Spring 2015 Week 1 Module 3

Digital Circuits and Systems

Boolean Theorems

*Shankar Balachandran**
Associate Professor, CSE Department
Indian Institute of Technology Madras

*Currently a Visiting Professor at IIT Bombay

DeMorgan's Theorem

$$\overline{(X_1 + X_2 + \ldots + X_n)} = \overline{X_1} \cdot \overline{X_2} \cdot \ldots \cdot \overline{X_n}$$

$$\overline{(X_1 \cdot X_2 \cdot \ldots \cdot X_n)} = \overline{X_1} + \overline{X_2} + \ldots + \overline{X_n}$$

Remember:

$$\overline{X \cdot Y} \neq \overline{X} \cdot \overline{Y}$$

$$\overline{X + Y} \neq \overline{X} + \overline{Y}$$

Proof Techniques (Example 1)

- Prove $x + (y \cdot z) = (x + y) \cdot (x + z)$

$$
\begin{aligned}
\text{RHS} &= x \cdot (x+z) + y \cdot (x+z) \\
&= x \cdot x + x \cdot z + y \cdot x + y \cdot z \\
&= x \cdot (1+z+y) + y \cdot z \\
&= x \cdot (1) + y.z \\
&= x + y.z
\end{aligned}
$$

Fairly Straightforward

Proof Techniques (Example 2)

- Prove $x + x \cdot y = x$

$$x + x \cdot y = x \cdot 1 + x \cdot y$$
$$= x \cdot (1 + y)$$
$$= x \cdot 1$$
$$= x$$

Introduce a constant

Proof Techniques (Example 3)

- Prove $x.y + y.z + x'.z = x.y + x'.z$

$$
\begin{aligned}
\text{LHS} &= x.y + y.z.1 + x'.z \\
&= x.y + y.z.(x+x') + x'.z \\
&= x.y + y.z.x + y.z.x' + x'.z \\
&= x.y\,(1 + z) + (y + 1)\,.x'.z \\
&= x.y + x'.z
\end{aligned}
$$

Introduce a constant and introduce a function

Boolean Techniques (Example 4)

- Simplify $x.y + x.y.z + y.z$

$$
\begin{aligned}
\text{LHS} &= x.y + x.y.z + y.z \\
&= x.y + x.y.z + x.y.z + y.z \\
&= x.y\,(1 + z) + y.z\,(x + 1) \\
&= x.y + y.z \\
&= y.(x+z)
\end{aligned}
$$

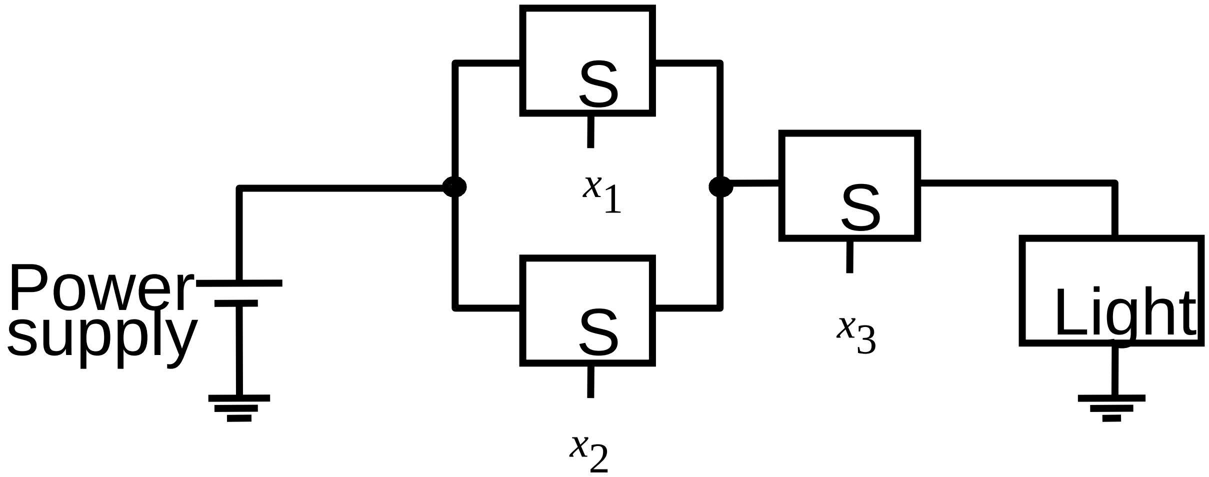
Introduced a copy of a term

A simpler derivation is there. Think about it!

Exercise : Prove the Two Variable Theorems Below

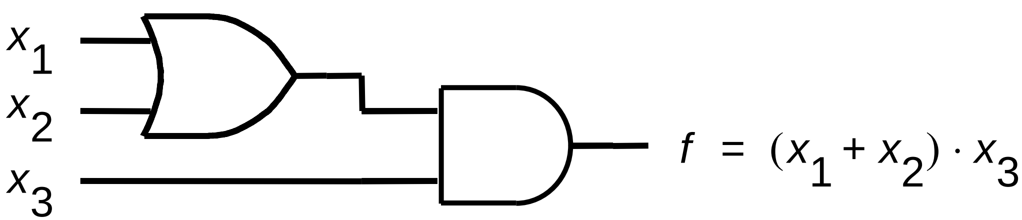
- Absorption
  - $x + (x \cdot y) = x$
  - $x \cdot (x + y) = x$
- Combining
  - $x \cdot y + x \cdot y' = x$
  - $(x + y) \cdot (x + y') = x$
- DeMorgan's Laws (You probably know how to use Venn Diagram to prove this)
  - $(x + y)' = x' \cdot y'$
  - $(x \cdot y)' = x' + y'$
- $x + (x' \cdot y) = x + y$
- $x \cdot (x' + y) = x + y$

Boolean Functions



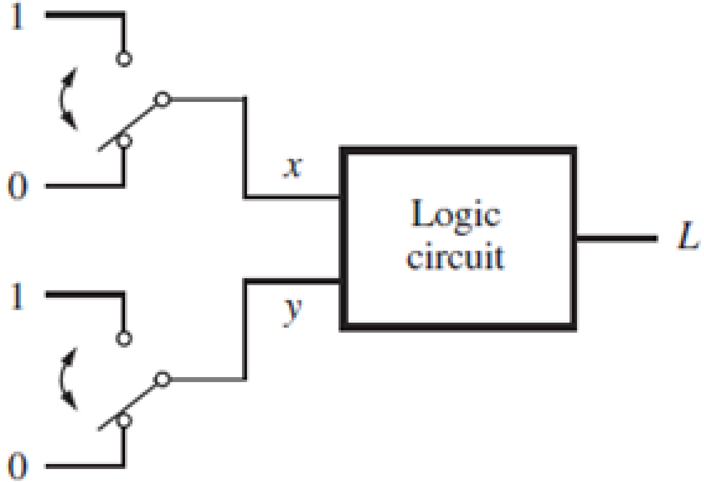
f = (x_1 OR x_2) AND (x_3)

= (x_1 + x_2) . x_3



Design Problem 1

- Design a circuit for staircase light control using a two-way switch
- If both switches are OFF or ON, light is off
- If one of them is ON and another is OFF, the light is ON



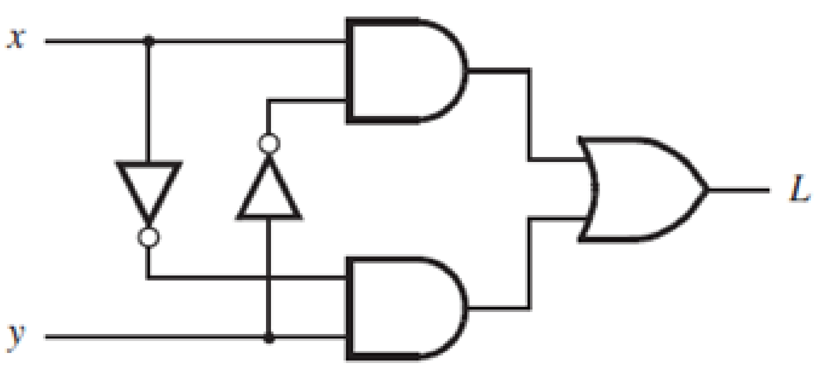
(a) Two switches that control a light

x	y	L
0	0	0
0	1	1
1	0	1
1	1	0

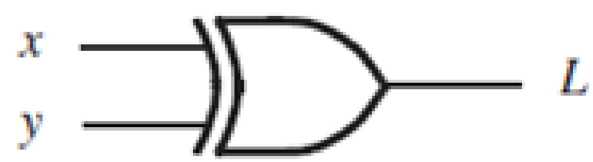
(b) Truth table

Boolean Expression

- Lamp is ON when
  - Either x is ON and y is OFF
  - Or x is OFF and y is ON
- L = x . y’
  + x’ . y



(c) Logic network



(d) XOR gate symbol

Design Problem 2

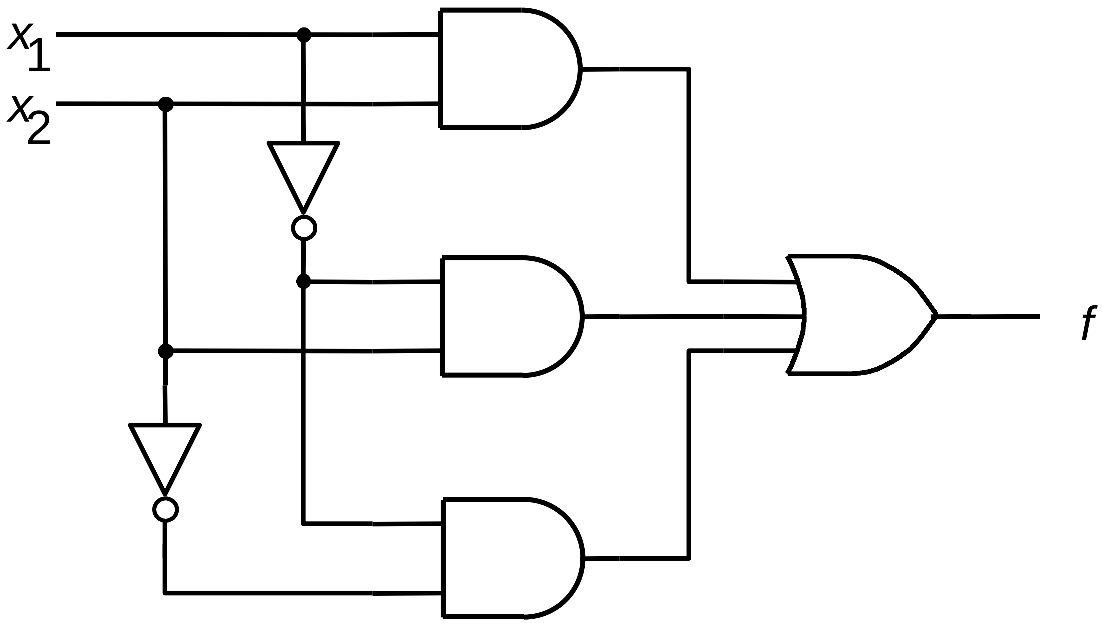
- Design a circuit which realizes the following truth table

x_1	x_2	$f(x_1, x_2)$
0	0	1
0	1	1
1	0	0
1	1	1

f = ?

Design Problem 2 (contd.)

- $f = x_1'.x_2' + x_1'.x_2 + x_1.x_2$



Power of Simplification

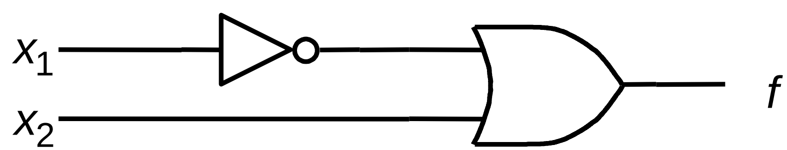
- $f = x_1'.x_2' + x_1'.x_2 + x_1.x_2$

  $= x_1'.x_2' + x_1'.x_2 + x_1'.x_2 + x_1.x_2$

  $= x_1'.(x_2' + x_2) + (x_1' + x_1).x_2$

  $= x_1'.1 + 1.x_2$

  $= x_1' + x_2$



Minimal-cost realization

Positional Number System

- Decimal: Uses base 10

First column starts at "1"

Note: base 10

$\times 10^2$	$\times 10^1$	$\times 10^0$
7	5	6
Hundreds	Tens	Ones

$$= \begin{array}{c} 7 \times 10 \times 10 + \\ 5 \times 10 + \\ 6 \times 1 \end{array}$$

- Binary Number: Uses base 2

Note: base 2

$\times 2^2$	$\times 2^1$	$\times 2^0$
1	1	1
2^2	2^1	2^0

$$= \begin{array}{c} 1 \times 2 \times 2 + \\ 1 \times 2 + \\ 1 \times 1 \end{array}$$

Binary to Decimal

- Because humans work well with decimal, it is useful to know how to convert between binary and decimal:

$0000_2 = 0$	$1000_2 = 8$
$0001_2 = 1$	$1001_2 = 9$
$0010_2 = 2$	$1010_2 = 10$
$0011_2 = 3$	$1011_2 = 11$
$0100_2 = 4$	$1100_2 = 12$
$0101_2 = 5$	$1101_2 = 13$
$0110_2 = 6$	$1110_2 = 14$
$0111_2 = 7$	$1111_2 = 15$

Binary to Decimal (contd.)

8	4	2	1
1	1	0	1

=

1x8 + 1x4 + 1x1 =
8 + 4 + 1 =
13

To convert to decimal, represent the column values in decimal

128	64	32	16	8	4	2	1
1	0	0	1	1	1	0	1

=

1x128 + 1x16 + 1x8 + 1x4 + 1x1 =
128 + 16 + 8 + 4 + 1 =
157

End of Week 1: Module 3

Thank You