Problem Set 6: Analysis of the 2004 Sumatra-Andaman earthquake
Part 2: Analyzing the effects of rupture complexity and Earth heterogeneity

GEOS 626: Applied Seismology, Carl Tape

Assigned: February 29, 2016 — Due: March 9, 2016

Last compiled: October 16, 2017

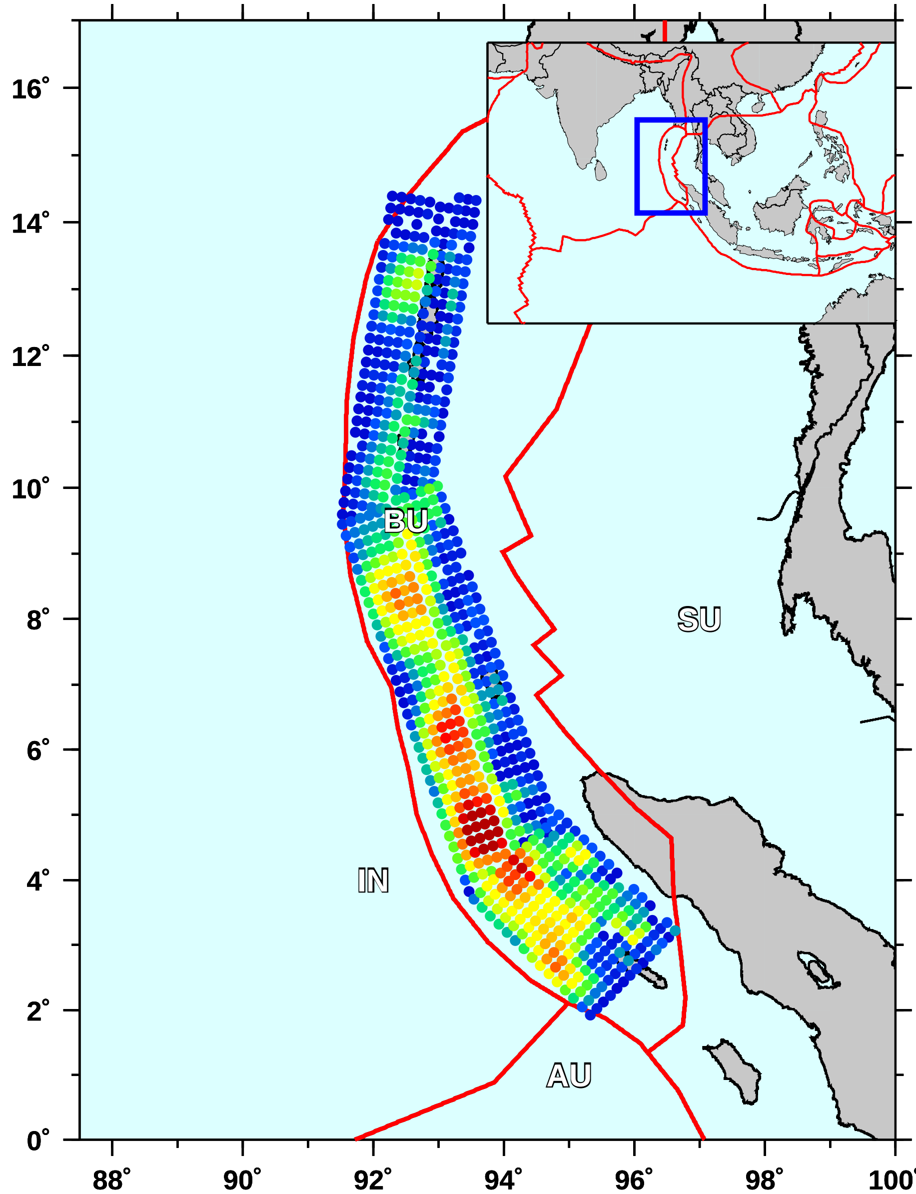

Figure 1: Rupture model for the 2004-12-26 M_{w} 9.2 Sumatra earthquake. This model, produced by Chen Ji, is a modified version of the one originally presented as Model III in *Ammon et al.* (2005, Figure 5c). The color corresponds to the seismological moment associated with each patch (red = large). Plate boundaries are from *Bird* (2003): AU = Australia, IN = India, BU = Burma, SU = Sunda.

Overview and Instructions

The purpose of this problem set is to handle a large data set of seismograms and to extract some useful scientific information about the earthquake source (and Earth structure). As you will see, much of the challenge is in representing the seismic waveforms in a sensible manner; this includes judging which seismograms are "bad," that is, not representative of the ground motion. A key part of this representation is determining what bandpass to use for the seismograms for each scientific question.

- This problem set utilizes some of scripts in `GEOTOOLS` for analyzing sets of waveforms. See `doc_startupB.pdf` to set up `GEOTOOLS`.
- Make sure you have completed:
  1. `lab_seismo_rs.pdf`, the lab for getting waveforms and plotting record sections. Be familiar with what `getwaveform.m` and `plotw_rs.m` are doing. Be sure you understand what a bandpass filter is.
  2. `lab_response.pdf`, the lab on instrument response
  3. `lab_sumatra.pdf`, the lab on sumatra waveforms
- The main scripts you will use are in your `seis2016` directory:
  - `sumatra_modes_template.m`
  - `sumatra_hf_template.m`
  - `sumatra_lf_template.m`
- Background reading:
  - instrument response and Fourier analysis: *Stein and Wysession* (2003, Ch. 6)
  - directivity: *Stein and Wysession* (2003, Section 4.3.2)
  - Sumatra earthquake: *Lay et al.* (2005); *Ammon et al.* (2005); *Park et al.* (2005); *Ni et al.* (2005); *Stein and Okal* (2007)
  - normal modes: *Stein and Wysession* (2003, Section 2.9) and *Dahlen and Tromp* (1998, Ch. 8). See also "Computational details" in Section 10.5.1 of DT.
  - PDFs of all referenced Sumatra papers can be found in one of two directories:

    `/home/admin/databases/SUMATRA/papers/`
    `/home/admin/databases/SUMATRA/papers/SCIENCE_2005/`
- To get a better feel for normal mode eigenfunctions, check out this website by Ruedi Widmer–Schnidrig:

  `http://stuplots.geophys.uni-stuttgart.de/~plots/Modes/modes.html`

  To get a better feel for spherical harmonic functions, try out `ylmplots.m`.

- To find the index of a certain station with an array of stations names `sta`, try something like this:

  ```
  imatch = find(strcmp('SBA',sta)==1)
  ```

  Or try out the commands `wkeep` and `wcut`.

Problem 1 (4.0). Splitting of normal mode frequencies

The template script is `sumatra_modes_template.m`. See note [1].

1. (0.5) Recall the modes spectrum that we computed for station CAN that emulated Figure 1 of *Park et al.* (2005). Having done the homework on toroidal modes, you should now have a better understanding of what the modes peaks are.

   Run `modes_PREMobs.m` to see the observed modes that were used in constructing PREM back in 1980 (*Dziewonski and Anderson*, 1981). Keep in mind that these observations were identified from dozens of different earthquakes and hundreds of different stations. The dispersion plot (Figure 2) identifies the frequency range 0.2–1.0 mHz that is used in Figure 1 of *Park et al.* (2005), which is a spectrum of the **vertical component** of ground motion for one station for one earthquake.

   (a) (0.1) List the toroidal modes, 0.2–1.0 mHz, that were observed in 1980.
   List them in increasing frequency.

   (b) (0.1) List the spheroidal modes, 0.2–1.0 mHz, that were observed in 1980.

   (c) (0.1) Describe the differences between what was observed in 1980 and what is observed in Figure 1 of *Park et al.* (2005).

   (d) (0.2) Using Ruedi Widmer–Schnidrig's interactive website (above), check out the eigenfunctions for ${}_0\mathsf{S}_2$, ${}_1\mathsf{S}_2$, and ${}_2\mathsf{S}_2$. Describe the differences qualitatively. Explain two reasons why ${}_2\mathsf{S}_2$ will be difficult to observe.

2. (0.5) Run `sumatra_modes.m` and examine the stations that were cut from the analysis. Describe three "errors" in these records, and list at least one station associated with each error.

3. (0.5)

   (a) (0.4) Set `iload = 0` in `sumatra_modes.m`, then modify `ipick` (or use `wkeep.m`) and plot the stations you selected in `lab_sumatra.pdf`. To save paper, use something like a 5×4 subplot.

   (b) (0.1) Compute the amplitude of the ${}_0\mathsf{S}_0$ peak for all your stations.
   List the median value.

[1] In this problem, our spectra are computed from the calibration-applied seismograms; the complete instrument response has not been deconvolved, as we did in the earlier homework. This should not impact our main findings, since we are not computing the *relative* amplitudes between modes within a given spectrum.

4. (0.5) "Stacking" refers to summing similar functions in order to enhance the signal-to-noise ratio. Use the function `w2fstack.m` to generate a stack of your spectra.

   (a) (0.3) Include a plot of the stacked spectrum over two ranges: 0.2–10 mHz and 0.2–1.0 mHz.

   (b) (0.2) Qualitatively, describe the meaning of this spectrum, e.g., what is the physical explanation for the spikes? It may be helpful to recall the modes homework.

5. (0.5) Several peaks in the Sumatra spectrum are clearly split; the central peak is the "degenerate" frequency ($m = 0$), and the split peaks are known as "singlets" ($m = -l, \ldots, l$).

   (a) (0.2) Use `w2fstack.m` to generate a stack of ${}_0\mathsf{S}_2$, and include this plot.

   Hint: Use a new frequency range.

   (b) (0.1) Label the peaks ${}_0\mathsf{S}_2^{-2}$, ..., ${}_0\mathsf{S}_2^{2}$ on the plot.

   (c) (0.2) Measure the frequency of the central peak, ${}_0f_2^0$ ($m = 0$), and the spacing between peaks, Δf. Assuming a linear model, write an expression for the singlet frequency ${}_0f_2^m(m)$.

6. (0.5) Make a plot with ${}_0\mathsf{S}_2$ spectra sorted by station latitude.

   Qualitatively, how does the relative sizes of the singlet peaks vary as a function of latitude?

   See *Stein and Okal* (2007, Figure 3) for additional background.

7. (0.5) Now consider the variation in ${}_0\mathsf{S}_2$ with source-station distance.

   (a) (0.3) What pattern might you expect to see? (Hint: Think about the nodal lines for this mode.)

   (b) (0.2) Make a plot with ${}_0\mathsf{S}_2$ spectra sorted by source-station distance. Qualitatively, how does the relative sizes of the singlet peaks vary as a function of source-station distance?

8. (0.5) Table 5 of *Dziewonski and Anderson* (1981) lists the observed frequencies of spheroidal modes; these are also read in by `modes_PREMobs.m`.

   (a) (0.2) See if you can identify the peaks for ${}_0\mathsf{S}_0$, ${}_1\mathsf{S}_0$, and ${}_2\mathsf{S}_0$ in the stacked Sumatra spectrum.

   Hint: You will want to zoom into certain regions of the stacked spectrum.

   (b) (0.3) What is special about these peaks/modes and why?

   See *Stein and Wysession* (2003, p. 106) for background.

Problem 2 (4.0). Directivity

1. (0.0) Examine the `sumatra_hf_template.m` and make sure you understand how to do certain operations for plotting record sections. It would be helpful to check what each parameter in `plotw_rs.m` does (type `open plotw_rs`).

2. (1.0) We will repeat the analysis of *Ni et al.* (2005) but using even more simplifications than they did. **Read *Ni et al.* (2005) carefully before you begin.** We will assume the following:

   - The Earth is flat.
   - The travel time between the fault and any station is encapsulated with the simple velocity $v = 11$ km/s. This is the mean velocity for stations between $30^\circ < \Delta < 85^\circ$ of the source, assuming Jeffrey-Bullen P travel times and arc distances (not distances along the P wave ray path).
   - All stations are "far" from the fault, such that the distance from the station to the starting point is approximately equal to the distance from the station to the stopping point.

   The apparent rupture time as measured on a seismogram is given by (*Stein and Wysession*, 2003, Section 4.3.2)

   $$T_r(\alpha) = L\left(\frac{1}{v_r} - \frac{\cos(\alpha - \alpha_0)}{v}\right) \tag{1}$$

   where v_r is the rupture velocity, v is the velocity of the medium (11 km/s), L is the fault length, and α is the azimuth to the station, and α_0 is the rupture direction. In answering the following questions, note that **no numbers are needed (only algebra).**

   (a) (0.1) What is the actual rupture time?

   (b) (0.1) What α will produce a T_r that is the actual rupture time?

   (c) (0.1) What α will produce a minimum $T_r(\alpha)$, $T_{\min}$?

   (d) (0.1) What α will produce a maximum $T_r(\alpha)$, $T_{\max}$?

   (e) (0.1) The range of T_r is given by $T_{\max} - T_{\min}$.
   What is the range, considering variations in α only?

   (f) (0.2) What is $\overline{T}_r$, the azimuthal average of T_r?
   Hint: Integration is needed.

   (g) (0.3) Show that, with our assumptions, Eq. 1 can be written in terms of only $T_{\min}$, $T_{\max}$, α, and α_0.
   Hint: Equation (1) is an equation with 6 unknowns: T_r, L, v_r, v, α, α_0. Your equations for $T_{\min}$ and $T_{\max}$ give you two additional equations with two additional unknowns ($T_{\min}$, $T_{\max}$). You are asked to write an equation with 5 unknowns (including T_r). Therefore you start with a system of 3 equations with 8 unknowns, and you can

reduce this to 1 equation with 5 unknowns. This is algebra, so no numbers should appear anywhere.

3. (1.2) Modify `sumatra_hf_template.m` to reproduce *Ni et al.* (2005, Figure 1d), and include a plot of your record section.

   Notes:

   - See the earlier homework on these waveforms.
   - Note that the epicentral distances must be $30° < \Delta < 85°$, so specify `stasub = deg2km([30 85])` when requesting waveforms.
   - Use at least 10 stations with high-quality seismograms. (Think about what stations would be needed to reproduce the plot in *Ni et al.* (2005). Be sure to include CAN from before.)
   - See the template code for how to align seismograms.
   - You do not need to compute the smoothed envelopes—the bandpassed seismograms are enough. (Though computing the envelopes is easy to do; see previous homework solutions.)

4. (0.4) List three stations with values near T_{min} and three stations with values near T_{max}.

   List the station name, distance (Δ), azimuth (α) for each set.

5. (0.4) Using your data, estimate T_{min} and T_{max}, then calculate the following quantities:

   (a) rupture direction, α_0

   (b) rupture time, $\overline{T}_r$ (show work)

   (c) rupture length, L (show work)

   (d) rupture velocity, v_r (show work)

6. (1.0)

   (a) (0.4) Modify `sumatra_lf_template.m` to generate a record section that shows that the directivity can also be inferred from the relative amplitudes of R1 and R2, the "minor orbit" and "major orbit" Rayleigh wave arrivals[2]. See Figure S1 of *Ammon et al.* (2005) as a guide.

   Note: You may find the command `extract` and `max` useful for quantifying the amplitude ratio, though quantification is not required. If you do quantify the ratios, use the log-scaled quantity $\ln(A_{R1}/A_{R2})$, where A_{R1} is the amplitude of the filtered R1 wave.

   (b) (0.3) Explain your results.

   (c) (0.3) Is your rupture direction (α_0) different for the high-frequency P wave estimate than it is for the long-period Rayleigh wave estimate. Why might this be the case? Hint: Think about Figure 1.

[2]See Figure 2.7-3 of *Stein and Wysession* (2003).

Problem 3 (2.0). Miscellaneous

In this problem, you will find it helpful to use an appropriate input for `stasub` so that `getwaveform.m` will return only a subset of stations/waveforms.

1. (1.0) Modify `sumatra_lf_template.m` to show the maximal displacement (not velocity) of some "near-source" stations. See Figure S12 of *Ammon et al.* (2005) for reference and checking.

   (a) (0.4) Include plots of seismograms (or a record section) for calibration applied (`iprocess=1`).

   (b) (0.4) Include plots of seismograms (or a record section) for instrument deconvolved (`iprocess=2`).

   (c) (0.2) How different are the waveform shapes and amplitudes?

   (d) (0.0) On the basis of your results, can you infer whether the seismograms in *Ammon et al.* (2005) were instrument deconvolved or not?

   Notes:

   - Note: Let us assume that *Ammon et al.* (2005) used the label H1 to denote the E channel (nominally east) and H2 to denote the N channel (nominally north).
   - Converting to displacement is trivial (see `plotw_rs.m` or `getwaveform.m`). If you use `plotw_rs.m`, then you may want to output the modified (i.e., filtered) `w` to analyze.

2. (0.5)

   (a) (0.3) Modify `sumatra_hf_template.m` to reproduce the azimuthal record section of P waves shown in Figure S11 of *Ammon et al.* (2005). (Plot with one column, not two.) Order the seismograms starting with the direction of the rupture direction (`azstart` parameter in `plotw_rs.m`). Hint: The variable `nfac` will change the amplitudes of the plotted waveforms.

   (b) (0.2) Qualitatively describe the variations in the P waves as a function of azimuth from the rupture direction.

3. (0.5)

   Modify `sumatra_hf_template.m` to filter the BHZ time series from 4–8 Hz. I suggest loading the entire data set and browsing all the stations, one seismogram at a time. Recall what we saw for CAN in the earlier homework.

   (a) (0.3) Show a plot and identify other stations (if any) that exhibit high-frequency, post-rupture signals that are similar to those identified at CAN.

   (b) (0.2) Is there anything systematic about the other stations?
   Provide your interpretation of these signals.

Problem

Approximately how much time *outside of class and lab time* did you spend on this problem set? Feel free to suggest improvements here.

References

Ammon, C. J., et al. (2005), Rupture process of the 2004 Sumatra-Andaman earthquake, *Science*, *308*, 1133–1139.

Bird, P. (2003), An updated digital model of plate boundaries, *Geochem. Geophy. Geosyst.*, *4*, 1027, doi:10.1029/2001GC000252.

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Dziewonski, A., and D. Anderson (1981), Preliminary reference Earth model, *Phys. Earth Planet. Inter.*, *25*, 297–356.

Lay, T., et al. (2005), The great Sumatra-Andaman earthquake of 26 December 2004, *Science*, *308*, 1127–1133.

Ni, S., D. Helmberger, and H. Kanamori (2005), Energy radiation from the Sumatra earthquake, *Nature*, *434*, 582.

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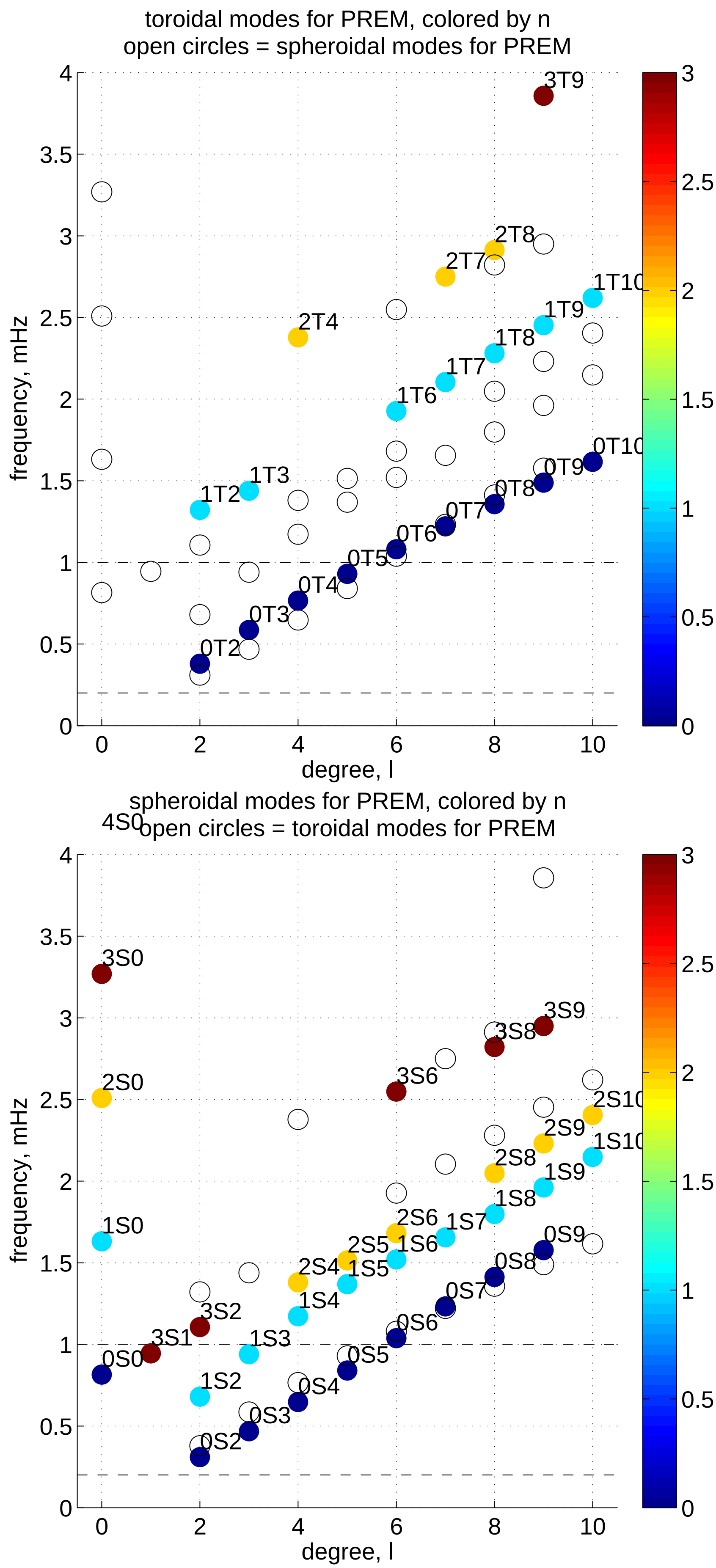
Stein, S., and M. Wysession (2003), *An Introduction to Seismology, Earthquakes, and Earth Structure*, Blackwell, Malden, Mass., USA.



(a

(b

Figure 2: Observations of (a) toroidal and (b) spheroidal modes used in making PREM (*Dziewonski and Anderson*, 1981). The horizontal dashed lines mark the frequency range 0.2–1.0 mHz. These figures are generated from `modes_PREMobs.m`.