Q: $x = \begin{bmatrix} 2 \\ 3 \end{bmatrix}$, $A = \begin{bmatrix} a_1 & a_2 \\ a_3 & a_4 \end{bmatrix}$, $f(x) = 0.5 * x^T A x$

Given A is symmetric, prove that $\nabla f(x) = Ax$

<u>Assume</u> $x = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$

$$x^T A = [x_1 \;\; x_2] \begin{bmatrix} a_1 & a_2 \\ a_3 & a_4 \end{bmatrix}$$

$$= [(a_1 x_1 + a_3 x_2) \;\; (a_2 x_1 + a_4 x_2)]$$

$$x^T A x = [(a_1 x_1 + a_3 x_2) \;\; (a_2 x_1 + a_4 x_2)] \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

$$\therefore f(x) = (a_1 x_1^2 + (a_2 + a_3) x_1 x_2 + a_4 x_2^2) \, 1/2$$

$$\nabla f(x) = \begin{bmatrix} 2a_1 x_1 + (a_2 + a_3) x_2 \\ 2a_4 x_2 + (a_2 + a_3) x_1 \end{bmatrix} \times 1/2$$

$\because$ 'A' is symmetric, $a_2 = a_3$

$$\therefore \nabla f(x) = \begin{bmatrix} 2a_1 x_1 + 2a_2 x_2 \\ 2a_4 x_2 + 2a_3 x_1 \end{bmatrix} 1/2$$

<u>substituting</u> $x_1 = 2$, $x_2 = 3$,

$$\nabla f(x) = \begin{bmatrix} 2a_1 + 3a_2 \\ 2a_3 + 3a_4 \end{bmatrix} = Ax = \begin{bmatrix} a_1 & a_2 \\ a_3 & a_4 \end{bmatrix} \begin{bmatrix} 2 \\ 3 \end{bmatrix}$$

$$= \begin{bmatrix} 2a_1 + 3a_2 \\ 2a_3 + 3a_4 \end{bmatrix}$$

$$\therefore \boxed{\nabla f(x) = Ax}$$