Problem Set 4: Analysis of the 2004 Sumatra-Andaman earthquake
Part 1: Instrument response and spectral analysis

GEOS 626: Applied Seismology, Carl Tape

Assigned: February 8, 2016 — Due: February 22, 2016

Last compiled: October 16, 2017

The goals of this problem set are:

1. to practice deconvolving the instrument response from a raw seismogram recorded in "counts"
2. to introduce you to the frequency dependence of the seismic wavefield, especially with regard to seismological investigations of earthquake sources and Earth structure

Instructions

- Make sure you have done the lab (`lab_response.pdf`).
- The main scripts you will use are
  - `CAN_response_template.m`
  - `CAN_noise_template.m`
  - `CAN_P_template.m`
  - `CAN_bp_template.m`

  Make a copy of each of these files. The label "CAN" is for a station at Canberra, Australia, featured in *Park et al.* (2005, Figure 1).
- The Global CMT catalog is at `www.globalcmt.org`
- Background reading:
  - instrument response and Fourier analysis: *Stein and Wysession* (2003, Ch. 6)
  - Sumatra earthquake: (*Lay et al.*, 2005; *Ammon et al.*, 2005; *Park et al.*, 2005; *Ni et al.*, 2005)
  - PDFs of all referenced Sumatra papers can be found in one of two directories:

    `/home/admin/databases/SUMATRA/papers/`
    `/home/admin/databases/SUMATRA/papers/SCIENCE_2005/`
- In your answers, please include lines of your computer code if:
  - you are unable to get the code "to work"
  - you think that the code lines are useful for conveying your solution

Problem 1 (0.5). Complex numbers and functions

- The best way to think of complex numbers is to sketch a circle (radius r) in the complex plane, with the x-axis the real component and the y-axis the imaginary component.

$$
\begin{aligned}
z &= re^{i\theta} \\
z &= (r\cos\theta) + i(r\sin\theta) \\
z &= a + bi
\end{aligned}
$$

- The forward and inverse Fourier transforms are defined as as[1]

$$
\begin{aligned}
\mathcal{F}\left[h(t)\right] &= H(\omega) = \int_{-\infty}^{\infty} h(t)\, e^{-i\omega t}\, \mathrm{d}t && (1) \\
\mathcal{F}^{-1}\left[H(\omega)\right] &= h(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} h(\omega)\, e^{i\omega t}\, \mathrm{d}\omega\ . && (2)
\end{aligned}
$$

- A complex-valued function, such as the instrument response, is written as

$$
\begin{aligned}
I(\omega) &= A(\omega)\, e^{i\phi(\omega)} && (3) \\
&= A(\omega)\cos\phi(\omega) + i\, A(\omega)\sin\phi(\omega) && (4)
\end{aligned}
$$

  The first version is conceptually more useful, since $I(\omega)$ is represented as two real-valued functions for amplitude $A(\omega)$ and phase $\phi(\omega)$.

  $A(\omega)$ is clearly the amplitude in the sense that

$$
|I(\omega)| = \left(I(\omega)I(\omega)^*\right)^{1/2} = \left(A(\omega)\, e^{i\phi(\omega)} A(\omega)^*\, e^{-i\phi(\omega)}\right)^{1/2} = A(\omega) \tag{5}
$$

1. (0.1) Let z be a complex number and z^* its complex conjugate. Derive the following expressions in two different ways: (1) $z = re^{i\theta}$, (2) $z = a + bi$.

   (a) $z_1 z_2$ (derive new r and θ)

   (b) z_1/z_2 (derive new r and θ)

   (c) $(z_1 z_2)^* = z_1^* z_2^*$

   (d) $|z| = \sqrt{z^* z}$

2. (0.1)

   (a) Using integration by parts ($\int u\,dv = [u\,v] - \int v\,du$), show that

$$
\mathcal{F}\left[\dot{h}(t)\right] = i\omega\, \mathcal{F}\left[h(t)\right]\ , \tag{6}
$$

   where $\dot{h}(t) = dh/dt$.

   Hint: You can assume that $h(t)$ is zero at $\pm\infty$. This is a reasonable assumption for a seismogram: the signal is at the noise level before the earthquake, and long after the earthquake, the effects of attenuation bring the signal back down to the noise level.

   (b) What are the units of $H(\omega)$?

[1]These are the Fourier conventions used in *Dahlen and Tromp* (1998, p. 109) and *Stein and Wysession* (2003, Section 6.4.2).

3. (0.1) Equation (6) holds for the first derivative of $h(t)$, which we denote as either $\dot{h}(t)$ or $h^{(1)}(t)$. The expression can be generalized for the nth derivative, which is (using two different notations)

$$\mathcal{F}\left[h^{(n)}(t)\right] = (i\omega)^n \mathcal{F}[h(t)] \tag{7}$$

$$H_n(\omega) = (i\omega)^n H(\omega) \tag{8}$$

Let the amplitude spectrum of $H(\omega)$ be $A(\omega)$,

$$A(\omega) = |H(\omega)|, \tag{9}$$

and the amplitude spectrum of $H_n(\omega)$ be $A_n(\omega)$.

Starting with Equation (8), show that

$$A_n(\omega) = \omega^n A(\omega) \tag{10}$$

4. (0.1) Amplitude spectra are almost always plotted in $\log_{10}$-$\log_{10}$ space, with $\log_{10}\omega$ on the x-axis and $\log_{10} A$ on the y-axis. We will use the natural log and ignore the unnecessary complication of using log-base-10. We will derive the relationship between

   (a) the slope in log-log space of an amplitude spectrum, $A(\omega)$, for an input function $h(t)$

   (b) the slope in log-log space of an amplitude spectrum, $A_n(\omega)$, for an input function $h^{(n)}(t)$, where $h(t) = h^{(0)}(t)$ and $h^{(n)}(t)$ is the nth derivative of $h(t)$

   The key step is to make the coordinate substitution

$$u = \ln\omega, \tag{11}$$

   then consider the function $g_n(u)$ that represents $\ln A_n(\omega)$. Recognizing that $\omega = e^u$ (and $\omega^n = e^{nu}$), the function we are interested in is obtained from taking the log of $A_n(\omega) = \omega^n A(\omega)$:

$$g_n(u(\omega)) = \ln A_n(\omega) = \ln[\omega^n A(\omega)] \tag{12}$$

$$g_n(u) = \ln[e^{nu} A(e^u)] \tag{13}$$

$$= \ln(e^{nu}) + \ln(A(e^u)) \tag{14}$$

$$= nu + g_0(u) \tag{15}$$

$$g_n'(u) = n + g_0'(u) \tag{16}$$

   where the prime ($'$) represents differentiation with respect to u.

   (a) Describe the meaning of Equation (16), starting with $h(t)$.

   (b) Amplitude spectra are often represented by piecewise linear functions in log-log space (e.g., *Stein and Wysession*, 2003, Figs. 6.3-6 and 6.6-8). Consider a linear spectrum (alternatively, this could be thought of as one segment of a spectrum).

$$\ln A = m \ln\omega + b. \tag{17}$$

   with slope m. Let $A(\omega) = A_0(\omega)$ represent the amplitude spectrum for a displacement time series, $h(t)$. Let $A(\omega)$ be flat ($m = 0$).

   What is the slope of the amplitude spectrum for velocity, $A_1(\omega)$, in log-log space?

   Hint: Write Equation (17) as $g_0(u)$.

(c) Now let $A(\omega) = A_0(\omega)$ have slope $m = -2$ in log-log space.
What is the slope of the acceleration spectrum, $A_2(\omega)$, in log-log space?

Problem 2 (3.5). Deconvolving the instrument response

1. (0.0) Run `CAN_response.m`. The first time you run this, it will compute and save the Fourier transform of a seismogram. This will take about 5 minutes.

   Note: make sure that you run this from your `seis2016` directory, since the file will be saved there.

2. (0.5) Analyze the raw 10-day time series of the Sumatra earthquake recorded at station CAN (Canberra, Australia), which is shown in Figure 1. We will use the notation $c(t)$ (c is "counts") to represent the time series.

   (a) (0.0) What is the duration of the seismogram?

   (b) (0.0) What is the time step?

   (c) (0.0) What is the Nyquist frequency?

   (d) (0.0) Zoom in on the y-limits with `ylim(3e4*[-1 1])`
   What is the approximate period (in seconds and in days) of the most conspicuous oscillation?

   (e) (0.1) Find the largest aftershock in the second half of the record. Use the example code to extract the absolute time, then list the location, magnitude, and origin time of the event from a catalog search in `www.globalcmt.org`.
   Hint: the magnitude is $M_\mathrm{w} > 6$.

   (f) (0.0) Does this aftershock have the same mechanism as the mainshock?

   (g) (0.4) In summary, identify and interpret the main features in the seismogram.
   Hint: noise is a feature.

3. (1.0)

   (a) (0.3) Plot the full amplitude spectrum, $|C(\omega)|$; use `axis tight` to see the full limits.

   (b) (0.3) Plot $|C(\omega)|$ using log-log scaling (`loglog`).
   Label the frequency window $[0.2, 1.0]$ mHz (note mHz, not Hz).

   (c) (0.2) Interpret the largest-amplitude spikes in the spectrum.

   (d) (0.1) What is the maximum allowable frequency and why?

   (e) (0.1) What is the minimum (non-zero) allowable frequency and why?
   What is the corresponding period?

4. (0.8)

   (a) (0.3) Plot the amplitude spectrum (do *not* use any log scaling) $|C(\omega)|$ over the frequency window $[0.2, 1.0]$ mHz.

   (b) (0.2) Label each of the peaks that you see (e.g., *Park et al.*, 2005).

   (c) (0.2) Why are some of the peaks "split" but one is not?

   (d) (0.1) What is the significance of some of the smallest peaks that you identified?

5. (0.4) Using the same frequencies that you used to define the full $C(\omega)$, evaluate the instrument response to acceleration, $I_a(\omega)$. Hint: see lab on instrument response.

   Plot $I_a(\omega)$ as an amplitude spectrum over the full range of frequencies. Use log-log scaling.

6. (0.4) The raw seismogram is a convolution of the ground acceleration, $x_a(t) = \ddot{x}(t)$, with the instrument response, $i_a(t)$. Written in the time domain and frequency domain, this is:

$$x_a(t) * i_a(t) = c(t) \tag{18}$$

$$X_a(\omega) I_a(\omega) = C(\omega) \tag{19}$$

   (a) (0.3) Deconvolve (or "remove") the instrument response from the raw spectral seismogram to obtain the spectral acceleration. Show your lines of code and plot of $|X_a(\omega)|$.

   (b) (0.1) Plot $|X_a(\omega)|$ over the frequency range $[0.2, 1.0]$ mHz (again, no log scaling), and compare it with $|C(\omega)|$. What is the effect of the deconvolution on the relative amplitudes of the peaks?

7. (0.4) Does your spectrum $|X_a(\omega)|$ look different from the one in *Park et al.* (2005)? Why might this be the case? In other words, what "choices" were taken that could influence some of the details in the spectrum?

Problem 3 (2.0). Spectral analysis of noise

Run `CAN_noise.m`. This will generate a time series at CAN for 10 days prior to the Sumatra earthquake.

1. (0.2) Using the GCMT catalog, list the magnitude, location, and origin time of the conspicuous event.

2. (0.3) Extract **a multi-day time series that does not contain any earthquake-like signals**. Include this plot. What is the dominant period?

   Tip: use the command `plot(w,'xunit','h')` to plot in hours.

3. (1.5)

   (a) (1.0) Now use your earthquake-free time series. Following the procedure in Problem 2, deconvolve the instrument response. Plot the amplitude spectrum of acceleration using log-log scaling and the frequency limits 10^{-4} Hz to 10^{1} Hz.

   Tip: If you want a smoother spectral curve, try something like

   `Habs_smooth = 10.^smooth(log10(Habs),1000,'moving');`

   where `Habs` is the amplitude spectrum $|H(\omega)|$.

   (b) (0.2) On your plot, mark the ocean microseismic period ranges 5–8 s and 10–16 s that are mentioned in *Shearer* (2009, Section 11.2).

   (c) (0.2) Qualitatively, how does your spectrum compare with Figure 11.6 of *Shearer* (2009)?

   (d) (0.1) Based on this observation, and considering all possible periods, what period range at CAN will provide the highest signal-to-noise ratio?

Problem 4 (4.0). Spectral analysis of the Sumatra earthquake recorded at Canberra

- For this problem we can analyze the raw records, since we are interested here in the basic characteristics of the amplitude spectra, but not the actual values. In other words, you do not need to deconvolve the instrument response.
- Note: For this problem, make sure you are not loading any pre-stored spectra; here the spectra can be computed quickly on the fly.

1. (1.5) **Direct arrival**. Return to `CAN_response.m` in Problem 2. Analyze the direct arrival by extracting (use `extract`) 1 hour before the centroid origin time to 3 hours after, where the centroid origin time is
   `otime_cmt = 7.323070424652778e5` (serial days in Matlab).
   (a) (0.7) Plot the amplitude spectrum (log-log) and mark the two ocean microseism frequency intervals.
   (b) (0.2) What is the approximate frequency and period associated with the maximal amplitude?
   (c) (0.3) Discuss how your plot compares with the noise spectrum from Problem 3.
   (d) (0.3) Discuss how your plot compares with the spectrum from Problem 2.
   In your 3-hour spectrum can you identify any of the normal modes in the range $[0.2, 1.0]$ mHz?
2. (1.5) **P wave**. Adapt `CAN_P.m` to answer the following questions.
   (a) (0.7) Using Supplemental Figure S11 of *Ammon et al.* (2005) as a guide, extract "the" P-wave and include a plot. About how long is the P wave signal?
   (b) (0.4) Plot the amplitude spectrum of the P wave on a log-log plot with frequency ranging from 10^{-3} Hz to 10^{1} Hz. (Make sure you are working with the BHZ data.)
   (c) (0.4) If you had to estimate a corner frequency (*Stein and Wysession*, 2003, p. 267), what would it be? Justify your estimate in words or numbers.
3. (1.0) **High-frequency bandpass**. Adapt `CAN_bp.m` to answer the following questions.
   (a) (0.8) Estimate the rupture duration by analyzing the high-pass filtered seismogram at CAN, after *Ni et al.* (2005). Include your plot and explain your interpretation.
   If you want to prepare for a future homework problem, then follow the filtering steps in *Ni et al.* (2005) by using `filtfilt`, `hilbert`, and `smooth` in this order.
   (b) (0.2) What else do you notice in the filtered seismogram?

Problem

Approximately how much time *outside of class and lab time* did you spend on this problem set? Feel free to suggest improvements here.

References

Ammon, C. J., et al. (2005), Rupture process of the 2004 Sumatra-Andaman earthquake, *Science*, *308*, 1133–1139.

Dahlen, F. A., and J. Tromp (1998), *Theoretical Global Seismology*, Princeton U. Press, Princeton, New Jersey, USA.

Lay, T., et al. (2005), The great Sumatra-Andaman earthquake of 26 December 2004, *Science*, *308*, 1127–1133.

Ni, S., D. Helmberger, and H. Kanamori (2005), Energy radiation from the Sumatra earthquake, *Nature*, *434*, 582.

Park, J., et al. (2005), Earth's free oscillations excited by the 26 December 2004 Sumatra Andaman earthquake, *Science*, *308*, 1139–1144.

Shearer, P. M. (2009), *Introduction to Seismology*, 2 ed., Cambridge U. Press, Cambridge, UK.

Stein, S., and M. Wysession (2003), *An Introduction to Seismology, Earthquakes, and Earth Structure*, Blackwell, Malden, Mass., USA.

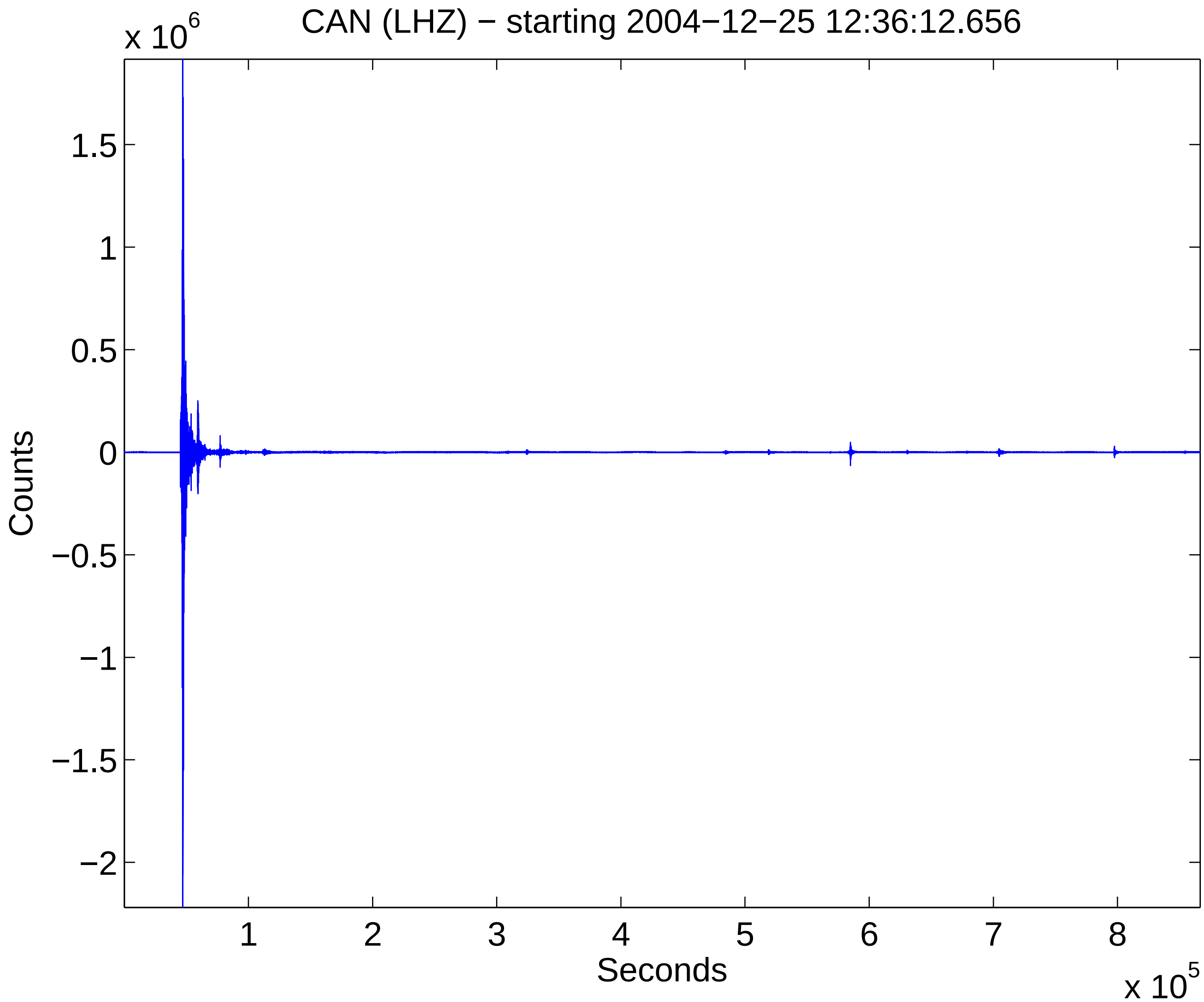

Figure 1: Figure from `CAN_response_template.m`.