Spring 2015 Week 1 Module 4

Digital Circuits and Systems

Minterms, Maxterms SoP and PoS forms

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Some Definitions

- A ***literal*** is a complemented or uncomplemented boolean variable.
  - *Examples:* a and $\bar{a}$ are distinct literals. $\bar{a}+cd$ is not.

- A ***product term*** is a single literal or a logical product (AND) of two or more literals.
  - *Examples:* $a, \bar{a}, ac, \bar{a}cd, aa\bar{a}b$ are product terms; $\bar{a}+cd$ is not a product term.
- A ***sum term*** is a single literal or a logical sum (OR) of two or more literals.
  - *Examples:* $a, \bar{a}, a+c, \bar{a}+c+d$ are sum terms; $\bar{a}+cd$ is not a sum term.

Some Definitions

- A ***normal term*** is a product or sum term in which no variable appears more than once.
  - *Examples:* $a, \bar{a}, a+c, \bar{a}cd$ are normal terms; $\bar{a}+a, \bar{a}a$ are not normal terms.
- A ***minterm*** of *n* variables is a normal product term with *n* literals. There are 2^n such product terms.
  - *Examples of 3-variable minterms:* $\bar{a}bc, abc$
  - *Example:* $\bar{a}b$ is not a 3-variable minterm.
- A ***maxterm*** of *n* variables is a normal sum term with *n* literals. There are 2^n such sum terms.
  - *Examples of 3-variable maxterms:* $\bar{a}+b+c, a+b+c$

Some Definitions

- A ***sum of products (SOP)*** expressions is a set of product (AND) terms connected with logical sum (OR) operators.
  - *Examples:* $a, \bar{a}, ab+c, \bar{a}c+bde, a+b$ are SOP expressions.

- A ***product of sum (POS)*** expressions is a set of sum (OR) terms connected with logical product (OR) operators.
  - *Examples:* $a, \bar{a}, a+b+c, (\bar{a}+c)(b+d)$ are POS expressions.

Some Definitions

- The ***canonical sum of products (CSOP)*** form of an expression refers to rewriting the expression as a sum of minterms.
  - *Examples for 3-variables:* $\bar{a}bc + abc$ is a CSOP expression; $\bar{a}b + c$ is not.
- The ***canonical product of sums (CPOS)*** form of an expression refers to rewriting the expression as a product of maxterms.
  - *Examples for 3-variables:* $(\bar{a}+b+c)(a+b+c)$ is a CPOS expression; $(\bar{a}+b)c$ is not.
- There is a close correspondence between the truth table and minterms and maxterms.

DeMorgan's Theorem (revisited)

$$\overline{(X_1 + X_2 + \ldots + X_n)} = \overline{X_1} \cdot \overline{X_2} \cdot \ldots \cdot \overline{X_n}$$

$$\overline{(X_1 \cdot X_2 \cdot \ldots \cdot X_n)} = \overline{X_1} + \overline{X_2} + \ldots + \overline{X_n}$$

Complement of Sum of Products is equivalent to Product of Complements.

Complement of Product of Sums is equivalent to Sum of Complements.

Minterms

- A minterm can be defined as a product term that is 1 in exactly one row of the truth table.
- *n* variable minterms are often represented by *n*-bit binary integers.

- How to associate minterms with integers?
  - State an ordering on the variables
  - Form a binary number
    - Set bit *i* of the binary number to 1 if the i^{th} variable appears in the minterm in an uncomplemented form
    - Set bit *i* to 0 if the variable appears in the complemented form.

Minterm Examples

- Assume a 3-variable expression,

$$F(x,y,z)=\bar{x}\,\bar{y}\,\bar{z}+\bar{x}\,y\,z+x\,y\,z$$

$$\begin{aligned}
\bar{x}\,\bar{y}\,\bar{z} &= minterm\,000 = m_{000} = m_0\\
\bar{x}\,y\,z &= minterm\,011 = m_{011} = m_3\\
x\,y\,z &= minterm\,111 = m_{111} = m_7
\end{aligned}$$

$$\begin{aligned}
F(x,y,z) &= \bar{x}\,\bar{y}\,\bar{z}+\bar{x}\,y\,z+x\,y\,z\\
&= m_0+m_3+m_7\\
&= \sum(m_0,m_3,m_7)\\
&= \sum(0,3,7)
\end{aligned}$$

Maxterms

- A maxterm can be defined as a sum term that is 0 in exactly one row of the truth table.
- *n* variable maxterms are also represented by *n*-bit binary integers.
- How to associate maxterms with integers?
  - State an ordering on the variables
  - Form a binary number
    - Set bit *i* of the binary number to 0 if the i^{th} variable appears in the maxterm in an uncomplemented form
    - Set bit *i* to 1 if the variable appears in the maxterm in the complemented form.

Maxterm Examples

- Assume a 3-variable expression,

$$F(x,y,z)=(x+y+z)(x+y+\overline{z})(\overline{x}+y+z)$$

$$\begin{aligned}
x+y+z &= max\,term\,000 &= M_{000} &= M_0 \\
x+y+\overline{z} &= max\,term\,001 &= M_{001} &= M_1 \\
\overline{x}+y+z &= max\,term\,100 &= M_{100} &= M_4
\end{aligned}$$

$$\begin{aligned}
F(x,y,z) &= (x+y+z)(x+y+\overline{z})(\overline{x}+y+z) \\
&= M_0 \cdot M_1 \cdot M_4 \\
&= \prod(M_0, M_1, M_4) \\
&= \prod(0,1,4)
\end{aligned}$$

Summary of Minterms and Maxterms

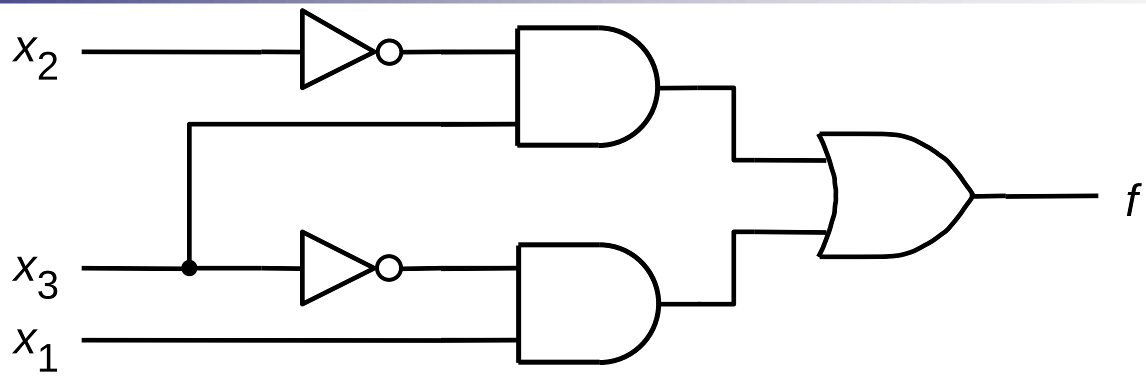
Row number	x_1	x_2	x_3	Minterm	Maxterm
0	0	0	0	$m_0 = \overline{x}_1\overline{x}_2\overline{x}_3$	$M_0 = x_1 + x_2 + x_3$
1	0	0	1	$m_1 = \overline{x}_1\overline{x}_2 x_3$	$M_1 = x_1 + x_2 + \overline{x}_3$
2	0	1	0	$m_2 = \overline{x}_1 x_2\overline{x}_3$	$M_2 = x_1 + \overline{x}_2 + x_3$
3	0	1	1	$m_3 = \overline{x}_1 x_2 x_3$	$M_3 = x_1 + \overline{x}_2 + \overline{x}_3$
4	1	0	0	$m_4 = x_1\overline{x}_2\overline{x}_3$	$M_4 = \overline{x}_1 + x_2 + x_3$
5	1	0	1	$m_5 = x_1\overline{x}_2 x_3$	$M_5 = \overline{x}_1 + x_2 + \overline{x}_3$
6	1	1	0	$m_6 = x_1 x_2\overline{x}_3$	$M_6 = \overline{x}_1 + \overline{x}_2 + x_3$
7	1	1	1	$m_7 = x_1 x_2 x_3$	$M_7 = \overline{x}_1 + \overline{x}_2 + \overline{x}_3$

A Sample Three Variable Function

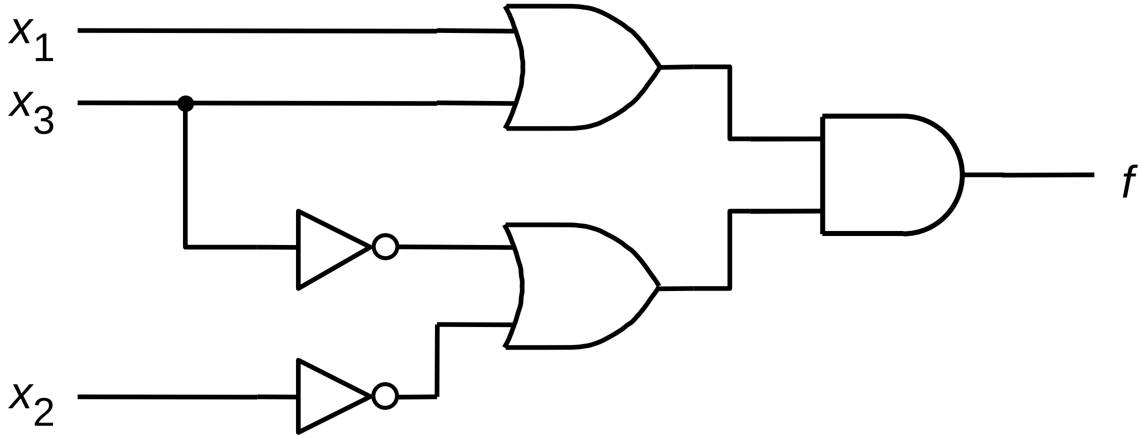
Row number	x_1	x_2	x_3	$f(x_1, x_2, x_3)$
0	0	0	0	0
1	0	0	1	1
2	0	1	0	0
3	0	1	1	0
4	1	0	0	1
5	1	0	1	1
6	1	1	0	1
7	1	1	1	0

$$f(x_1, x_2, x_3) = \sum(m_1, m_4, m_5, m_6) = \sum(1, 4, 5, 6)$$

$$f(x_1, x_2, x_3) = \prod(M_0, M_2, M_3, M_7) = \prod(0, 2, 3, 7)$$



(a) A minimal sum-of-products realization



(b) A minimal product-of-sums realization

End of Week 1: Module 4

Thank You