CH5440- Assignment - 2

CH18 B035

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3. a)

$$\bar{x} = \begin{bmatrix} 9 \\ 68 \\ 129 \end{bmatrix} \qquad S = \begin{bmatrix} 7 & 21 & 34 \\ 21 & 64 & 102 \\ 34 & 102 & 186 \end{bmatrix}$$

$\lambda_1 = 250.4$ $\lambda_2 = ?$ $\lambda_3 = ?$

$v_1 = ?$ $v_2 = ?$ $v_3 = ?$

$Sv_1 = \lambda_1 v_1$

let v_1 be $\begin{pmatrix} a \\ b \\ 1 \end{pmatrix}$

$7a + 21b + 34 = 250.4a$

$21a + 64b + 102 = 250.4b$

on solving.

$a = 0.1887$

$b = 0.5684$

$$\Rightarrow v_1 = \begin{pmatrix} 0.1887 \\ 0.5684 \\ 1 \end{pmatrix}$$

normalising

$$\Rightarrow v_1 = \begin{pmatrix} 0.1618 \\ 0.4876 \\ 0.8579 \end{pmatrix}$$

$Tr(S) = 7 + 64 + 186 = 250.4 + \lambda_2 + \lambda_3$

$|S| = 146 = \lambda_2 \times \lambda_3 \times 250.4$

$\lambda_2 + \lambda_3 = 6.6$

$\lambda_2\lambda_3 = 0.583$

$\Rightarrow \lambda_2 = 6.51$ $\lambda_3 = 0.089$

let v_2 be $\begin{pmatrix} a' \\ b' \\ 1 \end{pmatrix}$ let v_3 be $\begin{pmatrix} a'' \\ b'' \\ 1 \end{pmatrix}$

$S v_2 = \lambda_2 v_2$

$7a' + 21b' + 34 = 6.51a'$

$21a' + 64b' + 102 = 6.51b'$

$a' = -0.4538$

$b' = -1.608$

$$\begin{pmatrix} -0.4538 \\ -1.608 \\ 1 \end{pmatrix}$$

Normalising

$$\begin{pmatrix} -0.233 \\ -0.8258 \\ 0.5135 \end{pmatrix}$$

$S v_3 = \lambda_3 v_3$

$7a'' + 21b'' + 34 = 0.089a''$

$21a'' + 64b'' + 102 = 6.51b''$

$a'' = -44.96$

$b'' = 13.17$

$$\begin{pmatrix} -44.96 \\ 13.17 \\ 1 \end{pmatrix}$$

Normalising

$$\begin{pmatrix} -0.959 \\ 0.281 \\ 0.021 \end{pmatrix}$$

$\lambda = \quad 250.4 \quad 6.51 \quad 0.089$

$$v = \begin{bmatrix} 0.1618 & -0.233 & -0.959 \\ 0.4876 & -0.8258 & 0.281 \\ 0.8579 & 0.5135 & 0.021 \end{bmatrix}$$

b) atleast 95%

$$\text{Variance} = \frac{\sum^{k} \lambda_i}{\sum^{n} \lambda_i} = \frac{250.4}{250.4 + 6.51 + 0.089} = 97\%$$

$\Rightarrow$ 1 PC should be retained to maintain 95% of varaince.

c) One possible set of these linear relations

$\Rightarrow$ Corresponding to lowest two eigen values.

$$-0.959(m-9) + 0.281(SVL-68) + 0.021(HLS-129) = 0$$

$$\boxed{0.959m - 0.281\,SVL + 0.021\,HLS = -13.2068} \quad -(i)$$

$$-0.233(m-9) - 0.8258(SVL-68) + 0.5135(HLS-129) = 0$$

$$\boxed{0.233m + 0.8258\,SVL - 0.5135\,HLS = -7.99} \quad -(ii)$$

2 pairs

d) $X = [10.1 \quad 73 \quad 135.5]'$

$X_s = X - \bar{X} = [1.1 \quad 5 \quad 6.5]'$

$$X_s = \begin{bmatrix} 1.1 \\ 5 \\ 6.5 \end{bmatrix} \qquad v_1 = \begin{bmatrix} 0.1618 \\ 0.4876 \\ 0.8579 \end{bmatrix}$$

Scores Corresponding to largest eigen vector $= [1.1 \quad 5 \quad 6.5] \begin{bmatrix} 0.1618 \\ 0.4876 \\ 0.8579 \end{bmatrix}$

$= 8.192.$

$T_{score} = 8.192$

E) Using i & ii SVL = 73mm

$$0.959m - 0.281\,SVL - 0.021\,HLS = -13.186$$

$$0.233m + 0.8258\,SVL - 0.5135\,HLS = -7.99$$

$$0.959m - 0.021\,HLS = 7.327$$

$$0.233m - 0.5135\,HLS = -68.27$$

$$\boxed{m = 10.66\,gm \quad HLS = 137.84}$$

F) SVL = 73mm HLS = 135.5mm

Using Relation corresponding to Lowest eigen vector

$$0.959m - 0.281(73) - 0.021(135.5) = -13.186$$

$$\boxed{m = 10.607\,gm}$$