Spring 2015 Week 2 Module 8

Digital Circuits and Systems

Karnaugh Maps

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Karnaugh Maps

- Truth tables are a convenient form to represent equations but they don’t aid in the simplification of logic equations.
- *Karnaugh maps* (*K-maps*) are similar to TT’s and they lead to graphical methods for boolean expression simplification.
- A K-map is a multi-dimensional tabulation of function values.
- Each *minterm* is assigned an entry (a *cell*) in the table. The cell contains the value of the function for the corresponding minterm.

- 1 variable K-map: $f(a)$

	a	
	0	1
	m_0	m_1

OR

	0	m_0
a	1	m_1

- 2 variable K-map: f(a,b)

f(a,b) a \ b		b	
		0	1
	0	m_{00}	m_{01}
a	1	m_{10}	m_{11}

OR

f(a,b) b \ a		a	
		0	1
	0	m_{00}	m_{10}
b	1	m_{01}	m_{11}

- 3-variable K-map: *f(a,b,c)*

				b	b
	f(a,b,c) a \ bc	00	01	11	10
	0	m_{000}	m_{001}	m_{011}	m_{010}
a	1	m_{100}	m_{101}	m_{111}	m_{110}
			c	c	

OR

				a	a
	f(a,b,c) c \ ab	00	01	11	10
	0	m_{000}	m_{010}	m_{110}	m_{100}
c	1	m_{001}	m_{011}	m_{111}	m_{101}
			b	b	

- <u>4-variable K-map:</u> *f(a,b,c,d)*

	f(a,b,c,d) ab \ cd	00	01	11 (c)	10 (c)	
	00	m_{0000}	m_{0001}	m_{0011}	m_{0010}	
	01	m_{0100}	m_{0101}	m_{0111}	m_{0110}	b
a	11	m_{1100}	m_{1101}	m_{1111}	m_{1110}	b
a	10	m_{1000}	m_{1001}	m_{1011}	m_{1010}	
			d	d		

OR

	f(a,b,c,d) cd \ ab	00	01	11 (a)	10 (a)	
	00	m_{0000}	m_{0100}	m_{1100}	m_{1000}	
	01	m_{0001}	m_{0101}	m_{1101}	m_{1001}	d
c	11	m_{0011}	m_{0111}	m_{1111}	m_{1011}	d
c	10	m_{0010}	m_{0110}	m_{1110}	m_{1010}	
			b	b		

K-map Example 1

$$f = a + bc + \bar{d}$$

$f(a,b,c,d)$ ab \ cd	00	01	11	10
00	$\bar{d}$			$\bar{d}$
01	$\bar{d}$		bc	bc $\bar{d}$
11	a $\bar{d}$	a	a bc	a bc $\bar{d}$
10	a $\bar{d}$	a	a	a $\bar{d}$

$$f = a + bc + \bar{d}$$

$f(a,b,c,d)$ ab \ cd	00	01	11	10
00	1 $\bar{d}$			1 $\bar{d}$
01	1 $\bar{d}$		bc 1	bc 1 $\bar{d}$
11	a 1 $\bar{d}$	a 1	a bc 1	a bc 1 $\bar{d}$
10	a 1 $\bar{d}$	a 1	a 1	a 1 $\bar{d}$

K-map Example 1’

$$f = a + bc + \overline{d}$$

$f(a,b,c,d)$

cd \ ab	00	01	11	10
00	$\overline{d}$	$\overline{d}$	a $\overline{d}$	a $\overline{d}$
01			a	a
11		bc	a bc	a
10	$\overline{d}$	bc $\overline{d}$	a bc $\overline{d}$	a $\overline{d}$

Note: This K-map is drawn by swapping the placement of variable pairs ab and cd

$$f = a + bc + \overline{d}$$

$f(a,b,c,d)$ cd \ ab	00	01	11	10
00	1 $\overline{d}$	1 $\overline{d}$	a 1 $\overline{d}$	a 1 $\overline{d}$
01	0	0	a 1	a 1
11	0	bc 1	a bc 1	a 1
10	1 $\overline{d}$	bc 1 $\overline{d}$	a bc 1 $\overline{d}$	a 1 $\overline{d}$

K-map Example 2

$$f(w,x,y,z) = w\bar{z} + x\bar{y} + \bar{x}$$

$f(w,x,y,z)$

wx \ yz	00	01	11	10
00				
01				
11				
10				

K-map Example 2

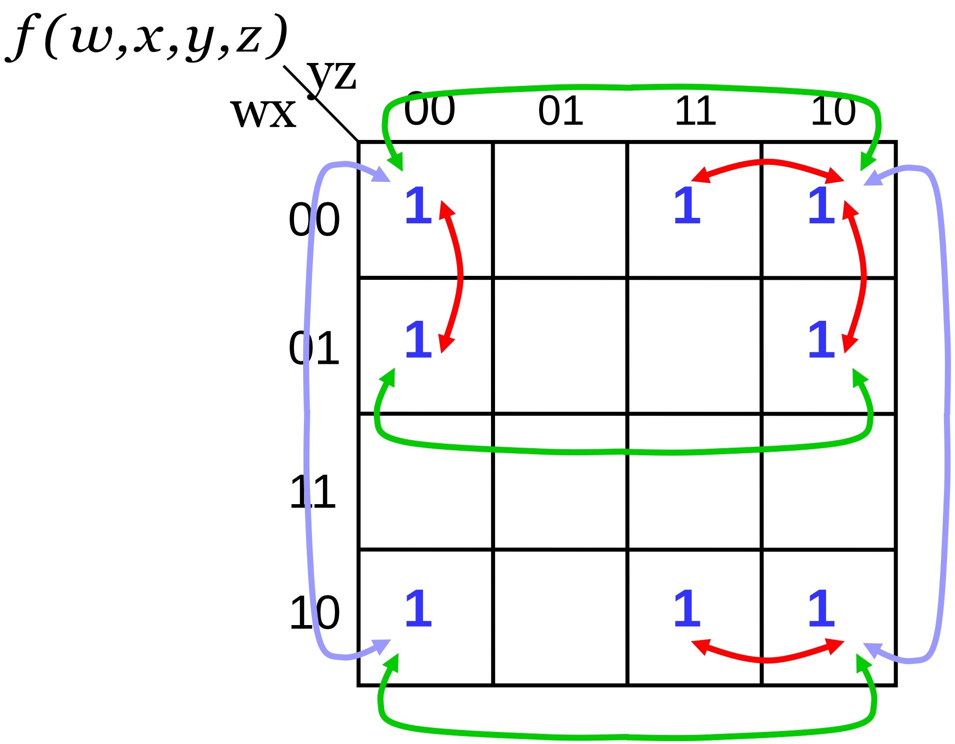
$$f(w,x,y,z) = w\bar{z} + x\bar{y} + \bar{x}$$

$f(w,x,y,z)$ wx \ yz	00	01	11	10
00	1 $\bar{x}$	1 $\bar{x}$	1 $\bar{x}$	1 $\bar{x}$
01	$x\bar{y}$ 1	$x\bar{y}$ 1		
11	$w\bar{z}$ $x\bar{y}$ 1	$x\bar{y}$ 1		$w\bar{z}$ 1
10	$w\bar{z}$ 1 $\bar{x}$	1 $\bar{x}$	1 $\bar{x}$	$w\bar{z}$ 1 $\bar{x}$

K-Map Properties

- Minterms mapped to any *two adjacent cells differ in exactly one bit* position

<u>Example</u> $f(w,x,y,z) = \sum(0,2,3,4,6,8,10,11)$



- The *sum of two minterms in adjacent cells* can be simplified to a single *product (AND) term with one less variable*.

<u>Example</u>

$f(a,b,c)$

a \ bc	00	01	11	10
0	1		1	1
1	1			1

If we combine adjacent minterms in the first column we get,

$$\bar{a}\bar{b}\bar{c} + a\bar{b}\bar{c} = \bar{b}\bar{c}\left(\bar{a} + a\right) = \bar{b}\bar{c}$$

That is, variable a is eliminated.

Example

$f(a,b,c)$

a \ bc	00	01	11	10
0	1		1	1
1	1			1

If we combine adjacent minterms like what is shown above we get,

$$\bar{a}bc + \bar{a}\,b\,\bar{c} = \bar{a}b\left(\bar{c} + c\right) = \bar{a}b$$

That is, variable c is eliminated.

End of Week 2: Module 8

Thank You