Theoretical Cosmology
Part II: Hot thermal Universe

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ICG PhD Lectures, November 2021

1. Introduction to Big-Bang cosmology	15 November
2. *Hot thermal Universe*	*19 November*
3. Inflation	22 November
4. Dark energy	26 November

Plan for today

1. *Thermodynamics in the early Universe*
2. *Ingredients of the thermal soup*
3. *Decoupling of interactions and freeze-out*
4. *Big-Bang Nucleosynthesis (BBN)*
5. *Cosmic Microwave Background (CMB)*

References

- *Big-Bang Cosmology*, Keith Olive and John Peacock, in *Review of Particle Physics* (http://pdg.lbl.gov/)
- *Big Bang Nucleosynthesis*, Brian Fields, Paolo Molaro, and Subir Sarkar, in *Review of Particle Physics*
- Past lecture notes by Kazuya Koyama, Vincent Vennin and Hans Winther

Statistical mechanics in equilibrium

- *Thermal equilibrium:* Frequent particle interactions. No net flow of thermal energy.

- *Density of states* of a particle with momentum between q and $q+dq$

$$dn_q = \frac{g}{2\pi^2} \frac{1}{e^{(E-\mu)/T} \pm 1} q^2 dq$$

Degrees of freedom

Chemical potential

Distribution function:
- : Bose-Einstein
+ : Fermi-Dirac
$E = \sqrt{m^2 + q^2}$

- *Number density* of the species i

$$n_i = \frac{N_i}{V} = \int dn_{q_i}$$

- *Energy density* for particle species i

$$\rho_i = \int E_i(q) dn_{q_i}$$

- *Pressure* of a perfect gas

$$P_i = \int \frac{q_i^2}{3E_i} dn_{q_i}$$

Entropy

- We start with first law of thermodynamics

$$dE = T\,dS - P\,dV + \textstyle\sum_i \mu_i\,dN_i$$

- In terms of densities $\rho = E/V$, $s = S/V$, $n_i = N_i/V$, this becomes:

$$dV\left(\rho + P - T\,s - \sum_i \mu_i n_i\right) + V\left(d\rho - T\,ds - \sum_i \mu_i dn_i\right) = 0$$

- This relation is valid for the whole system or a finite part. Evaluated within a region of constant volume

$$ds = \frac{1}{T}\left(d\rho - \sum_i \mu_i dn_i\right)$$

- Then evaluating over the entire system, we obtain the ***entropy density***

$$\boxed{s = \frac{\rho + P - \sum_i \mu_i n_i}{T}}$$

Conservation of entropy

$$ds = \frac{1}{T}\left(d\rho - \sum_i \mu_i dn_i\right) \qquad s = \frac{\rho + P - \sum_i \mu_i n_i}{T}$$

- Using these two equations, calculate how total entropy changes in time:

$$\frac{dS}{dt} = \frac{d(sV)}{dt} = \frac{V}{T}\frac{ds}{dt} + s\frac{dV}{dt}$$

$$= \frac{V}{T}\left(\frac{d\rho}{dt} - \sum_i \mu_i \frac{dn_i}{dt}\right) + \frac{1}{T}\left(\rho + P - \sum_i \mu_i n_i\right)\frac{dV}{dt}$$

$$= \frac{V}{T}\underbrace{\left[\frac{d\rho}{dt} + \frac{1}{V}\frac{dV}{dt}(P+\rho)\right]}_{=0} - \underbrace{\frac{1}{T}\sum_i \mu_i \frac{d\,n_i V}{dt}}_{=0}$$

- Continuity equation:

$$\dot{\rho} + 3\,\frac{\dot{a}}{a}(\rho + P) = 0$$

- Frequent interactions in equilibrium gas preserve the total # for each species:

$$dN_i = 0$$

- *Entropy of the thermal bath is conserved:*

$$\boxed{\frac{d(sa^3)}{dt} = 0}$$

- As Universe expands, interactions of some particles no longer reversed. Entropy conservation allows us to determine the resulting change in temperature in the thermal bath.

Relativistic particles

- At high temperatures $m,\, \mu \ll T$, particles are relativistic. Density of states for bosons reduces to ***Planck's distribution*** (blackbody).

$$dn_q|_{T\gg} = \frac{g}{2\,\pi^2}\frac{1}{\mathrm{e}^{q/T}\pm 1}\,q^2 dq$$

- In this limit, we can calculate the energy density of a given species

$$\rho_i = \frac{g_i\,\pi^2\,T^4}{30} \times \begin{cases} 1\,, & \text{bosons} \\ 7/8\,, & \text{fermions} \end{cases}$$

Stefan-Boltzmann law: $\rho \propto T^4$

- For multiple relativistic particle species in thermal equilibrium, we can write the ***total energy density*** as

$$\rho = \frac{g_{eff}\,\pi^2\,T^4}{30}$$

- ***Effective number of relativistic degrees of freedom***

$$g_{eff} \equiv \sum_B g_B + \frac{7}{8}\sum_F g_F$$

- ***Total pressure*** and **entropy density** in the relativistic regime is automatically related to density as $P = \frac{\rho}{3}$ and $s = \frac{3\,\rho}{4\,T}$.
- ***Number density*** of species i

$$n_i = \frac{g_i\,\zeta(3)\,T^3}{\pi^2} \times \begin{cases} 1\,, & \text{bosons} \\ 3/4\,, & \text{fermions} \end{cases}$$

Non-relativistic particles

- When the temperature drops below a particle's mass $T \ll m$, the distinction between fermions and bosons disappear and the density of states is given by the ***Maxwell-Boltzmann distribution***

$$dn_q = \frac{g}{2\,\pi^2} \frac{1}{e^{(E-\mu)/T} \pm 1} q^2 dq \quad \Rightarrow \quad dn_q\Big|_{NR} = \frac{g}{2\,\pi^2} e^{-(E-\mu)/T}\, q^2 dq$$

- Key quantity here is the ***number density of non-relativistic particles***

$$n = \int dn_q \simeq g \left(\frac{m\,T}{2\,\pi}\right)^{3/2} e^{-(m-\mu)/T}$$

- Since the ratio m/T increases with expansion, the # density of a massive particle decreases exponentially (***Boltzmann suppression***). Particles may still be in equilibrium, but they can only decay, not enough energy to create them.

- Energy density for non-relativistic species

$$\rho_i = m_i\, n_i$$

Temperature as an inverted clock

- Continuity equation for radiation ($P = \frac{\rho}{3}$) is solved by

$$\rho \propto a^{-4}$$

- On the other hand, $\rho \propto g_{eff} T^4$. For constant g_{eff}, this implies

$$T \propto a^{-1}$$

- *Temperature cools down as the universe expands*. Can use it to keep track of time.
- During radiation domination, using Friedmann equation:

$$\left(\frac{t}{\mathrm{s}}\right) \simeq \frac{2.4}{\sqrt{g_{eff}}} \left(\frac{\mathrm{MeV}}{T}\right)^2$$

- However, g_{eff} *does* change with time: as temperature decreases, more species become non-relativistic. For time estimations, we will neglect this effect.

Evolution of g_{eff}

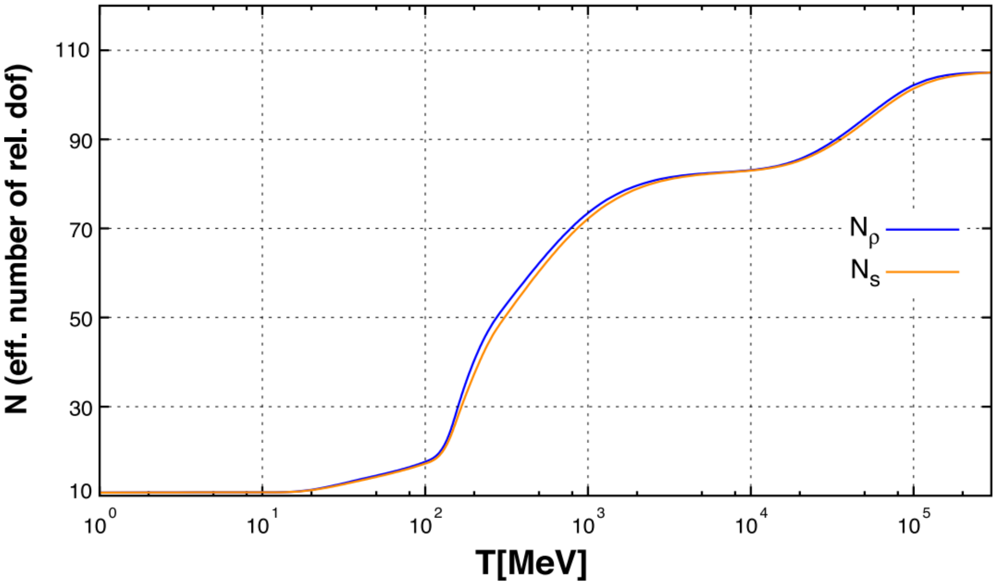
$$g_{eff} \equiv \sum_B g_B + \frac{7}{8}\sum_F g_F$$

- As the temperature cools down, particles become non-relativistic.

  → the effective # of relativistic degrees of freedom changes!

time ↑

Temperature	New Particles	$4\ g_{eff}$
$T < m_e$	γ's + ν's	29
$m_e < T < m_\mu$	$e^\pm$	43
$m_\mu < T < m_\pi$	$\mu^\pm$	57
$m_\pi < T < T_c{}^\dagger$	π's	69
$T_c < T < m_{\rm strange}$	π's + $u, \bar{u}, d, \bar{d}$ + gluons	205
$m_s < T < m_{\rm charm}$	$s, \bar{s}$	247
$m_c < T < m_\tau$	$c, \bar{c}$	289
$m_\tau < T < m_{\rm bottom}$	$\tau^\pm$	303
$m_b < T < m_{\rm W,Z}$	$b, \bar{b}$	345
$m_{W,Z} < T < m_{\rm Higgs}$	$W^\pm, Z$	381
$m_H < T < m_{\rm top}$	H^0	385
$m_t < T$	$t, \bar{t}$	427



From "Big Bang Cosmology", Olive and Peacock, PDG Reviews and Tables 2018

Decoupling and freeze-out

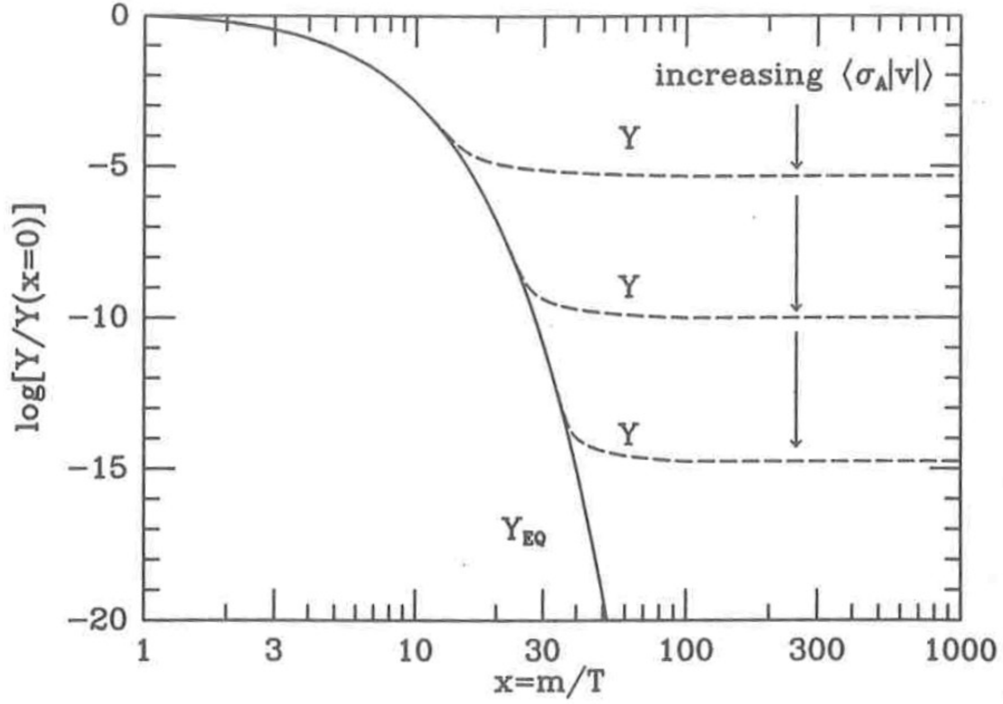
- When a particle becomes massive in thermal equilibrium, its number quickly decreases. However, they can also *fall out of equilibrium*.
- Thermal equilibrium is a consequence of the energy transfer in particle interactions. In an isolated fixed-sized box, equilibrium will continue indefinitely.
- Our "box" expands and cools. The cooling eventually starts to prevent interactions: *competition between expansion and particle physics processes*.
- *Interaction rate:* Γ. Time between consecutive interactions is Γ^{-1}. Provided that this time scale is much shorter than the Hubble time H^{-1}, processes are efficient.
- *Decoupling* happens when

$$\Gamma^{-1} > H^{-1}$$

In this case, the interaction takes longer than the age of the universe. The particles can no longer transfer energy through this specific interaction. If particles cannot take part in any interaction, they *freeze-out*. *No longer in thermal equilibrium.*

“Survival of the weakest”

If the particles become non-relativistic while in equilibrium, their number density decays exponentially. When their interactions decouple, they are no longer in equilibrium and they freeze-out.



From “Early Universe”, Kolb and Turner (1990)

The plot shows the comoving abundance of a particle normalised to its relativistic value. The dashed lines show the actual value, the solid line is the equilibrium value.

Particles with *weaker interactions* leave equilibrium *earlier* and end up with a *larger abundance*.

Boltzmann equation

- For an accurate description of the decoupling, we should keep track of the evolution of the distribution outside of equilibrium. The key equation is the ***Boltzmann equation***. For a particle species A

Distribution function

$$\frac{d f_A}{dt} = \hat{C}_A[f]$$

Collision terms

- If there are no collisions, the distribution function is conserved.
- Even if the particle is stable, it can still annihilate with its antiparticle. As an example, let's consider the process $A + \bar{A} \to B + \bar{B}$, where species A is non-relativistic and B is in thermal equilibrium. The Boltzmann equation in FLRW is

$$\dot{n}_A + \underbrace{3\,H\,n_A}_{\text{expansion}} = -\underbrace{<\sigma\, v>}_{\text{Particle physics}}\left(n_A^2 - n_{A,eq}^2\right)$$

σ Cross section
v Velocity
$<>$ Thermal average

$$\Gamma = n <\sigma\, v>$$

- For a non-relativistic particle, when right hand side becomes negligible compared to the expansion term (decoupling), value of n_A freezes to the equilibrium value at the freeze-out temperature.

Thermal history of the Universe

TeV GeV MeV (1 sec) eV (3000K) meV(3K)

Electro weak phase transition

Quark hadron phase transition

Neutrino decoupling

e+ e- annihilation

nucleosynthesis

Recombination

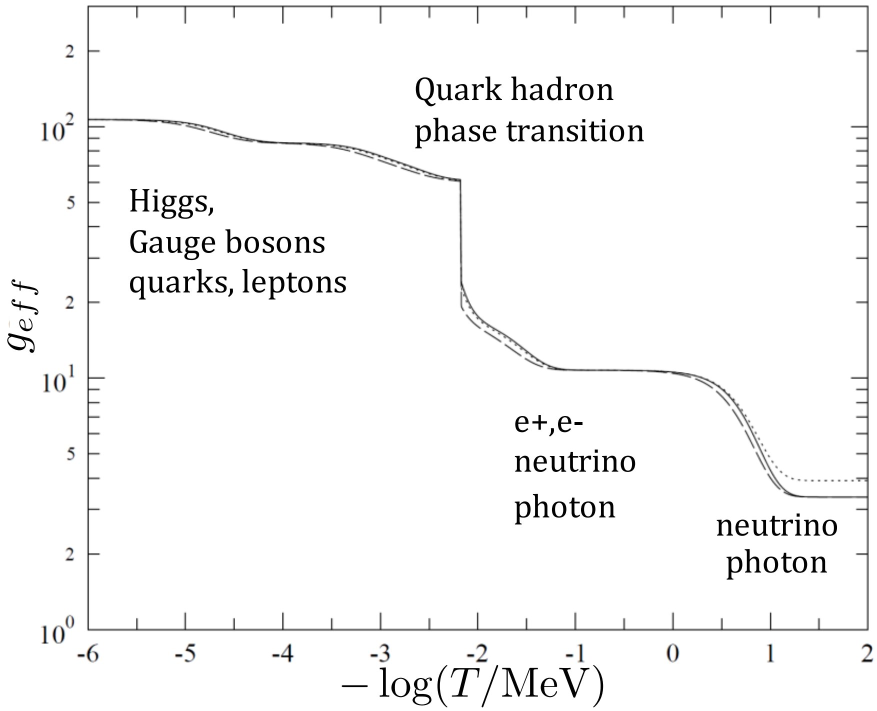
Photon decoupling

quark n,p He,p He,H

lepton e+,e- e-

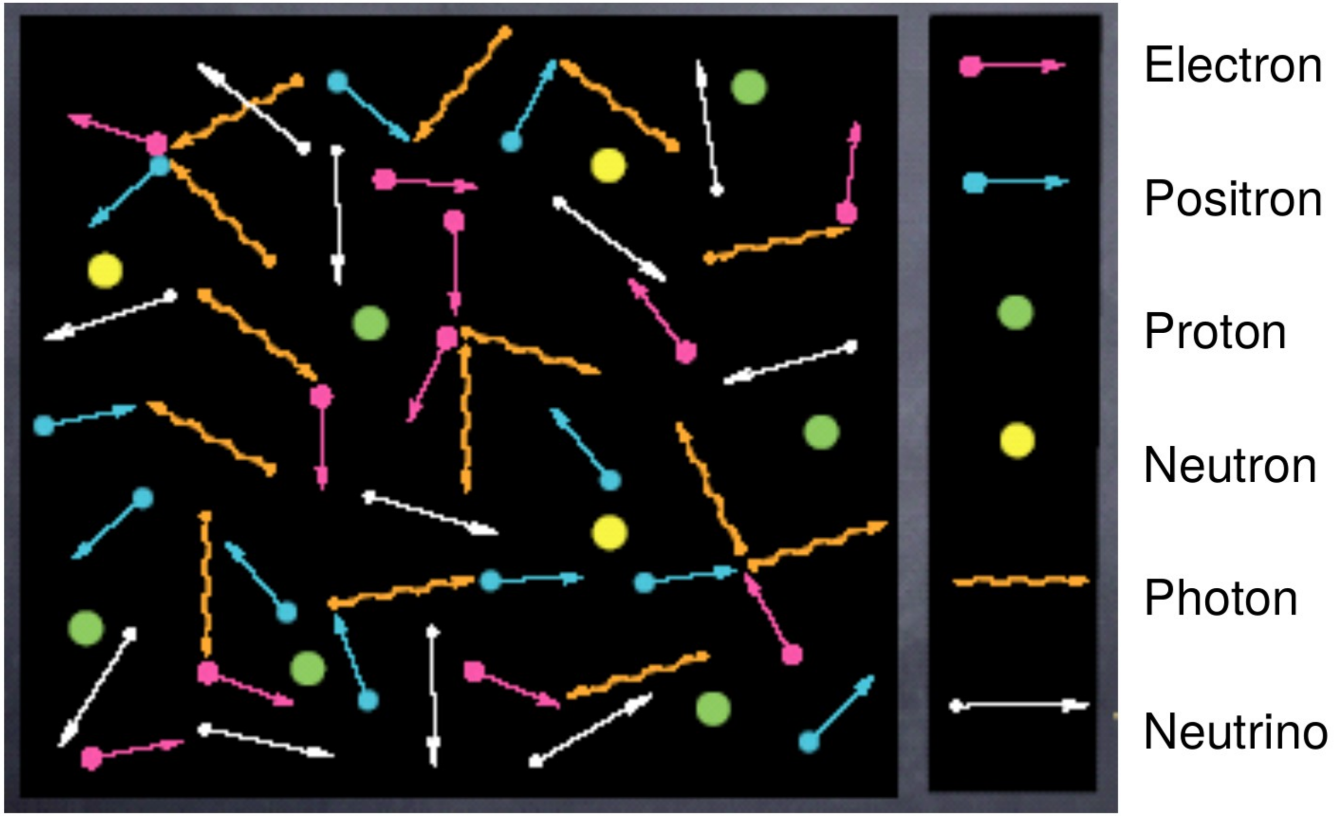
neutrino

photon



We pick our story up after the QCD phase transition.

Neutrino decoupling



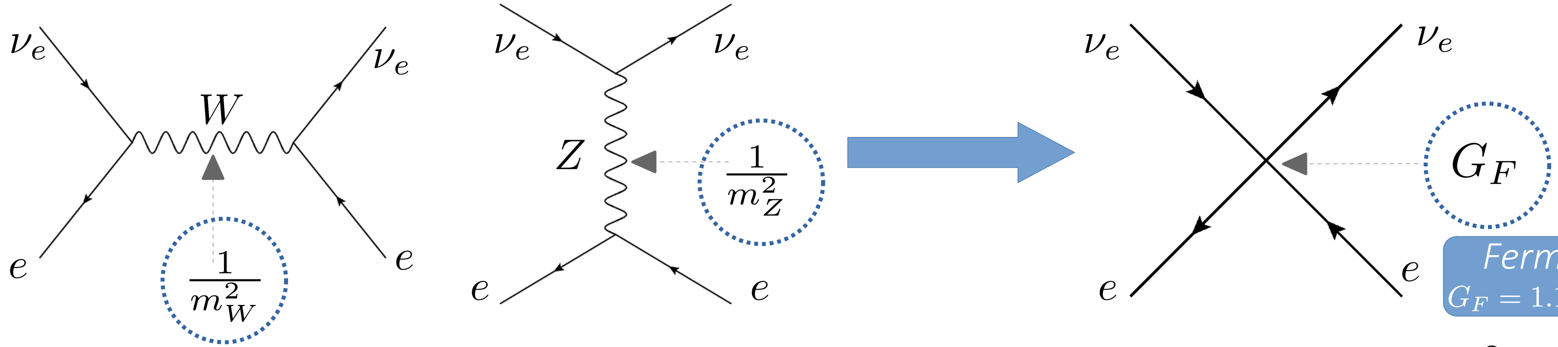
At this point in the thermal history, all that remains in the thermal bath are the light leptons, nucleons and photons.

Neutrino decoupling

- Neutrinos charged only under ***weak interactions***. The relevant processes are

$$e^- + e^+ \longleftrightarrow \nu_e + \bar{\nu}_e \quad , \quad e^- + \bar{\nu}_e \longleftrightarrow e^- + \bar{\nu}_e$$

- Interactions mediated by $W^\pm$ or Z bosons, but at temperatures $T \ll 100\mathrm{GeV}$ they can be approximated by a single tree level diagram



Fermi constant
$G_F = 1.16 \times 10^{-5}\mathrm{GeV}^{-2}$

- The cross section $\sigma \propto G_F^2$. From dimensional analysis $[\sigma] = [\text{area}] = [\text{mass}^{-2}] = -2$ and $[G_F^2] = -4$, we need two more mass dimensions. Since the only scale is the temperature, we estimate $\sigma = G_F^2 T^2$. For the interaction rate $[\Gamma] = +1$, we have

$$\Gamma = G_F^2 T^5$$

Interaction rate falls down as a^{-5}.
In a cold universe (e.g. now), it is very difficult to observe cosmological neutrinos.

Neutrino decoupling

- Estimated interaction rate for neutrinos $\Gamma = G_F^2 T^5$
- Expansion rate, up to $\mathcal{O}(1)$ coefficients

$$H^2 = \frac{8\pi G_N}{3}\rho = \frac{8\pi G_N}{3}\frac{g_{eff}\pi^2 T^4}{30} \simeq G_N T^4$$

- Neutrinos ***decouple*** when $\Gamma \sim H$, or

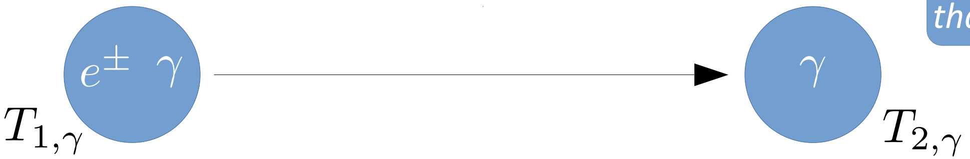
$$T \simeq \left(\frac{\sqrt{G_N}}{G_F^2}\right)^{1/3} = 1\,\mathrm{MeV} \Rightarrow \sim 1\,\mathrm{second}$$

- From now on, neutrinos travel freely since weak interactions no longer efficient. Their number density simply gets volume dilution factor a^{-3} from expansion, and their temperature decreases as a^{-1}.
- At MeV scale, nucleons already non-relativistic with mass $\sim 1\mathrm{GeV}$. They are still in the thermal bath, but their numbers have significantly decreased. We will come back to those later.
- We will next look at temperatures $T \sim m_e = 0.5\mathrm{MeV}$, when electrons and positrons become non-relativistic and start to annihilate.

Electron-positron annihilation

- At $T \gg m_e$, electron-positron pairs annihilate via EM interaction $e^- + e^+ \longleftrightarrow \gamma + \gamma$
- When $T < m_e$, photons no longer have sufficient energy to reverse process. All electron-positron pairs annihilate*. ***Thermal bath heats up***

*Lepton asymmetry: *More electrons than positrons. Leptogenesis?*



$$g_{eff,1} = 2 + \frac{7}{8}(2 \times 2) = \frac{11}{2}$$

$$g_{eff,2} = 2$$

Entropy conservation: $\boxed{\begin{array}{c} d(sa^3) = 0 \\ s \propto g_{eff} T^3 \end{array}}$

$$g_{eff,1}(a_1 T_{1,\gamma})^3 = g_{eff,2}(a_2 T_{2,\gamma})^3 \Rightarrow \boxed{a_2 T_{2,\gamma} = \left(\frac{11}{4}\right)^{1/3} a_1 T_{1,\gamma}}$$

- Meanwhile, since their freeze-out, the temperature of neutrinos decreased only due to expansion $a_1 T_{1,\gamma} = a_2 T_{2\,\nu}$.

- We can thus relate the two final temperatures: $\boxed{T_\gamma = \left(\frac{11}{4}\right)^{1/3} T_\nu \simeq 1.4\, T_\nu}$

Neutrinos today

- Although neutrinos decoupled, they continue to be relativistic. If this is still true today, we can estimate their present-day temperature.

$$T_{\nu,0} = \left(\frac{4}{11}\right)^{1/3} T_{\gamma,0} = 1.9\,\mathrm{K}$$

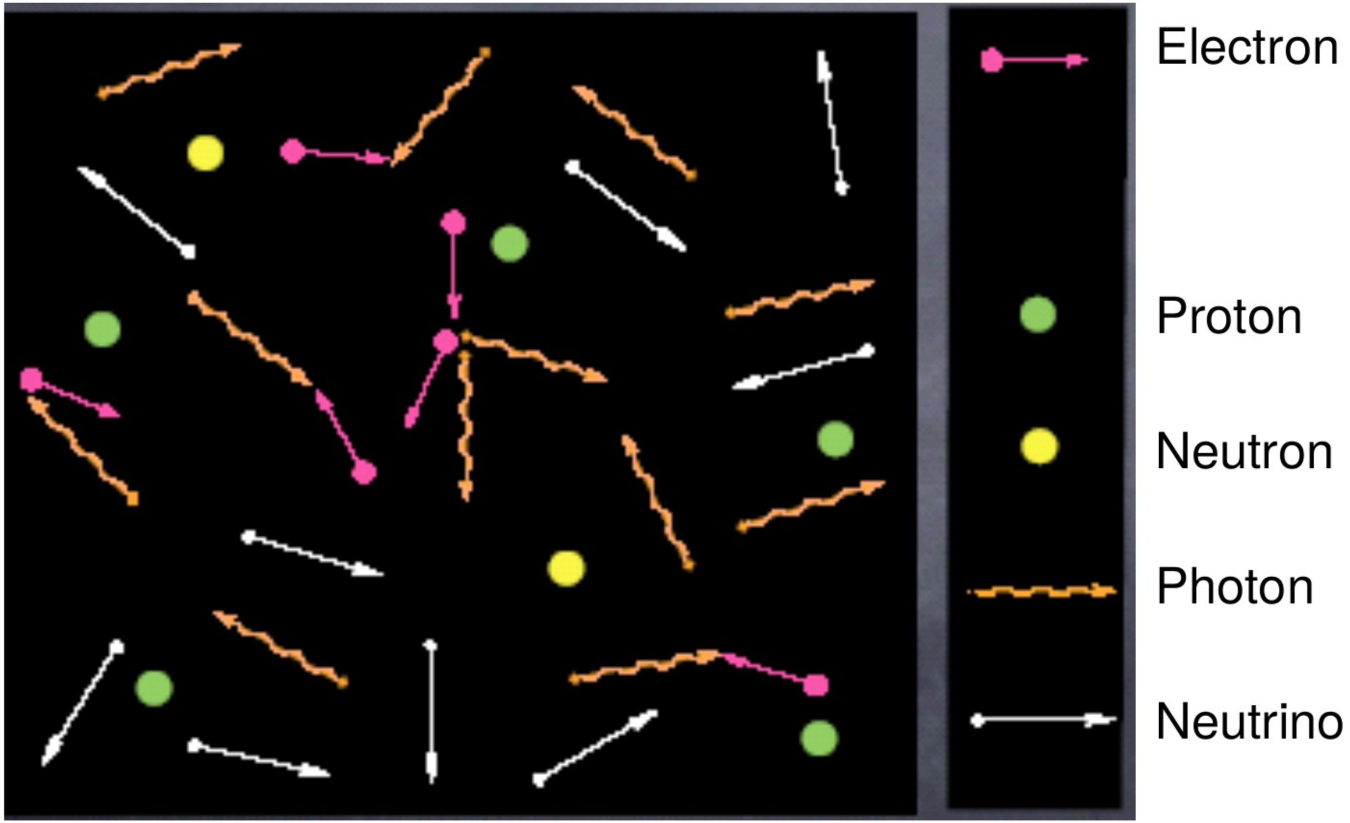
- Similarly, the total radiation density after electron annihilation should be modified to reflect the change in temperature

$$\rho_r = \frac{\pi^2}{30}\left[2 + \frac{21}{4}\left(\frac{T_\nu}{T_\gamma}\right)^4\right] T_\gamma^4 \simeq 1.68\rho_\gamma$$

- However, ***neutrinos have mass*** < 2 eV. We do not know the mass values, but we know from mass differences that at least one of the neutrino species are non-relativistic today

$$|\Delta m_{32}^2|^{1/2} > |\Delta m_{21}^2|^{1/2} > T_{\nu,0}$$

Big-Bang Nucleosynthesis (BBN)



Neutrinos have decoupled but are still relativistic. Most protons and neutrons annihilated, but due to baryon asymmetry a small but important amount remain. Similarly few electrons around due to lepton asymmetry. No positrons, anti-protons, anti-neutrons.

Baryon asymmetry

- Nucleons become non-relativistic at $T \sim \mathrm{GeV}$. However, they are still in thermal equilibrium until weak interactions decouple

In fact, protons stay in equilibrium longer since they are charged under EM.

$$p + \bar{p} \longrightarrow \gamma + \gamma \qquad n + \bar{n} \longrightarrow \gamma + \gamma$$

- If this is the whole story, there should be no nucleons left. Until the freeze-out of weak interactions, the Boltzmann suppression pulls the # density down

$$\mathrm{e}^{-\frac{m_N}{T_{FO}}} \simeq \mathrm{e}^{-\frac{\mathrm{GeV}}{\mathrm{MeV}}} \simeq \mathrm{e}^{-1000} \simeq 10^{-434}$$

m_N Nucleon mass
T_{F0} Freeze-out T

- Fortunately, there is an ***asymmetry*** between baryons and anti-baryons

$$\eta \equiv \frac{n_B - n_{\bar{B}}}{n_\gamma} \xrightarrow{T \ll \mathrm{GeV}} \eta = \frac{n_B}{n_\gamma}$$

Baryogenesis: "How did this asymmetry appear?"

- Number densities decrease with volume, so the ratio is roughly the same today. Using $\rho_{B,0} = m_N\, n_{B,0}$ and the expressions for thermal radiation $\rho_{\gamma,0} = \frac{\pi^4}{30\,\zeta(3)} n_{\gamma,0} T_0$

$$\eta = 6 \times 10^{-13} \left(\frac{T_0}{2.7\,\mathrm{K}} \right) \frac{\Omega_{B,0}}{\Omega_{\gamma,0}} \simeq 6 \times 10^{-10}$$

- At $T \gg \mathrm{GeV}$ for every billion photons there were a billion nucleons and anti-nucleons. At $T \sim \mathrm{GeV}$, a billion pair annihilated and only one nucleon remained!

Neutron decoupling

- After the nucleons become non-relativistic, and nucleon-antinucleon annihilations saturate, *but* before weak interactions decouple, they continue to interact via

$$n + \nu_e \longleftrightarrow e + p$$

- Nucleons have different masses. Ratio of their # densities not conserved!

$$\frac{n_n}{n_p} = \mathrm{e}^{-(m_n - m_p)/T} = \mathrm{e}^{\frac{-1.3\,\mathrm{MeV}}{T}}$$

$$\mu_n = \mu_p + \mu_e - \mu_\nu \simeq \mu_p$$

- *Fewer neutrons than protons*! However, neutrons decouple at $T \simeq 0.7\,\mathrm{MeV}$ and suppression freezes. Evaluating the ratio at this temperature $n_n/n_p \simeq 1/6$.
- Although $n \leftrightarrow p$ conversion has decoupled, ***neutrons are unstable*** and undergo free decay! Luckily, this is stopped when neutrons form deuterium ***bound states*** and become stable (otherwise, there would only be hydrogen in the Universe!). The additional decay pulls down the ratio to

$$\frac{n_n}{n_p} \simeq \frac{1}{7}$$

- With some neutrons around, elements heavier than H can form.

Nucleosynthesis

- At $T \gg \mathrm{MeV}$, nucleons form bound states, but energetic photons photo-dissociate them. Around $T \sim \mathrm{MeV}$, the nuclear binding energy becomes high enough to allow bound states to stay around longer.

B_Q	*Binding energy for Q*
$m_p\,,\, m_n$	*Free nucleon masses*
m_Q	*Mass of nucleus Q*
Z	*Atomic number*
A	*Mass number*

- ***Binding energy*** to form nucleus ${}^A_Z Q$ is

$$B_Q = Z\, m_p + (A - Z)\, m_n - m_Q > 0$$

- For $B_Q > T$ it becomes harder for the photons to break the nucleus. Typically, $B_Q \sim \mathcal{O}(\mathrm{MeV})$. Around $T \sim \mathrm{MeV}$ nuclei start to form.

$\mathrm{D}(^2\mathrm{H})$	$p\, n$
$^3\mathrm{H}$	$p\, n\, n$
$^3\mathrm{He}$	$p\, p\, n$
$^4\mathrm{He}$	$p\, p\, n\, n$
$^7\mathrm{Li}$	$p\, p\, p\, n\, n\, n\, n$
$^7\mathrm{Be}$	$p\, p\, p\, p\, n\, n\, n$

		Binding Energy
Stable but volatile →	^{2}H	2.2 MeV
Lifetime ~12 years →	^{3}H	8.5 MeV
Stable →	^{3}He	7.7 MeV
Stable →	^{4}He	28.3 MeV
Stable →	^{7}Li	39.2 MeV
Lifetime ~53 days →	^{7}Be	37.6 MeV
Stable but too far down →	^{12}C	92.2 MeV

Primordial Helium abundance

$$p + n \longrightarrow \mathrm{D} + \gamma \qquad {}^4\mathrm{He} + {}^3\mathrm{H} \longrightarrow {}^7\mathrm{Li} + \gamma \qquad n + {}^3\mathrm{He} \longrightarrow {}^4\mathrm{He} + \gamma$$

$$\mathrm{D} + \mathrm{D} \longrightarrow {}^3\mathrm{He} + n \longrightarrow {}^3\mathrm{H} + p \qquad {}^4\mathrm{He} + {}^3\mathrm{He} \longrightarrow {}^7\mathrm{Be} + \gamma \qquad p + {}^7\mathrm{Li} \longrightarrow {}^4\mathrm{He} + {}^4\mathrm{He}$$

$${}^3\mathrm{H} + \mathrm{D} \longrightarrow {}^4\mathrm{He} + n \qquad {}^7\mathrm{Be} + e^- \longrightarrow {}^7\mathrm{Li} + \nu_e \qquad n + {}^7\mathrm{Be} \longrightarrow {}^4\mathrm{He} + {}^4\mathrm{He}$$

- ^{4}He is special: quick to form (few steps), relatively high binding energy.
- ^{4}He processes with p, n or another ^{4}He are inefficient, since no stable nuclei with mass number 5 or 8.
- Density of ^{4}He is not big enough for ^{4}He+^{4}He+^{4}He processes (e.g. to produce ^{12}C)
- Safe to assume ***all neutrons end up in ^{4}He***. Defining primordial Helium abundance:

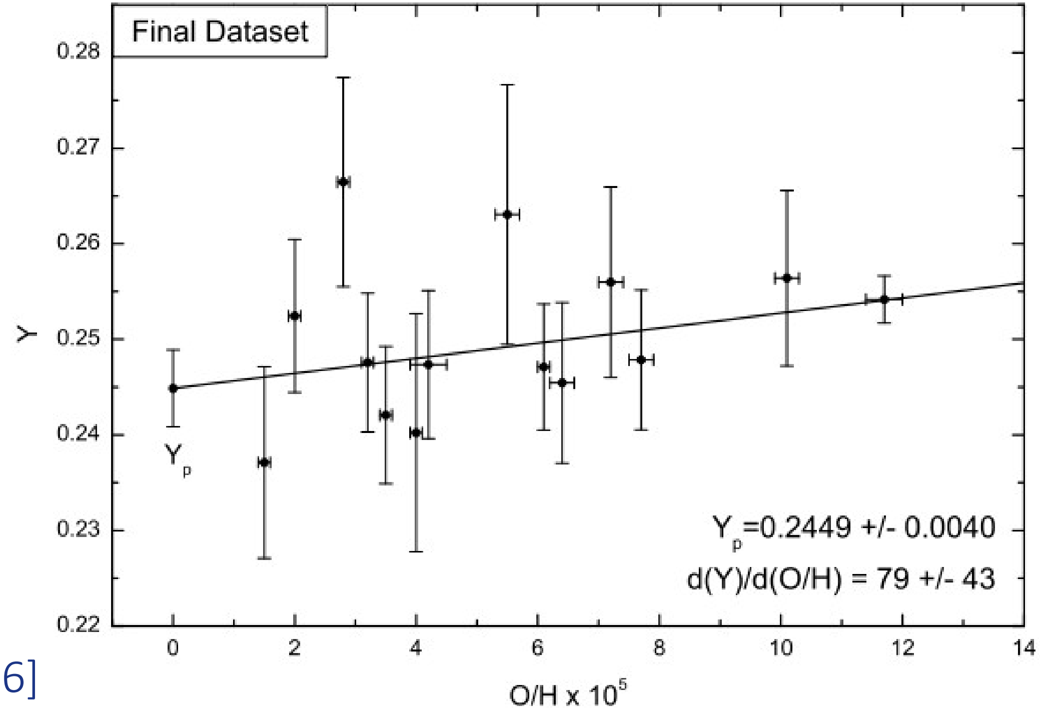
$$Y_p \equiv \frac{n_{N,He}}{n_N} = \frac{\text{\# of nucleons in } {}^4\text{He}}{\text{Total \# of nucleons}} = \frac{4 \times \frac{n_n}{2}}{n_n + n_p} = \frac{2\,\frac{n_n}{n_p}}{1 + \frac{n_n}{n_p}} = 0.25$$

$$\frac{n_n}{n_p} \simeq \frac{1}{7}$$

- The primordial abundance of other nuclei much smaller $(10^{-5} \div 10^{-10})$. BBN produces H, He and Li. Anything heavier is produced much later in stars.

Primordial ^{4}He abundance in the sky

- ^{4}He is also produced in the main sequence phase of stellar evolution.
- Luckily, stars also produce other things: “metals,” … like oxygen. Abundance vs metallicity. Regression to zero metallicity gives the primordial value.



- Current bound: $Y_p = 0.2449 \pm 0.0040$

Aver, Olive, Skillman 2015 [arXiv:1503.08146]

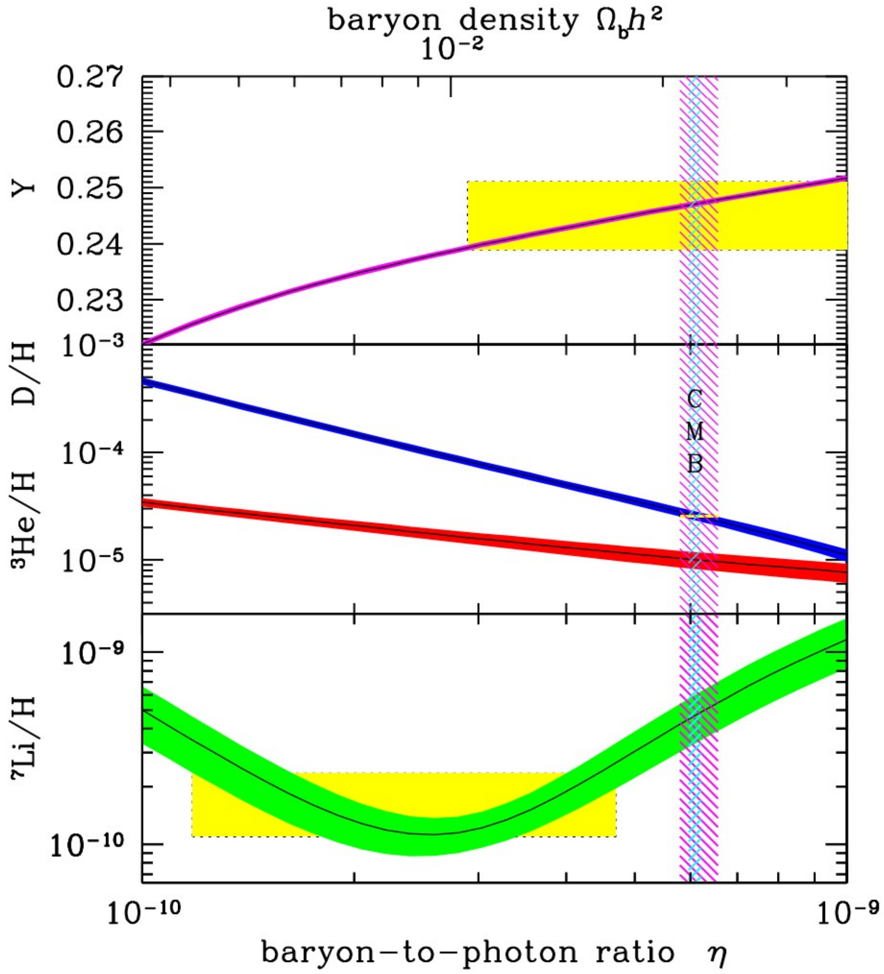
BBN Summary

- BBN starts at $t \sim 1\,\mathrm{s}\,(T \sim \mathrm{MeV})$ and ends at $t \sim 3\,\mathrm{min}\,(T \sim 100\,\mathrm{keV})$.

- Mostly good agreement between theoretical predictions and observations.

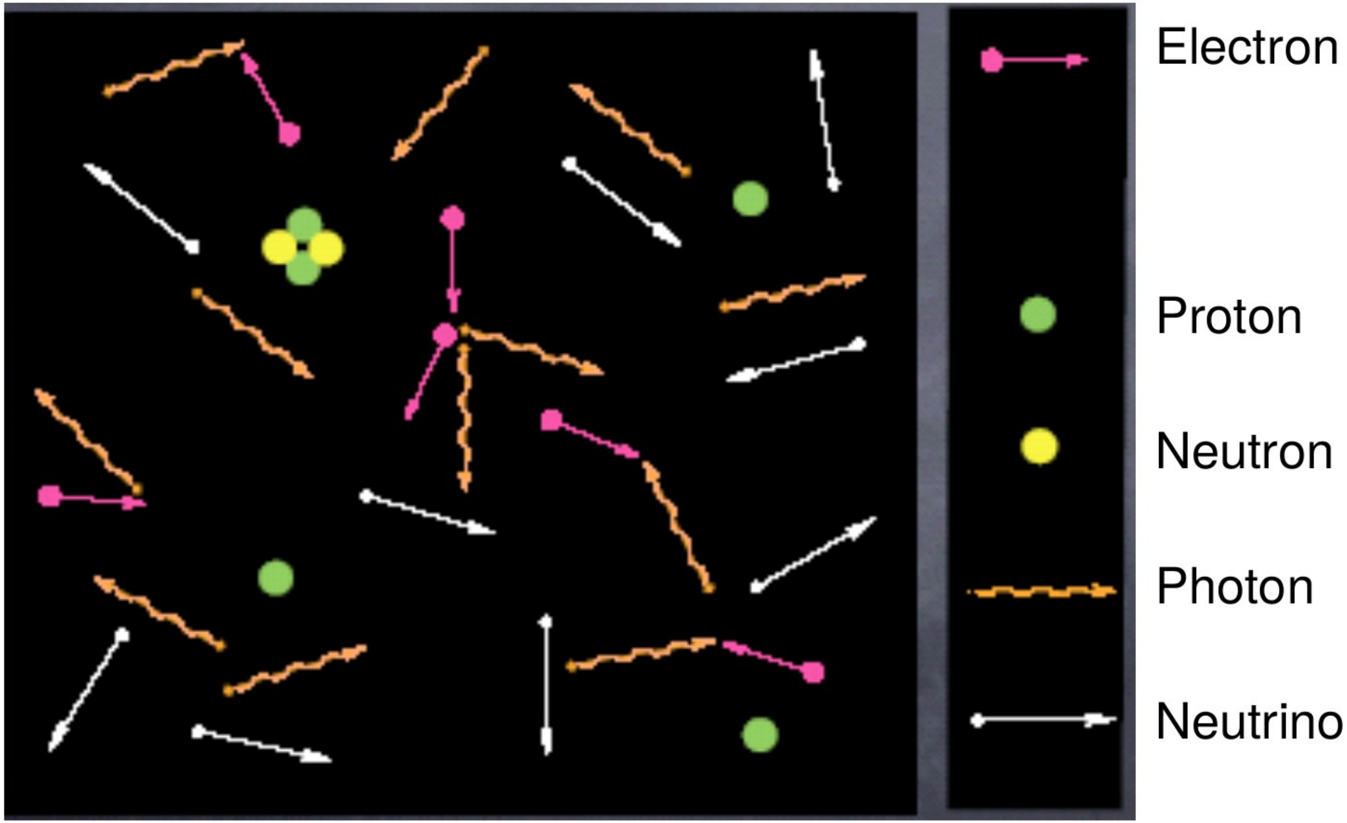
- D and ^{4}He constrains baryon/photon ratio. CMB provides an independent measure.

- Tension: ***Lithium problem***. Direct measurements of ^{7}Li inconsistent with baryon/photon ratio measured by CMB. Nuclear? Astrophysical? New physics?



Fields, Molaro, Sarkar (2017), from PDG review on BBN

Recombination and photon decoupling



At this stage, we have decoupled free neutrinos, interacting photon and electrons, and light nuclei.

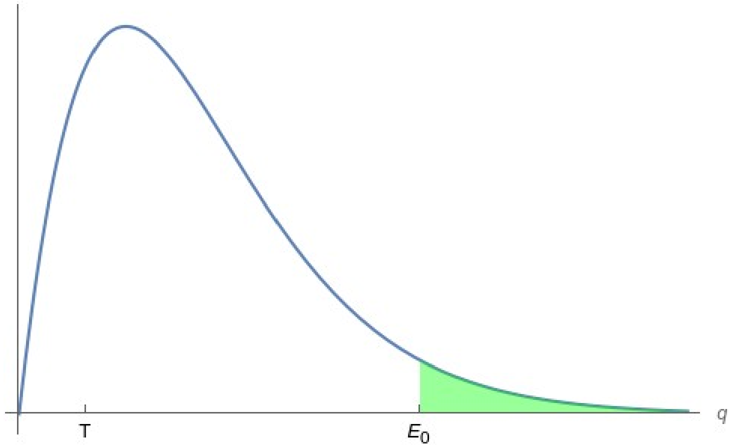
Recombination

- Binding energy of e^- in H $\Rightarrow$ $E_0 = 13.6\,\mathrm{eV}$
- When $T \gg E_0$ the Universe is ionised. EM interactions between electrons, protons and photons continue until $T \ll E_0$. e^- and p start to form H atoms (*recombination*).

$$p + e^- \longrightarrow H + \gamma$$

- As free charges are diluted, EM interactions start to decouple. This is the *last scattering surface (LSS)*, photons travel freely until today.
- However, this happens much later than $T \sim E_0$. There are 10^9 times more photons than protons and electrons. *The tail end of the distribution re-ionizes the atoms*.
- We can estimate an upper limit on the temperature of recombination. If the fraction of photons which have energy $> E_0$ falls below the baryon/photon ratio, recombination becomes efficient. The temperature where these are comparable:

$$\frac{\int_{E_0}^{\infty} p^2 dp\,[\mathrm{e}^{p/T} - 1]^{-1}}{\int_0^{\infty} p^2 dp\,[\mathrm{e}^{p/T} - 1]^{-1}} < 6 \times 10^{-10} \Rightarrow T \lesssim 0.5\,\mathrm{eV} \sim 5800\mathrm{K}$$



Ionisation fraction

- We can also keep track of the # of free electrons. Define the ***ionisation fraction***

$$x \equiv \frac{n_e}{n_p + n_H} = \frac{n_e}{\hat{n}_B}$$

n_H : Neutral Hydrogen density
$\hat{n}_B$: Baryon density (without Helium!)

- To avoid using individual chemical potentials, we calculate the ***Saha equation***

$$\frac{n_p n_e}{n_H} = \frac{\cancel{g_p g_e}}{\cancel{g_H}} \left(\frac{m_e \cancel{m_p} T}{2\pi \cancel{m_H}} \right)^{3/2} \mathrm{e}^{-[\underbrace{(m_p+m_e-m_H)}_{=E_0} - \underbrace{(\mu_p+\mu_e-\mu_H)}_{=0}]/T} = \left(\frac{m_e T}{2\pi} \right)^{3/2} \mathrm{e}^{-E_0/T}$$

$m_H \simeq m_p$
$g_p = g_e = g_H/2 = 2$

- Using charge conservation $n_e = n_p$, we can relate the ionisation function to Saha eq. as:

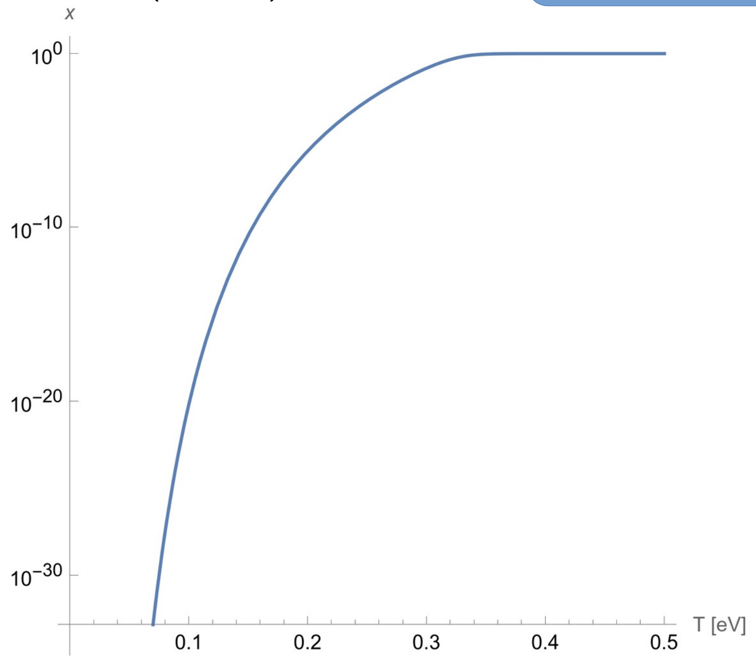
$$\frac{x^2}{1-x} = \frac{1}{\hat{n}_B} \left(\frac{m_e T}{2\pi} \right)^{3/2} \mathrm{e}^{-E_0/T}$$

- Since $\hat{n}_B = 0.75\,\eta\, n_\gamma$, we can use $\eta = 6 \times 10^{-10}$ and $n_\gamma = 0.24 \times T^3$

$$\frac{x^2}{1-x} = 2 \times 10^{17} \left(\frac{\mathrm{eV}}{T} \right)^{3/2} \mathrm{e}^{-13.6\mathrm{eV}/T}$$

- Recombination starts when $n_p \sim n_H$

→ two orders smaller than E_0 → $T \simeq 0.32\ \mathrm{eV}$



Last scattering surface

- Our estimates may be a bit off the mark, since we assumed that all H are formed in ground state. Taking into account that excited H will be re-ionised by photons, the estimates can be improved. *Recombination ends at temperature:*

$$T_{LSS} = 0.25\,\mathrm{eV} \simeq 3000\,\mathrm{K}$$

- *Thomson scattering* $e^- + \gamma \to e^- + \gamma$ decouples at the end of recombination. From then on, photons propagate freely.

- Redshift of LSS:

$$\frac{T_{LSS}}{T_{\gamma,0}} = \frac{a_0}{a_{LSS}} = z_{LSS} + 1 \Rightarrow \boxed{z_{LSS} \simeq 1100} \Rightarrow \boxed{t_{LSS} \sim 3 \times 10^5 \text{years}}$$

- Photons always relativistic, so the relic radiation, or the *cosmic microwave background (CMB)* obeys the Planck (black-body) distribution.

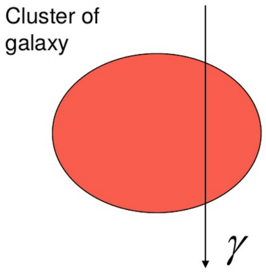
The perfect blackbody spectrum

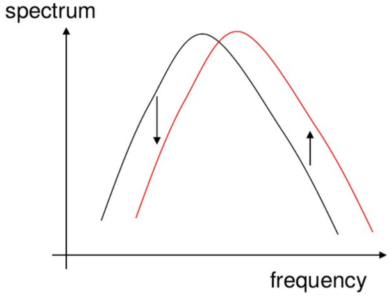
- Existence of CMB was predicted by Gamow (1946), Alpher and Herman (1948), who estimated its temperature off by an $\mathcal{O}(1)$ factor. Observationally identified in 1964 by Penzias and Wilson.

- The blackbody spectrum was measured by FIRAS instrument on the COBE satellite (predecessor of WMAP and Planck).

$$T_{\gamma,0} = 2.7\,K$$

- After photon decoupling, there is no process to bring photons back to equilibrium. ***Changes in the spectrum stay there for good***.
  e.g. Sunyaev-Zeldovich effect. High energy e^- in galaxy clusters distort the CMB.





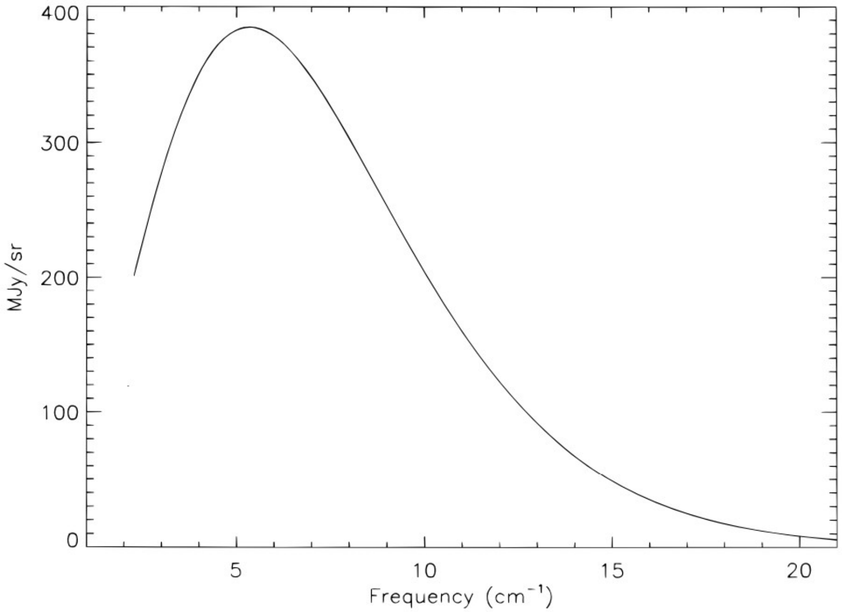
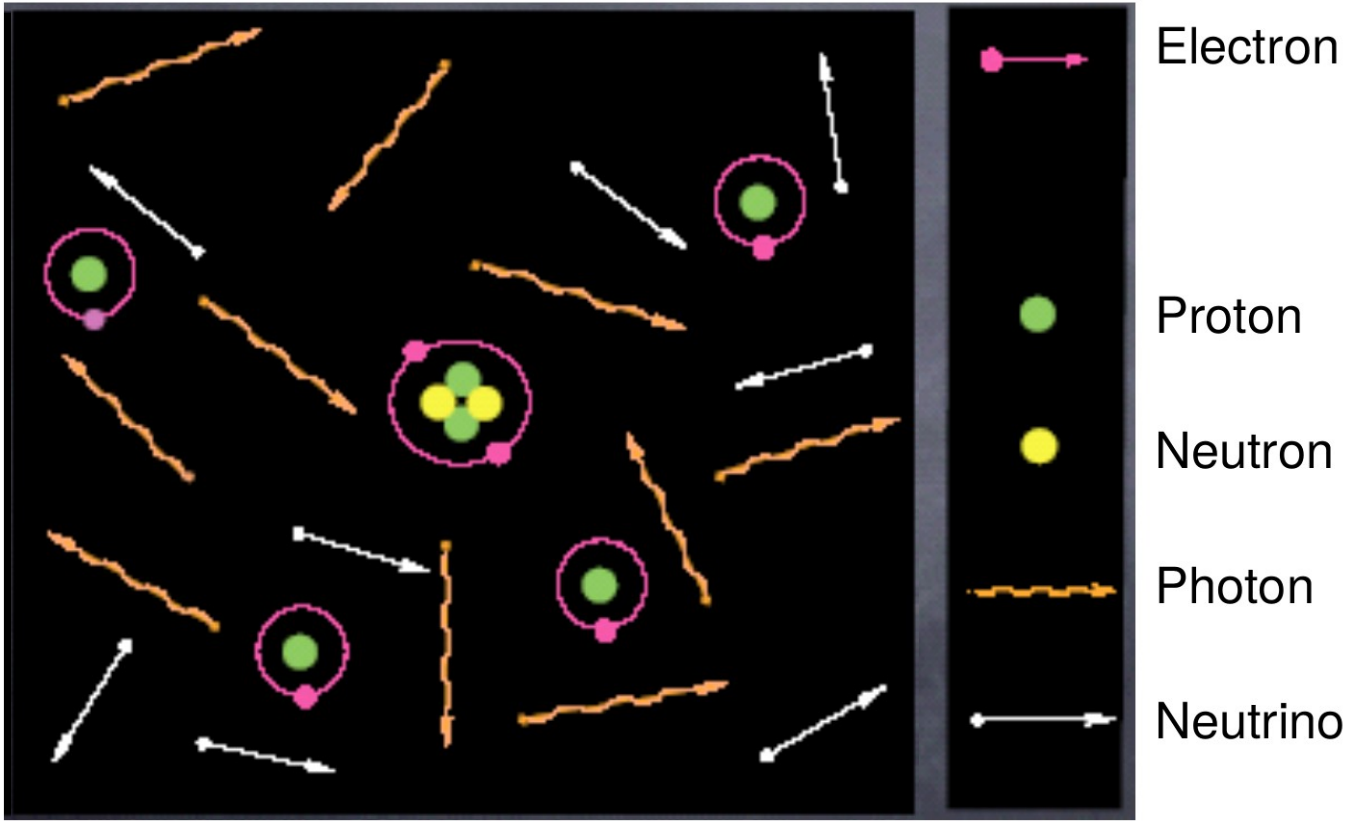


FIG. 4.—Uniform spectrum and fit to Planck blackbody (T). Uncertainties are a small fraction of the line thickness.

Fixsen et al 1996 [arXiv:astro-ph/9605054]

End of radiation era



This is how the Universe looks like after LSS. Photons and neutrinos are decoupled, and the Universe is now full of light elements. There is probably some *darkness* as well.
It turns out that the radiation era had already ended ***before*** *last scattering*.

Matter-Radiation Equality

- Radiation decreases as a^{-4} while non-relativistic matter a^{-3}. When does matter take over?
- Radiation density (assuming massless neutrinos)

$$\rho_r = \frac{\pi^2}{30}\left[2 + \frac{21}{4}\left(\frac{4}{11}\right)^{4/3}\right] T^4 \simeq 1.1\, T^4$$

- If matter consists only of baryons

$$\rho_m = m_N\, n_B = m_N\, \eta\, n_\gamma \simeq (0.13\,\mathrm{eV})\, T^3$$

$$\cancel{T_{eq} = 0.13\,\mathrm{eV} \simeq 1500\,\mathrm{K}}$$

$$\cancel{z_{eq} \simeq 560}$$

~~Too late for structure formation~~

- Instead, let us use the present-day matter density as a reference

$$\rho_m = \Omega_{m,0}\rho_c \left(\frac{T}{T_0}\right)^3$$

$$\rho_r = 1.68 \times \Omega_{\gamma,0}\rho_c \left(\frac{T}{T_0}\right)^4$$

$$1 + z_{eq} = \frac{\Omega_{m,0}}{1.68 \times \Omega_{\gamma,0}} = \frac{0.3}{1.68 \times 5.38 \times 10^{-5}}$$

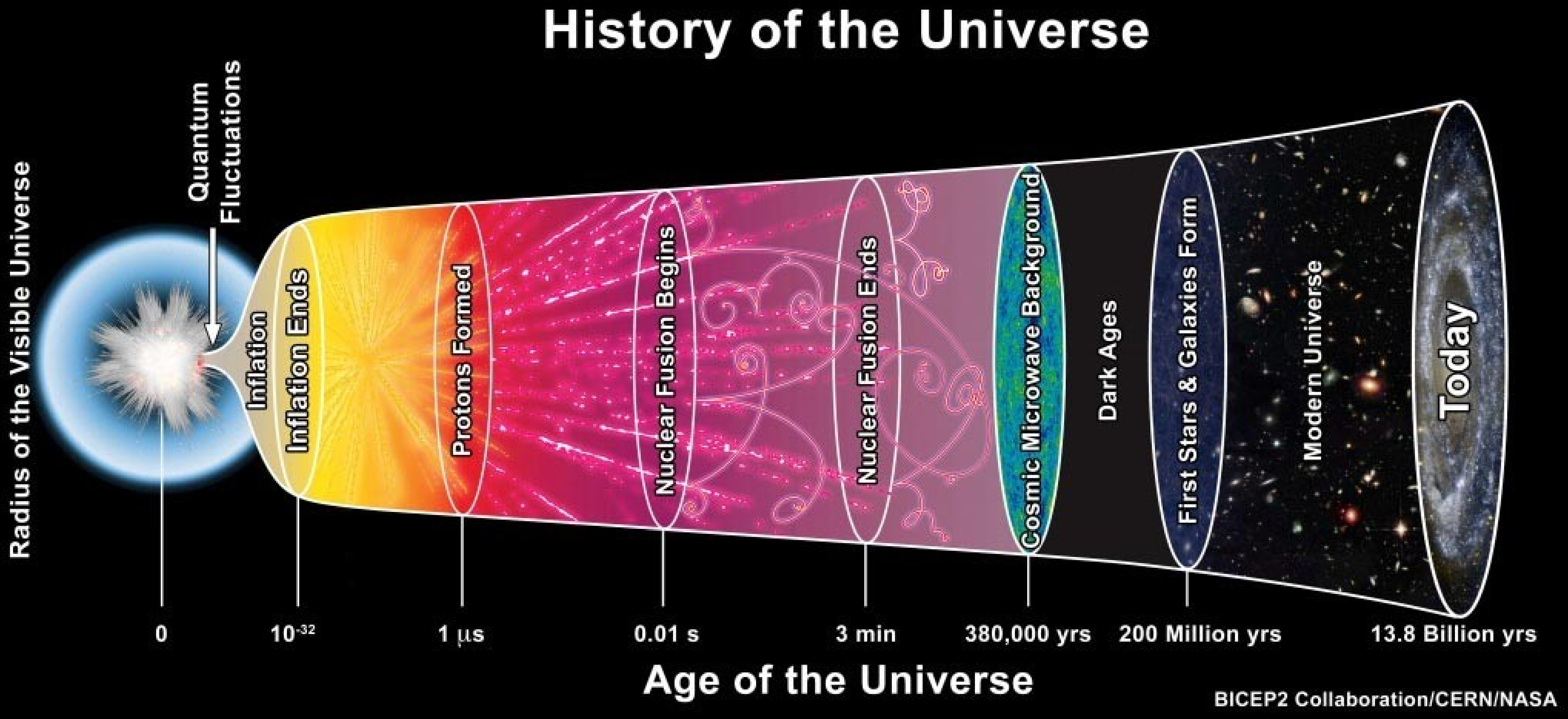
$$z_{eq} \sim 3300\,, \quad T_{eq} \simeq 0.8\,\mathrm{eV} \simeq 9,000\,\mathrm{K}\,, \quad t_{eq} \sim 50,000\,\mathrm{years}$$

- At the time of recombination, the universe was dominated by matter… dark matter.

Dark matter

- We know it exists, but we do not know what it is.
- ***Weak***: It interacts weakly (possibly only via gravity), so that it freezes-out long before baryons to have a sizeable abundance today.
- It should be ***cold***, i.e. massive, in order to avoid dissipative effects of ***free-streaming*** (damping of small scale inhomogeneities). Massive particles move slowly, free-streaming negligible.
- ***Structure formation is dark***: Matter perturbations grow during MD, but baryons frozen until LSS. Only after LSS, photon pressure becomes negligible and baryon perturbations can grow. CDM does not interact with photons, so it starts growing immediately at equality. After recombination, baryons fall into the potential wells of CDM.

Bertone, Tait (2018) [arXiv:1810.01668]


History of the Universe
Radius of the Visible Universe
Quantum Fluctuations
Inflation
Inflation Ends
Protons Formed
Nuclear Fusion Begins
Nuclear Fusion Ends
Cosmic Microwave Background
Dark Ages
First Stars & Galaxies Form
Modern Universe
Today
0
10-32
1 μs
0.01 s
3 min
380,000 yrs
200 Million yrs
13.8 Billion yrs
Age of the Universe
BICEP2 Collaboration/CERN/NASA