Spring 2015 Week 2 Module 7

Digital Circuits and Systems

Universality, Rearranging Truth Tables

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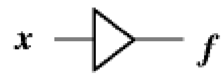
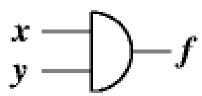
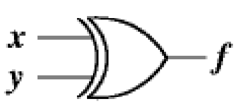
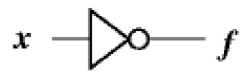
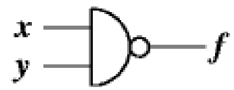
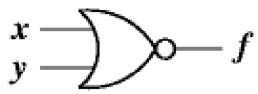
Summary of Digital Logic Gates

Gate	Schematic Symbol	Algebraic Function	Truth Table
BUFFER		$f = x$	x f 0 0 1 1
AND		$f = xy$	x y f 0 0 0 0 1 0 1 0 0 1 1 1
OR		$f = x + y$	x y f 0 0 0 0 1 1 1 0 1 1 1 1
XOR		$f = x \oplus y$	x y f 0 0 0 0 1 1 1 0 1 1 1 0

Summary of Digital Logic Gates

Gate	Schematic Symbol	Algebraic Function	Truth Table
NOT (Inverter)		$f = \overline{x}$	x f 0 1 1 0
NAND		$f = \overline{xy}$	x y f 0 0 1 0 1 1 1 0 1 1 1 0
NOR		$f = \overline{x + y}$	x y f 0 0 1 0 1 0 1 0 0 1 1 0
XNOR (Equivalence)		$f = \overline{x \oplus y}$ $= x \odot y$	x y f 0 0 1 0 1 0 1 0 0 1 1 1

AND/OR CIRCUITS

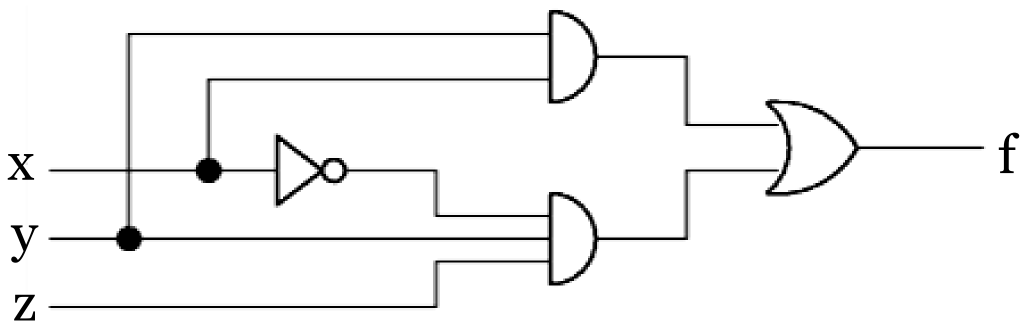
- The simplest type of combinational logic design consists of inverters, AND gates, and OR gates. This is known as an AND/OR circuit.

- An AND/OR circuit can be designed to implement any function by performing the following steps:

  1. Put the expression in SOP form
  2. Form complemented literals with inverters.
  3. Form product terms with AND gates.
  4. Sum the product terms with an OR gate

Example

$$f(x,y,z) = xy + \bar{x}yz$$



Exercise

Implement the function $f(x,y,z) = (x+\bar{y})(y+\bar{z})(x+\bar{x})$ using OR/AND logic

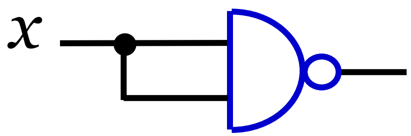
Universality

- All Boolean functions can be implemented using the set {AND, OR, NOT}
- Universal gates
  - Gates which can implement any Boolean function without the need to use any other type of gate
  - NAND and NOR are universal gates
- To show universality of a gate:
  - Show that AND, OR and NOT can be implemented using that gate

NAND Universality

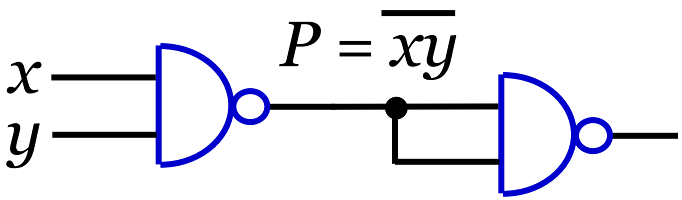
- AND, OR and NOT can be implemented using NAND only

NOT or **INV**



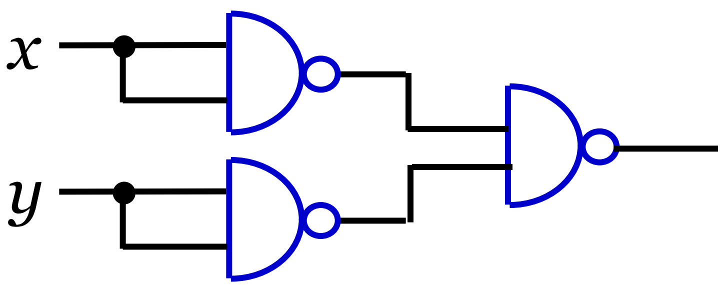
$$F = \overline{x.x} = \overline{x}$$

AND



$$F = \overline{\overline{xy}} = xy$$

OR



$$F = \overline{\overline{x}\,.\,\overline{y}} = \overline{\overline{x}} + \overline{\overline{y}} = x + y$$

Exercises

- Show that NOR gate is a universal gate also

- Is XOR a universal gate?
  - If so, show how {AND, OR, NOT} operations can be done using XOR gates only.
  - If not, show which operations can be done and which cannot be.

Boolean Expression ⇨ Truth Table

- To convert boolean expression to truth table:
  - Expand the expression into the minterms (i.e., canonical SOP form) and enter 1's in truth table rows (or, expand into canonical POS and enter 0's for each maxterm).

Example

$$f(x,y,z) = \overline{z} + yz$$
$$= \overline{z}(\overline{x} + x) + yz$$
$$= \overline{x}\,\overline{z} + yz + x\overline{z}$$
$$= \overline{x}\,\overline{z}(y + \overline{y}) + yz(x + \overline{x}) + x\overline{z}(y + \overline{y})$$
$$= \overline{x}y\overline{z} + \overline{x}\,\overline{y}\,\overline{z} + xyz + \overline{x}yz + xy\overline{z} + x\overline{y}\,\overline{z}$$
$$= \sum(0, 2, 3, 4, 6, 7)$$

x	y	z	f
0	0	0	
0	0	1	
0	1	0	
0	1	1	
1	0	0	
1	0	1	
1	1	0	
1	1	1	

Truth Table ⇨ Boolean Expression

- To convert a truth table to a boolean expression:
  - Write a canonical SOP expression that consists of all minterms (or write a canonical POS using maxterms) and then simplify the algebraic expression.

<u>Example</u>

$$f(x,y,z) = \sum(0,2,3,4,6,7)$$

$$\begin{aligned}
&= \bar{x}\bar{y}\bar{z} + \bar{x}y\bar{z} + \bar{x}yz + x\bar{y}\bar{z} + xy\bar{z} + xyz \\
&= \bar{x}\bar{z}\left(y + \bar{y}\right) + yz\left(x + \bar{x}\right) + x\bar{z}\left(y + \bar{y}\right) \\
&= \bar{x}\bar{z} + yz + x\bar{z} \\
&= \bar{z}\left(\bar{x} + x\right) + yz \\
&= \bar{z} + yz
\end{aligned}$$

Truth tables to Boolean Expression

- When the expressions get more complicated, simplification gets harder
  - You may miss out combinations
- More inputs, more the effort
- Systematic way to reduce effort
  - Karnaugh Maps

Rearranging Truth Tables

x	*y*	*z*	*f*
0	0	0	1
0	0	1	0
0	1	0	1
0	1	1	1
1	0	0	1
1	0	1	0
1	1	0	1
1	1	1	1

Minterms	*f(x,y,z)*
m0	1
m1	0
m2	1
m3	1
m4	1
m5	0
m6	1
m7	1

x\yz	*00*	*01*	*10*	*11*
0	1	0	1	1
1	1	0	1	1

x\yz	*00*	*01*	*11*	*10*
0	1	0	1	1
1	1	0	1	1

In General

x_1 \ x_2x_3	00	01	11	10
0	m_{000}	m_{001}	m_{011}	m_{010}
1	m_{100}	m_{101}	m_{111}	m_{110}

x_1 \ x_2x_3	00	01	11	10
0	m_0	m_1	m_3	m_2
1	m_4	m_5	m_7	m_6

End of Week 2: Module 7

Thank You