Studying the multi-depot split delivery cumulative capacitated vehicle routing problem: Application to the transportation of medical supplies in Wuhan China during the COVID-19 outbreak

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1 Aim, Background, and Significance

The vehicle routing problem (VRP) was first introduced by Dantzig and Ramser (1959) as the Truck Dispatching Problem, which has been extensively studied thereafter because of its high practicability in transportation logistics. The VRP is concerned about the determination of a set of routes for a fleet of vehicles such that each vehicle starts and ends at a single depot, while satisfying all the customers' requirements and operational constraints with the objective of minimising the total transportation cost. While the general assumption in most VRPs is profit-based such as the minimisation of the transportation cost, there are other objective functions such as providing a good service to the customers. Such applications appear in the context of humanitarian relief where the main objective is to reduce suffering and life losses.

In the classical VRP, the routes are designed by optimising the total route length. In such case, some commodities may be served significantly later than others to minimise the total cost. To better reflect priorities and to ensure equity and fairness, a customer-centric objective function is usually used. For instance in the cumulative capacitated VRP (CCVRP), the waiting time of a service system from the customer's point of view is considered. This type of objective function can be viewed as service-based rather than cost-based as in the traditional VRP. Campbell et al. (2008) demonstrate empirically that the optimal solutions could be significantly different for these two objectives, therefore using the VRP objective function to solve a problem where urgency is a concern is inappropriate. This motivates the proposed research topic on CCVRP.

The cumulative capacitated VRP (CCVRP) is a relatively new variant of the classical VRP. Here, the objective function is to minimise the sum of arrival times (min-sum)

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at the customers, subject to the vehicle capacity constraint. This problem has many real-world applications such as in humanitarian logistics for transferring people to a safer place after a natural disaster (e.g., tsunamis or earthquakes), delivering food, medical supplies and necessities to affected regions and dispatching ambulances to patients' locations. In the aftermath of natural disasters such as tsunamis or earthquakes, it is crucial to save as many lives or to alleviate suffering of the affected people in the shortest time possible.

In recent years, several regions around the world have fallen victim to large-scale natural disasters. This leads to increased attention on the effectiveness of humanitarian relief logistics. Clearly, the priority in search context is to provide timely response and quick recovery activities after a catastrophe. A relevant situation can be mentioned from the experience of the well-known Sichuan 2008 earthquake in China which prompted immediate demand for emergency supplies such as food, water, blankets, cots, flashlights, tents, sleeping bags, and medical supplies. In this crisis situation, the response operations started the delivery of essential commodities to more than 370,000 injured and 4.8 million homeless people. The emergency supplies were sent from several operational emergency response units. Multiple (split) deliveries were made to each demand location.

Another example is the recent COVID-19 pandemic, which was first identified in Wuhan, China. Data shows that there was a shortage of medical supplies threatening the frontline staff and patients in Wuhan hospitals at the early stage. Such medical supplies include masks, safety goggles, gloves, protective suits, medical equipment and medicines that are essential for the medical staff and patients. Several organizations, such as Hubei Charity Federation and the Red Cross Society of China's Wuhan branch, teamed up to deliver medicals supplies to out-of-stock clinics and health facilities from multiple donations and operational Emergency Response Units. It is significant that if these medical supplies can be delivered promptly to the hospitals, the medical workers could be more adequately protected and more patients were to be treated. A renowned news channel also reported on the mismanagement in aid allocation by the Red Cross Society of China's Wuhan branch. The source stated that the aforementioned non-governmental organization has allocated 16,000 masks to a small private hospital in Wuhan, capital of the Hubei province, but only 3,000 masks to the largest public hospital in the city where most people infected with the novel coronavirus are to be treated.[1] This indicates poor management in cooperation for shared delivery arrangement among participating organisations and also in logistics delivery planning.

[1]China Daily: NGO fails to mask problems with its poor excuses. https://www.chinadaily.com.cn/a/202002/03/WS5e376fe8a310128217274437.html

Since the aim of such emergency situations is not profit-oriented, there is a need for considering a more pragmatic routing planning problem, namely the CCVRP which aims to minimise the sum of arrival times at customers. However, the fact that the CCVRP only deals with a single depot and each customer is visited exactly once, is less practical in the context of large scale humanitarian logistic where multiple depots and deliveries (split) are often involved. With this in mind, we propose an extension and a more relevant variant, the multi-depot split delivery CCVRP (MDSDCCVRP). To our knowledge, this is the first research that considers this new proposed problem. Since this is a pioneering work on the MDSDCCVRP that combines the split delivery and multiple depot on the CCVRP, we provide a brief literature review on related studies for the CCVRP, MDSDVRP, multiple depot VRP (MDVRP) and split delivery VRP (SDVRP).

The literature on the CCVRP is relatively scarce and very recent despite its practical importance. Ngueveu et al. (2010) are among the first to solve the CCVRP. The authors proposed a metaheuristic method using two memetic algorithms (MA) and tested them on the classical VRP instances from Christofides et al. (1979) and Travelling Repairman Problem instances from Salehipour et al. (2008). Later, an adaptive large neighbourhood search (ALNS) heuristic was presented by Ribeiro and Laporte (2012) who discussed seven removal strategies and three insertion strategies to solve the instances from Christofides et al. (1979) and Golden et al. (1998). It is reported that the ALNS outperforms the MA in terms of solution quality and computational times. Ke and Feng (2013) also attempted to solve the CCVRP with a two-phase metaheuristic in which the first phase focuses on partitioning the customers by using inter-route moves whereas the second phase attempts to optimise each individual route. The first exact algorithm for the CCVRP was developed more recently by Lysgaard and Wøhlk (2014), where a branch-and-cut-and-price method was adopted. Instances from the classical VRP with up to 69 vertices are solved optimally and lower bounds are also provided for the others.

Although the MDSDVRP has numerous applications, the literature for it is also very limited. As far as we are concerned, there are only two papers that studied the MDSDVRP. Gulczynski et al. (2011) were the first to define the problem and develop an integer programming-based heuristic for it. New test instances were also generated and results were reported. Later, Ray et al. (2014) extended this problem by investigating a centralized model and a heuristic algorithm for solving the variant. The main difference between the two studies is that the former considers pre-defined depots whereas the latter establishes depots from the set of customer points and the goal is then to minimise the combined depot establishment and routing cost. In this study, we will consider pre-defined depot locations as this is the common situation in humanitarian logistics where aid is delivered from designated headquarters or warehouses to health facilities. Furthermore, the MDSDCCVRP is different from the MDSDVRP in terms of its objective function as it focuses on minimising the total waiting times of the customers. It is significant that the MDSDCCVRP is more complicated to solve than the MDSDVRP because of its special properties as explained in Section 2.

In the traditional VRP, vehicles are dispatched from a single depot to service the demands of customers, where each customer is visited exactly once. For the MDVRP, vehicles are dispatched from several depots, while for the SDVRP, a customer's demand can be split among vehicles on different routes. There is a large literature devoted to modelling these two variants of the VRP. The MDVRP was introduced 35 years ago whereas the SDVRP was first proposed 20 years ago. Early works on MDVRP mainly focused on exact algorithms such as mathematical models and branch-and-bound methods. With the advancement of computers and the increasing popularity of metaheuristics, there were more studies using genetic algorithms, tabu search, simulated annealing, large or variable neighbourhood procedures to efficiently solve the problem. Moreover, the hybridisations of different procedures have also been proposed and their effectiveness has been proved by using benchmark instances.

The split delivery vehicle routing problem (SDVRP) was introduced by Dror and Trudeau (1989, 1990). Split loads typically occur when transporting a large quantity of the same commodity (e.g., bikes) or when delivering or collecting a divisible product (e.g., liquids or waste) (Archetti and Speranza, 2012). In such situations, a customer may be serviced multiple times such that the demand is fulfilled. The resulting split delivery VRP is a relaxation of the capacitated VRP but is more complicated to solve. It has a notable larger solution space since splitting the delivery among different vehicles is subject to combinatorial explosion. It has been shown that there is a potential to reduce the total cost (by at most 50%; Archetti et al. (2006)) by splitting customer demands among vehicles (Dror and Trudeau, 1989, 1990) for problems in which all demands are less than the capacity of a vehicle. With split deliveries, the problem can be solved when individual customer demand is greater than the vehicle capacity. Even when all customer demands are less than the vehicle capacity, it may be beneficial both in terms of the number of routes and distance travelled with split deliveries (Archetti et al., 2014). Because of a lack of an efficient route-based decomposition, due to the customer demands which act as linking constraints, exact methods (Archetti et al., 2014) struggle to optimally solve instances of a size (e.g., 50–100 deliveries). The first heuristics for the SDVRP were developed by Dror and Trudeau (1989, 1990). They used a two-stage algorithm that incorporated k-split interchanges and route additions. Later, different metaheuristic methods have been used to solve the problem. Some of them include the tabu search algorithm (Archetti et al., 2006), record-to-record travel algorithm (Chen and Golden, 2007), an iterated local search heuristic (Silva et al., 2015), a priori split strategy (Chen et al., 2017) and adaptive large neighbourhood search (Gu et al., 2019).

As the VRP is a NP-hard problem (Lenstra and Kan, 1981), exact algorithms are only efficient for small problem instances. Heuristics and metaheuristics are often more suitable for practical applications, because real-life problems are considerably larger in scale (e.g., a company may need to supply thousands of customers from dozens of depots with numerous vehicles and subject to a variety of constraints). Therefore in this research, we propose the mathematical formulation for the MDSDVRP and incorporate powerful valid inequalities to relax some constraints aiming to solve the instances as far as possible. The results from the exact method can be used as lower bound for the metaheuristic algorithm which we propose in Section 3.

In addition, during an emergency, there is often limited time for saving lives. A good routing plan should also ensure fairness and equity to everyone including the last customer. Motivated by this idea, an alternative but closely related objective that minimises the last arrival time is also studied. We refer to this variant as the min-max MDSDC-CVRP. The main difference between the min-sum and the min-max version is that the previous evaluates all routes whereas the latter considers the bottleneck route with the latest arrival time only. Since the min-max objective only considers the longest route with the latest arrival time, several optimal solutions with the same objective function value could be obtained (Golden et al., 2014). This property can be made use of as the decision-maker will have the added flexibility in selecting the next appropriate optimal configuration based on other non-quantifiable measures if need be.

It is worth noting that there are two definitions of the maximum arrival time in the literature (Golden et al., 2014). These include (a) the time when the last location (customer) is served (Campbell et al., 2008), and (b) the time when the vehicle arrives at the depot (Van Hentenryck et al., 2010). Definition (a) is useful in the context of delivery where there is no time pressure for the vehicle to return to the depot after serving the last customer, whereas (b) is more suitable for pick-up problems including emergency evacuation situations. In this study, we consider (a) as the problem investigated is a delivery problem.

For example, Figure 1 illustrates the difference between the VRP, the min-sum CCVRP and the min-max CCVRP of a single depot. The objective value for the VRP is 72, while the min-sum CCVRP is 163 and the min-max CCVRP is 21. The figure indicates that the total waiting times for all customers is 163 units of time. It is also shown in the figure that it takes 21 units of time for the vehicle in R_1 to arrive at its last customer. Note that for the VRP, the objective value remains unchanged if the route is considered in reversed direction but not for the case of CCVRP. For CCVRP, the direction of the route is important as a reversed route gives different result as will be explained in Section 2.

Our previous study (Sze et al., 2017) suggests that the solution of the classical VRP is poor in terms of the min-sum and min-max objective values, thus it is not suitable to be used when the arrival time at the customer point is crucial. In managerial decision making, the min-sum would be preferable if one aims to reduce the overall customers' arrival times. On the other hand, if the situation requires that the last customer must be served before the deadline, especially for the case where there is a high penalty for lateness, the min-max objective would provide promising and practical results. Therefore we intend to investigate both the min-sum and the min-max MDSDCCVRP in this research and compare the results of these two.

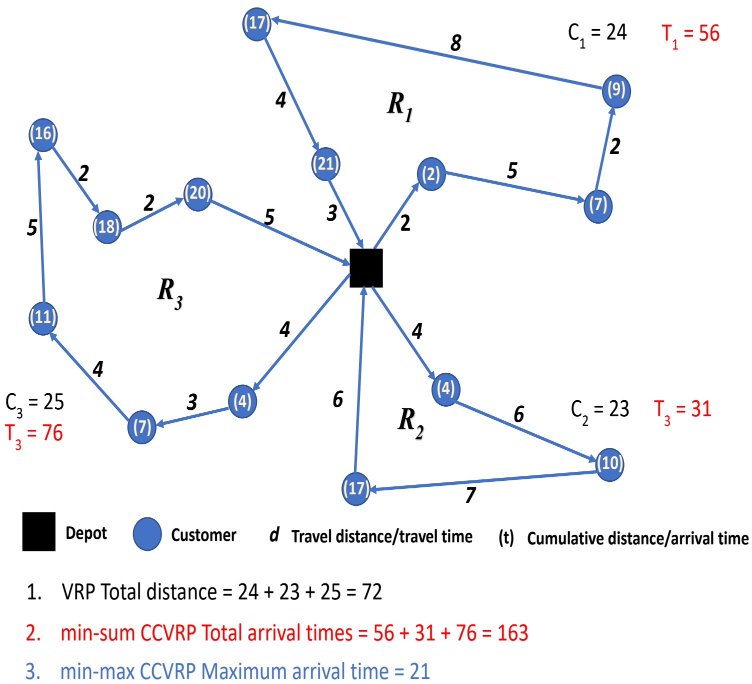

Figure 1: An illustration example highlighting the difference between the objective values for the VRP, min-sum CCVRP and min-max CCVRP.

The innovative aspects of this project lie in three points. First, the MDSDCCVRP is different from most VRP variants as it considers **the minimisation of customers' waiting times** while others focus on the reduction of the total cost. Second, we are the **first to introduce the MDSDCCVRP and to propose the mathematical formulations and exact solution methods for the problem (both min-sum and min-max versions). Third, the metaheuristic model proposed for the problem is also state-of-the-art with the incorporation of multiple intelligent aspects** such as learning, data structure and effective local search operators. The model will be tested on solving the medical supplies transportation problem in Wuhan during the COVID-19 outbreak.

The contribution of this research is threefold:

(i) To introduce the mathematical formulations for both the min-sum and the min-max MDSDCCVRP and to solve them optimally using the exact method incorporating some powerful valid inequalities;

(ii) To propose an effective metaheuristic algorithm for both the min-sum and the min-max MDSDCCVRP;

(iii) To test the mathematical formulations and the proposed metaheuristic method on solving a real data set of the medical supplies transportation problem in Wuhan during the COVID-19 outbreak.

2 Explanation of the MDSDCCVRP

Here, we provide the explanation for the two variants of MDSDCCVRP studied in this project, namely the min-sum and the min-max versions.

2.1 The min-sum MDSDCCVRP

Given a set of vehicles at multiple depots and a group of customers located at distinct locations with demand values, the objective of the MDSDCCVRP is to determine the routes for the vehicles to visit each of the customers at least once in which the total arrival times at the customers is minimised. Split deliveries can occur, both within the vehicles dispatched from the same or different depots. In addition, if the demand of a customer is larger than the vehicle capacity, a split is necessary for the order to be satisfied. For the case when customer's demand is smaller than the vehicle capacity, split deliveries allow customers to be served more than once and the number of routes and total arrival times at the customers may be minimised. Considering the transportation of medical supplies in Wuhan during the COVID-19 outbreak, it is a typical problem with multiple depots (warehouses) and each customer (medical facility) receives goods from one or more warehouses (i.e. split delivery).

Figure 2 illustrates an example of the MDSDCCVRP with three depots and 13 customers. The vehicle capacity is 200 and the total demands of all customers is 600, three vehicles are needed in this case (i.e. $\lceil \frac{600}{200} \rceil = 3$). It is shown that split deliveries occur at customers A and C. The demand of customer A is split between depots 1 and 2 whereas the demand of customer C is shared between depots 1 and 3. The delivery of customer A is split because its demand exceeds the vehicle capacity of 200. On the other hand, demand C is split for better serving of both vehicles 1 and 2.

There are three important properties of the CCVRP that make the problem different from the VRP, as highlighted next.

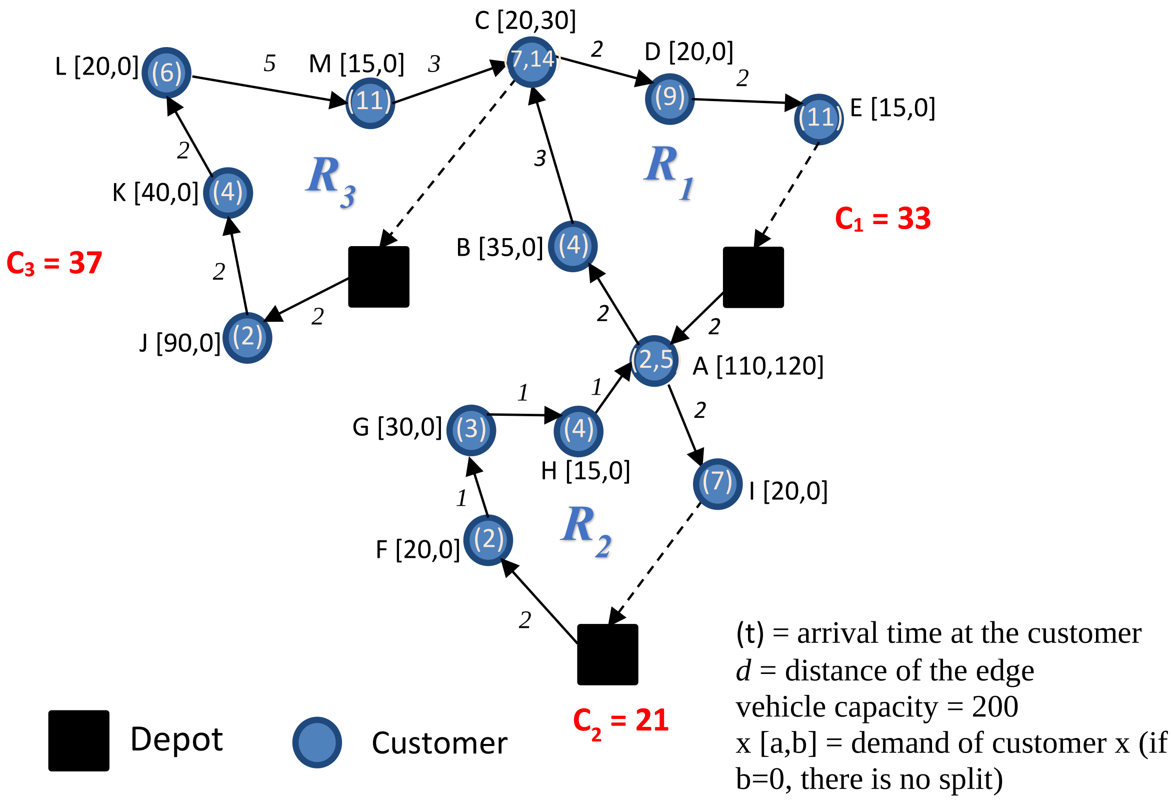

Figure 2: An illustration example of the MDSDCCVRP.

***Property 1*: The increase of $|R|$ results in a smaller objective function value**

One important difference between the classical VRP and the CCVRP lies in the effect of the number of routes, $|R|$ on the solution value of the problems. For the former, because of the triangular inequality, a lower cost is achieved when a group of customers is served by a single vehicle under the same route before it returns to the depot. However for the CCVRP, it is better to send one vehicle directly to each customer, so that the total arrival times at the customers is minimised. Figure 3 provides an example of the effect of the number of routes on the solution value for the two variants, assuming that each edge has a unit distance.

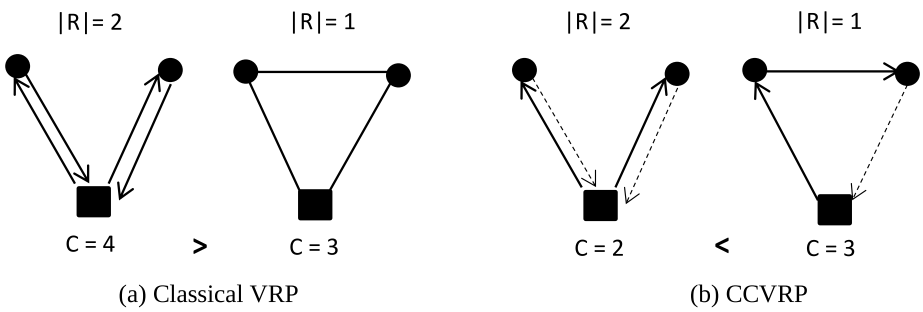

Figure 3: The effect of the number of routes in the classical VRP and the CCVRP.

Let n be the total number of customers, when there are enough vehicles such that $|R| \geq n$, the optimal solution for the CCVRP is obtained by assigning one customer directly to each route so that they can be served at the earliest time possible. On the other hand, when $|R| < n$, some customers need to be grouped together and served on the same route. Therefore, it is clear that the objective function value decreases when the number of routes increases and vice versa. This property imposes a big challenge in the split delivery case as split will reduce the number of routes but the objective values of the CCVRP will then be increased. For this reason, the number of routes for the MDSDCCVRP needs to be fixed to make sure that all customers' orders can be fulfilled but the objective value remains reasonable.

***Property 2*: There are exactly min$(|R|, n)$ routes for the CCVRP optimal solution**

For the case when there is an unlimited number of vehicles available, the number of routes in the optimal solution of the CCVRP is simply the total number of customers, n. Obviously for any variants of the VRP, it is impractical to set the routes to be equal to the number of customers. Therefore, we will initially define the number of routes $|R|$ using the lower bound: $|R| = \lceil \sum_{i \in V'} q_i/Q \rceil$ where q_i is the demand for customer i , Q is the vehicle capacity and V' is the customer set .

***Property 3*: The cost of a route could be different when the route is reversed**

The third property of the CCVRP is that the cost of a route may be different when it is reversed. This is because the arrival time at a customer is affected by all the preceding customers in the route. For example in Figure 4, the original cost of the route is 43, and increased to 97 when the route is inverted. Besides, a reduced cost is obtained when the shorter edges are visited first followed by the longer ones as illustrated in the same figure. It is therefore necessary to compute the cost of the reversed route to check whether or not the route is worth reversing.

2.2 The min-max MDSDCCVRP

The min-max MDSDCCVRP is a related variant of the min-sum version in which the objective is the minimisation of the arrival time at the last customer while other constraints remain the same. In other words, the min-max objective aims to minimise the route with the largest cost. To better differentiate the three variants of the min-sum, the min-max MDSDCCVRP and the MDSDVRP, the cost of a route is defined as follows:

(a) min-sum MDSDCCVRP – the total cumulative distances (i.e., sum of arrival times) at the customers excluding the last vertex (the depot),

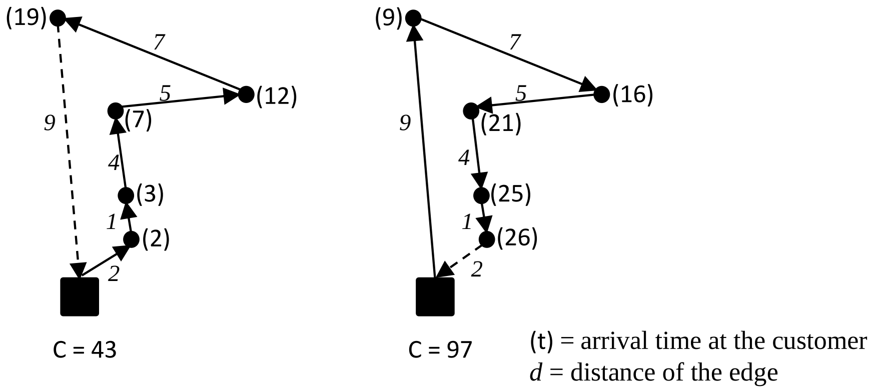

Figure 4: An example of a reversed route in the CCVRP.

(b) min-max MDSDCCVRP – the maximum distance travelled (i.e., the arrival time at the last customer) excluding the return trip from the last customer to the depot,

(c) min-max MDSDVRP – the maximum distance travelled for the complete round-trip (i.e., the arrival time at the depot).

It is worth noting that the min-max MDSDCCVRP is different from the min-max MDSDVRP in terms of the inclusion of the last edge in the route. In other words, the former aims to reduce the cumulative distance at the last customer whereas the latter intends to minimise the total distance of the longest route including the return trip from the last customer back to the depot. In the context of humanitarian relief, it is more important to deliver aid supplies promptly to the victims than sending the vehicles back to the depot timely.

Since the return trip to the depot is excluded, the min-max MDSDCCVRP can also be seen as the counterpart of the min-max open MDSDVRP, although the number of vehicles is predefined for the former. In addition, the open routes (the min-max MDSDCCVRP) instead of the closed ones (the min-max MDSDVRP) may affect the solutions to some extent.

To better illustrate the difference between the min-max MDSDCCVRP and the min-max MDSDVRP, an example is provided in Figure 5. Here, route R_2 is the longest for the former, and route R_1 for the latter. Therefore, the min-max CCVRP intends to reduce the arrival time at the last customer (customer 'b') in route R_2.

Note that another version of the min-max MDSDCCVRP would be the minimisation of the route with the largest sum of arrival times (i.e., the largest cost for the min-sum MDSDCCVRP). However in practice, it is more meaningful to minimise the arrival time at the last customer as defined in min-max MDSDCCVRP than the route with the largest total cumulative cost as in the min-sum version. For example in Figure 5, for

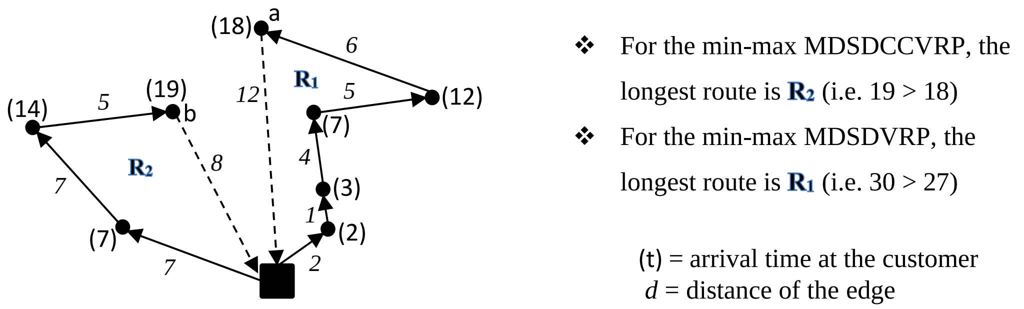

Figure 5: An illustration example highlighting the difference between the min-max MDSDCCVRP and the min-max MDSDVRP.

the case of the min-sum MDSDCCVRP, the longest route is R_1 (sum of arrival times for $R_1 = 42$ versus 40 for route R_2). If the aforementioned version is adopted, route R_1 will be chosen. Nevertheless, the vehicle arrival time at the last customer in route R_1 (customer 'a') is 18, which is smaller than the last customer in route R_2 (arrival time at customer 'b' = 19). This implies that customer 'b' actually waits longer than customer 'a'. In terms of practicality, the min-max MDSDCCVRP considered in this project aims to minimise the arrival time of the vehicle at the last customer, which is consistent with the one discussed by Campbell et al. (2008).

3 The Proposed Research Methodology

The proposed research methodology can be divided into three phases, namely (i) introducing of the mathematical formulation and solving using the exact approach, (ii) developing of the metaheuristic model to solve the problem and (iii) testing both the mathematical formulation and metaheuristic model on the transportation of medical supplies in Wuhan COVID-19 data. The details of these phases are explained here.

3.1 Mathematical formulation for the MDSDCCVRP

The MDSDCCVRP is defined on a complete directed graph $G = (V, E)$, where $V = V' \cup D$ is the vertex or node set that contains all customer and depot nodes. Vertex set $D = \{1, 2, ..., m\}$ represents the set of m uncapacitated depots whereas vertex set $V' = \{1, 2, ..., n\}$ represents the customers to be served. The edge set $E(i, j) : i, j \in V, i < j$ denotes the edges connecting all the vertices. Each edge $(i, j) \in E$ is associated with a distance d_{ij}. It is assumed that the distance is equivalent to the travel time and cost, therefore they are used interchangeably throughout this proposal. It is assumed that travel times are symmetric and satisfy the triangle inequality.

Each customer $i \in V'$ is associated with a non-negative demand, q_i and can be visited more than once. The limited set of homogeneous vehicles is defined by R and each vehicle has capacity Q. The quantity of the demand of customer i delivered by the