

Theoretical Cosmology

Part IV: Dark Energy

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ICG PhD Lectures, November 2021

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|---------------------------------------|--------------------|
| 1. Introduction to Big-Bang cosmology | 15 November |
| 2. Hot thermal Universe | 19 November |
| 3. Inflation | 22 November |
| 4. <i>Dark energy</i> | <i>26 November</i> |

Plan for today

1. *Observational evidence for acceleration*
2. *Cosmological constant problem*
3. *Alternative explanations*
4. *Dynamical DE, Modified gravity – challenges*
5. *Growth history and observational tests*

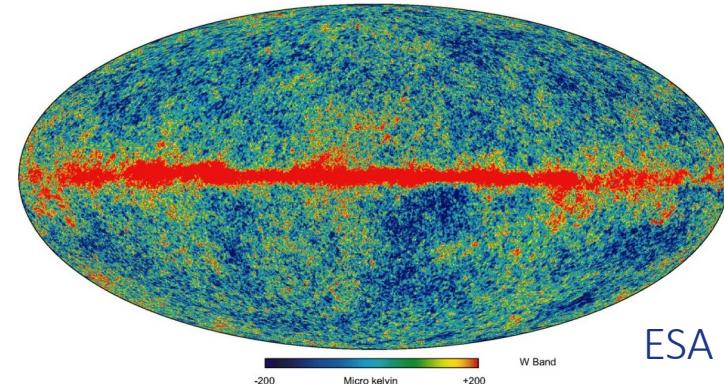
References

- Kazuya Koyama's 2016 lecture notes (especially the observation section)
- *Dark Energy*, David Weinberg and Martin White, in *Review of Particle Physics*.
- *Everything You Always Wanted To Know About The Cosmological Constant Problem (But Were Afraid To Ask)*, Jérôme Martin, arXiv:1205.3365
- Several research papers, references included in slides.

Basic Assumptions

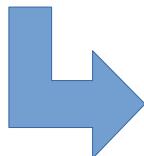
1. Cosmological Principle

- *Isotropy*: CMB photons with temperature fluctuations $\frac{\Delta T}{T} \sim 10^{-5}$



ESA Planck

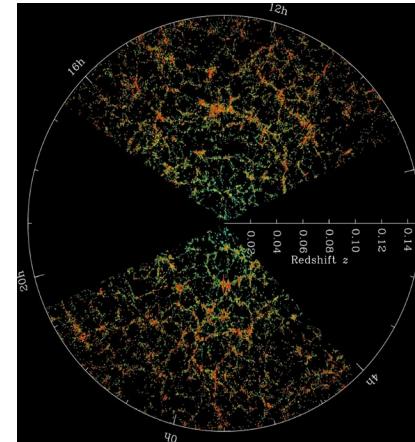
- *Homogeneity*: Galaxy distribution



FLRW metric:

$$ds^2 = -dt^2 + a(t)^2 ds_3^2$$

Constant curvature space
(open, closed, flat)



SDSSIII

Basic Assumptions

2. General Relativity (GR)

- Equations of motion

$$G_{\mu\nu} \equiv \underbrace{R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R}_{\text{geometry}} = \underbrace{8\pi G_N T_{\mu\nu}}_{\text{matter}}$$

- Matter

$$T_{\mu\nu} = (\rho + P)u_\mu u_\nu + P g_{\mu\nu}, \quad u_\mu = (1, 0, 0, 0)$$

- Contracted Bianchi identity

$$\nabla^\mu G_{\mu\nu} = 0 \quad \rightarrow \quad \nabla^\mu T_{\mu\nu} = 0$$

Basic Assumptions

3. Matter contents of the Universe

- All matter components (baryons+CDM, radiation, ...) independently satisfy their individual conservation equation. No interaction between different components

$$\dot{\rho}_i + 3H(1+w_i)\rho_i = 0, \quad w_i = \frac{P_i}{\rho_i} \quad \rightarrow \rho_i \propto a^{-3(1+w_i)}$$

- Matter evolution is determined by the equation of state

$$w_r = \frac{1}{3}, \quad w_m = 0, \quad w_{DE} = ?$$

- Dimensionless density parameters

$$\Omega_i = \frac{8\pi G_N \rho_i}{3H^2}, \quad \Omega_K = -\frac{K}{a^2 H^2} \quad \rightarrow \sum_i \Omega_i + \Omega_K = 1$$

What do we measure?

- Assumption (1) implies

$$ds_3^2 = d\chi^2 + f_K(\chi)^2(d\theta^2 + \sin\theta d\phi^2), \quad f_K(\chi) \equiv \frac{1}{\sqrt{-K}} \sinh(\sqrt{-K}\chi)$$

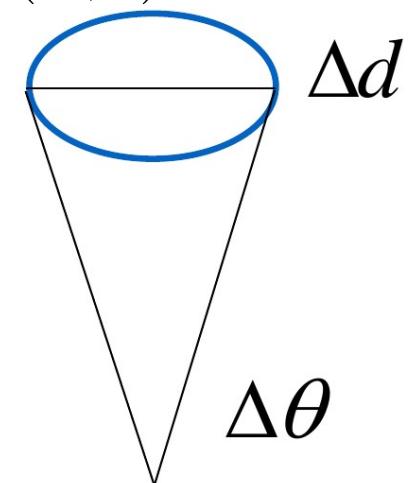
- We detect light

$$ds^2 = -dt^2 + a^2(t)ds_3^2 = 0$$

- Light emitted at time t and comoving distance χ , seen today at $(t_0, 0)$

$$\chi = \int_t^{t_0} \frac{dt'}{a(t')} = \frac{1}{a_0} \int_0^z \frac{dz}{H}$$

Redshift:
 $z \equiv \frac{a_0}{a} - 1$



- Luminosity distance and angular distance

$$d_L = f_K(\chi)(1+z), \quad d_A = \frac{d_L}{(1+z)^2}$$

Cosmological model

- Model based on assumptions (2) and (3)

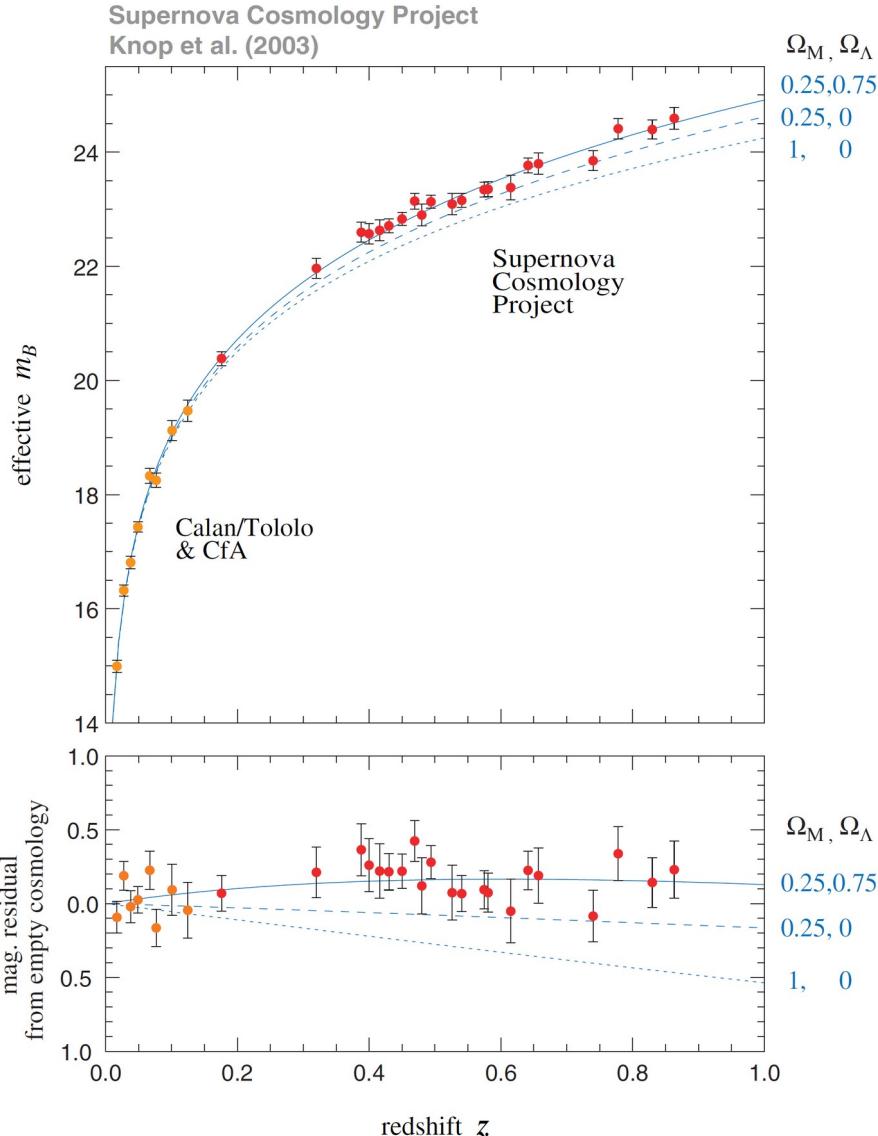
$$\frac{H(z)^2}{H_0^2} = \Omega_{m,0}(1+z)^3 + \Omega_{r,0}(1+z)^4 + \Omega_{K,0}(1+z)^2 + \Omega_{DE,0}(1+z)^{3(1+w_{DE})}$$

- Λ CDM model

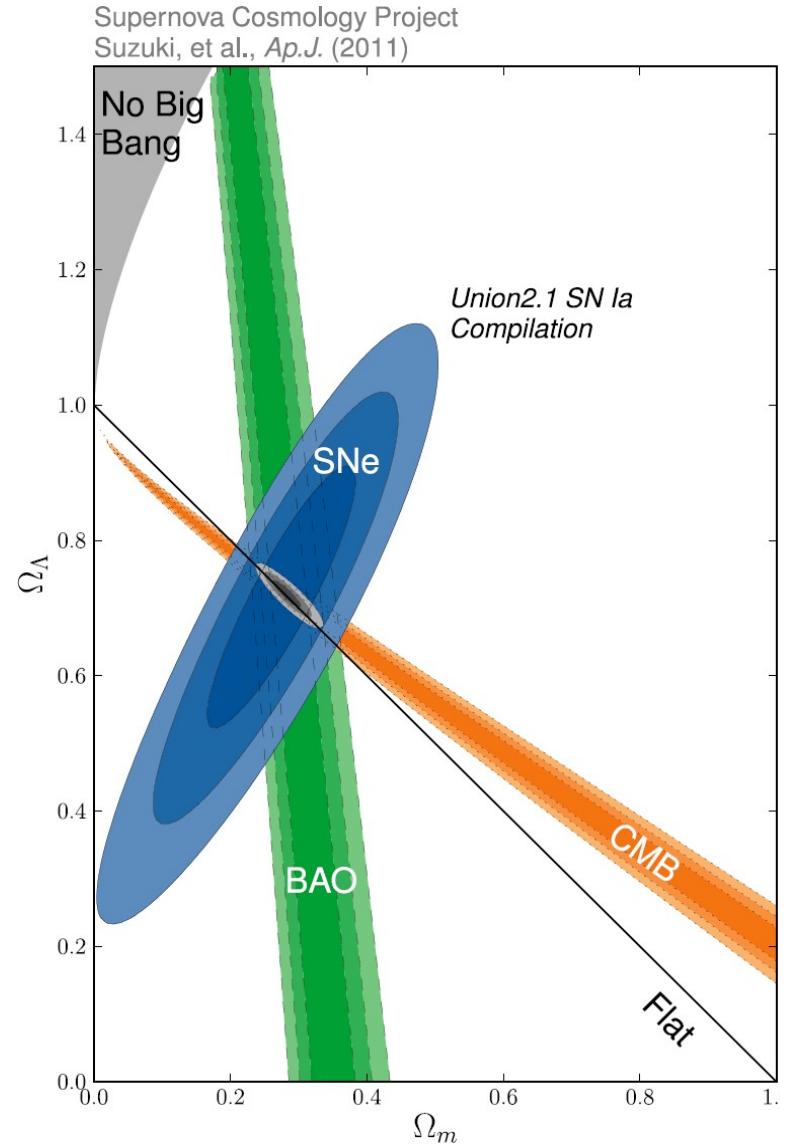
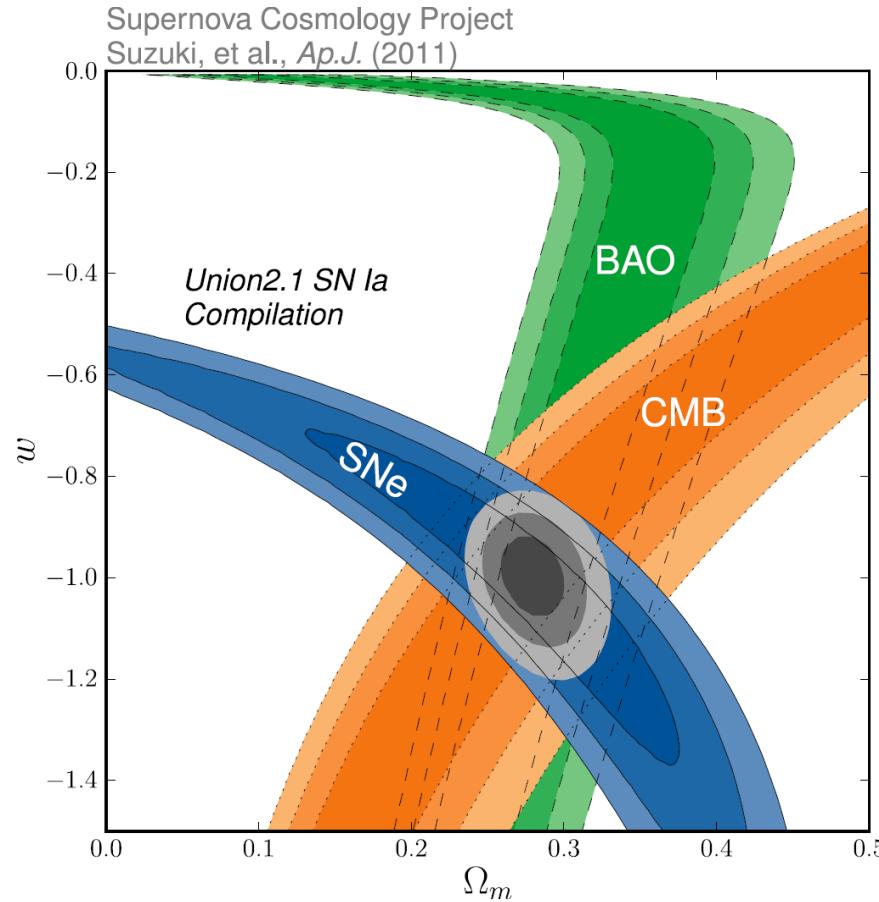
$$w_{DE} = -1, \quad \Omega_{DE,0} = \Omega_\Lambda, \quad (\Omega_{r,0} = 5 \times 10^{-5})$$

- Distance measurements

- Supernovae d_L
- CMB d_A
- BAO d_A



Concordance



Adding cosmological constant to theory

- Action

$$S = \frac{1}{16\pi G} \int d^4x \sqrt{-g} (R - 2\Lambda)$$

- Einstein's equations

$$G_{\mu\nu} = 8\pi G T_{\mu\nu} - \Lambda g_{\mu\nu}$$

$$\begin{aligned} H^2 &= \frac{8\pi G_N}{3} \rho - \frac{K}{a^2} + \frac{\Lambda}{3} \\ \frac{\ddot{a}}{a} &= -\frac{4\pi G_N}{3} (\rho + 3P) + \frac{\Lambda}{3} \end{aligned} \qquad \left(\rho_\Lambda = \frac{\Lambda}{8\pi G_N} = -P_\Lambda \right)$$

- Cosmological constant is unaffected by the expansion. When it dominates (which is now) the universe accelerates $\ddot{a} > 0$.

What is the problem then?

- ✓ Λ CDM works well to explain observations
- ✓ c.c. can be implemented into General Relativity with minimal effort
- How big (or small) is the observed c.c.?

$$\rho_\Lambda = \frac{\Lambda}{8\pi G_N}$$

- Let's get some numbers. We are in natural units, so using

$$G_N = \frac{1}{8\pi M_{Pl}^2} \sim 10^{-38} \text{GeV}^{-2} \quad \text{and} \quad H_0 \sim 10^{-42} \text{GeV}$$

we get

$$\rho_\Lambda = \frac{3H_0^2}{8\pi G_N} \sim 10^{-47} \text{GeV}^4 \sim (10^{-3} \text{eV})^4$$

- c.c. looks like vacuum energy from QFT. Let's compare.

Dimensionful quantity!
Is it small or large?

How big is vacuum energy?

- Quantum fields have zero-point energy $E_0 = g_i \frac{\omega}{2} = \frac{g_i}{2} \sqrt{p^2 + m_i^2}$
- Energy of the vacuum

$$\rho_{vac} = \frac{1}{2} \sum_i g_i \int_0^\infty \frac{d^3 p}{(2\pi)^3} \sqrt{p^2 + m_i^2}$$

- Cannot integrate to infinity, but set a UV cut-off at $p = p_{max} \gg m$

$$\rho_{vac} = \sum_i g_i \left[\frac{p_{max}^4}{16\pi^2} + \frac{m_i^2 p_{max}^2}{16\pi^2} + \frac{m_i^4}{64\pi^2} \log\left(\frac{m_i^2 e^{1/2}}{4p_{max}^2}\right) \right] + \mathcal{O}\left(\frac{m_i}{p_{max}}\right)$$

- Quartic divergence! *Naïve approach*: new physics at $p_{max} = M_{Pl}$ provides a natural cutoff?

$$\rho_{vac} \sim M_{Pl}^4 = 10^{121} \rho_\Lambda^{obs}$$

Disagreement at 121 orders!

- We can do better than this. The cutoff is *not physical*. Renormalisation can tame the divergence.

of degrees of freedom
 $g > 0$ (boson)
 $g < 0$ (fermion)

Vacuum energy is big, but *how* big?

- Imposing Lorentz invariance of the vacuum state, the 1-loop renormalised vacuum energy is

$$\rho_{vac}^{ren} = \sum_i g_i \frac{m_i^4}{64\pi^2} \log \left(\frac{m_i^2}{\mu^2} \right)$$

Renormalisation scale. Can estimate it to be the mean of the photon's and graviton's energy in supernovae observations $\mu \simeq 3 \times 10^{-25} \text{ GeV}$

Koksma, Prokopec 1105.6296

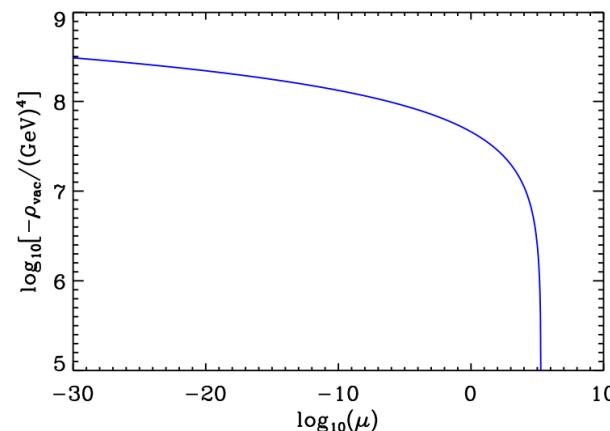
- Contributions:

- Top quark $m_t \sim 173 \text{ GeV}$, $g_t = -12 \rightarrow \Delta\rho_{vac} \sim -2 \times 10^9 \text{ GeV}^4 = -10^{56} \rho_\Lambda^{obs}$
- EW phase transition $\Delta\rho_{vac} \sim -10^8 \text{ GeV}^4 = -10^{55} \rho_\Lambda^{obs}$
- QCD phase transition $\Delta\rho_{vac} \sim 10^{-2} \text{ GeV}^4 = 10^{45} \rho_\Lambda^{obs}$

- Including all contributions

$$\rho_{total} = \rho_{bare} + \sum_i g_i \frac{m_i^4}{64\pi^2} \log \frac{m_i^2}{\mu^2} + \rho_{EW} + \rho_{QCD} + \dots$$

Martin 1205.3365



Is vacuum energy real?

- Example: Casimir effect.

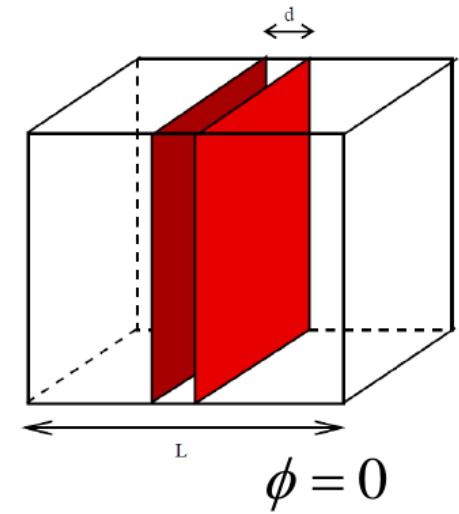
Boundary in \hat{x} direction: momentum discrete

$$\phi(\vec{x}) = \phi(\vec{x} + L \vec{n}), \quad \vec{p} = \left(\frac{n\pi}{d}, p_y, p_z \right), \quad n = 1, 2, 3 \dots$$

- Zero-point energy $\frac{\omega}{2} = \frac{1}{2} \sqrt{\left(\frac{n\pi}{d}\right)^2 + p_y^2 + p_z^2}$

- Total zero-point energy per unit area

$$\frac{E(d)}{A} = \sum_{n=1}^{\infty} \int \frac{dp_y dp_z}{(2\pi)^2} \left[\frac{1}{2} \sqrt{\left(\frac{n\pi}{d}\right)^2 + p_y^2 + p_z^2} \right] F_{reg}(a)$$



Regulator:

$$F_{reg}(a) = e^{-a\omega}$$

- Total energy $E_{tot}(d)$ diverges as $a \rightarrow 0$.
- Force between the two plates is finite

$$F = -\frac{1}{A} \frac{\partial E_{tot}(d)}{\partial d} = -\frac{\pi^2}{240 d^4}$$

Confirmed by measurements

Old cosmological constant problem

- In QFT (flat spacetime), we don't deal with zero-point energy: Casimir force is determined by the change in energy, not its value.
- In GR, all matter, including vacuum energy curves spacetime.

$$\rho_{vac} \sim m_e^4 = (0.5 \text{ MeV})^4 \quad \rightarrow \quad H \sim \frac{m_e^2}{M_{Pl}} \sim (10^6 \text{ km})^{-1}$$

Old c.c. problem: Why doesn't the vacuum energy gravitate? Where did it go?

- What's wrong with fine-tuning? We have renormalised vacuum energy. Fix bare c.c. such that:

$$\rho_{bare} = \frac{\Lambda_{bare}}{8\pi G_N} = \rho_{obs} - \rho_{vac} \quad \rightarrow \quad S = \frac{1}{16\pi G} \int d^4x \sqrt{-g} (R - 2\Lambda_{bare})$$

- Vacuum energy very sensitive to high energy physics! At each loop order, the tuning is spoiled and have to re-tune.
→ Unstable under radiative corrections see e.g. Padilla 1502.05296
- Something is missing. New symmetry? UV modification of gravity?

New c.c. problem / coincidence problem

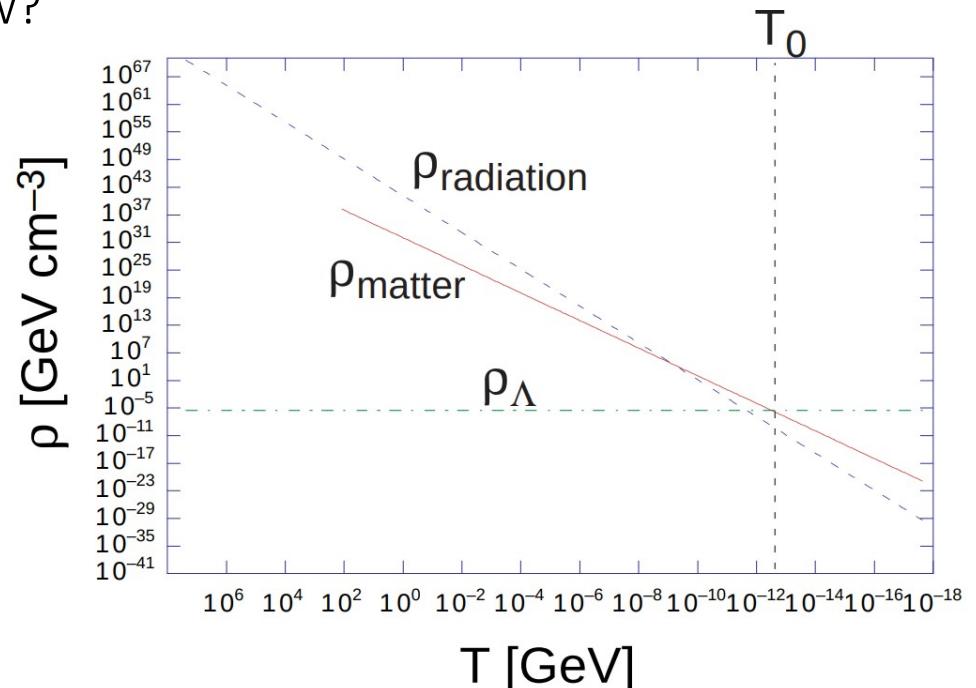
Let's assume somehow the UV physics resolves the old c.c. problem.

- **New c.c. problem:** why is c.c. not exactly zero, but very small? *Radiative corrections??*
- Anthropic principle? Cannot form galaxies with large Λ (no collapse)
- **Coincidence problem:** why $\Omega_\Lambda \sim \Omega_m$ now?

Example model that exploits

$$\rho_\Lambda \sim \mathcal{O} \left(\frac{M_{EW}^2}{M_p} \right)^4$$

Arkani-hamed et.al.
astro-ph/0005111



We actually don't know what Λ is!

- We know that vacuum energy exists. But it either doesn't gravitate the way it should in GR, or we are not calculating it correctly.
- Regardless of the c.c. problem, ***we do observe an acceleration***, whether it is caused by a (fine-tuned) vacuum energy or something else.
- Without a conclusive explanation, we can try to find other ways to approach this observation:
 - *Is the acceleration even real?*
 - *Alternative source, preferably protected against quantum corrections?*
- Since observations are quite conclusive, need to revisit our basic assumptions:
 1. Cosmological principle
 2. General Relativity
 3. Matter content of the universe

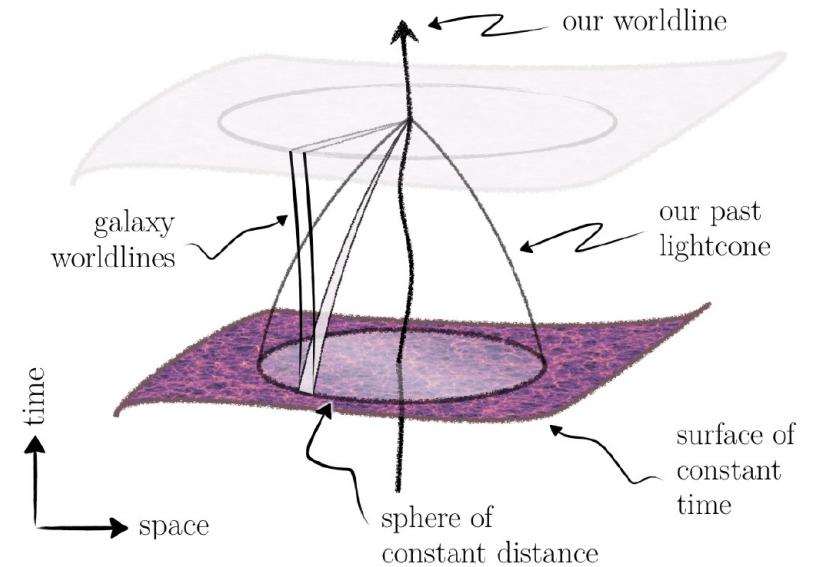
Revisiting assumption 1

- **Cosmological principle:** On large scales, Universe is homogeneous and isotropic.
- If all observers measure isotropic distance-redshift relation, then the space-time is FLRW.
- CMB and galaxy distributions are evidence of homogeneity/isotropy only around us.
- **Copernican principle:** We are not at a special location in the Universe
- Need Copernican principle to prove the cosmological principle, but hard to test.

See e.g.

Uzan 0912.5452

Ellis et al. "Relativistic Cosmology" (2012)



Clarkson arXiv:1204.5505

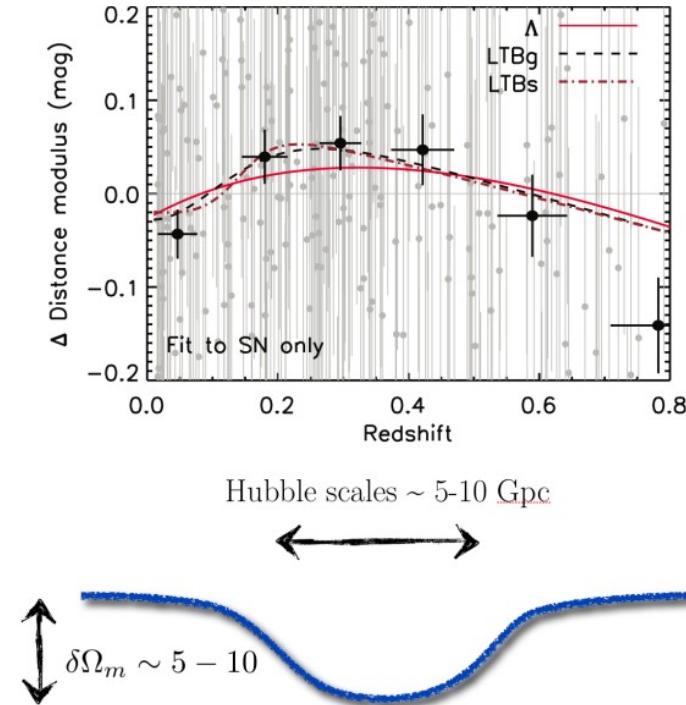
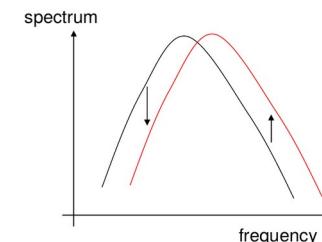
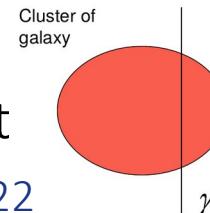
Inhomogeneous universe models

- *Void models*: If we are living inside a void with low densities, the expansion appears to be accelerating.
- Lemaître-Tolman-Bondi (LTB) model:

$$ds^2 = -dt^2 + \frac{a_{\parallel}^2(t, r)}{1 - K(r)r^2} dr^2 + a_{\perp}^2(t, r)r^2 d\Omega^2$$

- Can fit supernovae data exactly
- Combined with CMB, H_0 too low
- Overpredicts the amplitude of kinetic Sunyaev-Zeldovich effect

Bull, Clifton, Ferreira 1108.2222



Clarkson arXiv:1204.5505

- Not the only way to incorporate inhomogeneities

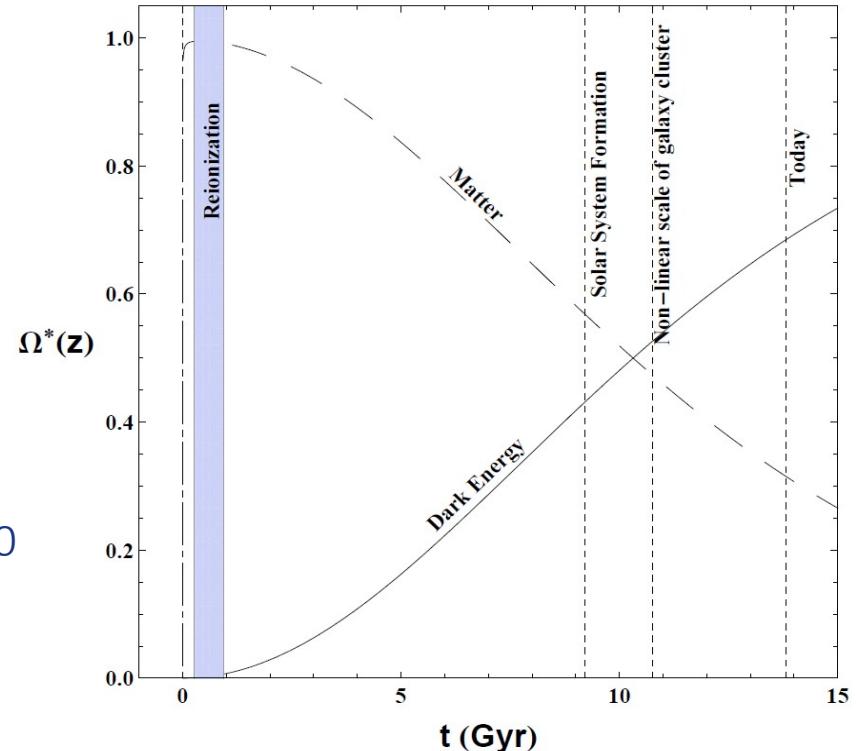
Back-reaction

- Late time Universe is inhomogeneous at small scales, perturbations non-linear.
But we still use FLRW at large scales.
- Can the acceleration be due to back-reaction of these inhomogeneities?
e.g. Buchert et al. 1505.07800
- If so, it might also provide an explanation for the coincidence problem.
- There is no consensus on how big is the effect from inhomogeneities. But it might be difficult to explain acceleration with non-linear inhomogeneities.

see e.g.

Giblin et al. 1608.04403

Tian et al. 2010.07274



Velten et al. arXiv:1410.2509

Revisiting assumption 2

- How do we know that general relativity is correct?
 - *Observations:* GR is tested to very high accuracies by solar system experiments and pulsar timing measurements
 - *Theoretical:* GR + Λ is the unique metric theory in 3+1D that gives second order equations of motion.
- C. Will's Living Review (2014)
arXiv:1403.7377
- Lovelock (1971)

Solar system tests

- Post-Newtonian parameter

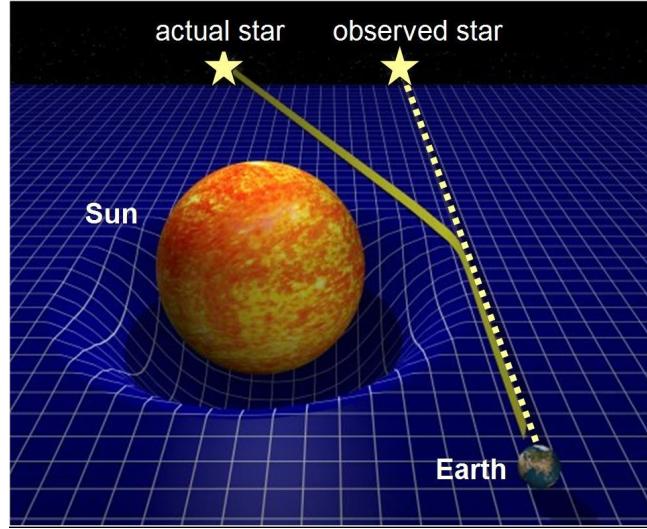
$$g_{00} = -1 + 2G_N \frac{M}{r}$$

$$g_{ij} = \delta_{ij} \left(1 + 2\gamma G_N \frac{M}{r} \right)$$

γ : How much spatial curvature produced by unit rest mass?

$\gamma = 0$: "Newtonian"

$\gamma = 1$: GR



- Deflection of light

$$\theta = 2(1 + \gamma) \frac{M_\odot}{r} = \frac{1 + \gamma}{2} \theta_{GR}$$

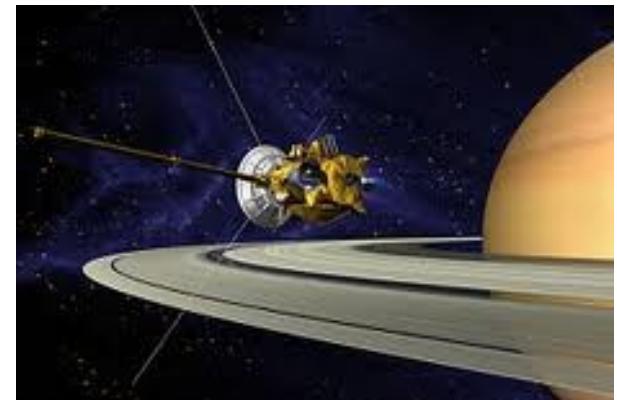
$$\theta = (0.99992 \pm 0.00023) \times 1.75'' \implies \gamma - 1 = (-1.7 \pm 4.5) \times 10^{-4}$$

(VLBI)

- Shapiro time delay

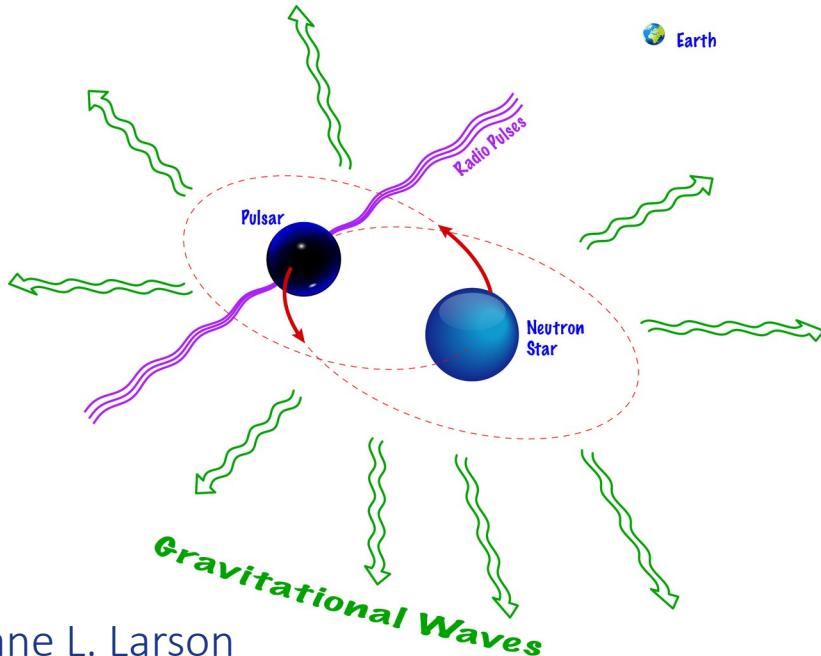
$$\Delta t = (1.00001 \pm 0.0001) \times \Delta t_{GR} \implies \gamma - 1 = (2.1 \pm 2.3) \times 10^{-5}$$

(Cassini probe)

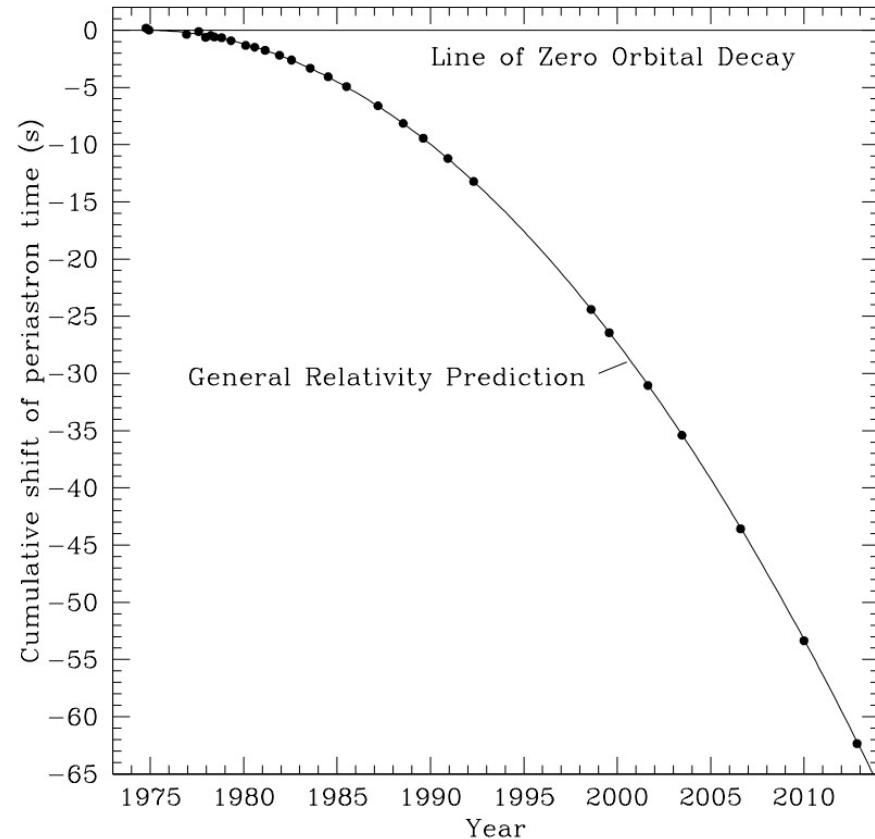


Pulsar Timing

- Hulse & Taylor binary (pulsar + neutron star)
1993 Nobel Prize in Physics
- Orbital decay due to gravitational wave emission perfectly agrees with GR prediction.



Credit: Shane L. Larson



Weisberg, Huang 1606.02744

Pulsar Timing

- PSR J0737-3039A/B
Post-Keplerian parameters

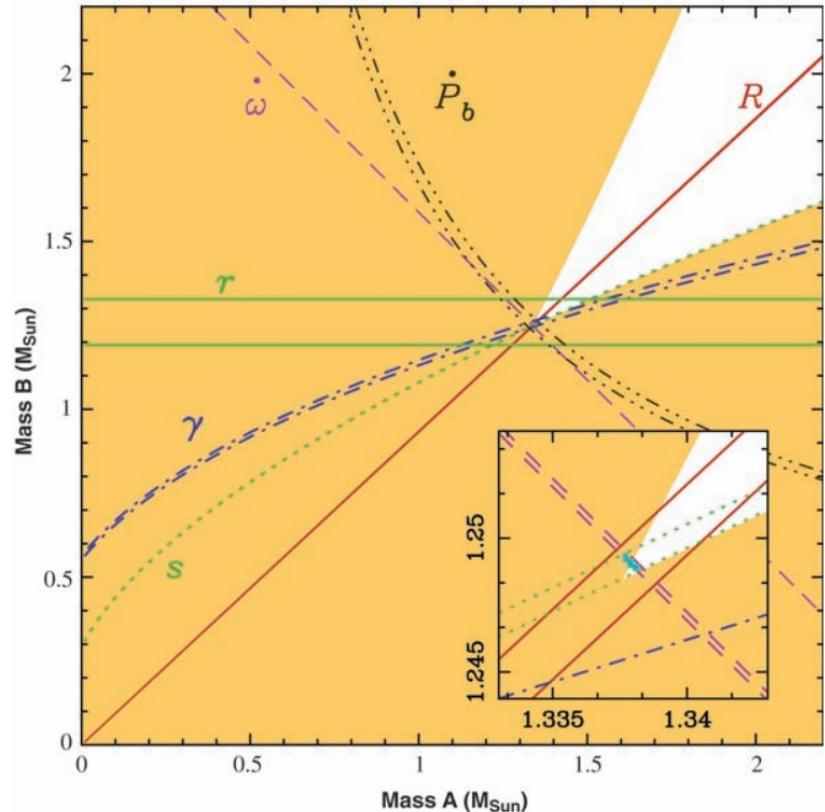
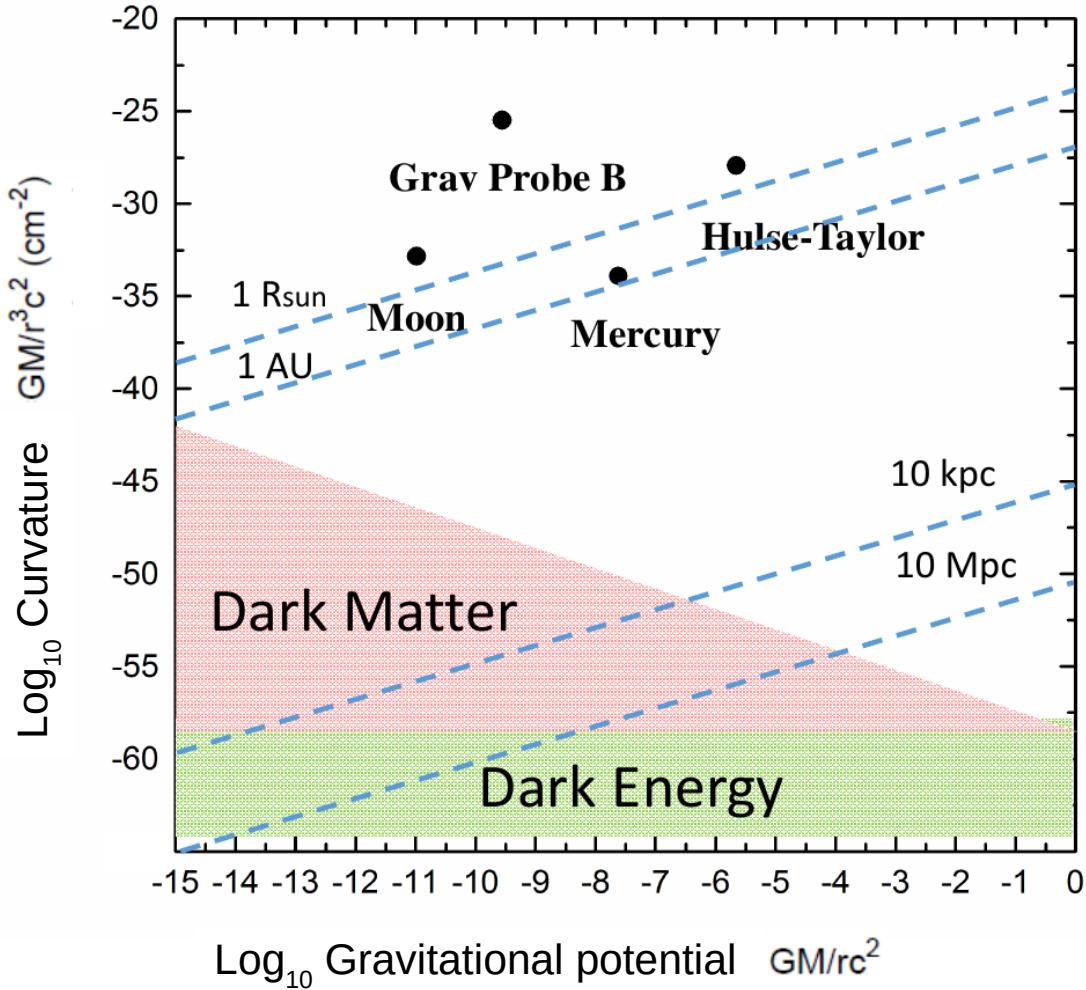


Fig. 1. Graphical summary of tests of GR parameters. Constraints on the masses of the two stars (A and B) in the PSR J0737-3039A/B binary system are shown; the inset is an expanded view of the region of principal interest. Shaded regions are forbidden by the individual mass functions of A and B because $\sin i$ must be ≤ 1 . Other constraining parameters are shown as pairs of lines, where the separation of the lines indicates the measurement uncertainty. For the diagonal pair of lines labeled as R , representing the mass ratio derived from the measured semimajor axes of the A and B orbits, the measurement precision is so good that the line separation becomes apparent only in the inset. The other constraints shown are based on the measured PK parameters interpreted within the framework of general relativity. The PK parameter ω describes the relativistic precession of the orbit, γ combines gravitational redshift and time dilation, and \dot{P}_b represents the measured decrease in orbital period due to the emission of gravitational waves. The two PK parameters s and r reflect the observed Shapiro delay, describing a delay that is added to the pulse arrival times when propagating through the curved space-time near the companion. The intersection of all line pairs is consistent with a single point that corresponds to the masses of A and B. The current uncertainties in the observed parameters determine the size of this intersection area, which is marked in blue and reflects the achieved precision of this test of GR and the mass determination for A and B.

Tests of General Relativity



Curvature:

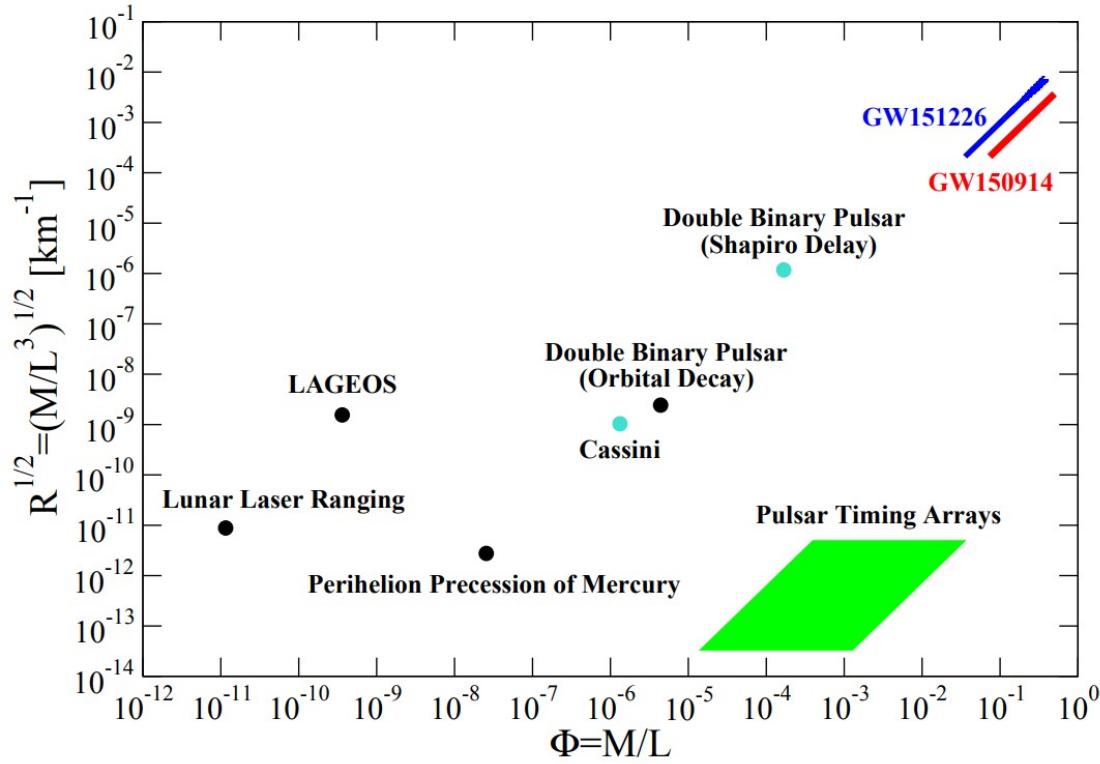
$$R = \frac{G_N M}{r^3}$$

Potential:

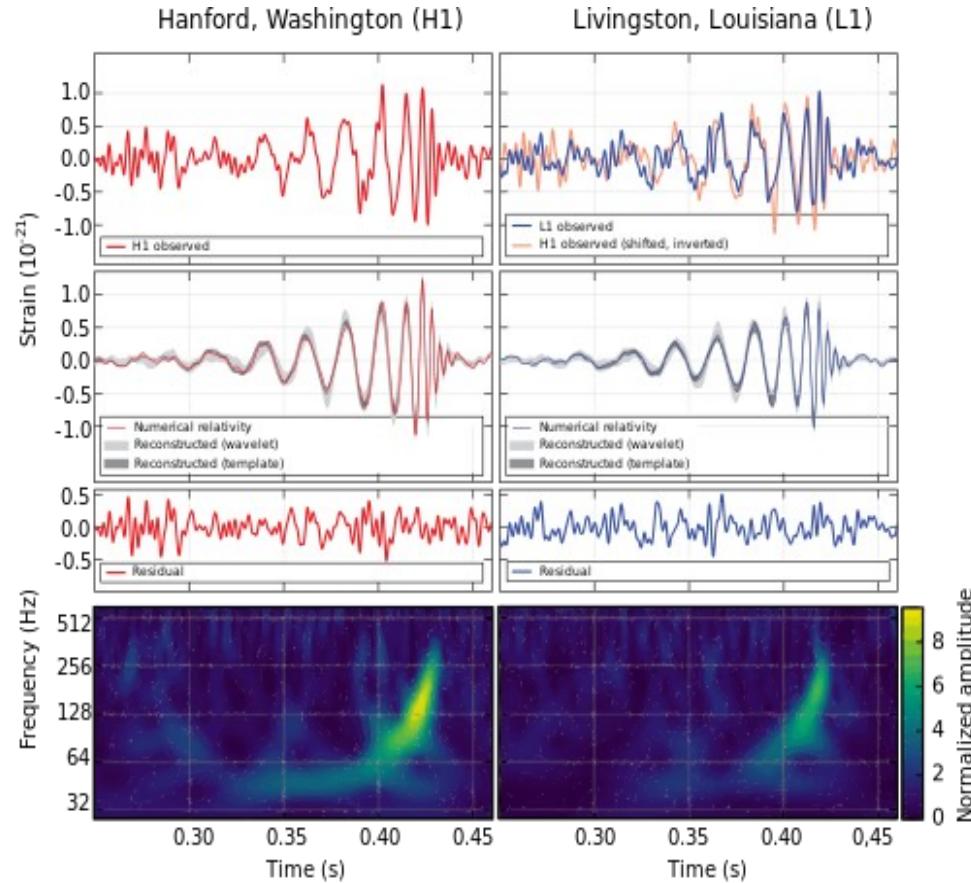
$$\Phi = \frac{G_N M}{r}$$

Psaltis, Living Review (2008) arXiv:0806.1531
also see: Baker et al (2015) arXiv:1412.3455

Tests of General Relativity



Yunes et al. (2016) arXiv:1603.08955



LIGO collaboration

Modifying GR?

- Everything seems to be great with GR. Both observationally and theoretically.
- **However**, we are extrapolating GR well beyond the scales that have been confirmed observationally.
- Theoretically, GR has short-comings at short distances (it is non-renormalisable). Would it be possible that it needs to be tweaked at large distances too?



More detailed
discussion to follow

Revisiting assumption 3

- Can we have some fluid other than a cosmological constant?
 - For the cosmological background, all information is in $w_{DE} = \frac{P_{DE}}{\rho_{DE}}$
 - ➡ extend to $w_{DE} = w_{DE}(z)$ ➡ **Dynamical Dark Energy**
- We can use this as a *phenomenological model*. But what are the candidates for $w(z)$ and how can they be justified theoretically?
- How can we distinguish between DE models and modifications of GR?

$$G_{\mu\nu} + G_{\mu\nu}^{MG} = 8\pi G_N (T_{\mu\nu} + T_{\mu\nu}^{DE})$$

Introducing beyond- Λ CDM effects

1. **Phenomenological model:** represent feature via new parameters in select observables. Allows quantitative evaluation of data and detection of new effects.

e.g. dark energy with varying eos: $w_{DE}(z) = w_0 + w_a \frac{z}{z+1}$

2. **Theoretical framework:** consistent theoretical representation of an effect via new dof

e.g. $w_{DE}(z)$ \rightarrow dynamical scalar field

- The two approaches complement each other. Parametrisation grants the tools to interpret data, to make a discovery. Theory provides justification for the parameters, while imposing consistency conditions, sharpening the analysis with viability priors

e.g. no phantom crossing: $w_{DE}(z) > -1$

- In 2, difficult to associate a parameter/dof with a single observable/scale. In a consistent theory, a new theory parameter/dof may contaminate observables.

Wanted effect
(accelerated expansion)



Unwanted effects
(spoils solar system experiments, gives large contributions to CMB, unstable, predicts faster than light travel, breaks all known rules of physics)

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Dynamics of dark energy
Copeland et al. hep-th/0603057

Quintessence: $w_{DE}(t)$ from a scalar field

- In field theory, explicit t dependence $\xleftarrow{\text{VEV}}$ VEV of a (clock) field
- We know that (at least one) scalar field exists

$$S = \frac{M_{Pl}^2}{2} \int d^4x \sqrt{-g} R + \int d^4x \sqrt{-g} \mathcal{L}_\phi$$

with $\mathcal{L}_\phi = \int d^4x \sqrt{-g} \left[-\frac{1}{2} g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi - V(\phi) \right]$

- Energy-momentum tensor

$$T_{\mu\nu}^\phi \equiv -\frac{2}{\sqrt{-g}} \frac{\delta(\sqrt{-g}\mathcal{L}_\phi)}{\delta g^{\mu\nu}} = \partial_\mu \phi \partial_\nu \phi + g_{\mu\nu} \mathcal{L}_\phi$$

- Scalar field equation of motion

$$\frac{\delta S}{\delta \phi} = 0 \quad \xrightarrow{\text{VEV}} \quad \nabla^\mu \nabla_\mu \phi - V'(\phi) = 0 \quad \longleftrightarrow \quad \nabla^\mu T_{\mu\nu}^\phi = 0$$

Quintessence: Background

- Uniform scalar field $\phi = \phi(t)$.
- Energy density and pressure

$$\rho_\phi = \frac{1}{2} \dot{\phi}^2 + V$$

$$P_\phi = \frac{1}{2} \dot{\phi}^2 - V$$

- Scalar field equation

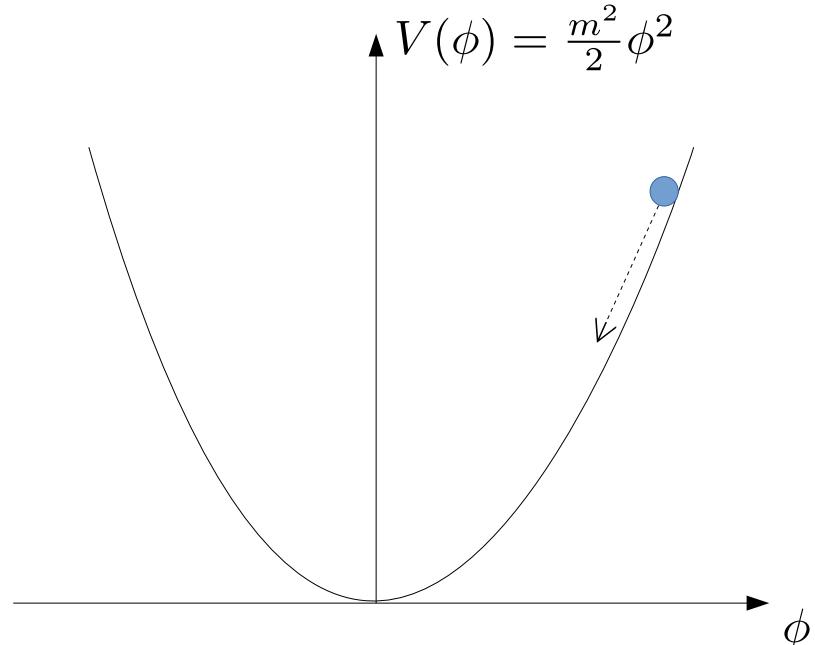
$$\ddot{\phi} + 3H\dot{\phi} + V'(\phi) = 0$$

$$\updownarrow$$

$$\dot{\rho}_\phi + 3H(\rho_\phi + P_\phi) = 0$$

- For accelerated expansion, scalar field needs to slowly roll down the potential

$$\dot{\phi}^2 \ll V(\phi) \quad \Rightarrow \quad w_\phi = \frac{P_\phi}{\rho_\phi} \simeq -1$$



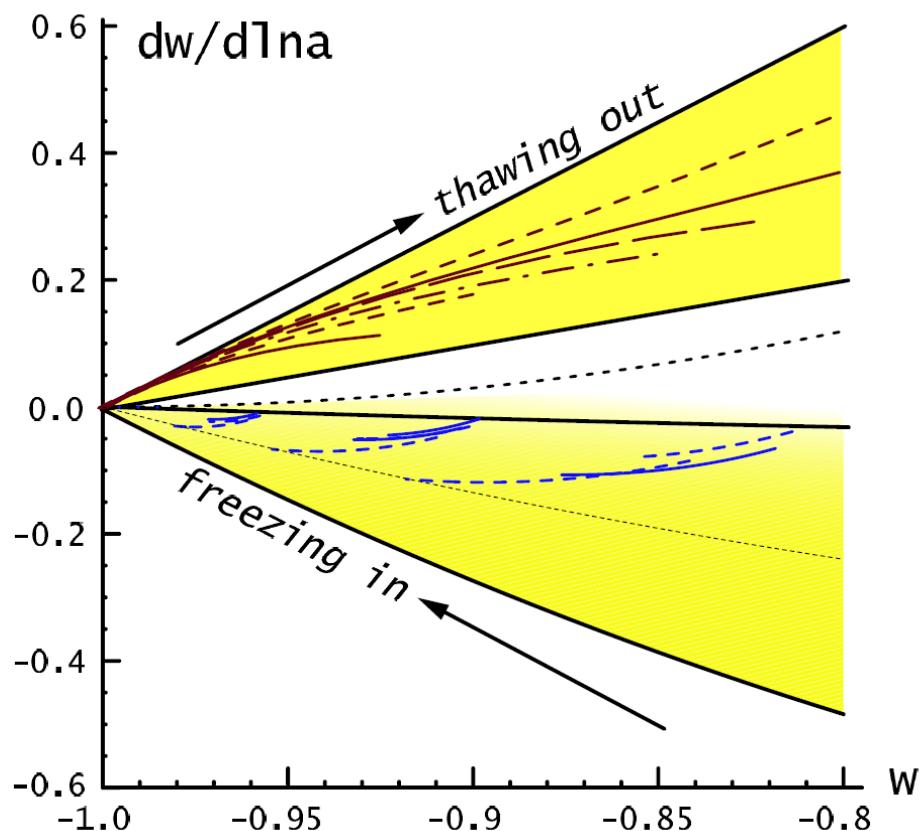
Quintessence: Potentials

- **Freezing models:** Scalar field dynamics freezes at late time approaching $w_\phi \simeq -1$. Example:

$$V(\phi) = M^4 \left(\frac{M}{\phi} \right)^n$$

- **Thawing models:** Scalar initially held at $w_\phi \simeq -1$ by Hubble friction. Starts to roll down towards to minimum recently (similar to inflation). Example:

$$V(\phi) = M^4 \cos^2 \left(\frac{\phi}{f} \right)$$



Quintessence: Challenges

- The main challenge is to control quantum corrections to the potential. Consider a massive scalar field

$$V(\phi) = \frac{m^2}{2} \phi^2$$

- Constant energy density sustained by slow roll

$$\epsilon = \frac{M_{Pl}^2}{2} \left(\frac{V'}{V} \right)^2 = \frac{2 M_{Pl}^2}{\phi^2} \ll 1 \quad \rightarrow \quad \phi \gg M_{Pl} \sim 10^{18} \text{GeV}$$

- On the other hand, we have

$$V(\phi_0) = \rho_{DE} = 10^{-48} \text{GeV}^4 \quad \rightarrow \quad m = \left(\frac{2 \rho_{DE}}{\phi^2} \right)^{1/2} \ll 10^{-42} \text{GeV} \simeq H_0$$

- Correction to the potential:

$$\Delta V = \frac{\lambda}{4} \phi^4 < \rho_{DE} \quad \rightarrow \quad \lambda \ll 10^{-120}$$

These small numbers are very difficult to protect against quantum corrections

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Modified Gravity and Cosmology
Clifton et al. arXiv:1106.2476

Modifying gravity?

- We have to evade Lovelock's theorem (1971).

Lovelock's Theorem: GR + c.c. unique theory assuming:

- a) EOM originates from an action
- b) EOM has at most 2nd order derivatives
- c) 3+1 dimensions
- d) Invariance under general coordinate transformations
- e) Locality (i.e. no operators like $(\nabla^\mu \nabla_\mu)^{-1}$)
- f) The only field is the metric tensor

- A modified theory of GR should relax one of these. This lecture briefly focusses only on breaking (f), as it *partially* covers breaking all other assumptions except (a).

Scalar-Tensor Theories: Horndeski class

- Extension of Lovelock's theorem: most general ***2nd order*** scalar-tensor theory

$$\mathcal{L}_H = \sum_{n=2}^5 \mathcal{L}_n$$

$$\mathcal{L}_2 = K(\phi, X)$$

$$\mathcal{L}_3 = -G_3(\phi, X) \square \phi$$

$$\mathcal{L}_4 = G_4(\phi, X) R + G_{4,X} [(\square \phi)^2 - (\nabla_\mu \nabla_\nu \phi)^2]$$

$$\mathcal{L}_5 = G_5(\phi, X) G^{\mu\nu} \nabla_\mu \nabla_\nu \phi - \frac{1}{6} G_{5,X} [(\square \phi)^3 - 3(\square \phi)(\nabla_\mu \nabla_\nu \phi)^2 + 2(\nabla_\mu \nabla_\nu \phi)^3]$$

$$X \equiv \frac{g^{\mu\nu}}{2} \partial_\mu \phi \partial_\nu \phi$$

- Some examples:

K-essence

$$K = K(X), G_4 = \frac{M_{Pl}^2}{2}$$

$$G_3 = G_5 = 0$$

Dynamical DE

Covariant Galileon

$$K, G_3 \propto X, G_5 \propto X^2$$

$$G_4 = \frac{M_{Pl}^2}{2} + (...) X^2$$

Self-acceleration

Fab Four

$$G_2 = 2V''_{\text{John}}(\phi)X^2 - V^{(3)}_{\text{Paul}}(\phi)X^3 + 6V''_{\text{George}}(\phi)X + 8V^{(3)}_{\text{Ringo}}(\phi)X^2(3 - \ln(|X|)),$$

$$G_3 = 3V'_{\text{John}}(\phi)X - \frac{5}{2}V''_{\text{Paul}}(\phi)X^2 + 3V'_{\text{George}}(\phi) + 4V^{(3)}_{\text{Ringo}}(\phi)X(7 - 3\ln(|X|)),$$

$$G_4 = V_{\text{John}}(\phi)X - V'_{\text{Paul}}(\phi)X^2 + V_{\text{George}}(\phi) + 4V''_{\text{Ringo}}(\phi)X(2 - \ln(|X|)),$$

$$G_5 = -3V_{\text{Paul}}(\phi)X - 4V'_{\text{Ringo}}(\phi)\ln(|X|).$$

Self-tuning

- Scalar-Tensor class is actually larger: ***Beyond Horndeski*** and ***DHOST*** (*degenerate*) terms.

MG dark energy after multi-messenger astronomy

- GW170817/GRB170817A : BNS merger, measure time delay between GW and GRB ~ 1.74 s.

$$-3 \times 10^{-15} < \frac{c_{GW}}{c} - 1 < 0.7 \times 10^{-15}$$

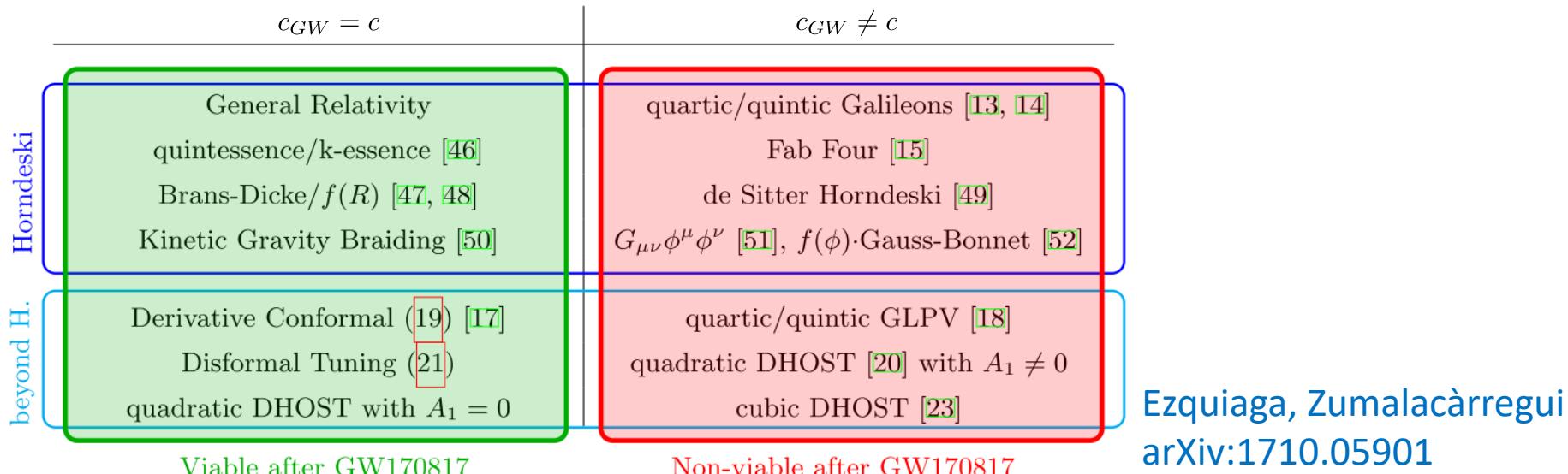
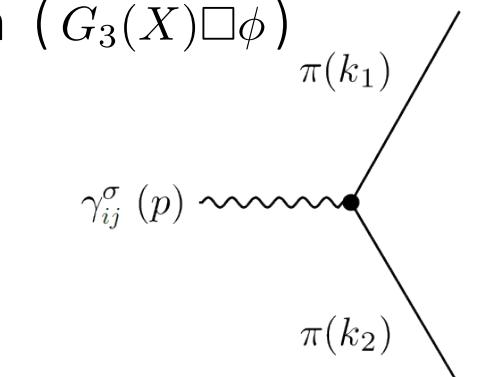


FIG. 2: Summary of the viable (left) and non-viable (right) scalar-tensor theories after GW170817. Only simple Horndeski theories, $G_{4,x} \approx 0$ and $G_5 \approx \text{constant}$, and specific beyond Horndeski models, conformally related to $c_g = 1$ Horndeski or disformally tuned, remain viable.

Instabilities from GW

- Many DE models from S-T class are excluded by the constraint on the speed of light.
- There's more: $\delta g_{ij} \partial_i \delta\phi \partial_j \delta\phi$ interactions in e.g. cubic Galileon ($G_3(X) \square\phi$) can lead to other unwanted effects:
 - GW → DE + DE decay Creminelli et al. 1809.03484
 - Instability induced by GW Creminelli et al. 1910.14035
- With these restrictions, all left from the scalar-tensor class:

$$\mathcal{L} = P(\phi, X) + C(\phi, X)R + \frac{6C_{,X}(\phi, X)^2}{C(\phi, X)} \phi^{;\mu} \phi_{;\mu\nu} \phi_{;\lambda} \phi^{;\nu\lambda}$$



Assuming theories still valid at GW energies. See de Rham, Melville 1806.09417

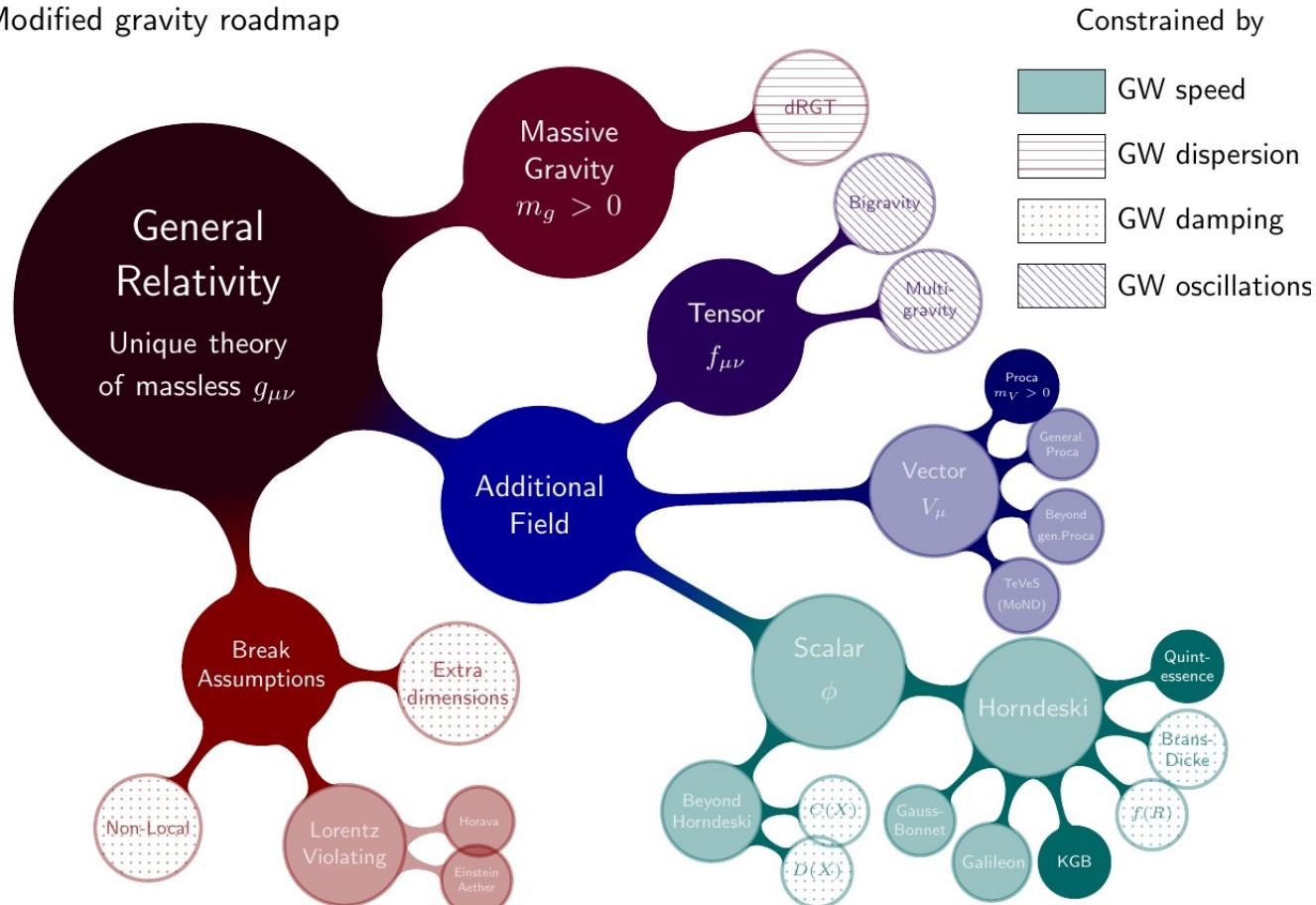
Only quintessence/k-essence and simple modifications e.g. f(R), Brans-Dicke.

Beyond the scalar-tensor class?

- **Scalar-tensor** class is a convenient framework for modified gravity. Most observable quantities depend on scalar perturbations (e.g. potentials, growth rate ...).
- Despite all the developments, we are left with a simple extension of c.c. that *cannot* be falsified! Does not address c.c. problem. ***True modifications of gravity?***
- More complex modifications of GR (with additional vector or tensor modes) also generate scalar perturbations. ***Does the S-T class describe the scalar sector of more general theories accurately?***
- There is evidence of departures from S-T class in more general frameworks. One example: ***general massive gravity theory class:*** *Gumrukcuoglu et al. 2003.11831*
 - Theory class with 6 free functions. Supports stable cosmology with an effective DE with $w < -1$, $\sim 1\%$ deviation in density contrast, may be probed in near future
 - Addresses the new cosmological constant problem
 - In the LCDM limit, still has massive GW, hence falsifiable

Modified gravity theories

Modified gravity roadmap



Constrained by

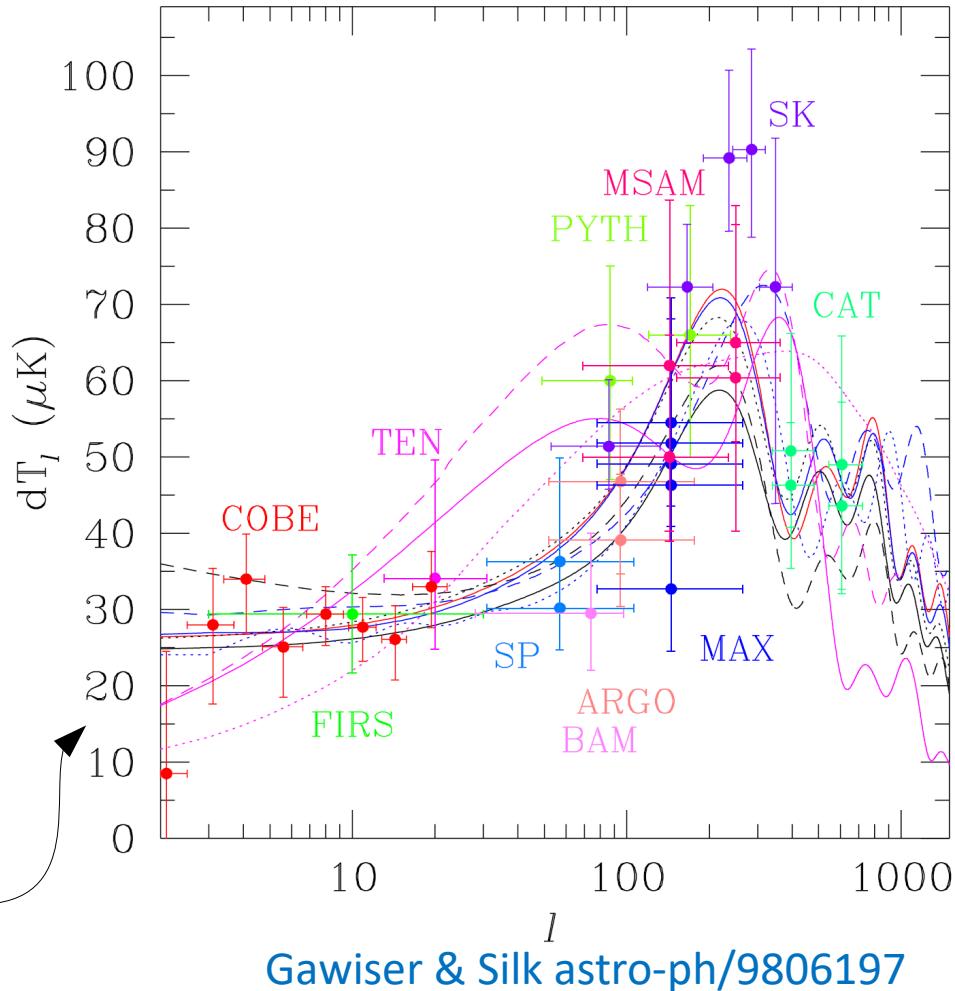
- GW speed
- GW dispersion
- GW damping
- GW oscillations

Discrimination between models

- There are many dark energy and modified gravity models, but we still do not have a compelling alternative to Λ CDM
- Observations may give us clues to answer some basic questions:
 - ➔ Is it a cosmological constant or a light degree of freedom?
 - ➔ Does it cluster?
 - ➔ Does it interact with other matter?

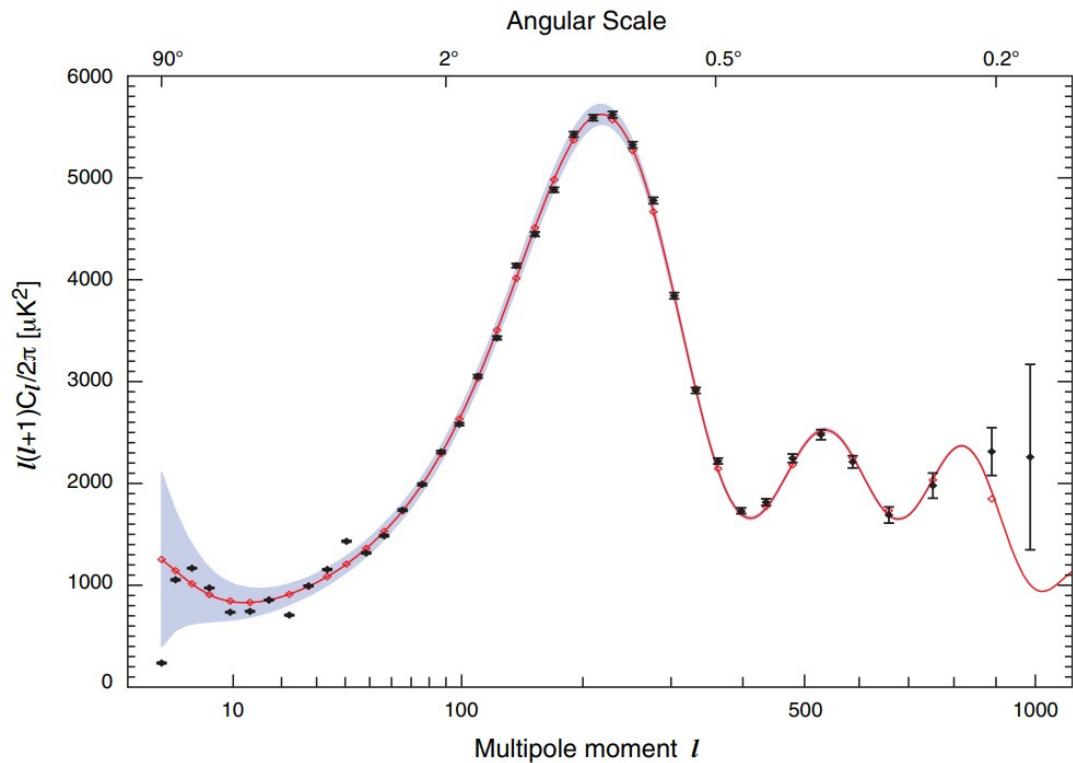
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Cosmology models
before WMAP

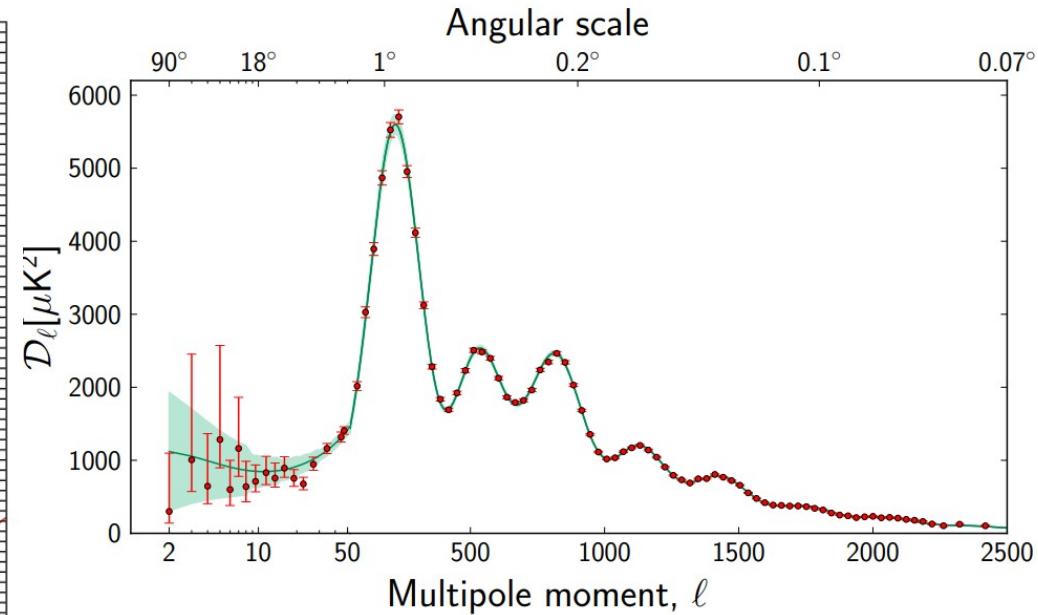


Discrimination between models

Only Λ CDM survived after WMAP



WMAP 3-year astro-ph/0603451



Planck 2013 1303.5075

Can a single model of DE/MG triumph after new observations?

How to test DE/MG models

- We will treat DE and MG models as a single fluid. Einstein's equations:

$$G_{\mu\nu} = 8\pi G (T_{\mu\nu}^m + E_{\mu\nu}) \quad \text{with} \quad E_{\mu\nu} \equiv T_{\mu\nu}^{DE} - G_{\mu\nu}^{MG} \quad \nabla^\mu (T_{\mu\nu}^m + E_{\mu\nu}) = 0$$

- In FLRW background, we can treat $E_{\mu\nu}$ like a perfect fluid

$$E^\mu{}_\nu = \begin{pmatrix} -\rho_E & & & \\ & P_E & & \\ & & P_E & \\ & & & P_E \end{pmatrix} \quad \text{with effective eos} \quad w_E = P_E/\rho_E$$

- We will only look at scalar perturbations from now on.
- Scalar metric perturbations in **longitudinal gauge** (=conformal Newtonian):

$$ds^2 = a^2(\tau) [-(1 + 2\Phi)d\tau^2 + (1 - 2\Psi)\delta_{ij}dx^i dx^j]$$

Longitudinal gauge:
(c.f. Inflation lecture)

$$E = B = 0$$

Conformal time:
 $a d\tau = dt$

- We expand perturbations into plane waves $e^{i\vec{k}\cdot\vec{x}}$.

$$\partial_i \leftrightarrow i k_i \quad \text{and} \quad \nabla^2 \leftrightarrow -k^2$$

Matter content

- Energy-momentum tensor for component I

$$T^{\mu}_{\nu,I} = \begin{pmatrix} -(\rho_I + \delta\rho_I) & (\rho_I + P_I)v_{i,I} \\ -(\rho_I + P_I)v_I^j & (P_I + \delta P_I)\delta_j^i + P_I \Pi^i_{j,I} \end{pmatrix}$$

- Conservation of *total* energy-momentum: $\nabla^\mu T_{\mu\nu} = \sum_I \nabla^\mu T_{\mu\nu,I} = 0$
- Let's assume matter and DE/MG component satisfy conservation independently.

$$\nabla^\mu T_{\mu\nu}^m = \nabla^\mu E_{\mu\nu} = 0$$

If not assumed, **interacting DE**: $\nabla^\mu E_{\mu\nu} = -\nabla^\mu T_{\mu\nu}^m = Q_\nu \neq 0$

Continuity eq.

$$\frac{d\delta\rho_I}{d\tau} + 3\mathcal{H}(\delta\rho_I + \delta P_I) = (\rho_I + P_I)(k^2 v_I + 3\Psi')$$

Euler eq.

$$-\left(\frac{d}{d\tau} + 4\mathcal{H}\right)[(\rho_I + P_I)v_I] = \delta P_I + \frac{2}{3}k^2 P_I \Pi_I + (\rho_I + P_I)\Phi$$

$$\mathcal{H} \equiv \frac{a'(\tau)}{a(\tau)} = a H$$

Velocity perturbation:
 $v^i = \partial^i v$

Anisotropic stress:

$$\Pi^i_j = \left(\frac{\delta_j^i}{3} \nabla^2 - \partial^i \partial_j\right) \Pi$$

Matter: $w_m = P_m/\rho_m = 0$

$\delta P_m = \Pi_m = 0$

DE/MG: $w_E = P_E/\rho_E$

$\delta P_E, \Pi_E$

Equations for linear perturbations

- Einstein's equations

Poisson's eq.

$$k^2 \Psi = -4\pi G a^2 (\rho_m \Delta_m + \rho_E \Delta_E)$$

$$\Phi - \Psi = 8\pi G a^2 P_E \Pi_E$$

Density contrast:

$$\Delta_I \equiv \frac{\delta \rho_I}{\rho_I} - \frac{3\mathcal{H}(\rho_I + P_I)}{\rho_I} v_I$$

- Conservation of energy-momentum for matter (sub-horizon scales $k/\mathcal{H} \gg 1$)

$$\left\{ \begin{array}{l} \Delta'_m + \mathcal{H} \theta_m = 0 \\ \theta'_m + \mathcal{H} \left(1 + \frac{\mathcal{H}'}{\mathcal{H}^2} \right) \theta_m = \frac{k^2}{\mathcal{H}} \Phi \end{array} \right.$$

Velocity divergence:

$$\theta_m \equiv -\frac{k^2}{\mathcal{H}} v_m$$

$$\Delta''_m + \mathcal{H} \Delta'_m = -k^2 \Phi$$

- Evolution of matter determined by the Newtonian potential
- DE/MG component affects the evolution through the Newtonian potential

➡ Evolution of structure is sensitive to the dark energy model.

Growth of structure

1. Λ CDM

- Dark component pure c.c. (no perturbation) $E_{\mu\nu} = -\Lambda g_{\mu\nu}$

$$k^2 \Psi = -4 \pi G a^2 \rho_m \Delta_m$$

$$\Phi - \Psi = 0$$

}

$$\Delta_m'' + \mathcal{H} \Delta_m' - 4 \pi G a^2 \rho_m \Delta_m = 0$$

- During MD, $a \propto \tau^2 \rightarrow \mathcal{H} = \frac{2}{\tau}$

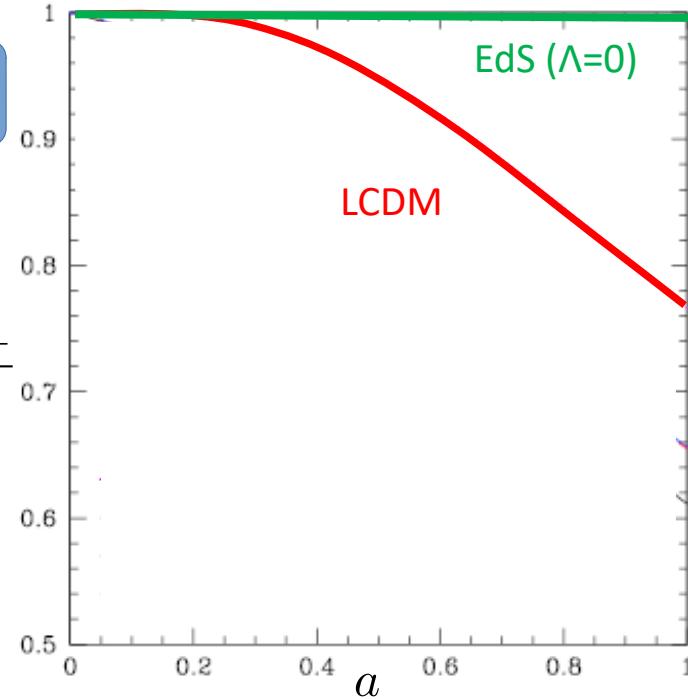
$$4 \pi G a^2 \rho_m = \frac{3 \mathcal{H}^2}{2} = \frac{6}{\tau^2}$$

Growing mode
 $D_+ \propto a$

$$\Delta_m = C_1 a + \frac{C_2}{a^{3/2}}$$

From now on,
 $a_0 = 1$

$$\frac{D_+}{a}$$



- At late times, due to c.c. gravity becoming weaker

$$\mathcal{H}^2 = \frac{8 \pi G a^2}{3} (\rho_m + \rho_\Lambda)$$

- Effect on growth only due to background expansion.

Growth of structure

2. Smooth DE

- Smoothness assumption \rightarrow Negligible perturbations $\delta\rho_E = \Pi_E = 0$ with $w_E = w_E(z)$

$$\Delta_m'' + \mathcal{H} \Delta_m' - 4\pi G a^2 \rho_m \Delta_m = 0$$

Same form
as c.c.

- Changing time to e-foldings $N = \log a$

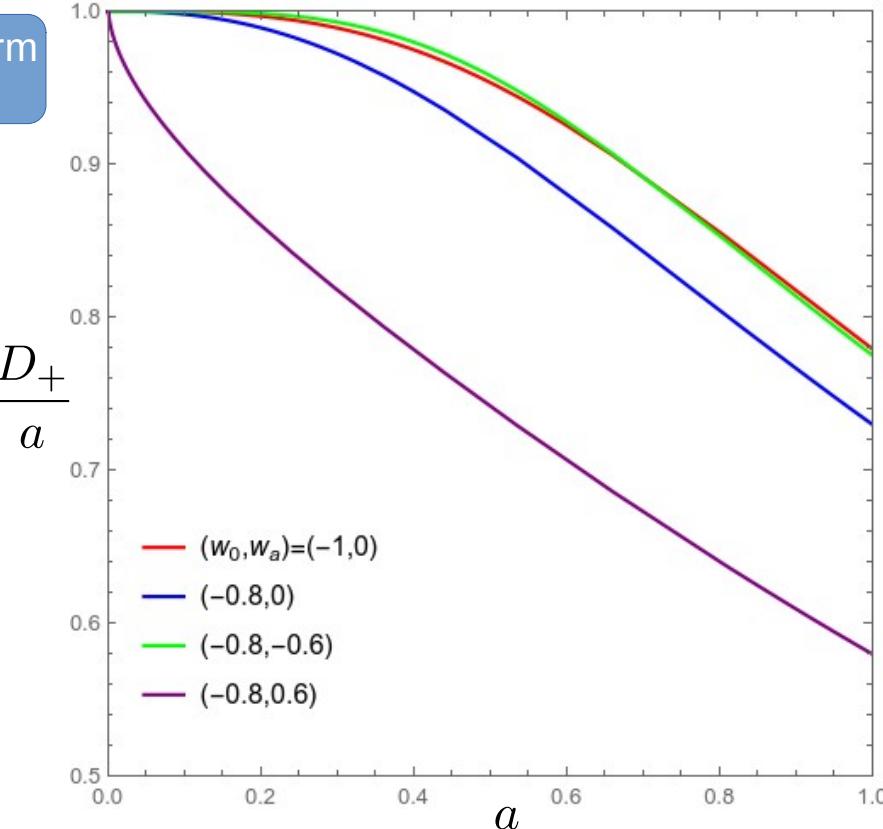
$$\ddot{\Delta}_m + \frac{1}{2} (1 - 3 \Omega_E w_E) \dot{\Delta}_m - \frac{3}{2} \Omega_m \Delta_m = 0$$

$$\dot{\Omega}_m = 3 w_E (1 - \Omega_m) \Omega_m$$

- Parameterize e.o.s. for low redshift:

$$w_E(a) = w_0 + w_a (1 - a)$$

- For a fixed $\Omega_{E,0}$, if $w_E > -1$, the DE density larger in the past, suppressing the growth compared to Λ CDM.

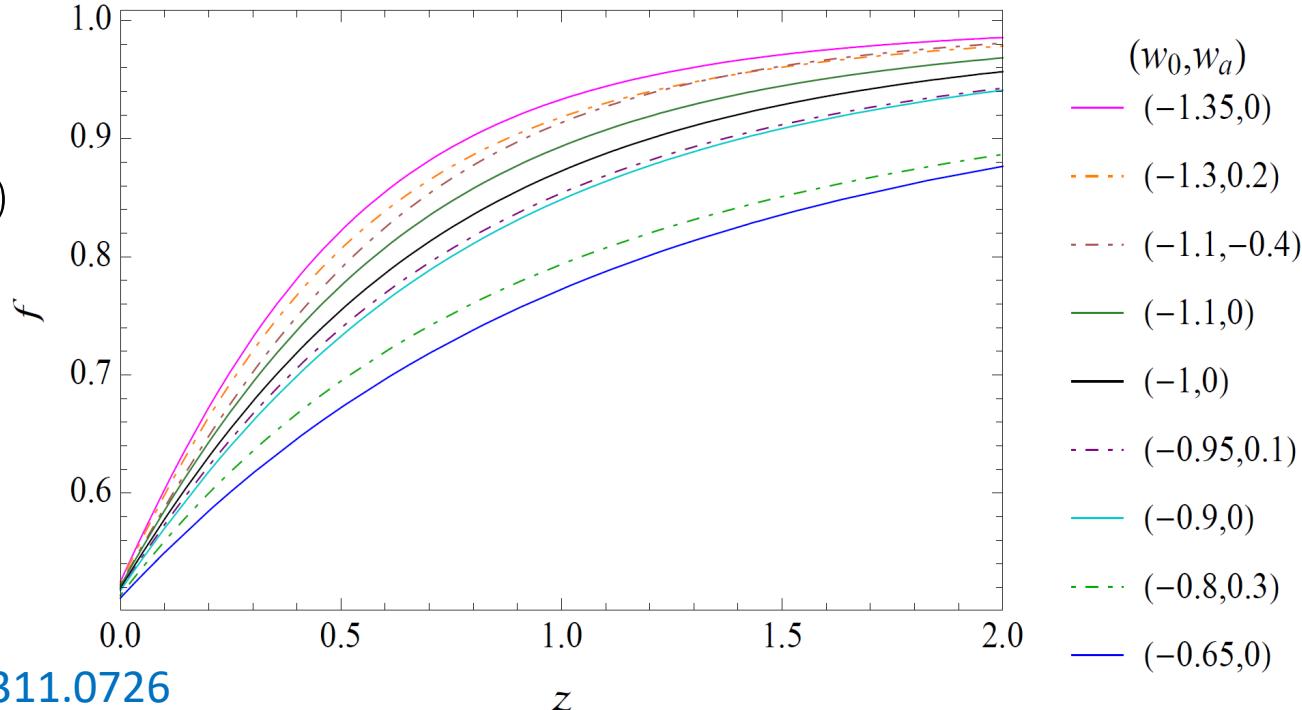


Growth rate for smooth DE

- We define logarithmic growth rate $f \equiv \frac{d \log \Delta_m}{d \log a} = \frac{\dot{\Delta}_m}{\Delta_m}$

$$\dot{f} + f^2 + \frac{1}{2} [1 - 3 w_E(1 - \Omega_m)] f = \frac{3}{2} \Omega_m$$

- Ansatz: $f = (\Omega_m)^\gamma$
- Best fit:
 $\gamma = 0.545 + 0.05(1 + w_E(z = 1))$
- γ is insensitive to DE eos.
- The growth rate f depends on w_E through Ω_m .
- Even when DE perturbations and anisotropic stress is accounted for, growth index still approx. constant.



Growth of structure

3. Clustering DE

- Reintroduce DE perturbations $\delta\rho_E \neq 0$, but keep $\Pi_E = 0$
- Consider toy model with non-zero sound speed

$$\delta P_E = c_{s,E}^2 \delta\rho_E$$

- Assume dark component dominates the universe (sub-horizon)

$$\Delta_E'' + \mathcal{H} \Delta_E' + c_{s,E}^2 (k^2 - k_J^2) \Delta_E = 0$$

Jeans wave-number

$$k_J \equiv \sqrt{\frac{4\pi G a^2 \rho_E}{c_{s,E}^2}}$$

- For $k > k_J$, DE pressure wins over gravitational pull and prevents growth.
- For $k < k_J$, gravity wins and DE clusters. This requires small $c_{s,E}^2$

Example: Scalar field with canonical kinetic term

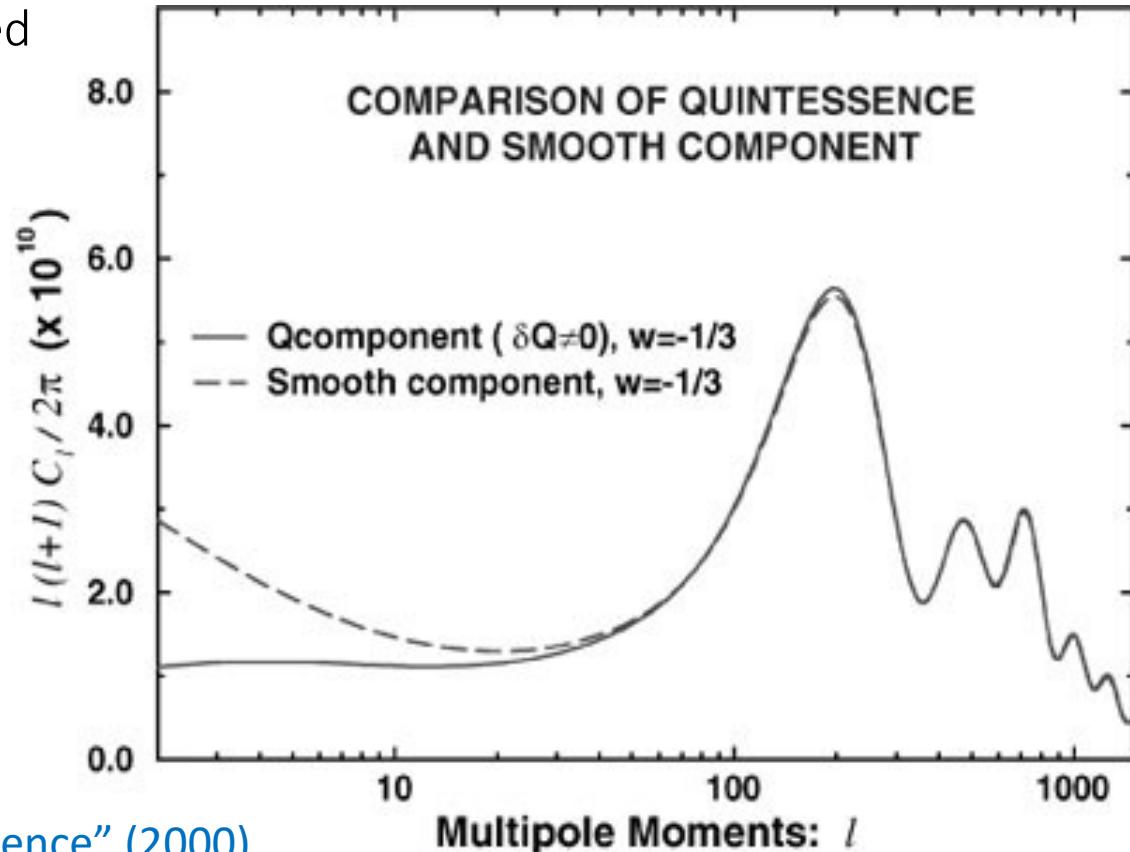
$$S_\phi = \int d^4x \sqrt{-g} \left(-\frac{1}{2} \partial_\mu \phi \partial^\mu \phi - V(\phi) \right) \rightarrow$$

$c_{s,\phi}^2 = 1 \rightarrow$ canonical scalar does not cluster at subhorizon scales. Can approximate as smooth DE $\delta\rho_\phi = \Pi_\phi = 0$

Growth of structure

3. Clustering DE

- Canonical scalar can be approximated to be smooth for subhorizon growth of structure, but we cannot ignore its perturbations entirely!
- Long wavelength modes can interact with large scale fluctuations in CDM and baryons.



Caldwell, "An introduction to quintessence" (2000)

Growth of structure

4. Clustering DE and MG

- Poisson's equation

$$k^2 \Psi = -4 \pi a^2 G \left(1 + \frac{\rho_E \Delta_E}{\rho_m \Delta_m} \right) \rho_m \Delta_m$$

- From point of view of dark matter, generic clustering DE acts like modification of GR

$$k^2 \Psi = -4 \pi a^2 G_{eff}(k, \tau) \rho_m \Delta_m$$

- Anisotropic stress

$$\Phi - \Psi = 8 \pi G a^2 P_E \Pi_E$$

- For scalar field matter: $\Pi_E = 0$, but it typically appears in modified gravity.

Zoology of DE/MG models

Λ CDM $\Lambda(\Omega_E)$

Smooth DE (Ω_E , w_E)
Quintessence

Clustering DE (Ω_E , w_E , $\delta\rho_E$)
K-essence

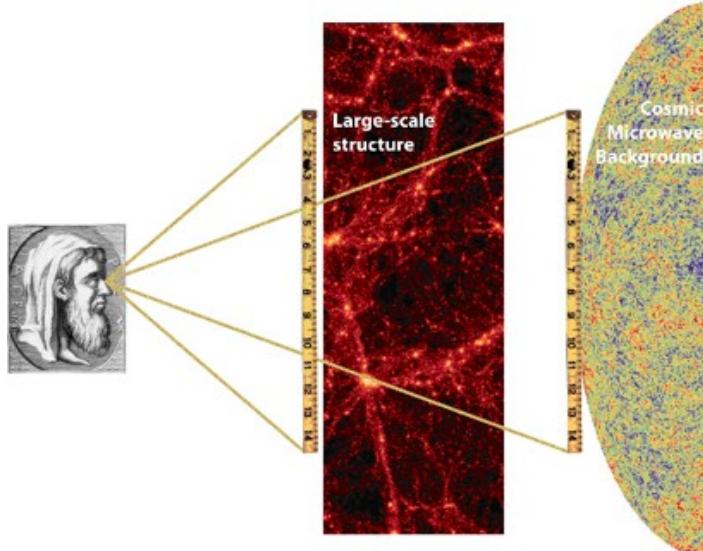
Modified gravity (Ω_E , w_E , $\delta\rho_E$, Π_E)
(screening mechanisms)

Interacting DE (Ω_E , w_E , $\delta\rho_E$, Π_E , Q_E^μ)
(violation of weak equivalence principle)

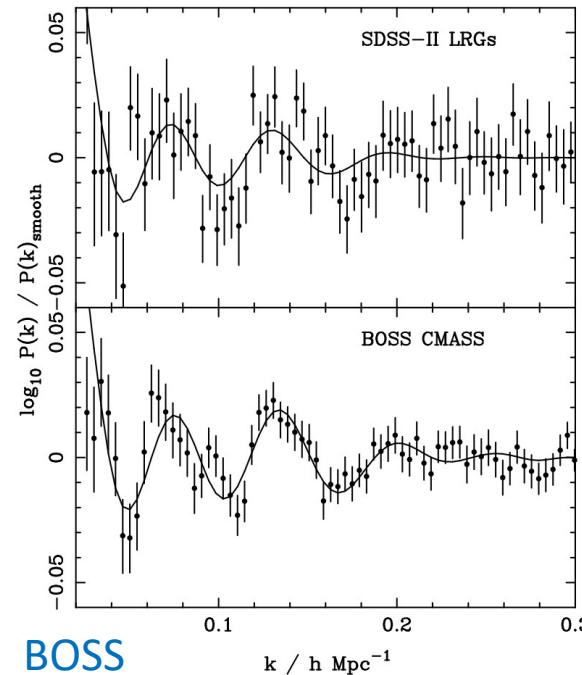
Observations

Background

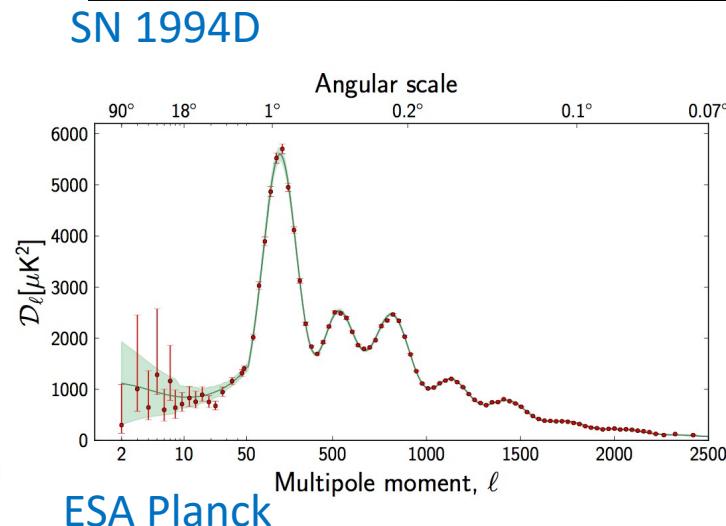
- Measurement of $H(z)$
 - Type 1a Supernovae: luminosity distance
 - CMB/BAO: angular diameter distance



ESA Euclid



BOSS



ESA Planck



SN 1994D

Observations

Weak lensing

$$ds^2 = a^2(\tau) [-(1 + 2\Phi)d\tau^2 + (1 - 2\Psi)\delta_{ij}dx^i dx^j]$$

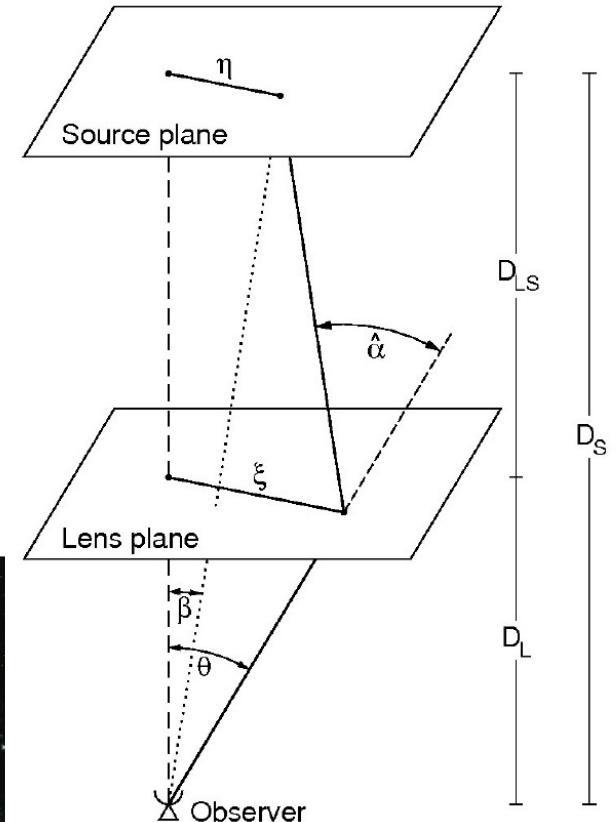
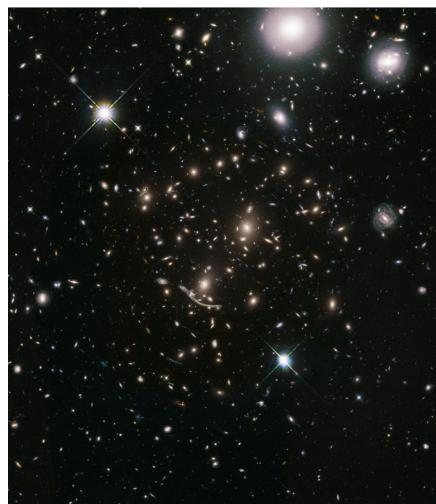
- Convergence (photons follow geodesics):

$$\kappa(\hat{n}) = \int d\chi \frac{(\chi_s - \chi)\chi}{\chi_s} \nabla_{\perp}^2 \phi_W(\tau_0 - \chi, \chi \hat{n})$$

Lensing Potential:

$$\phi_W \equiv \frac{\Phi + \Psi}{2}$$

Galaxy Cluster Abell 370
(HST)



Bartelmann & Schneider
astro-ph/9912508

Observations

CMB

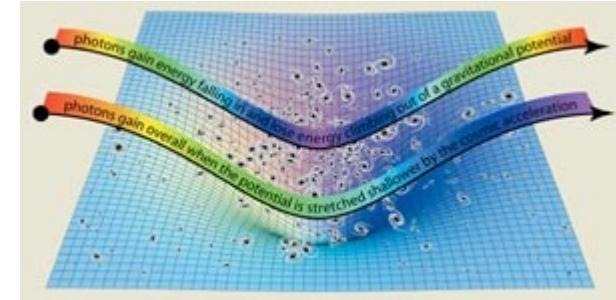
- *Late Integrated Sachs-Wolfe (ISW) effect:* Time variation of the gravitational potentials causes shift in photon temperature

$$\frac{\Delta T_{ISW}}{T}(\hat{n}) = 2 \int_0^{\chi_{LSS}} \frac{\partial \phi_W(\tau_0 - \chi, \chi \hat{n})}{\partial \tau} d\chi$$

- *Lensing:* CMB photons are also lensed.

$$\frac{\Delta T_{lensed}}{T}(\hat{n}) = \frac{\Delta T}{T}(\hat{n} + \vec{\nabla}\psi)$$

$$\psi(\hat{n}) = -2 \int d\chi \frac{(\chi_{LSS} - \chi)\chi}{\chi_{LSS}} \phi_W(\tau_0 - \chi, \chi \hat{n})$$



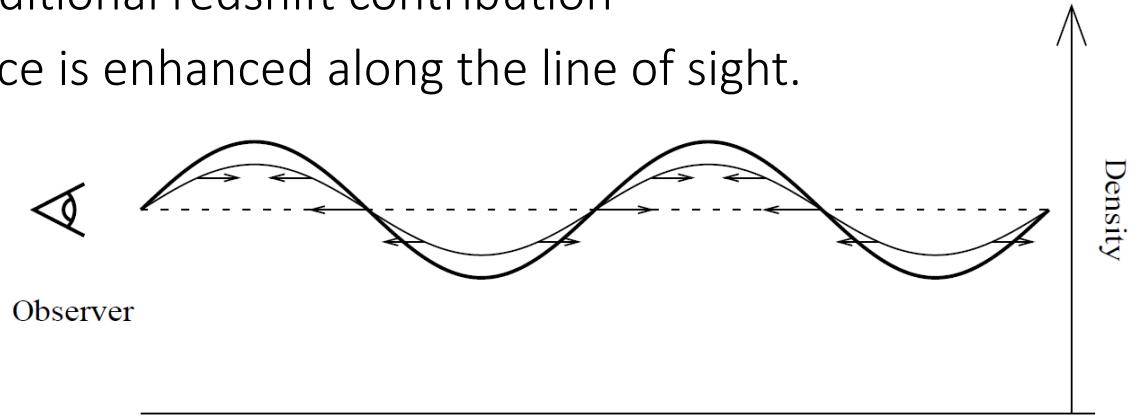
Observations

Redshift-space distortions

- Galaxies have peculiar velocities: additional redshift contribution
- Clustering of galaxies in redshift space is enhanced along the line of sight.
- The enhanced amplitude

$$\delta^s(k, \mu) = \Delta_m(k) - \mu^2 \theta(k)$$

$$\mu \equiv \hat{k} \cdot \hat{n}$$



Hamilton astro-ph/9708102

- Using the continuity equation $\Delta'_m = -\mathcal{H} \theta_m$, we relate it to the growth rate

Can determine Newtonian potential from Euler eq:

$$a k^2 \Phi = (a \mathcal{H} \theta_m)'$$

$$\delta^s(k, \mu) = \Delta_m(k) (1 + \mu^2 f)$$

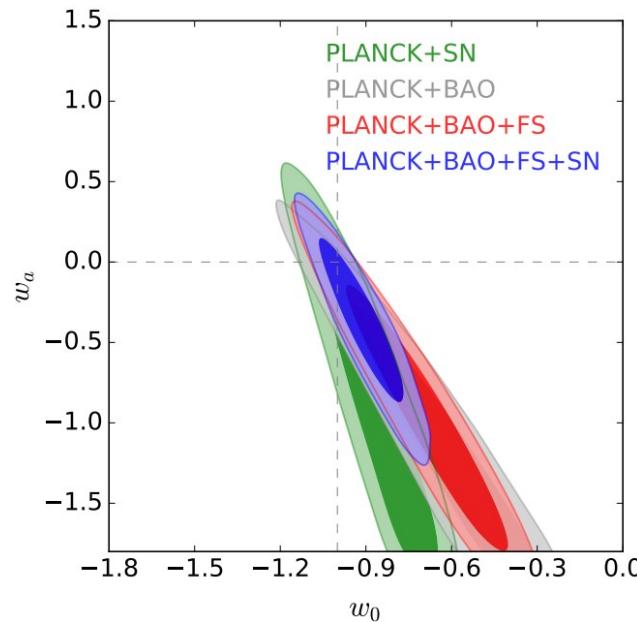
Growth rate:
 $f \equiv \frac{d \log \Delta_m}{d \log a}$

Testing DE/MG

Background expansion history

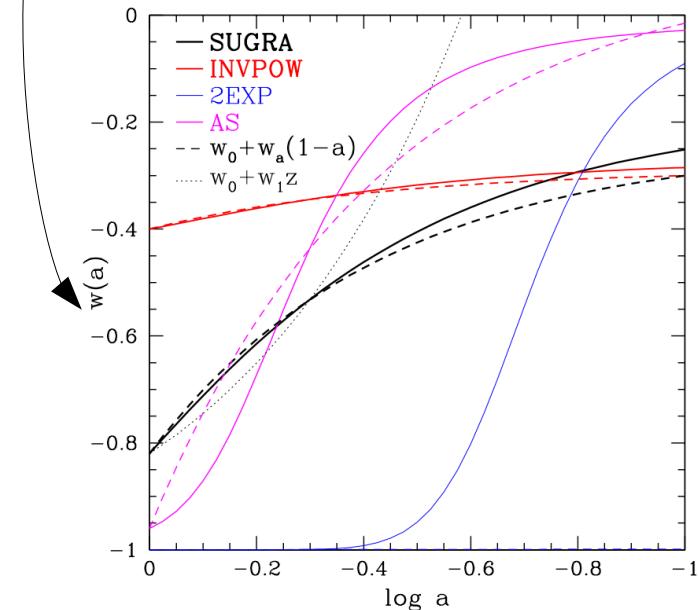
- Background expansion is solely determined by the DE equation of state

$$w_E(t) = w_0 + w_a(1 - a) + \mathcal{O}(1 - a)^2$$



BOSS Collaboration 1607.03155

- Some dynamical DE models and their reconstruction from linear parametrisation.
- Good agreement for slow moving scalar field models



Linder astro-ph/0311403

Testing DE/MG

Parametrisation for slowly rolling scalar DE

- For slow roll,

$$(1 + w_E) \simeq \frac{2}{3} \epsilon_V \Omega_E(a)$$

$$\epsilon_V \equiv \frac{M_{Pl}^2}{2} \left(\frac{V'}{V} \right)^2$$

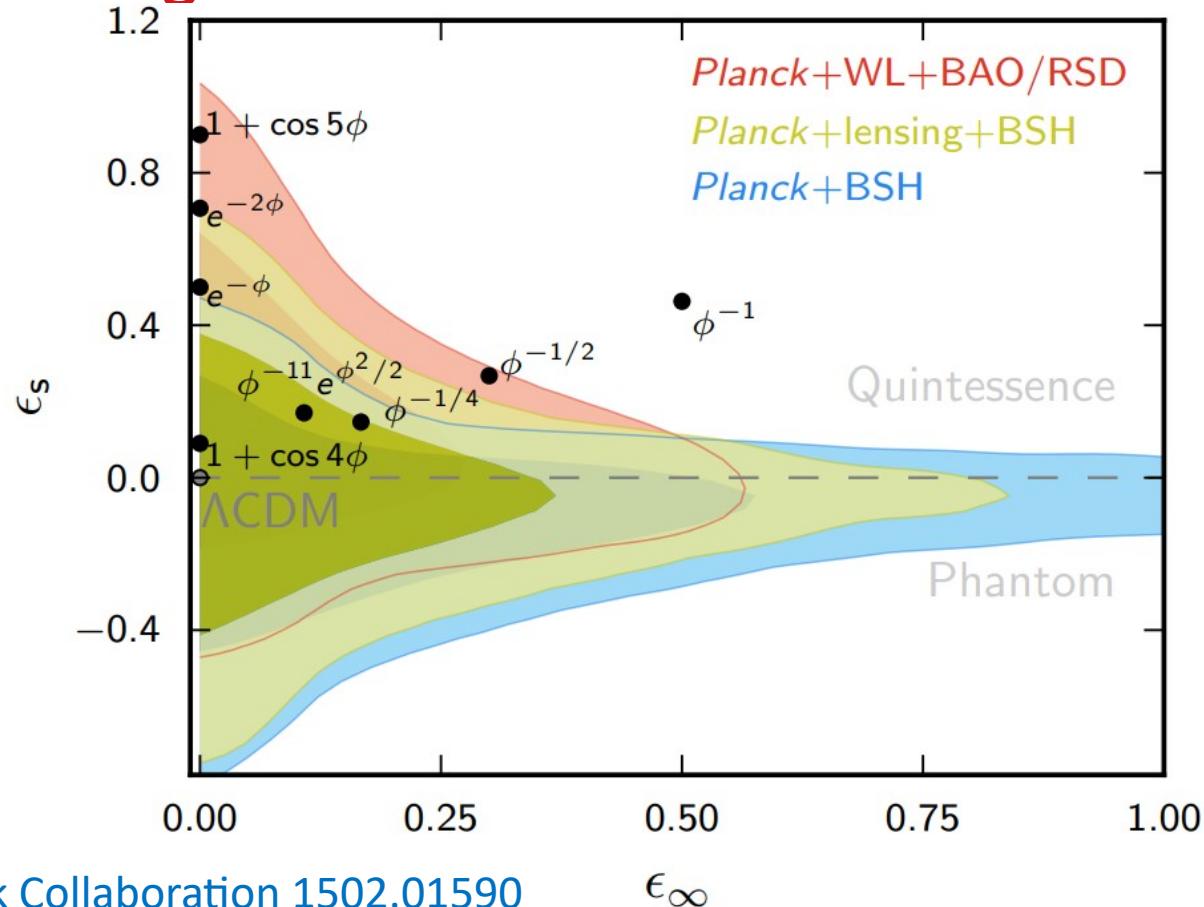
$$\Omega_E \equiv \frac{\rho_E}{\rho_E + \rho_m}$$

- Slope parameter:

$$\epsilon_s \equiv \epsilon_V \Big|_{\rho_m = \rho_E}$$

- $\epsilon_V \Omega_E \sim \text{constant}$ at early times:

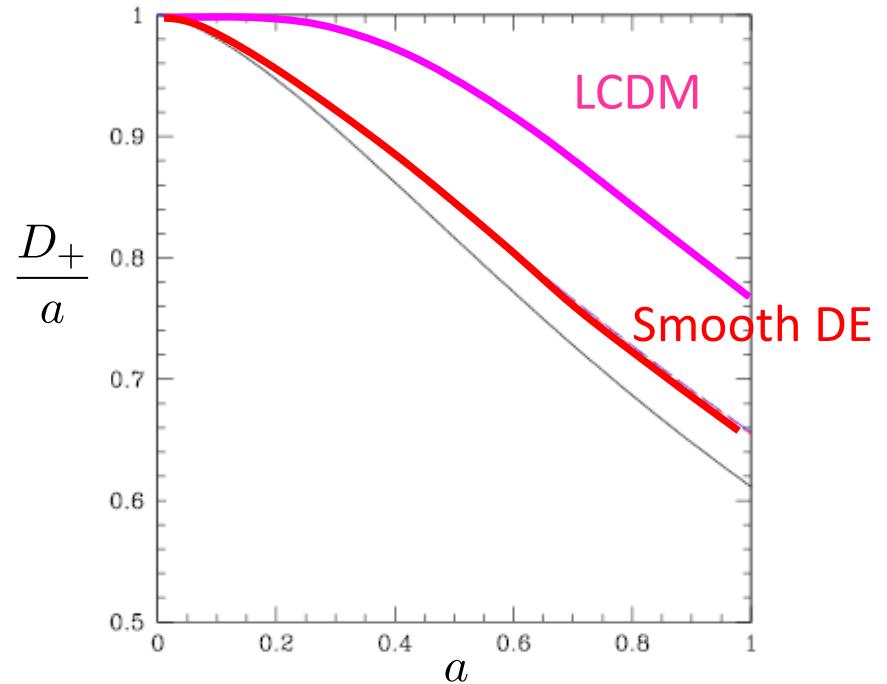
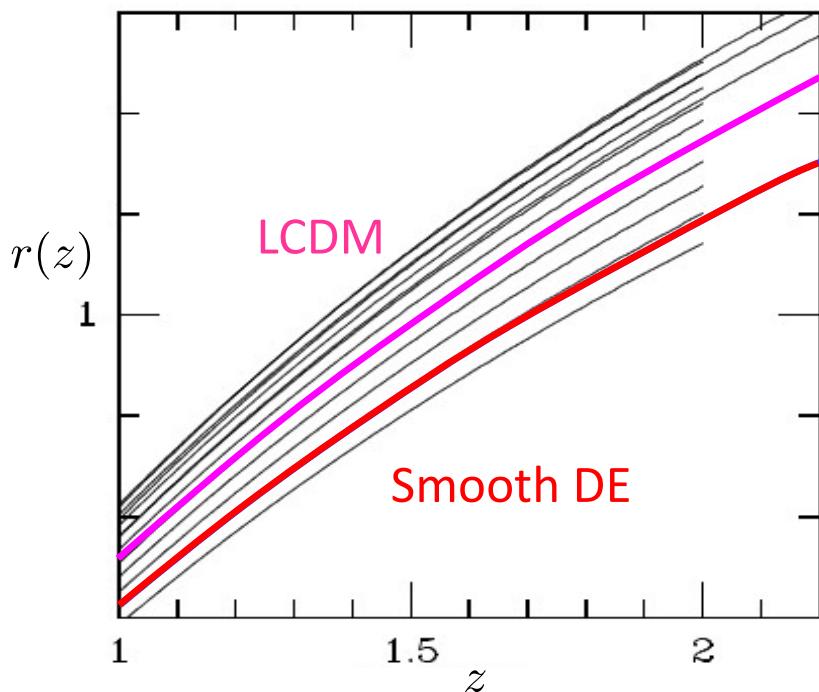
$$\epsilon_\infty \equiv \epsilon_V \Omega_E \Big|_{a \rightarrow 0}$$



Testing DE/MG

Degeneracies and consistency tests

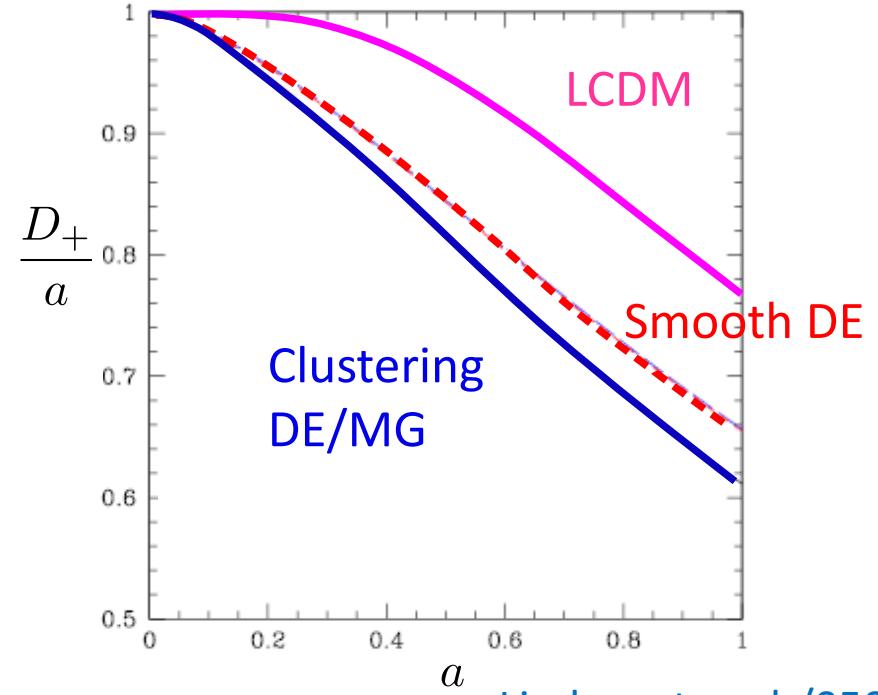
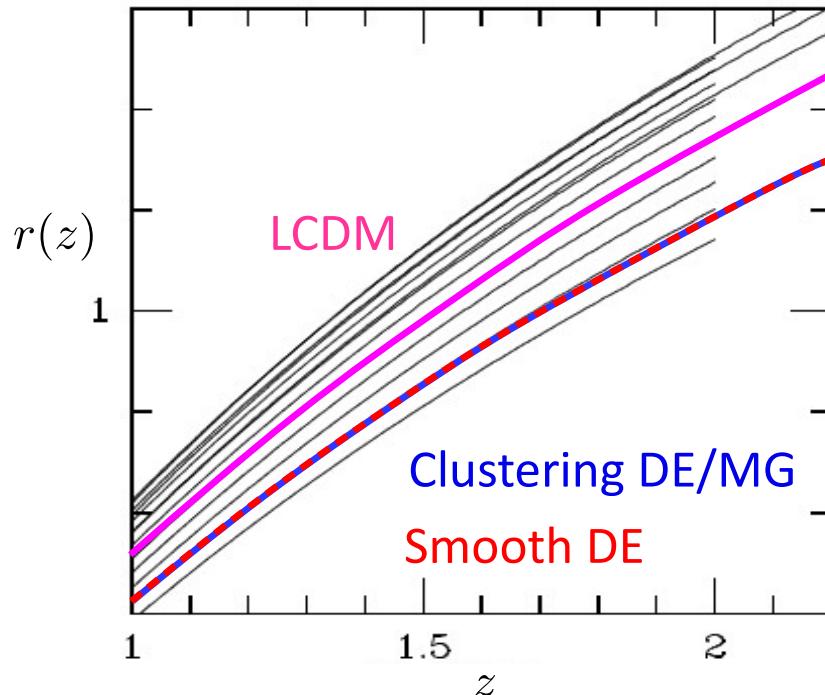
- Λ CDM and smooth DE models: 1:1 correspondance between BG expansion history and growth of structure. No degeneracy.



Testing DE/MG

Degeneracies and consistency tests

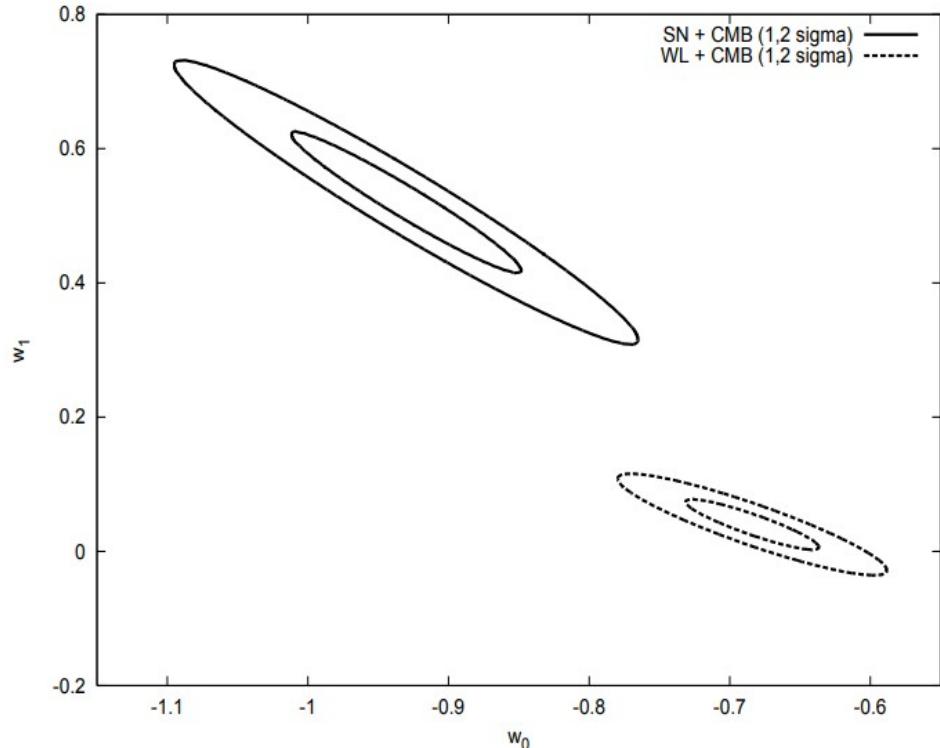
- Clustering DE and MG models: Structure is also controlled by $\delta\rho_E$. Can have exact same expansion history as a smooth DE model, with a different structure growth. Need to combine measurements to discriminate.



Testing DE/MG

Degeneracies and consistency tests

- Assume that the acceleration is *actually* due to clustering DE/MG, but we still try to fit the data using a smooth DE model



- Data generated using DGP model (MG)
- Fitting to a smooth DE model parametrised by $w(z) = w_0 + w_1 z$
- Inconsistent between two different data combinations!

Testing DE/MG

Degeneracies and consistency tests

Jain & Zhang 0709.2375
Song & Koyama 0802.3897

- A simple consistency relation: Using Friedmann and Poisson's equations

$$H^2 = \frac{8\pi G}{3} \rho_{tot}$$

$$k^2 \Psi = -4\pi G a^2 \rho_{tot} \delta_{tot}$$

Assumptions:

- Cosmological principle
- Conservation of CDM energy-momentum
- MG is a metric theory with Newtonian limit

- Can define the following unit quantity:

$$\alpha(k, t) \equiv \frac{2 k^2}{3 a^2 H^2} \frac{\Phi - (\Psi + \Phi)}{\delta_{tot}} = 1$$

weak lensing

galaxy distribution
 $\delta_g = b \delta_{tot}$

background

peculiar velocity
(redshift space distortions)
using $k^2 \Phi = \partial_t(H \theta_m)$

$$\rho_{tot} \equiv \sum_I \rho_I$$
$$\rho_{tot} \delta \rho_{tot} \equiv \sum_I \rho_I \delta \rho_I$$

In the comoving gauge:

$$\Delta_m \Big|_{B=v_m=0} = \delta_m$$

Challenge: Need to make all measurements at the same time and same location

Testing DE/MG

Model independent parametrisation

- For smooth DE, perturbation equations essentially same as Λ CDM.
- For clustering DE/MG, $\delta\rho_E$, δP_E , $\delta\Pi_E$ also play a role.
- Parametrise the effective Newton's constant and anisotropic stress

$$k^2\Phi = -4\pi G a^2 \mu(k, a) \rho_m \Delta_m$$

$$\Psi = \eta(k, a)\Phi$$

$$ds^2 = a^2(\tau) [-(1 + 2\Phi)d\tau^2 + (1 - 2\Psi)\delta_{ij}dx^i dx^j]$$

- Equivalently, we can parametrise the lensing potential

$$k^2\Phi = -4\pi G a^2 \mu(k, a) \rho_m \Delta_m$$

$$k^2 \frac{(\Phi + \Psi)}{2} = -4\pi G a^2 \Sigma(k, a) \rho_m \Delta_m$$

$$\Sigma = \frac{\mu(1 + \eta)}{2}$$

For smooth DE

$$\mu = \eta = \Sigma = 1$$

Project	Dates	Area/deg ²	Data	Spec- <i>z</i> Range	Methods
BOSS	2008–2014	10,000	Opt-S	0.3–0.7 (gals) 2–3.5 (LyαF)	BAO/RSD
KiDS	2011–2019	1500	Opt-I	—	WL/CL
DES	2013–2019	5000	Opt-I	—	WL/CL SN/BAO
eBOSS	2014–2018	7500	Opt-S	0.6–2.0 (gal/QSO) 2–3.5 (LyαF)	BAO/RSD
SuMIRE	2014–2024	1500	Opt-I	—	WL/CL
			Opt/NIR-S	0.8–2.4 (gals)	BAO/RSD
HETDEX	2017–2023	450	Opt-S	1.9 < <i>z</i> < 3.5 (gals)	BAO/RSD
DESI	2020–2025	14,000	Opt-S	0–1.7 (gals) 2–3.5 (LyαF)	BAO/RSD
LSST	2022–2032	20,000	Opt-I	—	WL/CL SN/BAO
<i>Euclid</i>	2022–2028	15,000	Opt-I	—	WL/CL
			NIR-S	0.7–2.2 (gals)	BAO/RSD
WFIRST	2025–2030	2200	NIR-I	—	WL/CL/SN
			NIR-S	1.0–3.0 (gals)	BAO/RSD

Table 28.1: A selection of major dark-energy experiments, based on Ref. [25]. Abbreviations in the “Data” column refer to optical (Opt) or near-infrared (NIR) imaging (I) or spectroscopy (S). For spectroscopic experiments, the “Spec-*z*” column lists the primary redshift range for galaxies (gals), quasars (QSOs), or the Lyman- α forest (LyαF). Abbreviations in the “Methods” column are weak lensing (WL), clusters (CL), supernovae (SN), baryon acoustic oscillations (BAO), and redshift-space distortions (RSD).