Spring 2015 Week 1 Module 5

Digital Circuits and Systems

Algebraic Minimization Examples

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Summary of Minterms and Maxterms

Row number	x_1	x_2	x_3	Minterm	Maxterm
0	0	0	0	$m_0 = \overline{x}_1\overline{x}_2\overline{x}_3$	$M_0 = x_1 + x_2 + x_3$
1	0	0	1	$m_1 = \overline{x}_1\overline{x}_2 x_3$	$M_1 = x_1 + x_2 + \overline{x}_3$
2	0	1	0	$m_2 = \overline{x}_1 x_2\overline{x}_3$	$M_2 = x_1 + \overline{x}_2 + x_3$
3	0	1	1	$m_3 = \overline{x}_1 x_2 x_3$	$M_3 = x_1 + \overline{x}_2 + \overline{x}_3$
4	1	0	0	$m_4 = x_1\overline{x}_2\overline{x}_3$	$M_4 = \overline{x}_1 + x_2 + x_3$
5	1	0	1	$m_5 = x_1\overline{x}_2 x_3$	$M_5 = \overline{x}_1 + x_2 + \overline{x}_3$
6	1	1	0	$m_6 = x_1 x_2\overline{x}_3$	$M_6 = \overline{x}_1 + \overline{x}_2 + x_3$
7	1	1	1	$m_7 = x_1 x_2 x_3$	$M_7 = \overline{x}_1 + \overline{x}_2 + \overline{x}_3$

Conversion between SOP and POS

- Conversion between $\sum$ and $\prod$ representations is easy. Assume an *n*-variable function so the minterm and maxterm lists that represent the function are subsets of {0,1,……,2^n –1}. It can be shown that the minterm indices and maxterm indices are complementary.

  That is, $\boxed{M_i = \overline{m_i}}$

  - *Example*: Assume a 3 variable expression $F(x,y,z)$.

$$\sum(1,4,7) = \prod(0,2,3,5,6)$$
$$m_1 + m_4 + m_7 = M_0 \cdot M_2 \cdot M_3 \cdot M_5 \cdot M_6$$

Algebraic Simplification

- To reduce circuit complexity and to maximize circuit performance, it is often necessary to write algebraic expressions in SOP or POS forms.

- The rules discussed earlier are used to do the simplification.

Example:

- Simplify to SOP form:

$$F(x,y,z) = \left(\overline{x}\left(y+\overline{z}\right)+\overline{z}\right)y$$

$$= \overline{x}\left(y+\overline{z}\right)y+\overline{z}\,y$$

$$= \overline{x}\,y\,y+\overline{x}\,\overline{z}\,y+\overline{z}\,y$$

$$= \overline{x}\,y+\overline{x}\,\overline{z}\,y+\overline{z}\,y\cdot\mathbf{1}$$

$$= \overline{x}\,y+\overline{z}\,y\left(\overline{x}+\mathbf{1}\right)$$

$$= \overline{x}\,y+\overline{z}\,y\cdot\mathbf{1}$$

$$= \overline{x}\,y+\overline{z}\,y$$

$$\boxed{F(x,y,z) = \overline{x}\,y+\overline{z}\,y}$$

Example:

- Write the following to canonical SOP (sum of minterms) form.

$$f(x,y,z) = \overline{x}y + \overline{z}y$$

$$= \overline{x}y(\overline{z} + z) + \overline{z}y(x + \overline{x})$$

$$= \overline{x}y\overline{z} + \overline{x}yz + xy\overline{z} + \overline{x}y\overline{z}$$

$$= \overline{x}yz + \overline{x}y\overline{z} + xy\overline{z}$$

$$= \overline{x}y\overline{z} + \overline{x}yz + xy\overline{z}$$

$$\boxed{f(x,y,z) = \sum(2,3,6)}$$

Example:

- Simplify to POS form:

$$
\begin{aligned}
f(x,y,z) &= xyz + \overline{x}y + xy\overline{z} \\
&= xy\left(z + \overline{z}\right) + \overline{x}y \\
&= xy + \overline{x}y \\
&= \left(x + \overline{x}\right)y \\
&= \mathbf{1} \cdot y \\
&= y
\end{aligned}
$$

Example:

- Simplify to SOP and POS forms.

$$
\begin{aligned}
(ab+c)(b+\bar{c}d) &= ab+ab\bar{c}d+bc+c\bar{c}d \\
&= ab+ab\bar{c}d+bc+0 \\
&= ab\left(1+\bar{c}d\right)+bc \\
&= ab+bc \qquad \ldots\ldots \quad \text{SOP form} \\
&= b\left(a+c\right) \qquad \ldots\ldots \quad \text{POS form}
\end{aligned}
$$

Example:

- Simplify to POS and expand to canonical POS (product of maxterms).

$$f(x,y,z) = xy + \bar{x}z$$
$$= (xy + \bar{x})(xy + z)$$
$$= (x + \bar{x})(y + \bar{x})(x + z)(y + z)$$
$$= (\bar{x} + y)(x + z)(y + z) \quad \text{........} \quad \text{POS form}$$

$$= (\bar{x} + y + z\bar{z})(x + y\bar{y} + z)(x\bar{x} + y + z)$$
$$= (\bar{x} + y + z)(\bar{x} + y + \bar{z})(x + y + z)(x + \bar{y} + z)(x + y + z)(\bar{x} + y + z)$$
$$= (x + y + z)(x + \bar{y} + z)(\bar{x} + y + z)(\bar{x} + y + \bar{z})$$

$$\boxed{f(x,y,z) = \prod(0,2,4,5)} \quad \text{......} \quad \text{Canonical POS form}$$

Example:

- Simplify to SOP form:

$$\overline{\left(\overline{w}+\overline{x}\right)y+z}+wxz = \left(\overline{\left(\overline{w}+\overline{x}\right)y}\cdot\overline{z}\right)+wxz$$

$$= \left(\left(\overline{\left(\overline{w}+\overline{x}\right)}+\overline{y}\right)\overline{z}\right)+wxz$$

$$= \left(\left(wx+\overline{y}\right)\overline{z}\right)+wxz$$

$$= \left(\left(w+\overline{y}\right)\left(x+\overline{y}\right)\overline{z}\right)+wxz$$

$$= \left(wx+w\overline{y}+x\overline{y}+\overline{y}\,\overline{y}\right)\overline{z}+wxz$$

$$= \left(wx\overline{z}+w\overline{y}\,\overline{z}+x\overline{y}\,\overline{z}+\overline{y}\,\overline{z}\right)+wxz$$

$$= wx\left(\overline{z}+z\right)+\overline{y}\,\overline{z}\left(w+x+1\right)$$

$$= wx+\overline{y}\,\overline{z}$$

End of Week 1: Module 5

Thank You