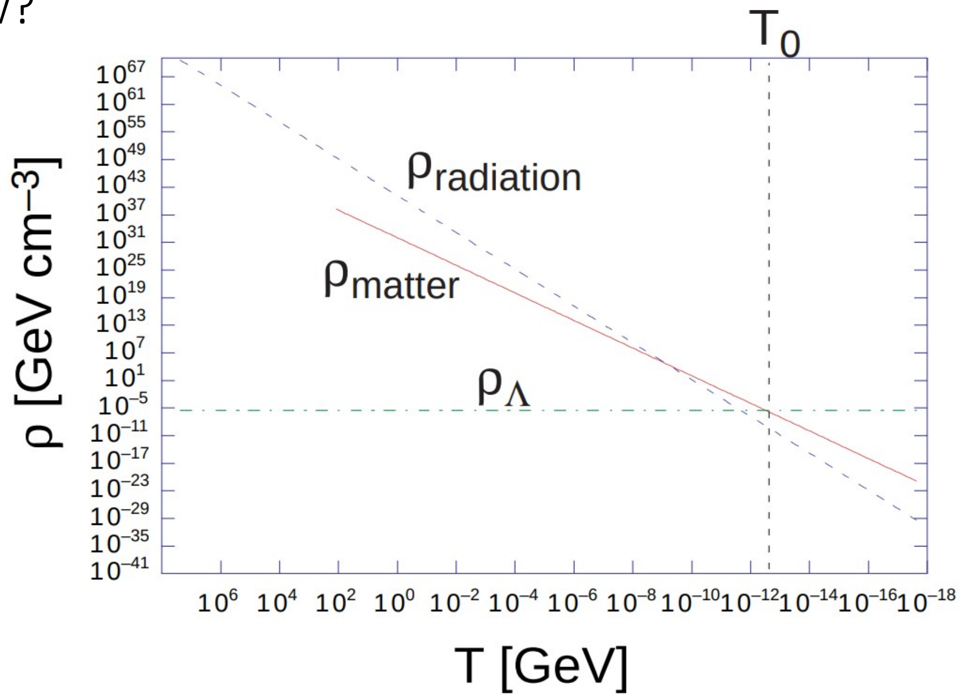

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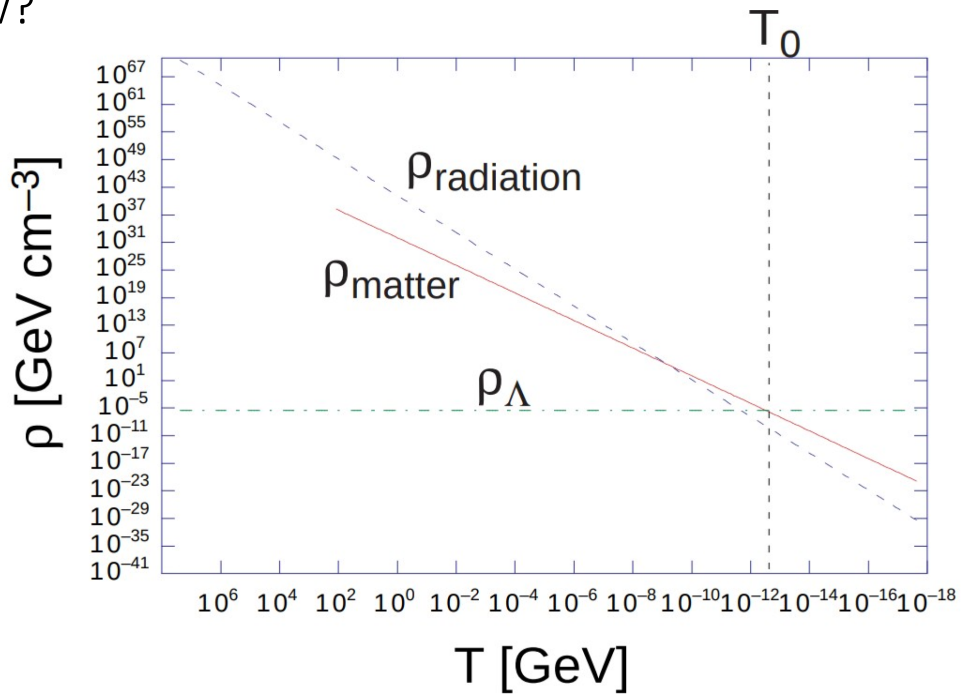
Example model that exploits

$$\rho_\Lambda \sim \mathcal{O}\left(\frac{M_{EW}^2}{M_p}\right)^4$$

Arkani-hamed et.al.
astro-ph/0005111



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- Since observations are quite conclusive, need to revisit our basic assumptions:
  1. Cosmological principle
  2. General Relativity
  3. Matter content of the universe

Revisiting assumption 1

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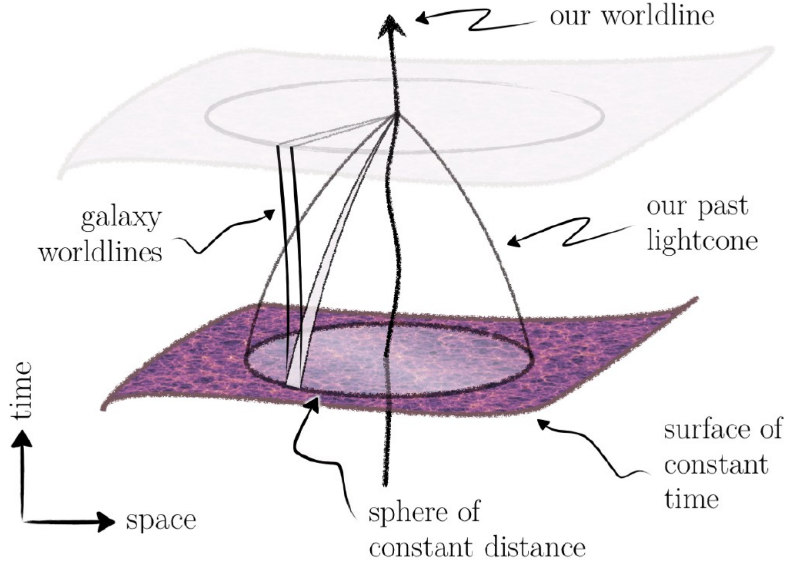

Clarkson arXiv:1204.5505

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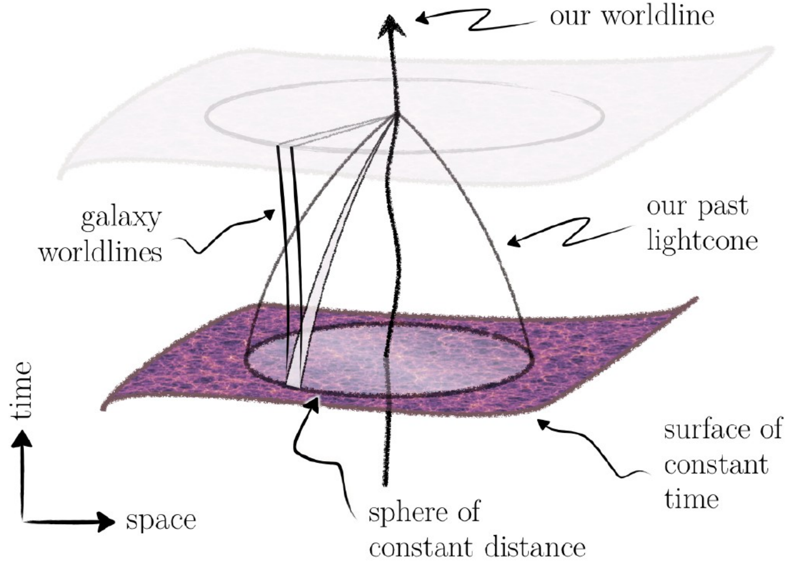
- *Copernican principle*: We are not at a special location in the Universe

- Need Copernican principle to prove the cosmological principle, but hard to test.

See e.g.
Uzan 0912.5452
Ellis et al. “Relativistic Cosmology” (2012)



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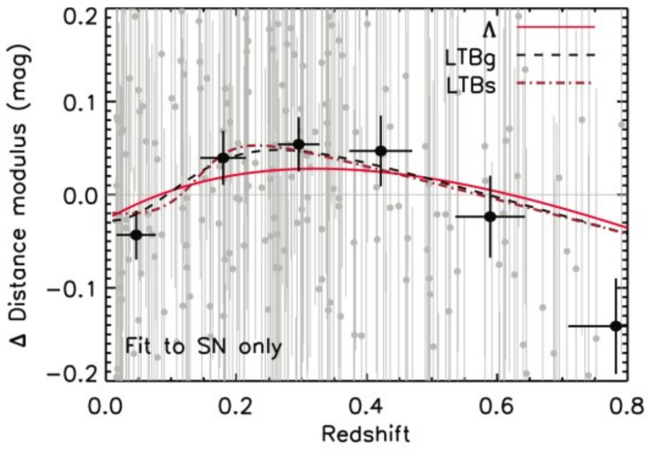

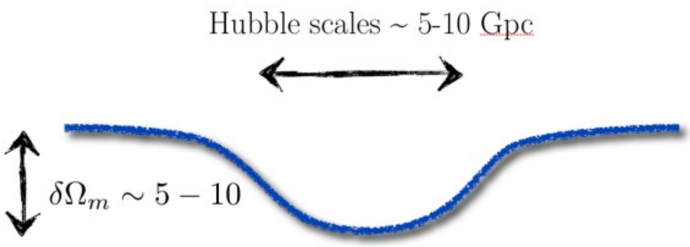

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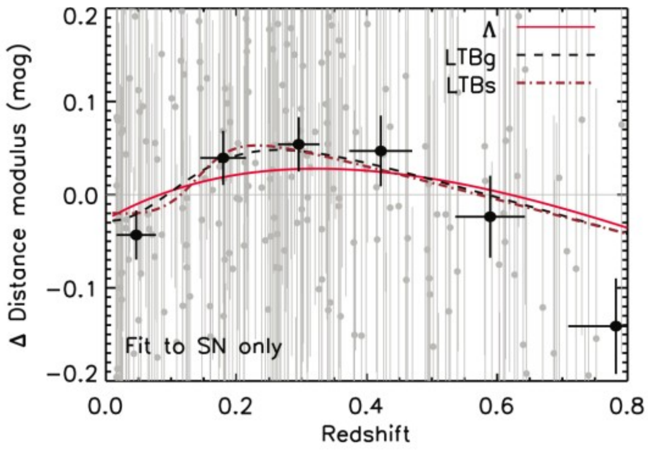

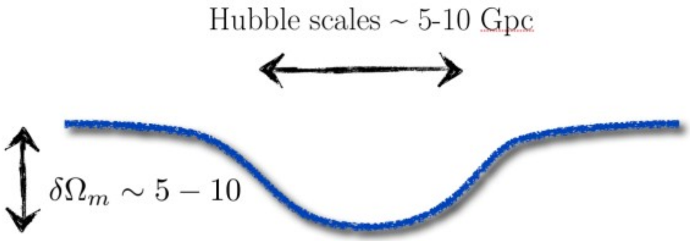

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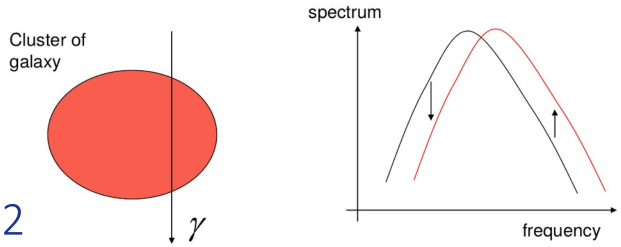

Bull, Clifton, Ferreira 1108.2222

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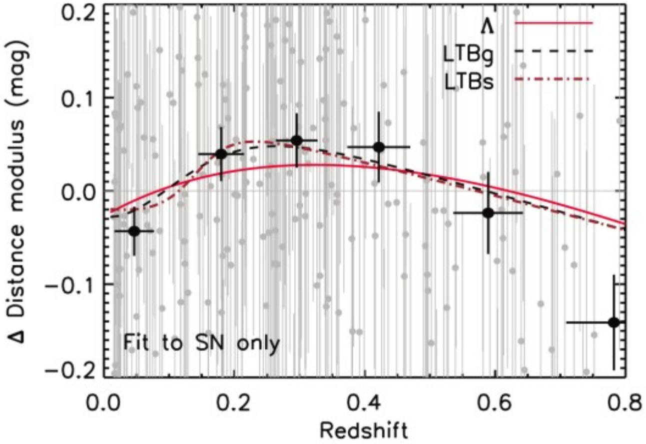

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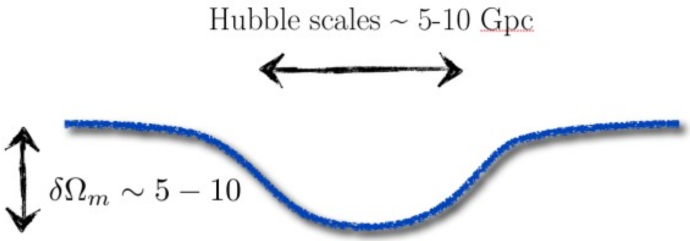

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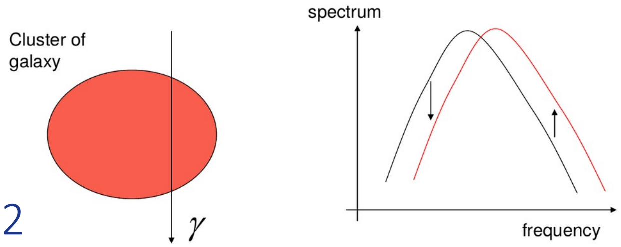


Bull, Clifton, Ferreira 1108.2222

- Not the only way to incorporate inhomogeneities

Back-reaction

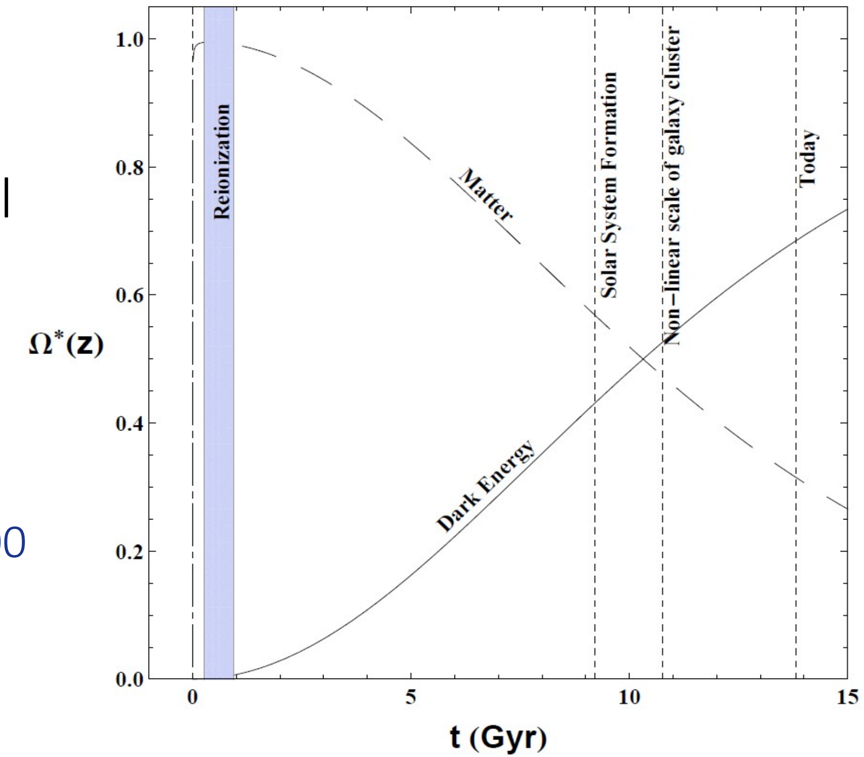
- Late time Universe is inhomogeneous at small scales, perturbations non-linear.
  But we still use FLRW at large scales.

- Can the acceleration be due to back-reaction of these inhomogeneities?
  e.g. Buchert et al. 1505.07800

- If so, it might also provide an explanation for the coincidence problem.

- There is no consensus on how big is the effect from inhomogeneities. But it might be difficult to explain acceleration with non-linear inhomogeneities.

see e.g.
Giblin et al. 1608.04403
Tian et al. 2010.07274



Velten et al. arXiv:1410.2509

Revisiting assumption 2

- How do we know that general relativity is correct?
  - *Observations*: GR is tested to very high accuracies by solar system experiments and pulsar timing measurements

C. Will's Living Review (2014)
arXiv:1403.7377

  - *Theoretical*: GR + Λ is the unique metric theory in 3+1D that gives second order equations of motion.

Lovelock (1971)

Solar system tests

- Post-Newtonian parameter

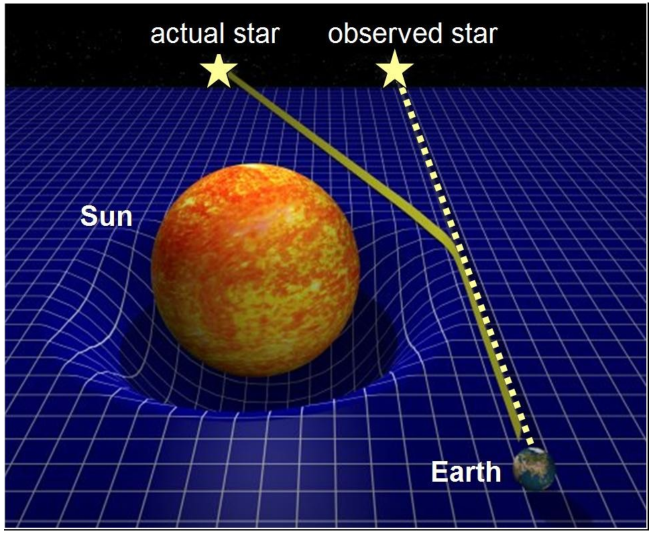
$$g_{00} = -1 + 2\,G_N\,\frac{M}{r}$$

$$g_{ij} = \delta_{ij}\left(1 + 2\,\gamma\,G_N\,\frac{M}{r}\right)$$

γ : *How much spatial curvature produced by unit rest mass?*

$\gamma = 0$: *" Newtonian"*

$\gamma = 1$: *GR*



- Deflection of light

$$\theta = 2\,(1+\gamma)\,\frac{M_\odot}{r} = \frac{1+\gamma}{2}\,\theta_{GR}$$

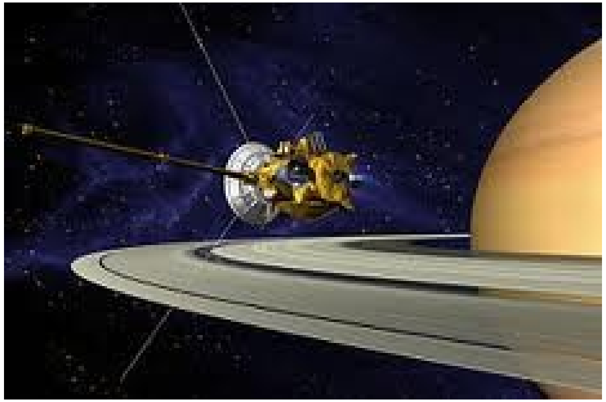
$$\theta = (0.99992 \pm 0.00023) \times 1.75'' \implies \gamma - 1 = (-1.7 \pm 4.5) \times 10^{-4}$$

(VLBI)

- Shapiro time delay

$$\Delta t = (1.00001 \pm 0.0001) \times \Delta t_{GR} \implies \gamma - 1 = (2.1 \pm 2.3) \times 10^{-5}$$

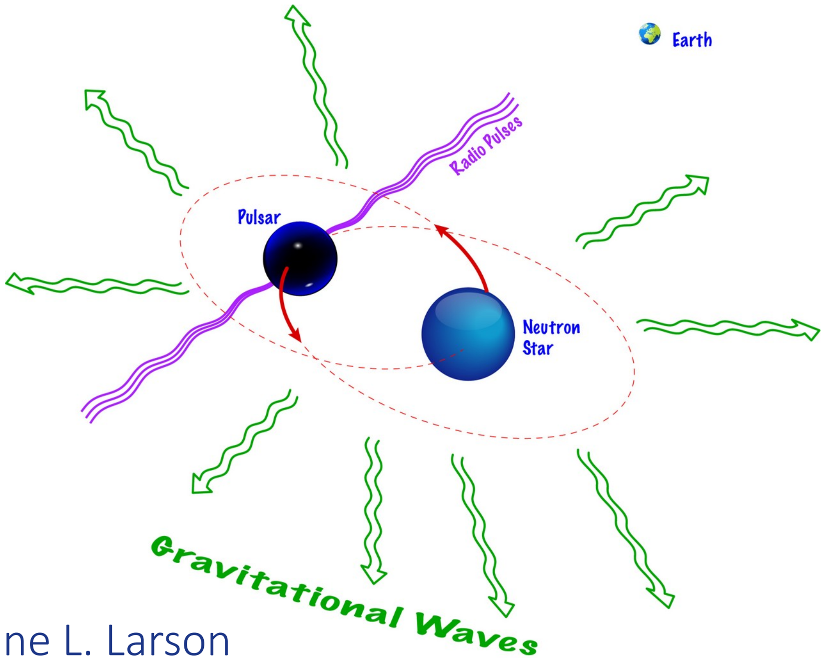
(Cassini probe)



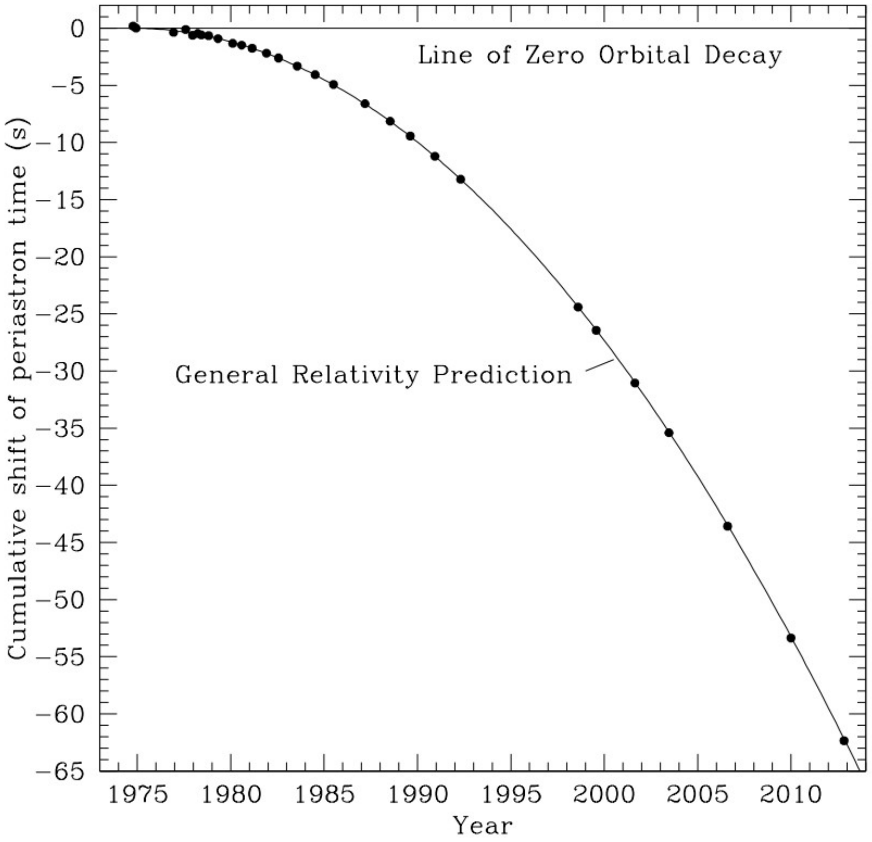
Pulsar Timing

- Hulse & Taylor binary (pulsar + neutron star)

  1993 Nobel Prize in Physics
- Orbital decay due to gravitational wave emission perfectly agrees with GR prediction.



Credit: Shane L. Larson



Weisberg, Huang 1606.02744

Pulsar Timing

- **PSR J0737-3039A/B**
  Post-Keplerian parameters

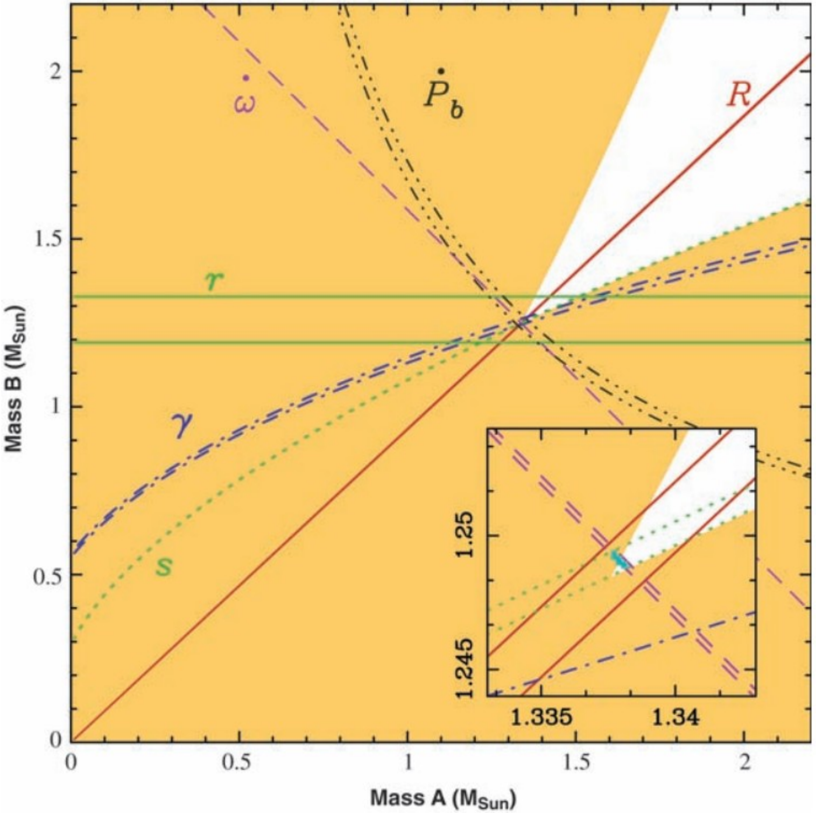
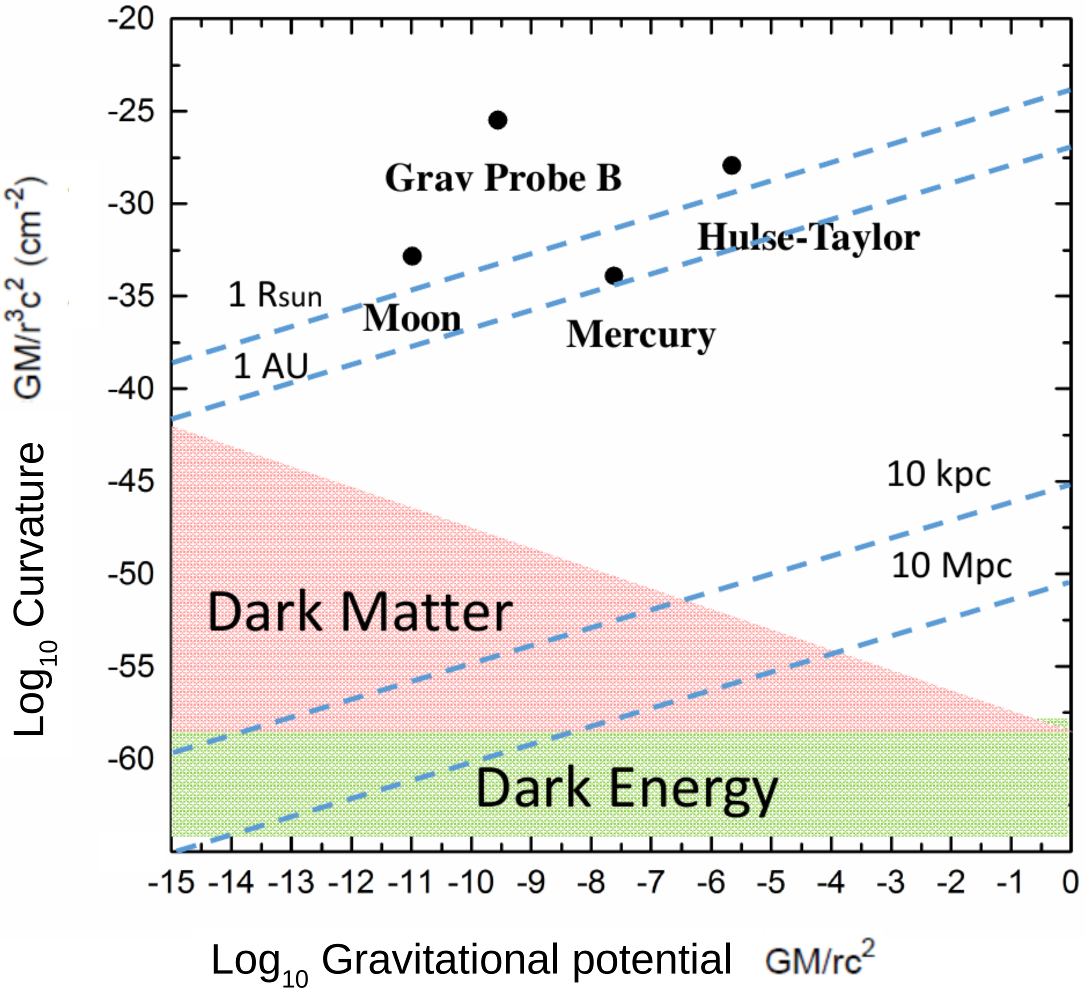


Fig. 1. Graphical summary of tests of GR parameters. Constraints on the masses of the two stars (A and B) in the PSR J0737-3039A/B binary system are shown; the inset is an expanded view of the region of principal interest. Shaded regions are forbidden by the individual mass functions of A and B because sin i must be ≤ 1. Other constraining parameters are shown as pairs of lines, where the separation of the lines indicates the measurement uncertainty. For the diagonal pair of lines labeled as R, representing the mass ratio derived from the measured semimajor axes of the A and B orbits, the measurement precision is so good that the line separation becomes apparent only in the inset. The other constraints shown are based on the measured PK parameters interpreted within the framework of general relativity. The PK parameter $\dot{\omega}$ describes the relativistic precession of the orbit, γ combines gravitational redshift and time dilation, and $\dot{P}_b$ represents the measured decrease in orbital period due to the emission of gravitational waves. The two PK parameters s and r reflect the observed Shapiro delay, describing a delay that is added to the pulse arrival times when propagating through the curved space-time near the companion. The intersection of all line pairs is consistent with a single point that corresponds to the masses of A and B. The current uncertainties in the observed parameters determine the size of this intersection area, which is marked in blue and reflects the achieved precision of this test of GR and the mass determination for A and B.

Kramer et al. astro-ph/0609417

Tests of General Relativity

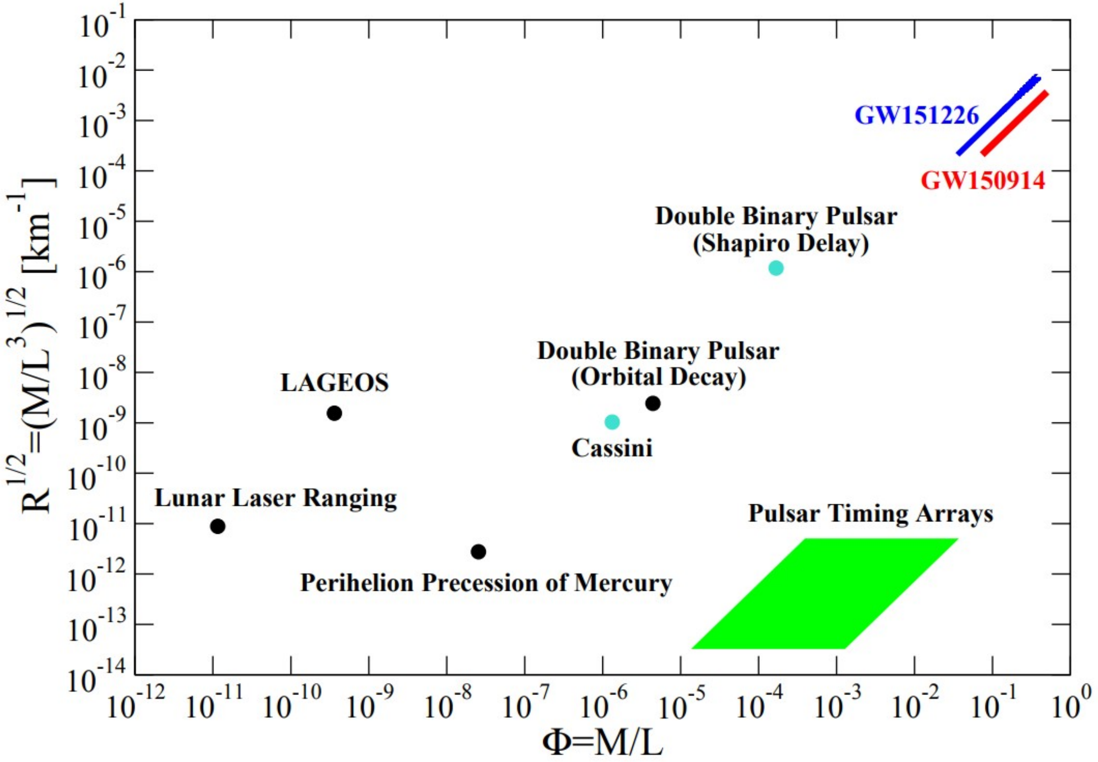


Curvature: $$R = \frac{G_N M}{r^3}$$

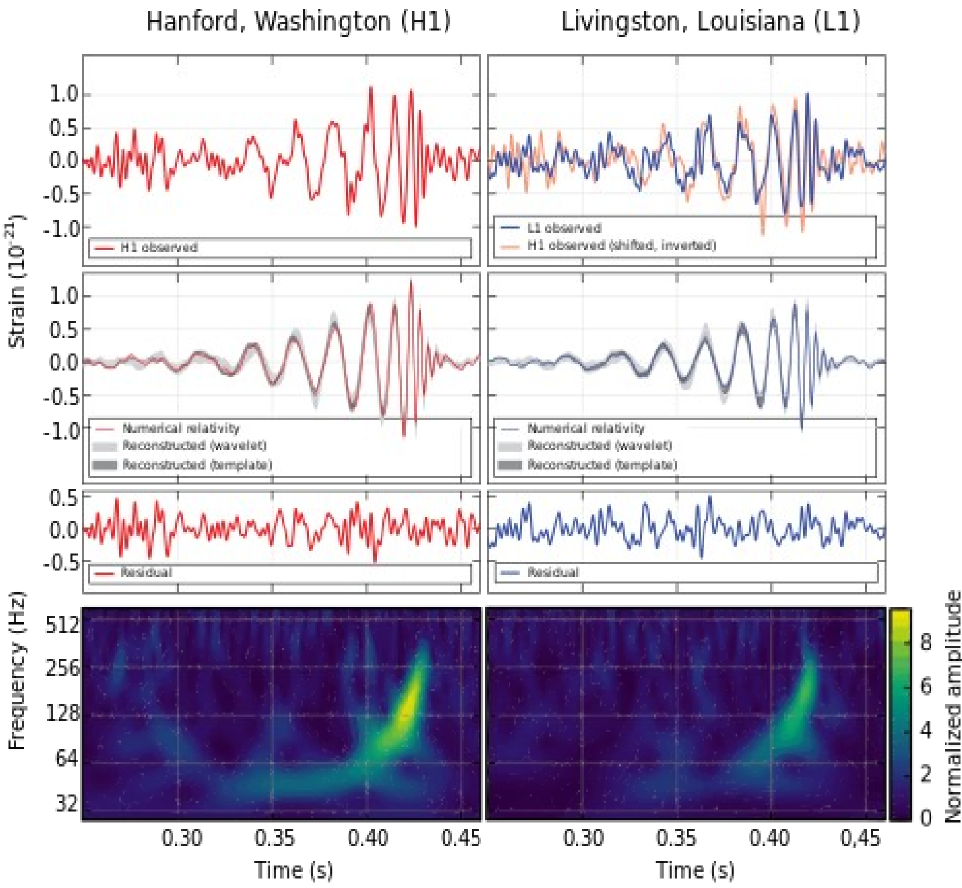
Potential: $$\Phi = \frac{G_N M}{r}$$

Psaltis, Living Review (2008) arXiv:0806.1531
also see: Baker et al (2015) arXiv:1412.3455

Tests of General Relativity



Yunes et al. (2016) arXiv:1603.08955



LIGO collaboration

Modifying GR?

- Everything seems to be great with GR. Both observationally and theoretically.

- *However*, we are extrapolating GR well beyond the scales that have been confirmed observationally.

- Theoretically, GR has short-comings at short distances (it is non-renormalisable). Would it be possible that it needs to be tweaked at large distances too?

More detailed discussion to follow

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- How can we distinguish between DE models and modifications of GR?

$$G_{\mu\nu} + G_{\mu\nu}^{MG} = 8\pi G_N \left(T_{\mu\nu} + T_{\mu\nu}^{DE}\right)$$

Introducing beyond-ΛCDM effects

1. *Phenomenological model*: represent feature via new parameters in select observables. Allows quantitative evaluation of data and detection of new effects.

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Wanted effect
(accelerated expansion)

⟺

Unwanted effects
(spoils solar system experiments, gives large contributions to CMB, unstable, predicts faster than light travel, breaks all known rules of physics

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Dynamics of dark energy
Copeland et al. **hep-th/0603057**

Quintessence: $w_{DE}(t)$ from a scalar field

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with $$\mathcal{L}_\phi = \int d^4x\,\sqrt{-g}\left[-\frac{1}{2}\,g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi - V(\phi)\right]$$

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with $$\mathcal{L}_\phi = \int d^4x\,\sqrt{-g}\left[-\frac{1}{2}\,g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi - V(\phi)\right]$$

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$$\Updownarrow$$

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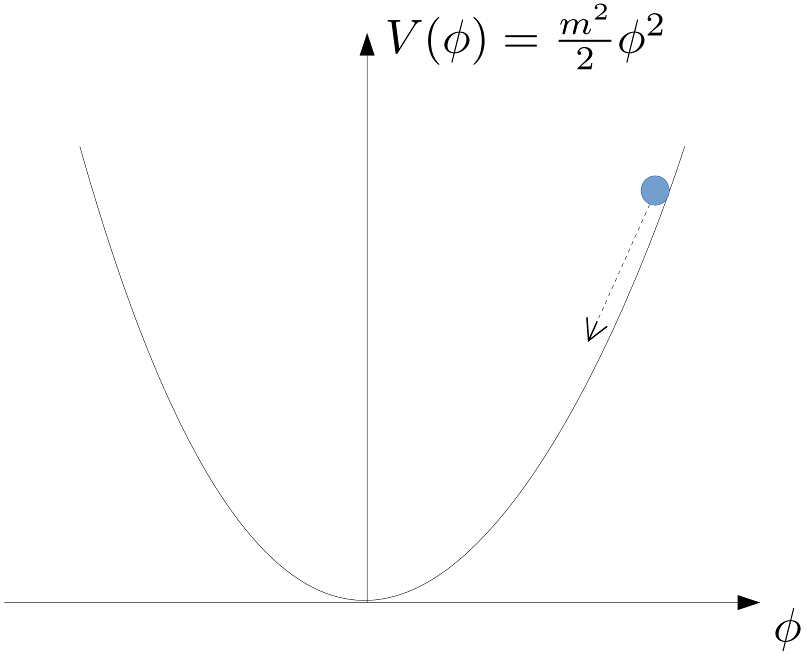
$$\dot{\rho}_\phi + 3\,H(\rho_\phi + P_\phi) = 0$$

- For accelerated expansion, scalar field needs to slowly roll down the potential

$$\dot{\phi}^2 \ll V(\phi) \quad \Longrightarrow \quad w_\phi = \frac{P_\phi}{\rho_\phi} \simeq -1$$



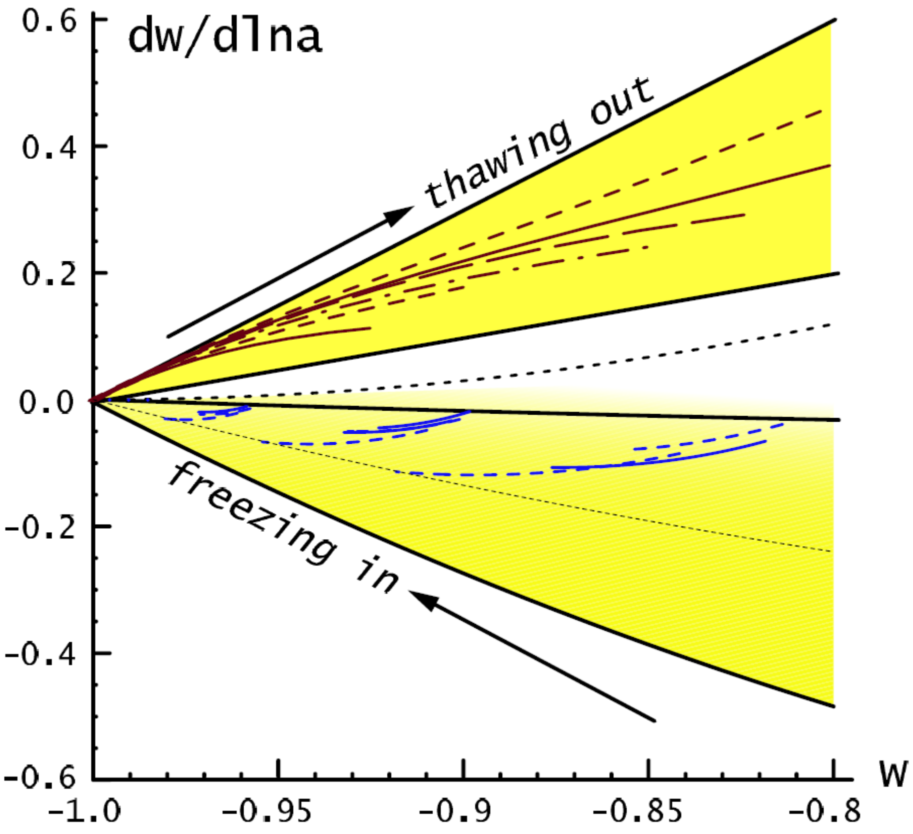
Quintessence: Potentials

- *Freezing models:* Scalar field dynamics freezes at late time approaching $w_\phi \simeq -$
Example:

$$V(\phi) = M^4 \left(\frac{M}{\phi}\right)^n$$

- *Thawing models:* Scalar initially held at $w_\phi \simeq -1$ by Hubble friction. Starts to roll down towards to minimum recently (similar to inflation). Example:

$$V(\phi) = M^4 \cos^2\left(\frac{\phi}{f}\right)$$



Linder & Caldwell astro-ph/0505494

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- Correction to the potential:

$$\Delta V = \frac{\lambda}{4}\,\phi^4 < \rho_{DE} \quad \Rightarrow \quad \lambda \ll 10^{-120}$$

These small numbers are very difficult to protect against quantum corrections

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Modified Gravity and Cosmology
Clifton et al. **arXiv:1106.2476**

Modifying gravity?

- We have to evade Lovelock's theorem (1971).

Lovelock's Theorem: GR + c.c. unique theory assuming:

a) EOM originates from an action

b) EOM has at most 2nd order derivatives

c) 3+1 dimensions

d) Invariance under general coordinate transformations

e) Locality (i.e. no operators like $(\nabla^\mu \nabla_\mu)^{-1}$)

f) The only field is the metric tensor

- A modified theory of GR should relax one of these. This lecture briefly focusses only on breaking (f), as it *partially* covers breaking all other assumptions except (a).

Horndeski 1974, independently discovered by Deffayet et al. 1103.3260

Scalar-Tensor Theories: Horndeski class

- Extension of Lovelock's theorem: most general ***2nd order*** scalar-tensor theory

$$\mathcal{L}_H = \sum_{n=2}^{5} \mathcal{L}_n$$

$$\begin{aligned}
\mathcal{L}_2 &= K(\phi, X) \\
\mathcal{L}_3 &= -G_3(\phi, X)\Box\phi \\
\mathcal{L}_4 &= G_4(\phi, X)\, R + G_{4,X}\left[(\Box\phi)^2 - (\nabla_\mu\nabla_\nu\phi)^2\right] \\
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- Some examples:

K-essence

$$K = K(X)\,,\; G_4 = \frac{M_{Pl}^2}{2}$$
$$G_3 = G_5 = 0$$

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$$K,\, G_3 \propto X\,,\; G_5 \propto X^2$$
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Self-tuning

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Self-tuning

- Scalar-Tensor class is actually larger: ***Beyond Horndeski*** and ***DHOST*** *(degenerate)* terms.

Gleyzes et al. 1404.6495, Langlois et al. 1510.06930, Ben Achour et al. 1608.08135

MG dark energy after multi-messenger astronomy

- GW170817/GRB170817A : BNS merger, measure time delay between GW and GRB $\sim$ 1.74 s.

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	$c_{GW} = c$	$c_{GW} \neq c$
Horndeski	General Relativity quintessence/k-essence [46] Brans-Dicke/$f(R)$ [47, 48] Kinetic Gravity Braiding [50]	quartic/quintic Galileons [13, 14] Fab Four [15] de Sitter Horndeski [49] $G_{\mu\nu}\phi^\mu\phi^\nu$ [51], $f(\phi)\cdot$Gauss-Bonnet [52]
beyond H.	Derivative Conformal (19) [17] Disformal Tuning (21) quadratic DHOST with $A_1 = 0$	quartic/quintic GLPV [18] quadratic DHOST [20] with $A_1 \neq 0$ cubic DHOST [23]
	Viable after GW170817	Non-viable after GW170817

Ezquiaga, Zumalacàrregui
arXiv:1710.05901

FIG. 2: Summary of the viable (left) and non-viable (right) scalar-tensor theories after GW170817. Only simple Horndeski theories, $G_{4,X} \approx 0$ and $G_5 \approx$ constant, and specific beyond Horndeski models, conformally related to $c_g = 1$ Horndeski or disformally tuned, remain viable.

Instabilities from GW

- Many DE models from S-T class are excluded by the constraint on the speed of light.

- There's more: $\delta\dot{g}_{ij}\,\partial_i\delta\phi\,\partial_j\delta\phi$ interactions in e.g. cubic Galileon ($G_3(X)\Box\phi$) can lead to other unwanted effects:
  - GW→ DE + DE decay   Creminelli et al. 1809.03484
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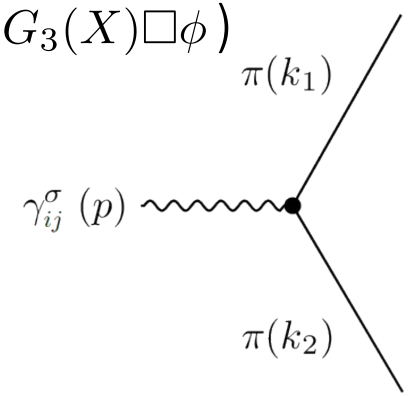

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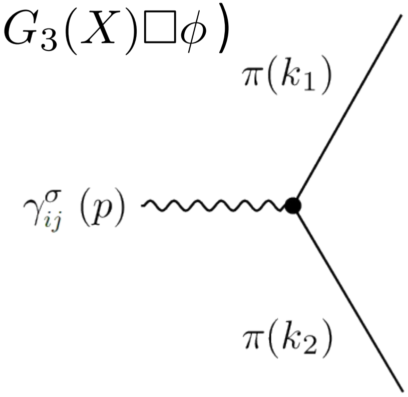

- With these restrictions, all left from the scalar-tensor class:

$$\mathscr{L} = P(\phi, X) + C(\phi, X)R + \frac{6C_{,X}(\phi, X)^2}{C(\phi, X)}\phi^{;\mu}\phi_{;\mu\nu}\phi_{;\lambda}\phi^{;\nu\lambda}$$

Assuming theories still valid at GW energies. See de Rham, Melville 1806.09417

Only quintessence/k-essence and simple modifications e.g. f(R), Brans-Dicke.

Beyond the scalar-tensor class?

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- More complex modifications of GR (with additional vector or tensor modes) also generate scalar perturbations. *Does the S-T class describe the scalar sector of more general theories accurately?*

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- More complex modifications of GR (with additional vector or tensor modes) also generate scalar perturbations. ***Does the S-T class describe the scalar sector of more general theories accurately?***

- There is evidence of departures from S-T class in more general frameworks. One example: ***general massive gravity theory class***: *Gumrukcuoglu et al. 2003.11831*

Beyond the scalar-tensor class?

- *Scalar-tensor* class is a convenient framework for modified gravity. Most observable quantities depend on scalar perturbations (e.g. potentials, growth rate ...).

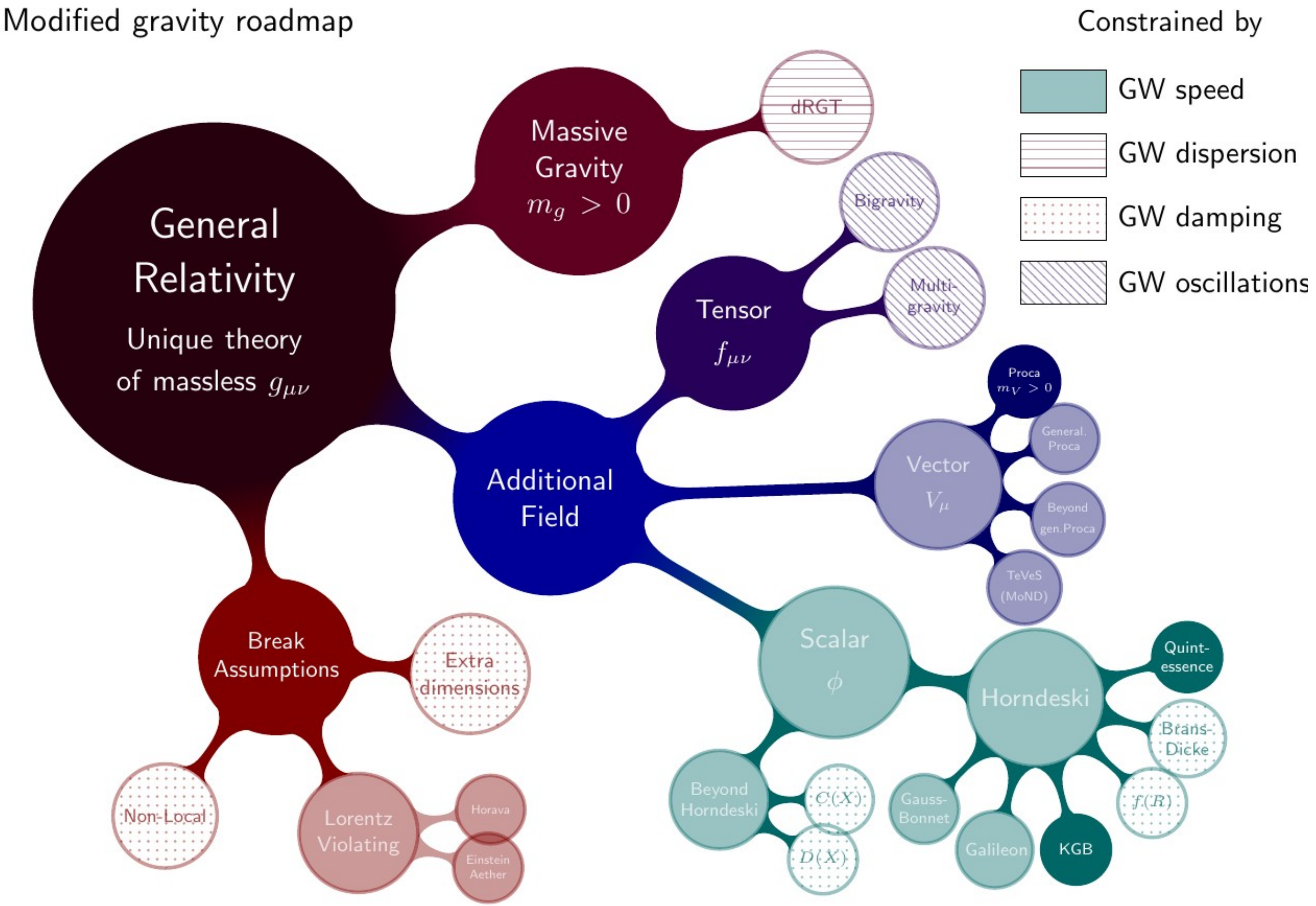
- Despite all the developments, we are left with a simple extension of c.c. that *cannot* be falsified! Does not address c.c. problem. ***True modifications of gravity***?

- More complex modifications of GR (with additional vector or tensor modes) also generate scalar perturbations. ***Does the S-T class describe the scalar sector of more general theories accurately?***

- There is evidence of departures from S-T class in more general frameworks. One example: ***general massive gravity theory class***: *Gumrukcuoglu et al. 2003.11831*
  - Theory class with 6 free functions. Supports stable cosmology with an effective DE with $w < -1$, ~1% deviation in density contrast, may be probed in near future
  - Addresses the new cosmological constant problem
  - In the LCDM limit, still has massive GW, hence falsifiable

Kenna-Allison et al. 2009.05405

Modified gravity theories



Ezquiaga, Zumalacàrregui arXiv:1807.09241

Discrimination between models

- There are many dark energy and modified gravity models, but we still do not have a compelling alternative to ΛCDM

- Observations may give us clues to answer some basic questions:

  ➔ *Is it a cosmological constant or a light degree of freedom?*
  ➔ *Does it cluster?*
  ➔ *Does it interact with other matter?*

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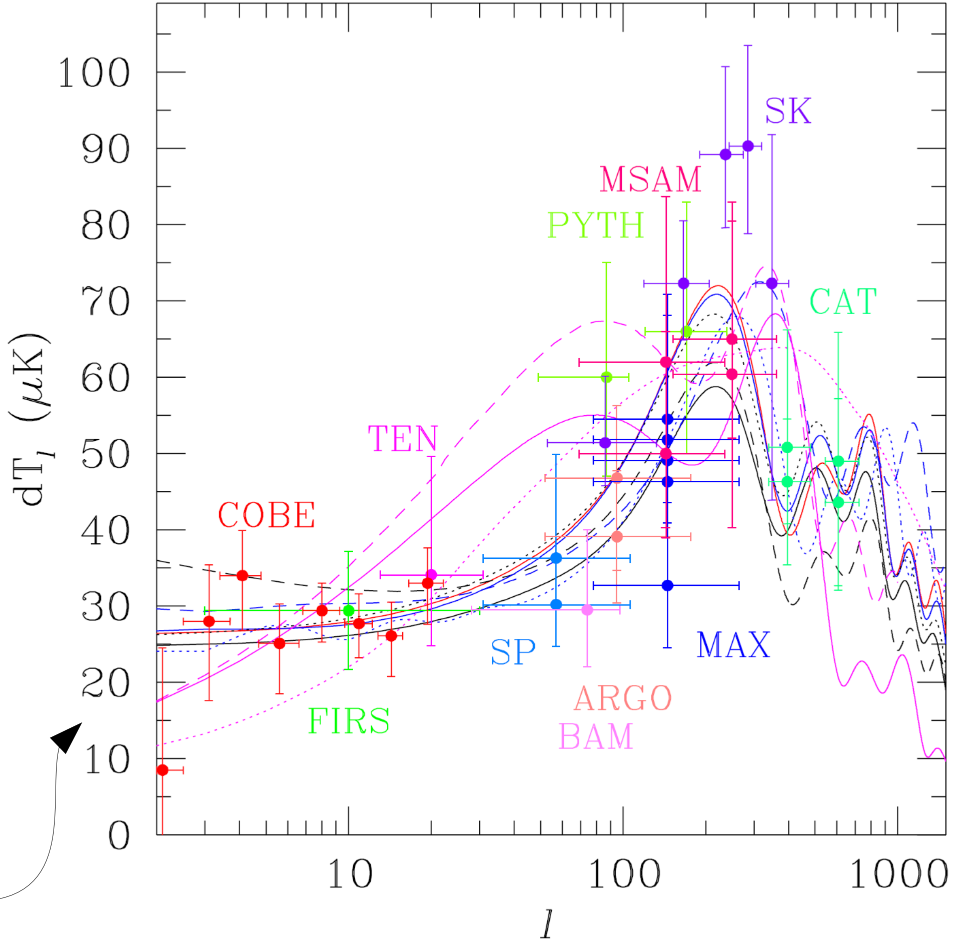
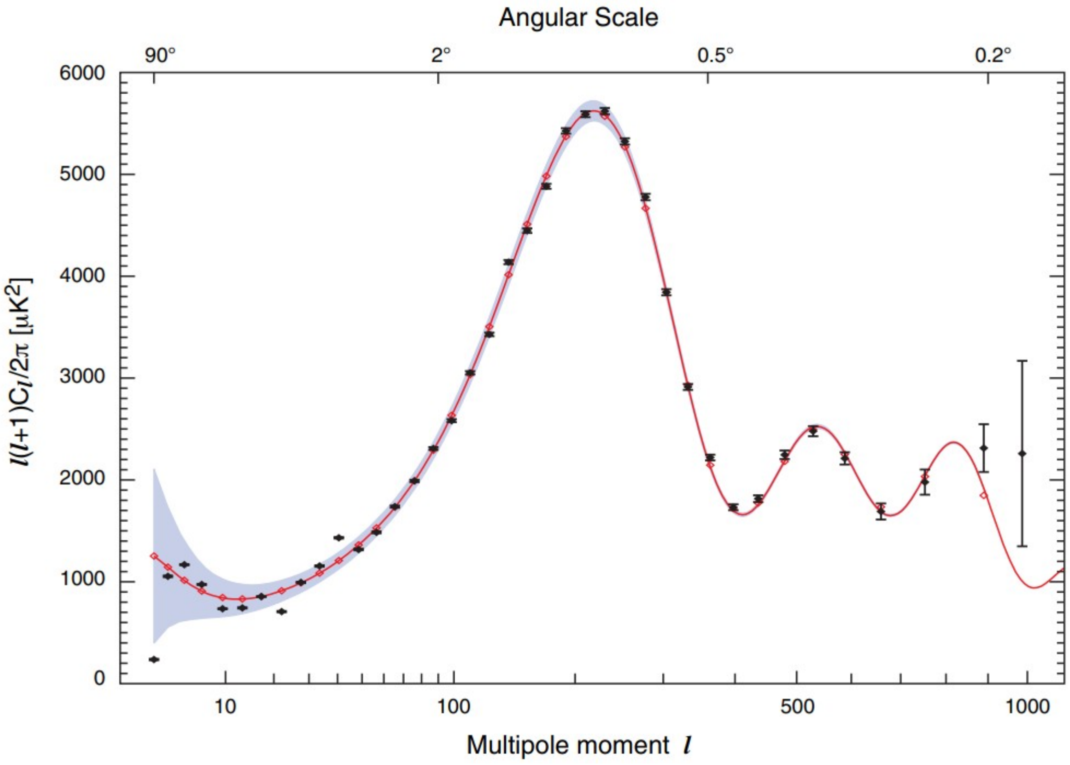
Cosmology models before WMAP

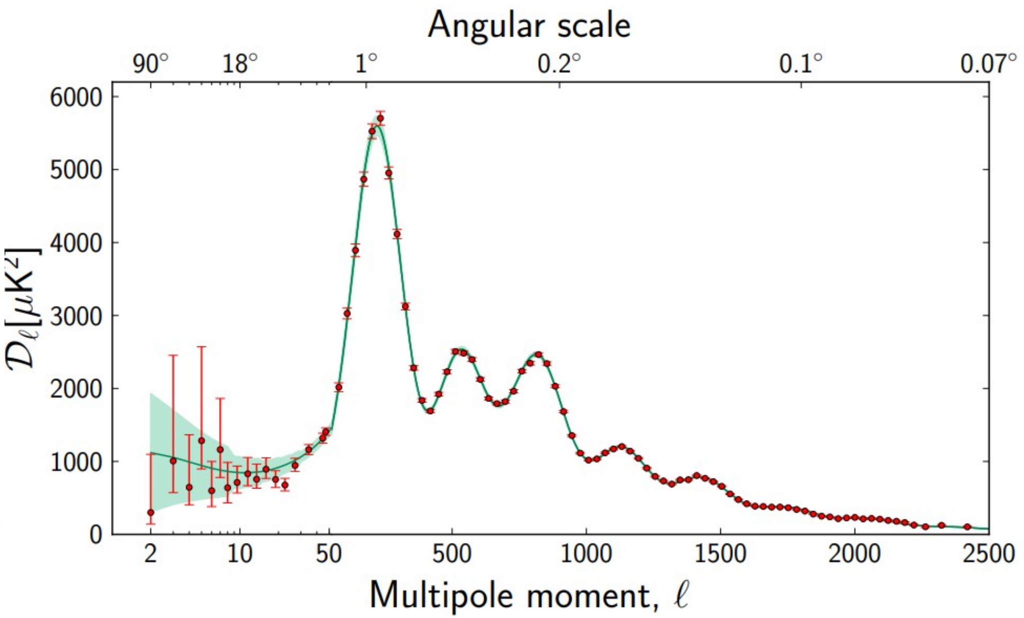


Gawiser & Silk astro-ph/9806197

Discrimination between models

Only ΛCDM survived after WMAP





Planck 2013 1303.5075

Can a single model of DE/MG triumph after new observations?

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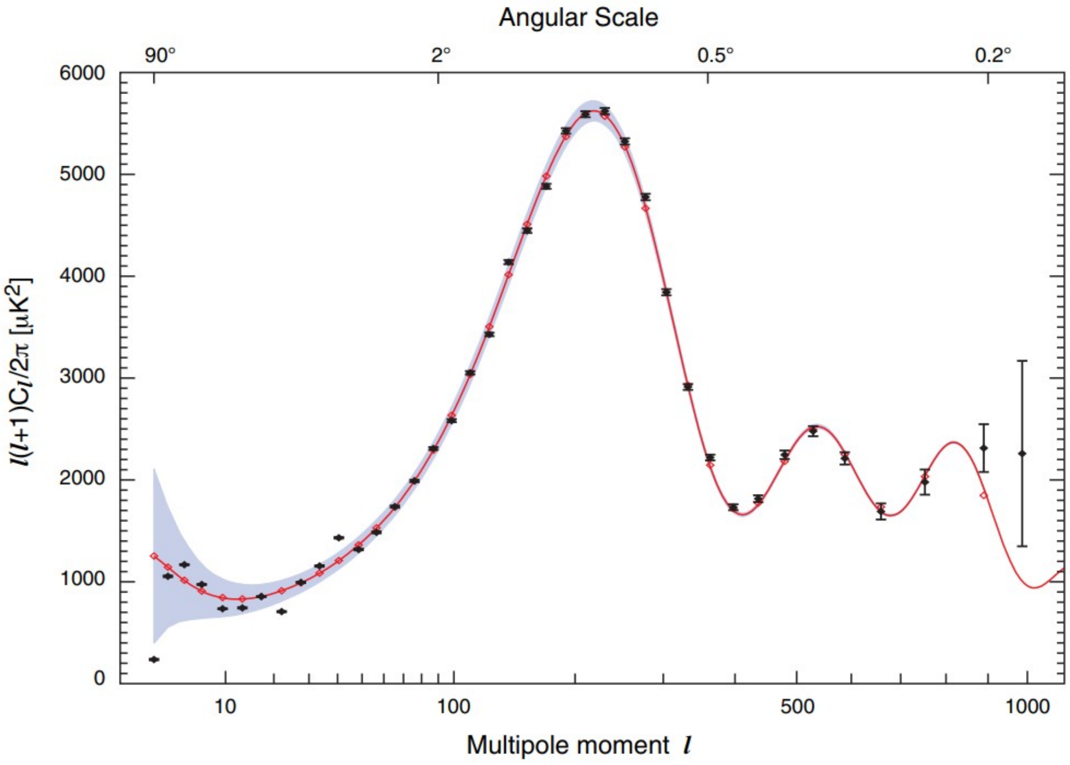
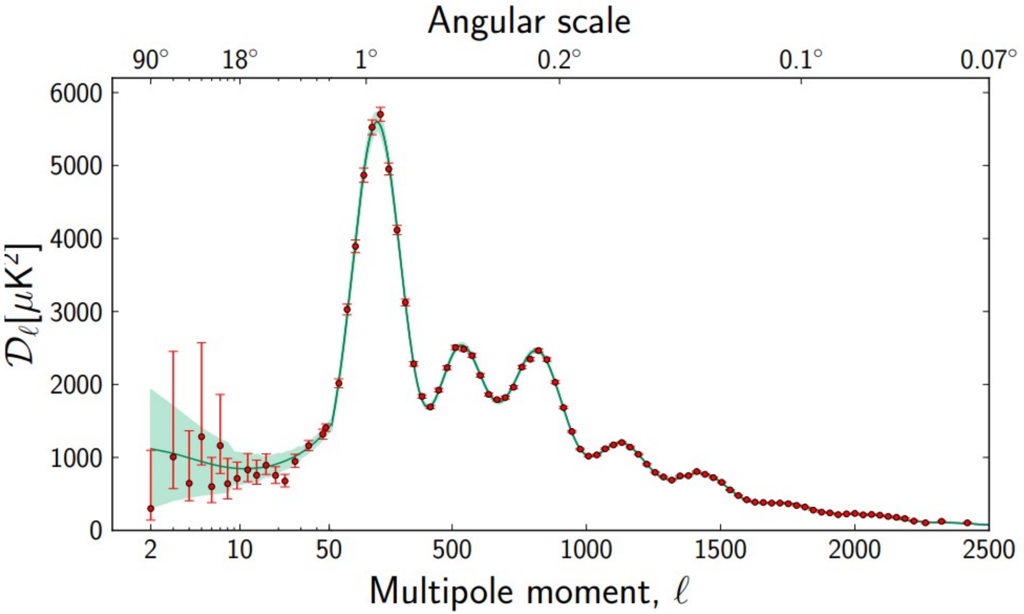


WMAP 3-year astro-ph/0603451



Planck 2013 1303.5075

Can a single model of DE/MG triumph after new observations?

How to test DE/MG models

- We will treat DE and MG models as a single fluid. Einstein's equations:

$$G_{\mu\nu} = 8\,\pi\,G\left(T^{m}_{\mu\nu} + E_{\mu\nu}\right) \qquad \text{with} \qquad E_{\mu\nu} \equiv T^{DE}_{\mu\nu} - G^{MG}_{\mu\nu} \qquad \boxed{\nabla^{\mu}\left(T^{m}_{\mu\nu} + E_{\mu\nu}\right) = 0}$$

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$$E^{\mu}{}_{\nu} = \begin{pmatrix} -\rho_E & & & \\ & P_E & & \\ & & P_E & \\ & & & P_E \end{pmatrix} \quad \text{with effective eos} \quad w_E = P_E/\rho_E$$

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Longitudinal gauge:
(c.f.Inflation lecture)

$$E = B = 0$$

Conformal time:

$$a\,d\tau = dt$$

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- We expand perturbations into plane waves $\mathrm{e}^{i\,\vec{k}\cdot\vec{x}}$.

$$\partial_i \leftrightarrow i\,k_i \quad \text{and} \quad \nabla^2 \leftrightarrow -k^2$$

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Matter content

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Continuity eq.
$$\frac{d\,\delta\rho_I}{d\tau} + 3\,\mathcal{H}\,(\delta\rho_I + \delta P_I) = (\rho_I + P_I)(k^2\, v_I + 3\,\Psi')$$

Euler eq.
$$-\left(\frac{d}{d\tau} + 4\,\mathcal{H}\right)[(\rho_I + P_I) v_I] = \delta P_I + \frac{2}{3} k^2 P_I\, \Pi_I + (\rho_I + P_I)\,\Phi$$

$$\mathcal{H} \equiv \frac{a'(\tau)}{a(\tau)} = a\, H$$

Matter: $w_m = P_m/\rho_m = 0$

$\delta P_m = \Pi_m = 0$

DE/MG: $w_E = P_E/\rho_E$

$\delta P_E\,,\ \Pi_E$

Equations for linear perturbations

- Einstein's equations

Poisson's eq.

$$k^2\Psi = -4\,\pi\,G\,a^2(\rho_m\,\Delta_m + \rho_E\,\Delta_E)$$

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$$\Delta_I \equiv \frac{\delta\rho_I}{\rho_I} - \frac{3\,\mathcal{H}\,(\rho_I + P_I)}{\rho_I}\,v_I$$

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Velocity divergence:
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- Evolution of matter determined by the Newtonian potential
- DE/MG component affects the evolution through the Newtonian potential

⇒ Evolution of structure is sensitive to the dark energy model.

Growth of structure

1. ΛCDM

- Dark component pure c.c. (no perturbation) $E_{\mu\nu} = -\Lambda g_{\mu\nu}$

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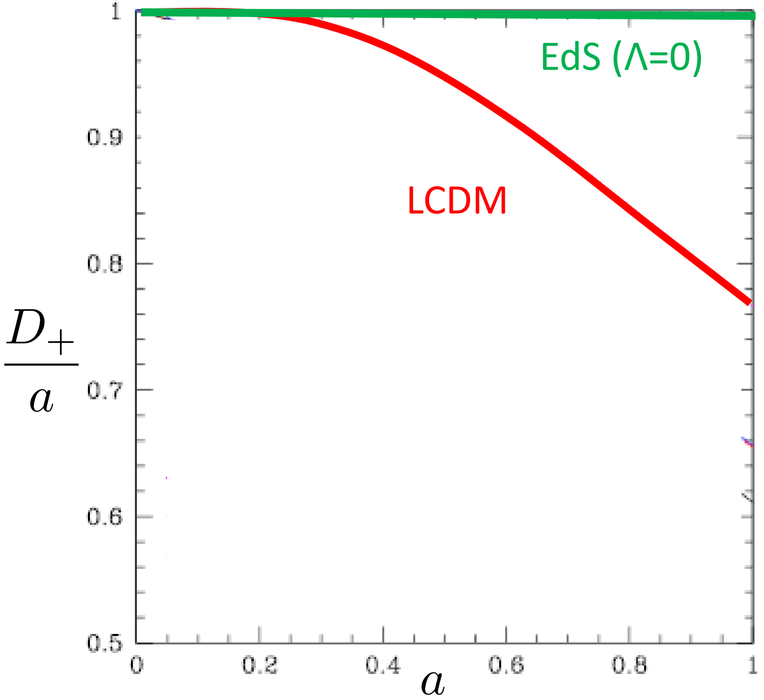

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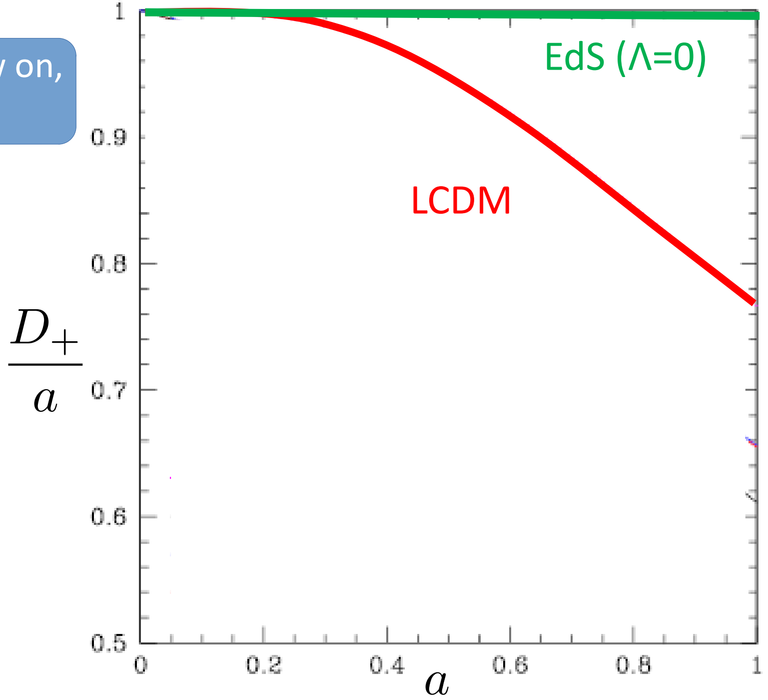

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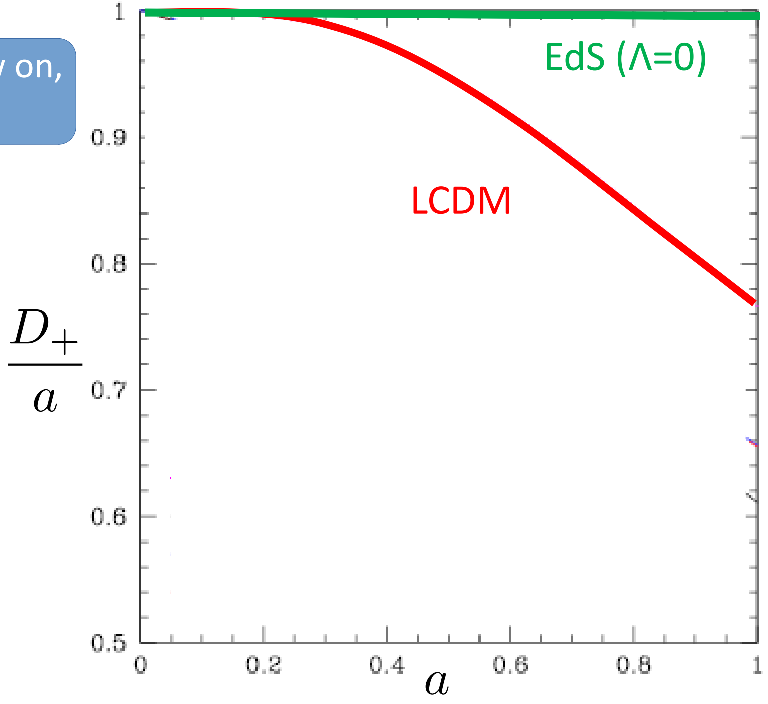
- Effect on growth only due to background expansion.



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- Smoothness assumption → Negligible perturbations $\delta\rho_E = \Pi_E = 0$ with $w_E = w_E(z)$

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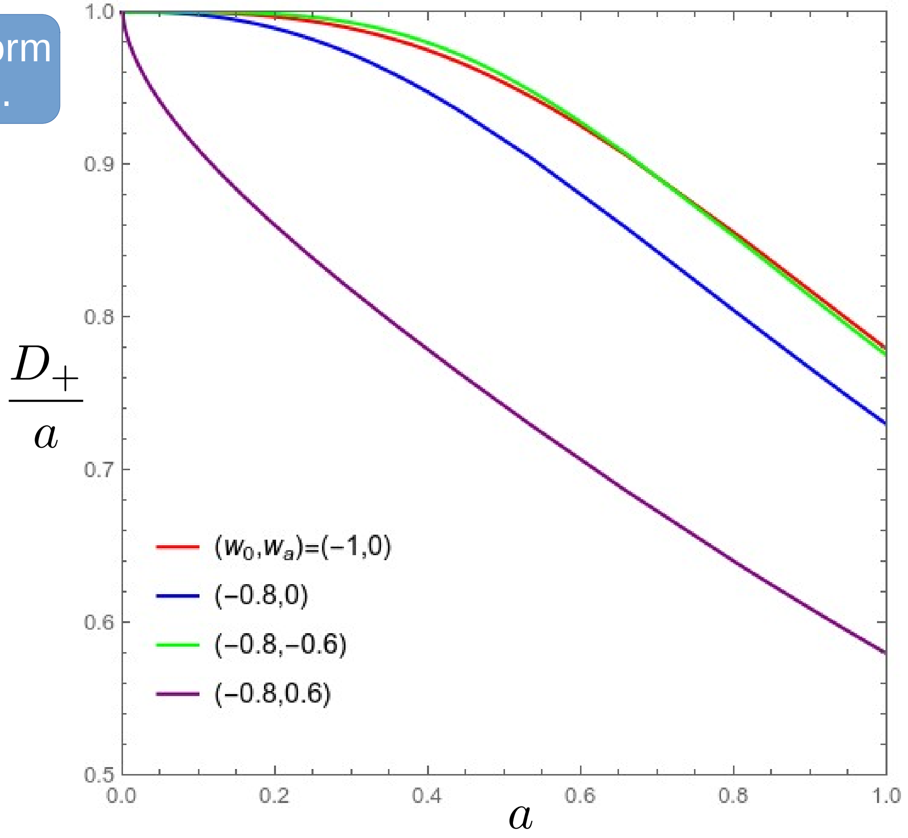
- For a fixed $\Omega_{E,0}$, if $w_E > -1$, the DE density larger in the past, suppressing the growth compared to ΛCDM.



Linder astro-ph/0305286

Growth rate for smooth DE

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$$\gamma = 0.545 + 0.05(1 + w_E(z=1))$$

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- γ is insensitive to DE eos.

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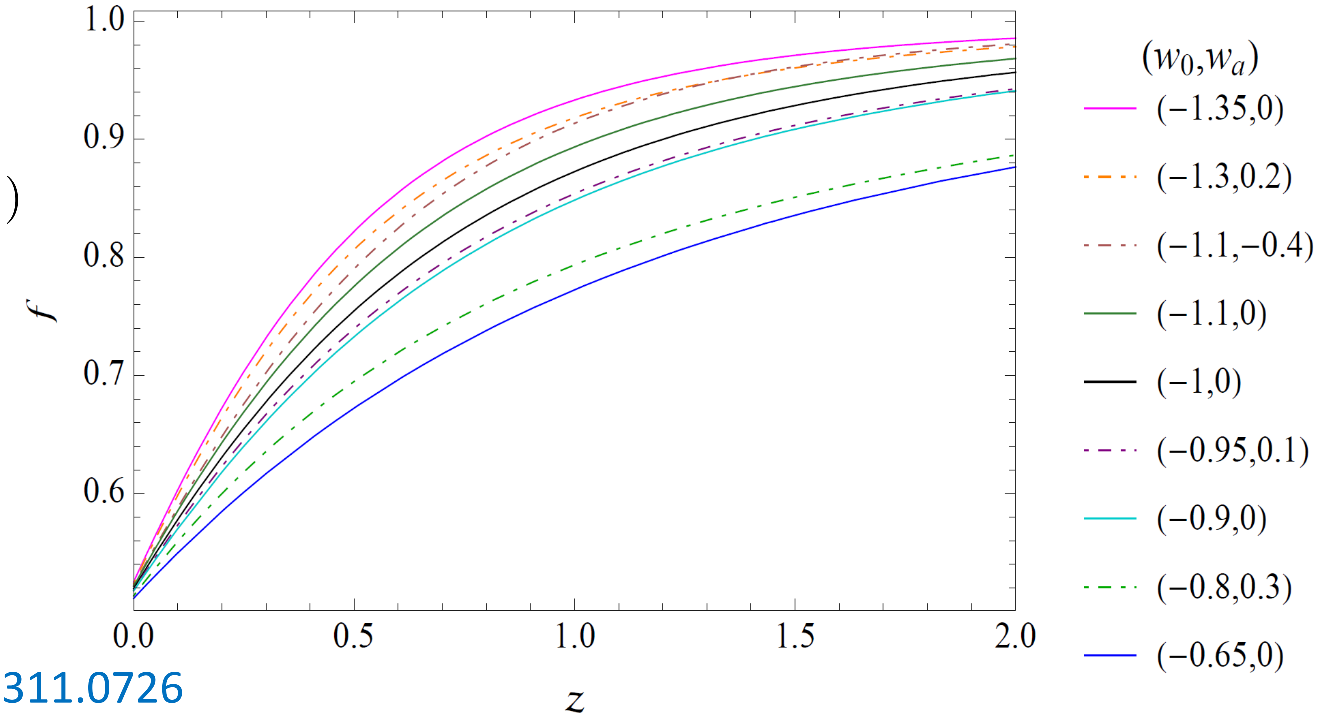

Dossett & Ishak 1311.0726

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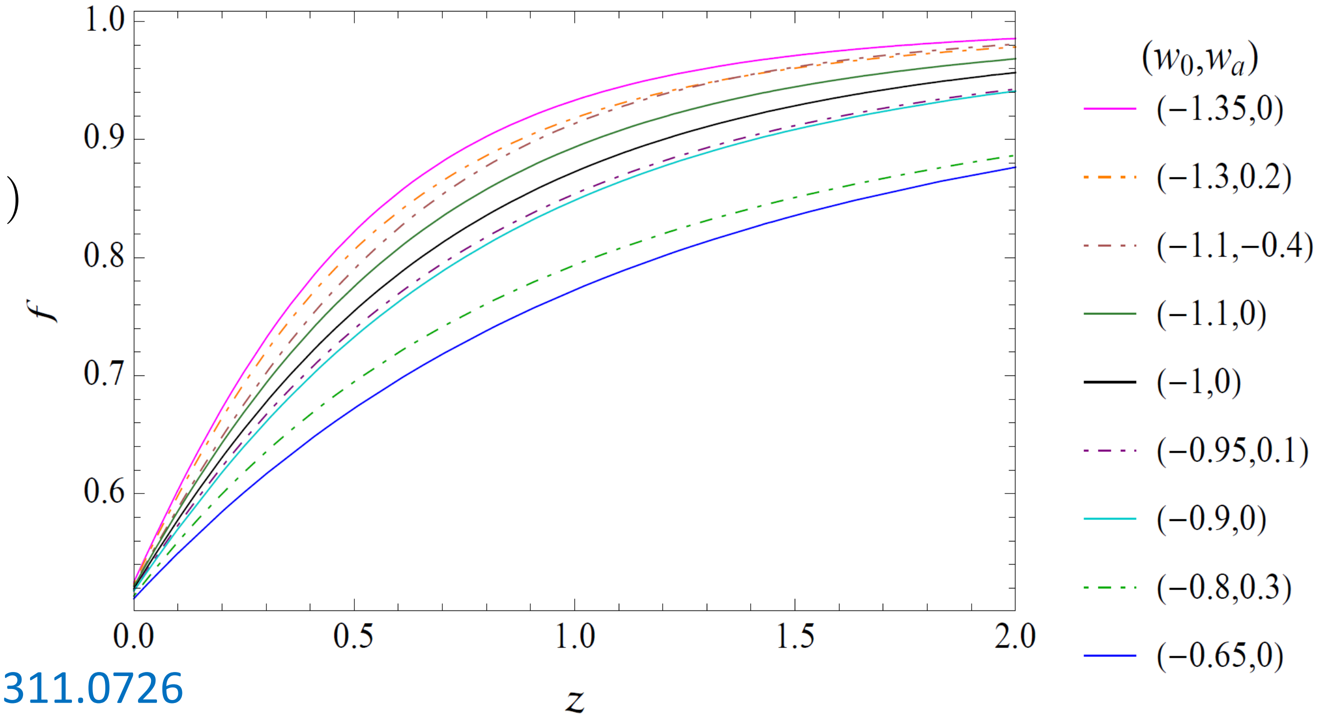
- Even when DE perturbations and anisotropic stress is accounted for, growth index still approx. constant.



Dossett & Ishak 1311.0726

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Example: Scalar field with canonical kinetic term

$$S_\phi = \int d^4x \sqrt{-g} \left(-\frac{1}{2} \partial_\mu \phi\, \partial^\mu \phi - V(\phi) \right) \Rightarrow$$

$c_{s,\phi}^2 = 1$ ⇒ canonical scalar does not cluster at subhorizon scales. Can approximate as smooth DE $\delta\rho_\phi = \Pi_\phi = 0$

Growth of structure

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- Canonical scalar can be approximated to be smooth for subhorizon growth of structure, but we cannot ignore its perturbations entirely!

Growth of structure

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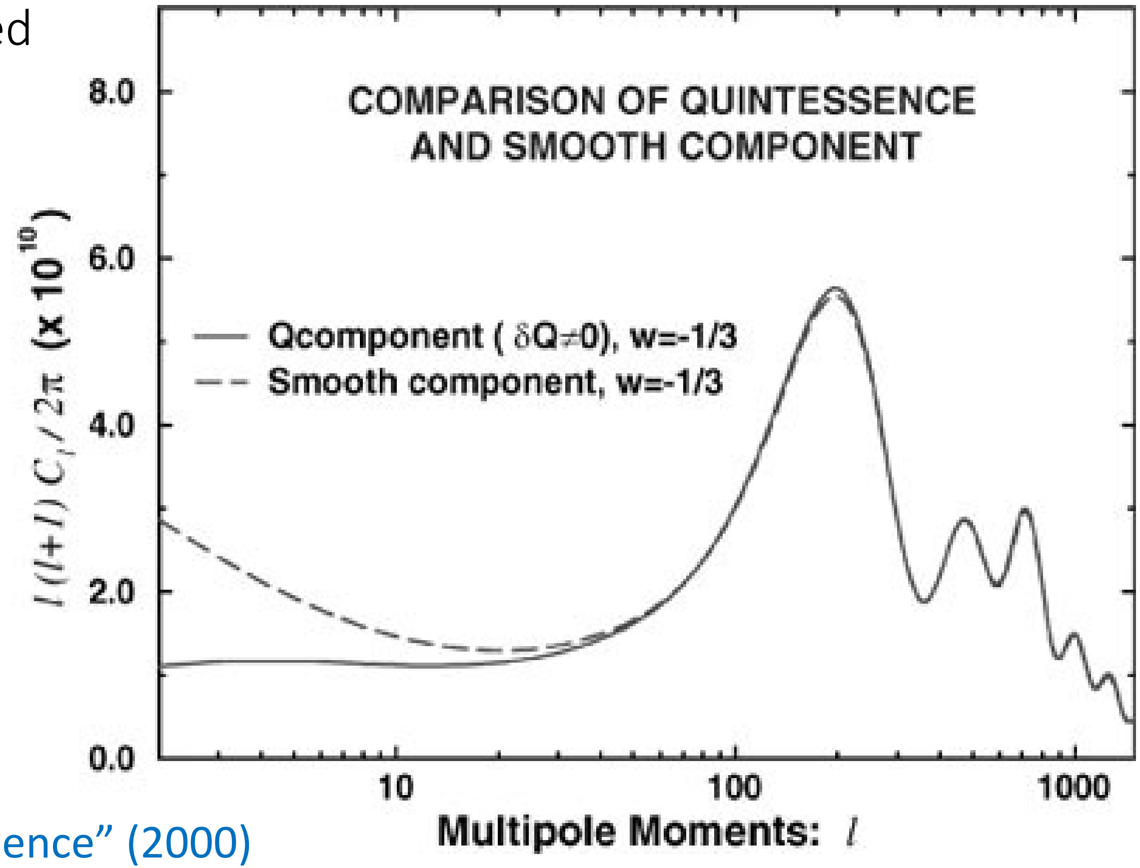
- Long wavelength modes can interact with large scale fluctuations in CDM and baryons.



Caldwell, “An introduction to quintessence” (2000)

Growth of structure

4. Clustering DE and MG

- Poisson's equation

$$k^2 \Psi = -4\,\pi\,a^2\,G\,\left(1 + \frac{\rho_E\,\Delta_E}{\rho_m\,\Delta_m}\right)\rho_m\,\Delta_m$$

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- Anisotropic stress

$$\Phi - \Psi = 8\,\pi G\;a^2\,P_E\,\Pi_E$$

- For scalar field matter: $\Pi_E = 0$, but it typically appears in modified gravity.

Zoology of DE/MG models

ΛCDM $\Lambda(\Omega_E)$

Clustering DE $(\Omega_E\,,\,w_E\,,\,\delta\rho_E)$
K-essence

Smooth DE $(\Omega_E\,,\,w_E)$
Quintessence

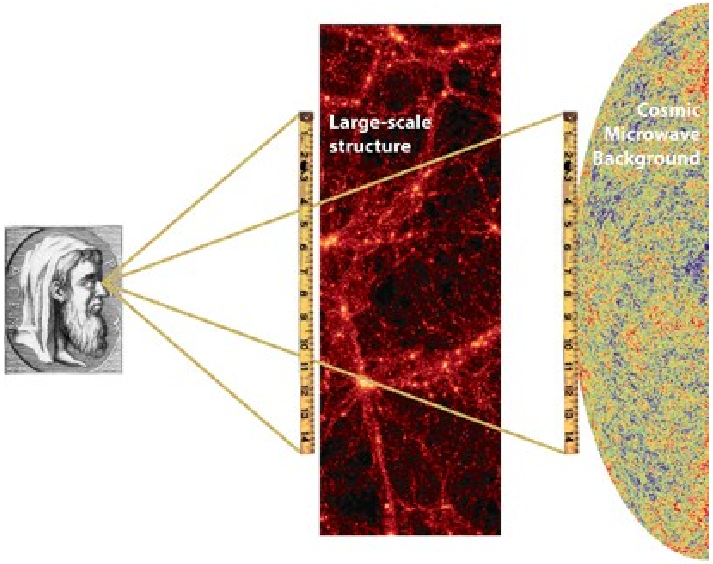
Modified gravity $(\Omega_E\,,\,w_E\,,\,\delta\rho_E\,,\,\Pi_E)$
(*screening mechanisms*)

Interacting DE $(\Omega_E\,,\,w_E\,,\,\delta\rho_E\,,\,\Pi_E\,,\,Q^{\mu}_E)$
(*violation of weak equivalence principle*)

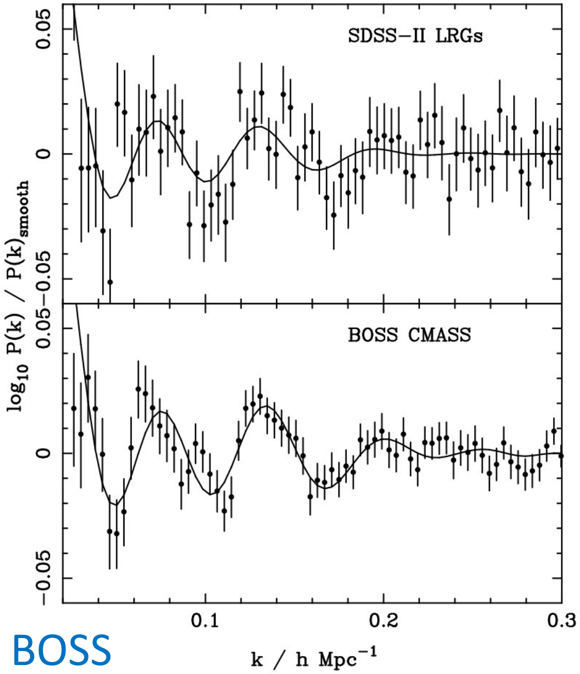
Observations

Background

- Measurement of $H(z)$
  - Type 1a Supernovae: luminosity distance
  - CMB/BAO: angular diameter distance

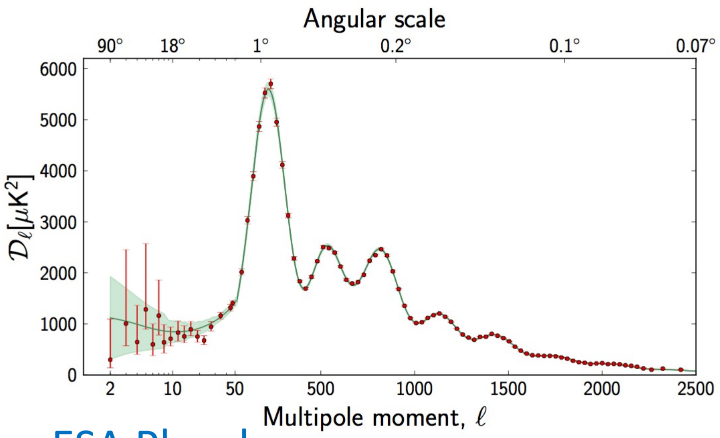


ESA Euclid



BOSS

SN 1994D



ESA Planck

Observations

Weak lensing

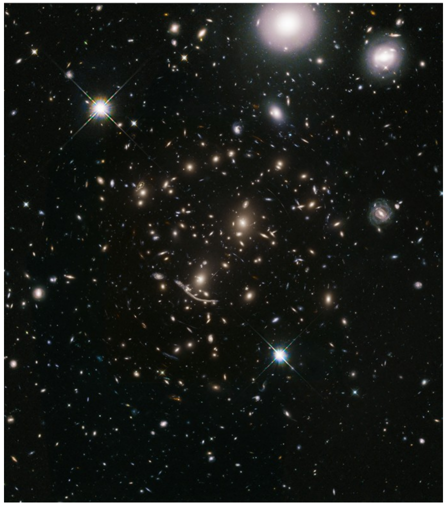
$$ds^2 = a^2(\tau)\left[-(1+2\,\Phi)d\tau^2 + (1-2\,\Psi)\delta_{ij}dx^i dx^j\right]$$

- Convergence (photons follow geodesics):

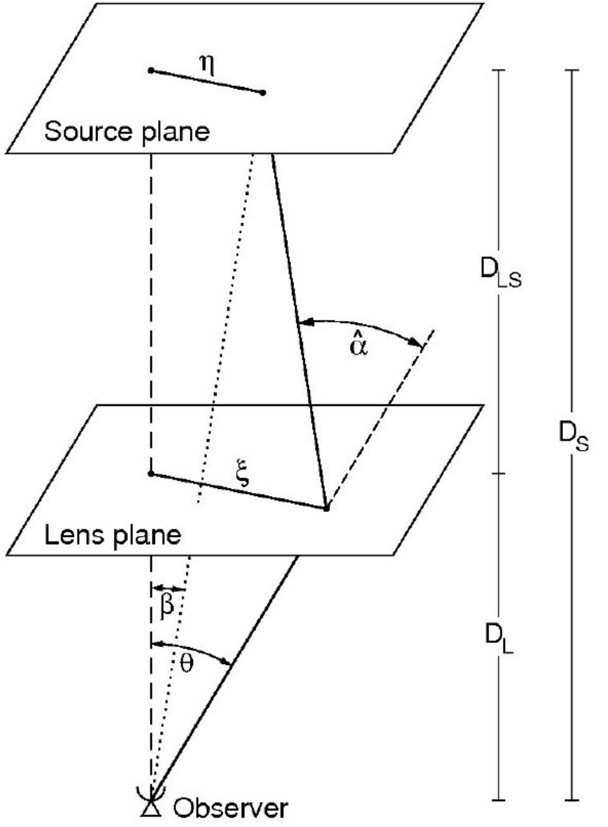
$$\kappa(\hat{n}) = \int d\chi \frac{(\chi_s-\chi)\chi}{\chi_s}\nabla_\perp^2\,\underbrace{\phi_W(\tau_0-\chi,\chi\,\hat{n})}$$

Lensing Potential:

$$\phi_W \equiv \frac{\Phi+\Psi}{2}$$



Galaxy Cluster Abell 370 (HST)



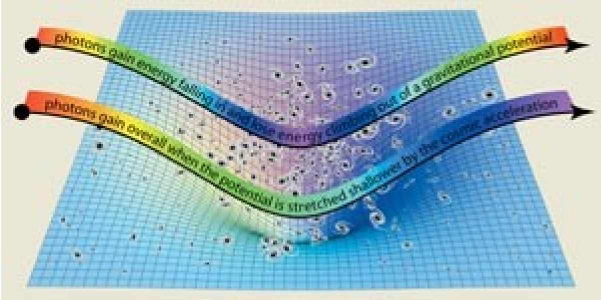
Bartelmann & Schneider astro-ph/9912508

Observations

CMB

- *Late Integrated Sachs-Wolfe (ISW) effect:* Time variation of the gravitational potentials causes shift in photon temperature

$$\frac{\Delta T_{ISW}}{T}(\hat{n}) = 2\int_0^{\chi_{LSS}} \frac{\partial \phi_W(\tau_0 - \chi, \chi\,\hat{n})}{\partial \tau}\,d\chi$$



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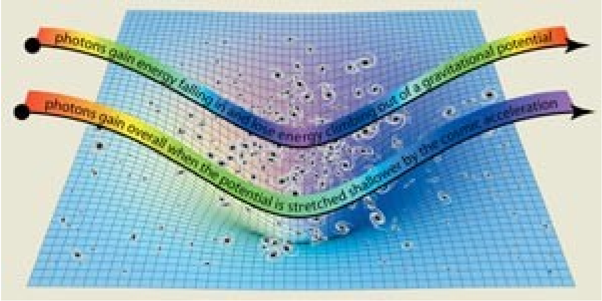


- *Lensing:* CMB photons are also lensed.

$$\frac{\Delta T_{lensed}}{T}(\hat{n}) = \frac{\Delta T}{T}(\hat{n} + \vec{\nabla}\psi)$$

$$\psi(\hat{n}) = -2\int d\chi \frac{(\chi_{LSS} - \chi)\chi}{\chi_{LSS}}\,\phi_W(\tau_0 - \chi, \chi\hat{n})$$

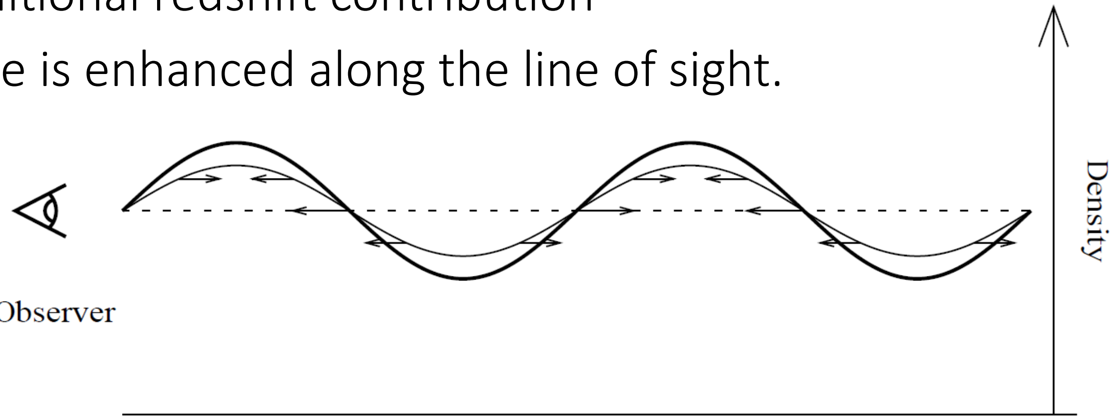
Observations

Redshift-space distortions

- Galaxies have peculiar velocities: additional redshift contribution
- Clustering of galaxies in redshift space is enhanced along the line of sight.
- The enhanced amplitude

$$\delta^s(k,\mu) = \Delta_m(k) + \mu^2\,\theta(k)$$

$$\boxed{\mu \equiv \hat{k}\cdot\hat{n}}$$



Hamilton astro-ph/9708102

- Using the continuity equation $\Delta_m' = -\mathcal{H}\,\theta_m$, we relate it to the growth rate

$$\delta^s(k,\mu) = \Delta_m(k)\left(1+\mu^2 f\right)$$

Growth rate:

$$f \equiv \frac{d\log\Delta_m}{d\log a}$$

Can determine Newtonian potential from Euler eq:

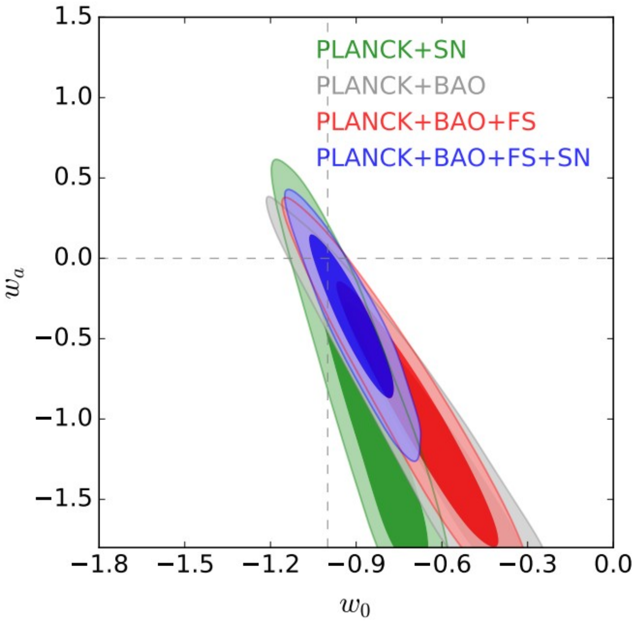
$$a\,k^2\Phi = (a\,\mathcal{H}\,\theta_m)'$$

Testing DE/MG

Background expansion history

- Background expansion is solely determined by the DE equation of state

$$w_E(t) = w_0 + w_a(1-a) + \mathcal{O}(1-a)^2$$



BOSS Collaboration 1607.03155

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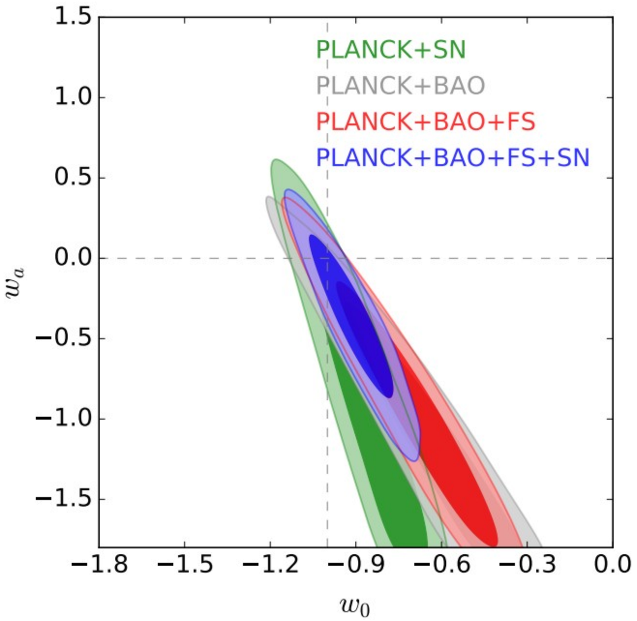

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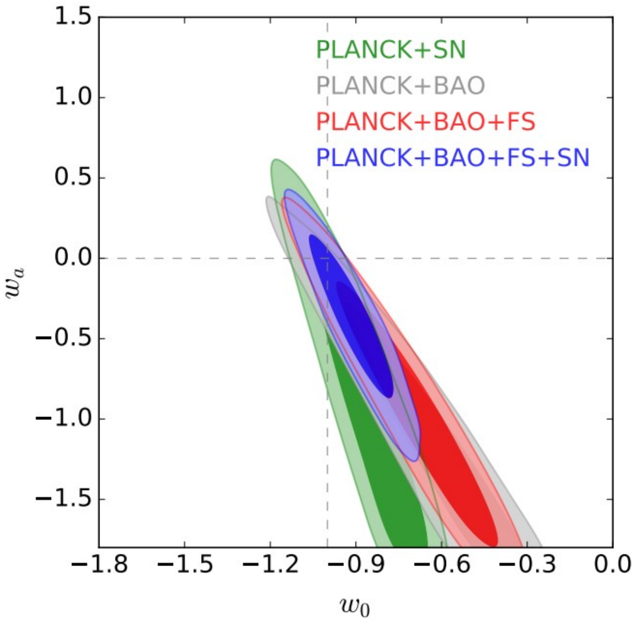
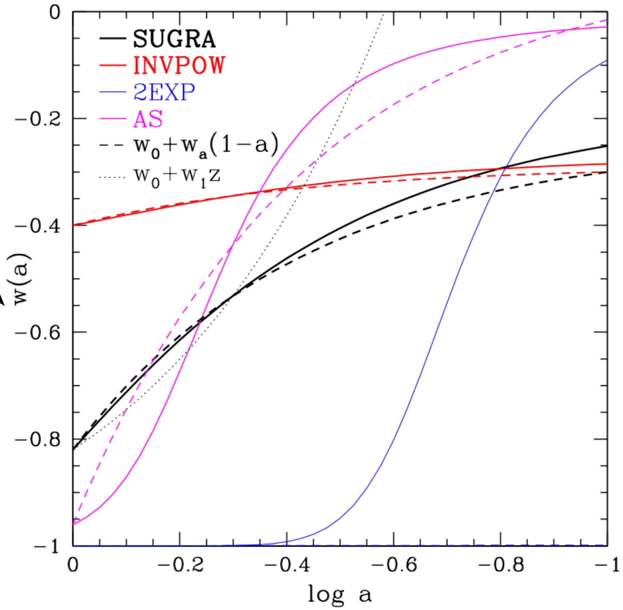


BOSS Collaboration 1607.03155

- Some dynamical DE models and their reconstruction from linear parametrisation.
- Good agreement for slow moving scalar field models



Linder astro-ph/0311403

Testing DE/MG

Parametrisation for slowly rolling scalar DE

- For slow roll,

$$(1 + w_E) \simeq \frac{2}{3}\,\epsilon_V \Omega_E(a)$$

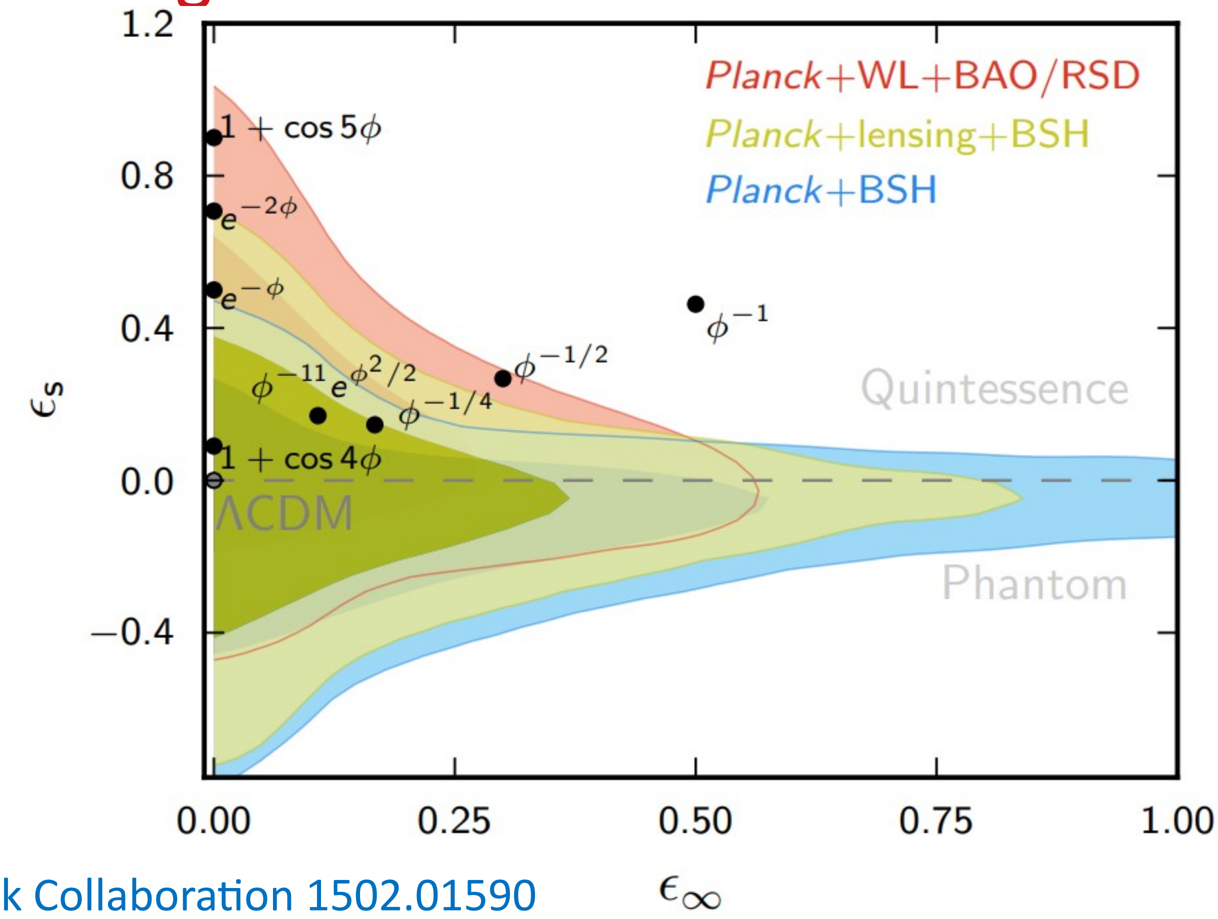
$$\boxed{\begin{aligned} \epsilon_V &\equiv \frac{M_{Pl}^2}{2}\left(\frac{V'}{V}\right)^2 \\ \Omega_E &\equiv \frac{\rho_E}{\rho_E + \rho_m} \end{aligned}}$$

- *Slope parameter:*

$$\epsilon_s \equiv \epsilon_V \Big|_{\rho_m = \rho_E}$$

- $\epsilon_V \Omega_E \sim$ constant at early times:

$$\epsilon_\infty \equiv \epsilon_V \Omega_E \Big|_{a \to 0}$$

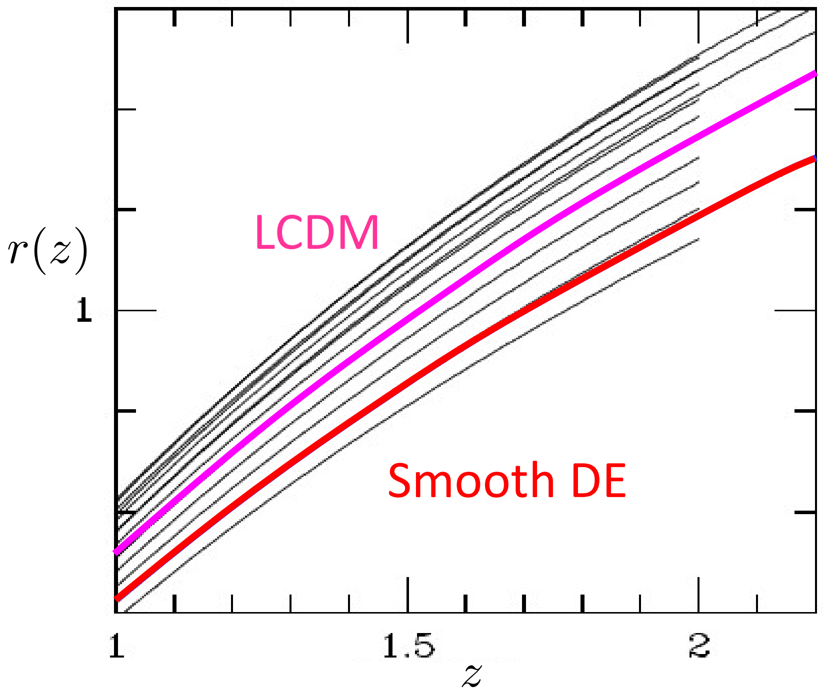


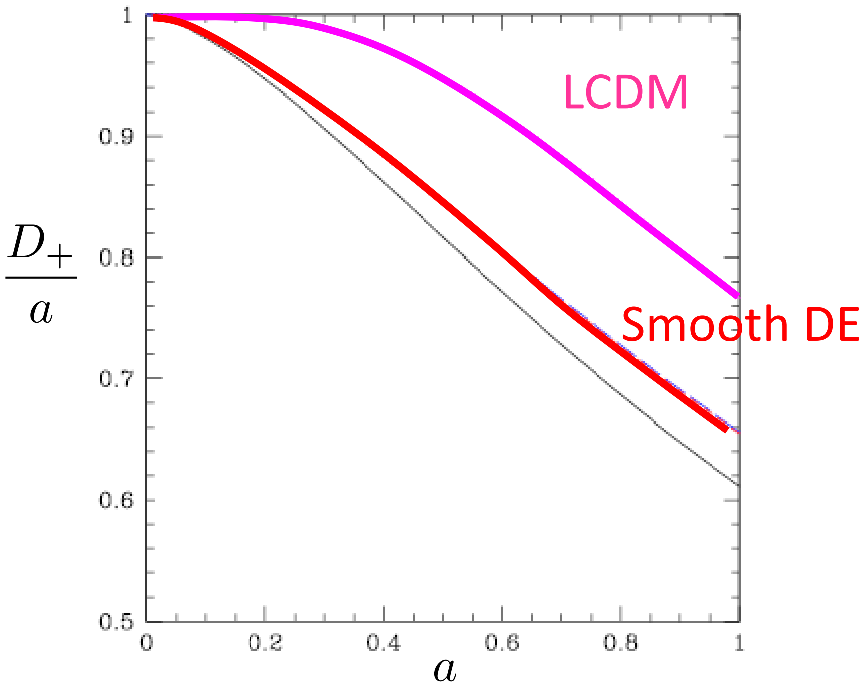
Planck Collaboration 1502.01590

Testing DE/MG

Degeneracies and consistency tests

- *<u>ΛCDM and smooth DE models:</u>* 1:1 correspondance between BG expansion history and growth of structure. No degeneracy.



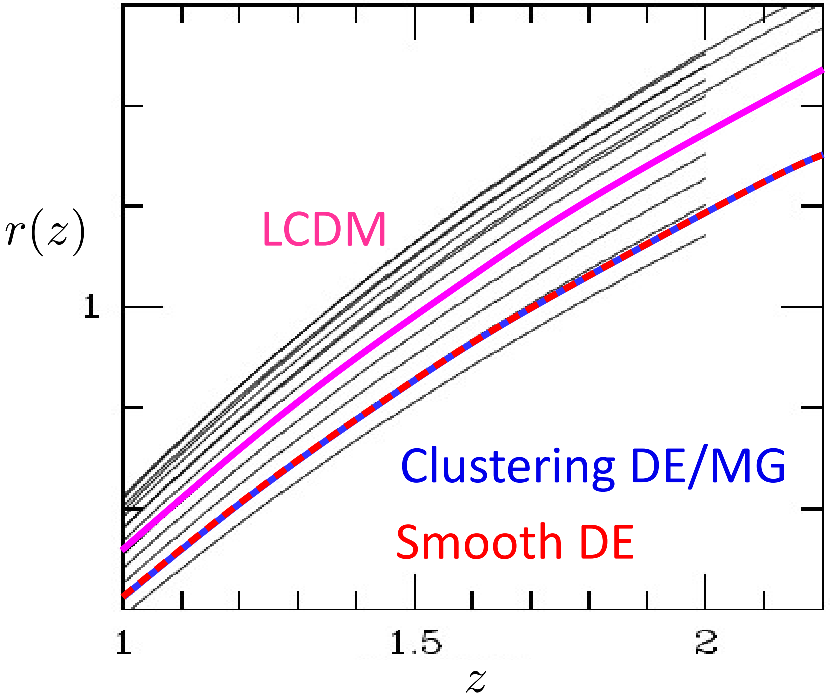


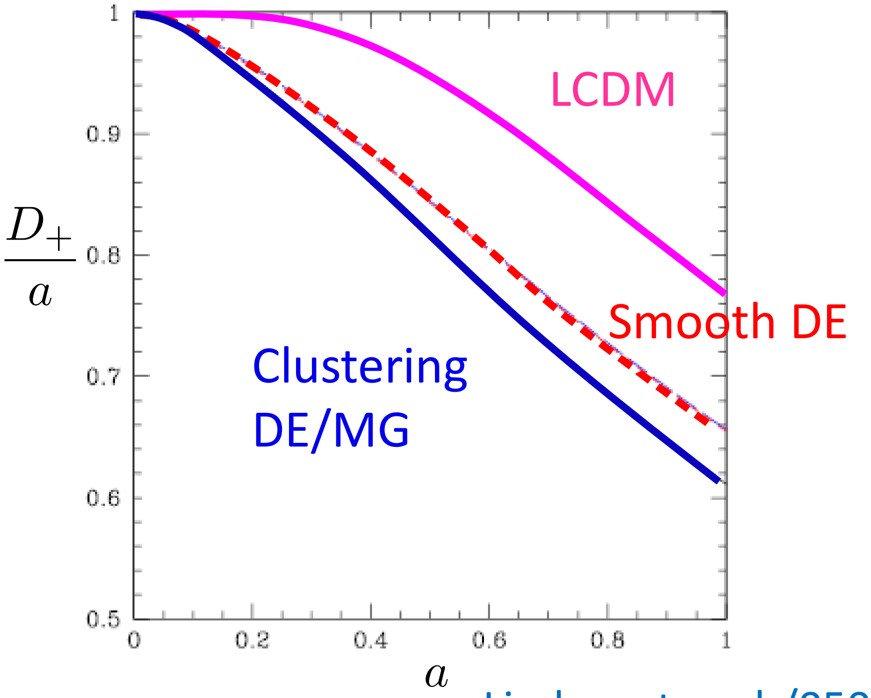
Linder astro-ph/0507263

Testing DE/MG

Degeneracies and consistency tests

- *Clustering DE and MG models:* Structure is also controlled by $\delta\rho_E$. Can have exact same expansion history as a smooth DE model, with a different structure growth.Need to combine measurements to discriminate.





Linder astro-ph/0507263

Testing DE/MG

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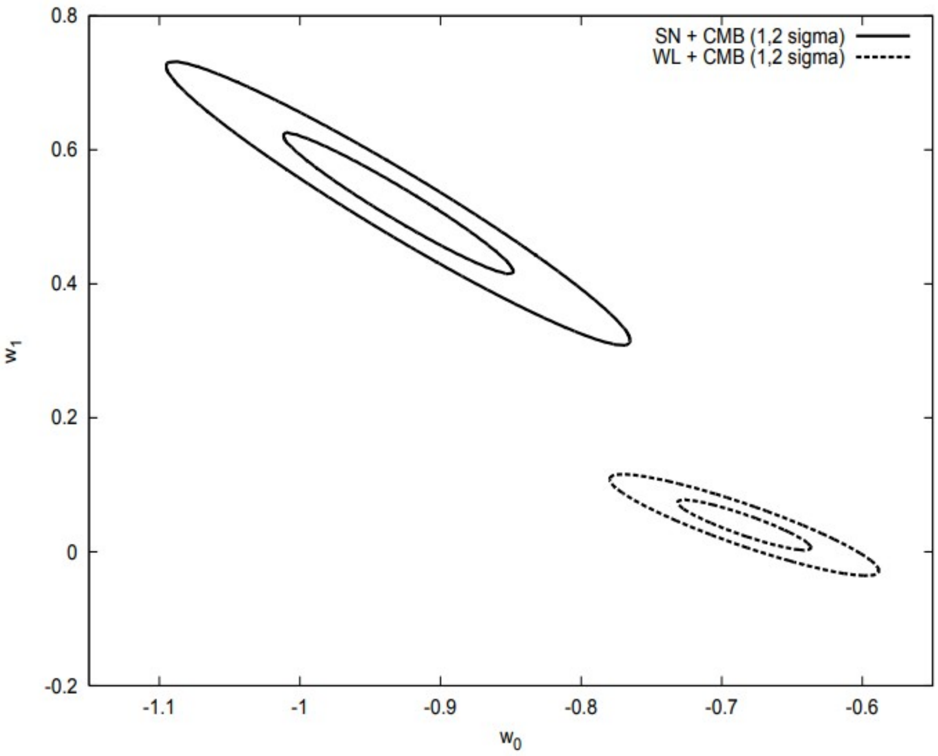
- Assume that the acceleration is *actually* due to clustering DE/MG, but we still try to fit the data using a smooth DE model



- Data generated using DGP model (MG)
- Fitting to a smooth DE model parametrised by $w(z) = w_0 + w_1 z$
- Inconsistent between two different data combinations!

Ishak et al. astro-ph/0507184

Testing DE/MG

Degeneracies and consistency tests

Jain & Zhang 0709.2375
Song & Koyama 0802.3897

- *A simple consistency relation:* Using Friedmann and Poisson's equations

$$H^2 = \frac{8\pi G}{3}\rho_{tot}$$

$$k^2\Psi = -4\pi G a^2 \rho_{tot}\delta_{tot}$$

Assumptions:
- Cosmological principle
- Conservation of CDM energy-momentum
- MG is a metric theory with Newtonian limit

$$\rho_{tot} \equiv \sum_I \rho_I$$

$$\rho_{tot}\delta\rho_{tot} \equiv \sum_I \rho_I \delta\rho_I$$

In the comoving gauge:

$$\Delta_m\Big|_{B=v_m=0} = \delta_m$$

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peculiar velocity
(redshift space distortions)

using $k^2\Phi = \partial_t(H\theta_m)$

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Challenge: Need to make all measurements at the same time and same location

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Model independent parametrisation

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$$k^2\Phi = -4\pi G\, a^2\, \mu(k,a)\, \rho_m\, \Delta_m$$

$$\Psi = \eta(k,a)\Phi$$

$$ds^2 = a^2(\tau)\,\left[-(1+2\,\Phi)d\tau^2 + (1-2\,\Psi)\delta_{ij}dx^i dx^j\right]$$

- Equivalently, we can parametrise the lensing potential

$$k^2\Phi = -4\pi G\, a^2\, \mu(k,a)\, \rho_m\, \Delta_m$$

$$k^2\frac{(\Phi+\Psi)}{2} = -4\pi G\, a^2\, \Sigma(k,a)\, \rho_m \Delta_m$$

$$\boxed{\Sigma = \frac{\mu\,(1+\eta)}{2}}$$

For smooth DE

$$\mu = \eta = \Sigma = 1$$

Project	Dates	Area/deg^2	Data	Spec-z Range	Methods
BOSS	2008–2014	10,000	Opt-S	0.3–0.7 (gals)	BAO/RSD
				2–3.5 (LyαF)	
KiDS	2011–2019	1500	Opt-I	——	WL/CL
DES	2013–2019	5000	Opt-I	——	WL/CL
					SN/BAO
eBOSS	2014–2018	7500	Opt-S	0.6–2.0 (gal/QSO)	BAO/RSD
				2–3.5 (LyαF)	
SuMIRE	2014–2024	1500	Opt-I		WL/CL
			Opt/NIR-S	0.8–2.4 (gals)	BAO/RSD
HETDEX	2017–2023	450	Opt-S	$1.9 < z < 3.5$ (gals)	BAO/RSD
DESI	2020–2025	14,000	Opt-S	0–1.7 (gals)	BAO/RSD
				2–3.5 (LyαF)	
LSST	2022–2032	20,000	Opt-I	——	WL/CL
					SN/BAO
Euclid	2022–2028	15,000	Opt-I		WL/CL
			NIR-S	0.7–2.2 (gals)	BAO/RSD
WFIRST	2025–2030	2200	NIR-I		WL/CL/SN
			NIR-S	1.0–3.0 (gals)	BAO/RSD

Table 28.1: A selection of major dark-energy experiments, based on Ref. [25]. Abbreviations in the "Data" column refer to optical (Opt) or near-infrared (NIR) imaging (I) or spectroscopy (S). For spectroscopic experiments, the "Spec-z" column lists the primary redshift range for galaxies (gals), quasars (QSOs), or the Lyman-α forest (LyαF). Abbreviations in the "Methods" column are weak lensing (WL), clusters (CL), supernovae (SN), baryon acoustic oscillations (BAO), and redshift-space distortions (RSD).

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