Convolutional Neural Networks for digit image classification

The popular MNIST handwritten digit image classification dataset contains 60'000 + 10'000 hand- written digits to be classified as a number between 0 and 9.

The sequence of (more and more complex) models defined below will eventually arrive at the one suggested in this tutorial.

Steps

1. Loading and plotting the mnist dataset
2. Setting the sceene for image classification
3. Defining and evaluating the baseline model
4. Adding Pooling
5. Adding a CNN Kernel
6. Adding another dense output layer

Loading and plotting the MNIST dataset

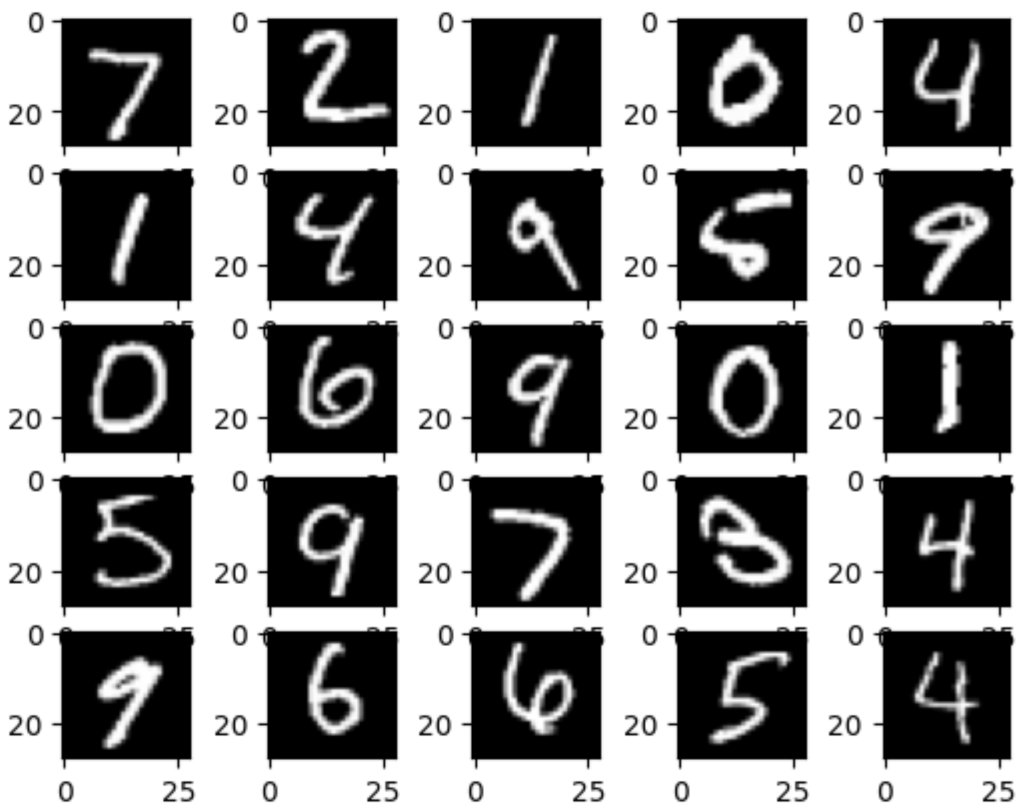
Imported some modules to set seeds

In [ ]:
```
import numpy as np
import random
import tensorflow as tf
```

In [ ]:
```
def set_seeds(seed=42):
    tf.random.set_seed(seed)
    np.random.seed(seed)
    random.seed(seed)
```

In [ ]:
```
from tensorflow.keras.datasets.mnist import load_data
from matplotlib import pyplot

set_seeds()
# load dataset
(x_train, y_train), (x_test, y_test) = load_data()

# summarize loaded dataset
print('Train: X=%s, y=%s' % (x_train.shape, y_train.shape))
print('Test: X=%s, y=%s' % (x_test.shape, y_test.shape))

# plot first few images
for i in range(25):
    # define subplot
    pyplot.subplot(5, 5, i+1)
    # plot raw pixel data
    pyplot.imshow(x_test[i], cmap=pyplot.get_cmap('gray'))

# show the figure
pyplot.show()
```

```
Train: X=(60000, 28, 28), y=(60000,)
Test: X=(10000, 28, 28), y=(10000,)
```



Setting the sceene for image classification

Fix necessary imports.

In [ ]:
```
from numpy import zeros
from numpy import unique
from numpy import argmax
from numpy import asarray
from tensorflow.keras import Sequential
from tensorflow.keras.layers import Dense
from tensorflow.keras.layers import Conv2D
from tensorflow.keras.layers import MaxPool2D
from tensorflow.keras.layers import Flatten
from tensorflow.keras.layers import Dropout
from tensorflow.keras.utils import plot_model
from tensorflow.keras.callbacks import EarlyStopping
from tensorflow.keras.metrics import Accuracy
```

Reshape data to have a single b/w channel.

In [ ]:
```
orig_shape = x_train.shape[1:]
x_train = x_train.reshape((x_train.shape[0], x_train.shape[1], x_train.shape[2], 1)
x_test = x_test.reshape((x_test.shape[0], x_test.shape[1], x_test.shape[2], 1))
in_shape = x_train.shape[1:]
print("Before: {0}".format(orig_shape))
print("After: {0}".format(in_shape))
```

```
Before: (28, 28)
After: (28, 28, 1)
```

Determine the number of classes.

In [ ]:
```
n_classes = len(unique(y_train))
print("Classes: {0}".format(n_classes))
```

```
Classes: 10
```

Normalize pixel values `int[0..255]` $\rightarrow$ `float32[0..1]`.

In [ ]:
```
x_train = x_train.astype('float32') / 255.0
x_test = x_test.astype('float32') / 255.0
```

Function to define the CNN model architecture and compile it with constant surrogate goal function (loss), evaluation metrics, and optimizer. We will vary depth, kernel_width, and pool_stride of the models.

In [ ]:
```
def make_model(depth, kernel_width, pool_stride, add_dense=False, dense_structure=[
    model = Sequential()
    model.add(Conv2D(depth, (kernel_width,kernel_width), activation='relu',input_sh
    model.add(MaxPool2D((pool_stride, pool_stride)))
    model.add(Flatten())
    if add_dense:
        for size in dense_structure:
            model.add(Dense(size, activation='relu'))
            model.add(Dropout(0.5))
    model.add(Dense(n_classes, activation='softmax'))
    model.compile(optimizer='adam', loss='sparse_categorical_crossentropy',metrics=
    return model
```

Define learning parameters for all models.

In [ ]:
```
EPOCHS = 10
BATCH_SIZE = 128
early_stopping = EarlyStopping(
    monitor='val_accuracy',
    verbose=1,
    patience=10,
    mode='max',
    restore_best_weights=True)
it = round(60000/BATCH_SIZE)
```

It takes {{it}} iterations to finish an epch.

Function to show the history of training.

In [ ]:
```
def plot_metrics(history):
    colors = pyplot.rcParams['axes.prop_cycle'].by_key()['color']
    metrics = ['loss', 'accuracy']
    pyplot.figure(figsize=(10,5))
    for n, metric in enumerate(metrics):
        name = metric.replace("_"," ").capitalize()
        pyplot.subplot(1,2,n+1)
        pyplot.plot(history.epoch, history.history[metric], color=colors[0],label='
        pyplot.plot(history.epoch, history.history['val_'+ metric],color=colors[0],
        pyplot.xlabel('Epoch')
        pyplot.ylabel(name)
        if metric == 'loss':
            pyplot.ylim([0, pyplot.ylim()[1] +0.1])
        else:
            pyplot.ylim([-0.1,1.1])
        pyplot.legend()
```

Baseline

We set `depth=1`, `kernel_width=1x1`, and `pool_stride=1`, i.e., no effective CNN.

In [ ]:
```
set_seeds()

baseline = make_model(1,1,1)
baseline.summary()
```

Model: "sequential_58"

Layer (type)	Output Shape	Param #
conv2d_118 (Conv2D)	(None, 28, 28, 1)	2
max_pooling2d_117 (MaxPooling2D)	(None, 28, 28, 1)	0
flatten_57 (Flatten)	(None, 784)	0
dense_81 (Dense)	(None, 10)	7,850

Total params: 7,852 (30.67 KB)

Trainable params: 7,852 (30.67 KB)

Non-trainable params: 0 (0.00 B)

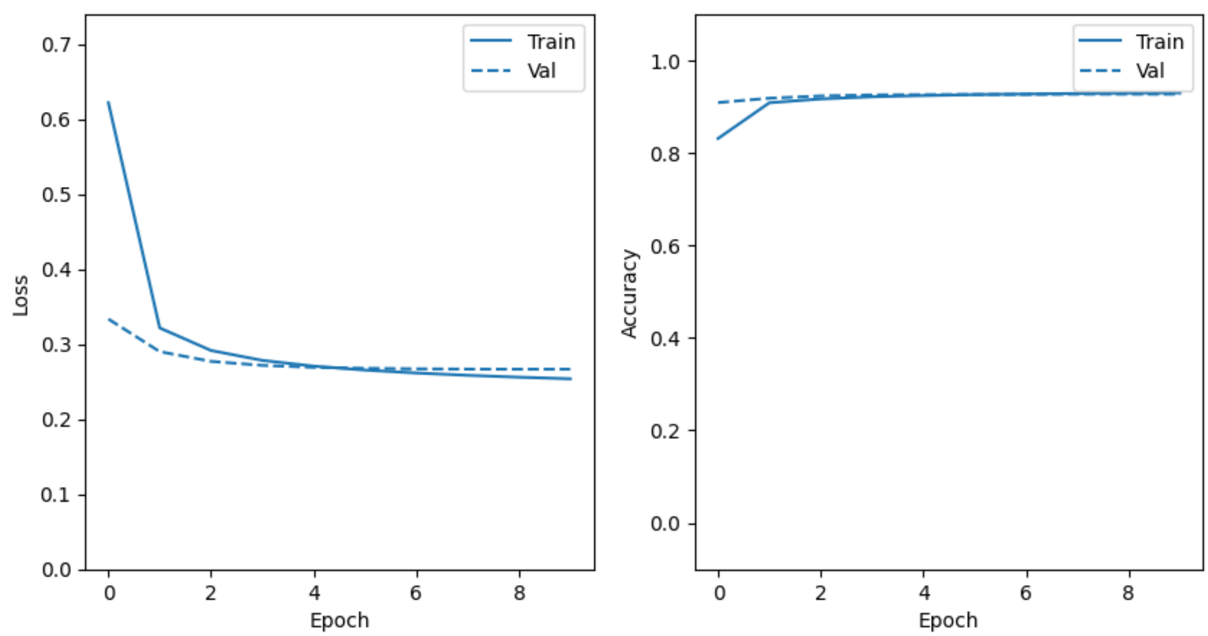
Note that the kernel has one weight and one bias, i.e., `Param = 2` for the CNN layer. The output of the CNN and the pooling layers are equal to their inputs.

We train the baseline model.

In [ ]:
```
baseline_history = baseline.fit(
    x_train,
    y_train,
    epochs=EPOCHS,
    callbacks = [early_stopping],
    validation_data=(x_test, y_test),
    batch_size=BATCH_SIZE)
```

```
Epoch 1/10
469/469 ━━━━━━━━━━━━━━━━━━━━ 1s 1ms/step - accuracy: 0.7030 - loss: 1.0291 - val_acc
uracy: 0.9084 - val_loss: 0.3335
Epoch 2/10
469/469 ━━━━━━━━━━━━━━━━━━━━ 1s 1ms/step - accuracy: 0.9049 - loss: 0.3351 - val_acc
uracy: 0.9178 - val_loss: 0.2901
Epoch 3/10
469/469 ━━━━━━━━━━━━━━━━━━━━ 1s 1ms/step - accuracy: 0.9146 - loss: 0.2972 - val_acc
uracy: 0.9227 - val_loss: 0.2771
Epoch 4/10
469/469 ━━━━━━━━━━━━━━━━━━━━ 1s 1ms/step - accuracy: 0.9206 - loss: 0.2821 - val_acc
uracy: 0.9249 - val_loss: 0.2718
Epoch 5/10
469/469 ━━━━━━━━━━━━━━━━━━━━ 1s 1ms/step - accuracy: 0.9236 - loss: 0.2736 - val_acc
uracy: 0.9255 - val_loss: 0.2692
Epoch 6/10
469/469 ━━━━━━━━━━━━━━━━━━━━ 1s 1ms/step - accuracy: 0.9260 - loss: 0.2680 - val_acc
uracy: 0.9258 - val_loss: 0.2678
Epoch 7/10
469/469 ━━━━━━━━━━━━━━━━━━━━ 1s 1ms/step - accuracy: 0.9273 - loss: 0.2638 - val_acc
uracy: 0.9259 - val_loss: 0.2672
Epoch 8/10
469/469 ━━━━━━━━━━━━━━━━━━━━ 1s 1ms/step - accuracy: 0.9285 - loss: 0.2605 - val_acc
uracy: 0.9266 - val_loss: 0.2668
Epoch 9/10
469/469 ━━━━━━━━━━━━━━━━━━━━ 1s 1ms/step - accuracy: 0.9290 - loss: 0.2579 - val_acc
uracy: 0.9267 - val_loss: 0.2667
Epoch 10/10
469/469 ━━━━━━━━━━━━━━━━━━━━ 1s 1ms/step - accuracy: 0.9299 - loss: 0.2556 - val_acc
uracy: 0.9266 - val_loss: 0.2667
Restoring model weights from the end of the best epoch: 9.
```

In [ ]:
```
plot_metrics(baseline_history)
```



We do not observe effective learning. We evaluate the trained model.

In my case, the learning was indeed effective

In [ ]:
```
loss, acc = baseline.evaluate(x_test, y_test, verbose=0)
print('Accuracy: %.3f' % acc)
```

```
Accuracy: 0.927
```

Use the model to make a prediction.

In [ ]:
```
xx = x_test[9]
xxx = asarray([xx])
yhat = baseline.predict(xxx)
argmax(yhat)
```

```
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 21ms/step1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 21ms/step
```

Out[ ]: 9

In [ ]:
```
def print_res(model):
    err = 0
    i_range = 10
    ys = zeros(i_range * i_range)
    class bcolors:
        FAIL = '\033[91m'
        ENDC = '\033[0m'
    for i in range(i_range):
        for j in range(i_range):
            idx = i*i_range+j
            image = x_test[idx]
            yhat = model.predict(asarray([image]))
            ys[idx] = argmax(yhat)
            print('%d ' % ys[idx], end = '')
        print()
    print("--")
    for i in range(i_range):
        for j in range(i_range):
            idx = i*i_range+j;
            y = y_test[idx]
            if y==ys[idx]:
                print('%d ' % y, end = '')
            else:
                err = err + 1
                print(f"{bcolors.FAIL}%d {bcolors.ENDC}" % y, end = '')
        print()
    return err
```

In [ ]:
```
err = print_res(baseline)
```

```
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
9
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
4
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
4
```

```
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 10ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
5
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 10ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
0
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
7
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 17ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 11ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 10ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 11ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
9
--
```

```
7 2 1 0 4 1 4 9 5 9
0 6 9 0 1 5 9 7 3 4
9 6 6 5 4 0 7 4 0 1
3 1 3 4 7 2 7 1 2 1
1 7 4 2 3 5 1 2 4 4
6 3 5 5 6 0 4 1 9 5
7 8 9 3 7 4 6 4 3 0
7 0 2 9 1 7 3 2 9 7
7 6 2 7 8 4 7 3 6 1
3 6 9 3 1 4 1 7 6 9
```

In [ ]:
```
err
```

Out[ ]: 4

The classic dense layer does perform good alone due to the fact that there are actually 7852 learnable parameters, i.e. 2 from the convolutional layer and 7850 from the dense layer. So a network with one hidden layer with the same size than the input layer would be sufficient either way. The target, therefore, is to reduce the parameters in the model while increasing the accuracy.

Baseline parameters:

- *Model Complexity (Paramereters) = 7852*
- *Accuracy: 0.927*
- *Test error in first 100 images: 4*

Adding Pooling

We set `depth=1`, `kernel_width=1x1`, and `pool_stride=2`, i.e., no effective CNN and we even squeeze the image size, which would result in a less complex model.

In [ ]:
```
set_seeds()
model1 = make_model(1,1,2, False)
model1.summary()
```

Model: "sequential_59"

Layer (type)	Output Shape	Param #
conv2d_119 (Conv2D)	(None, 28, 28, 1)	2
max_pooling2d_118 (MaxPooling2D)	(None, 14, 14, 1)	0
flatten_58 (Flatten)	(None, 196)	0
dense_82 (Dense)	(None, 10)	1,970

Total params: 1,972 (7.70 KB)

Trainable params: 1,972 (7.70 KB)

```
Non-trainable params: 0 (0.00 B)
```

The kernel has still one weight and one bias, i.e., `Param = 2` for the CNN layer. The output of the pooling layers is one quater (half in each dimension) of the CNN in- and output. The dense layer got 1970 weights.

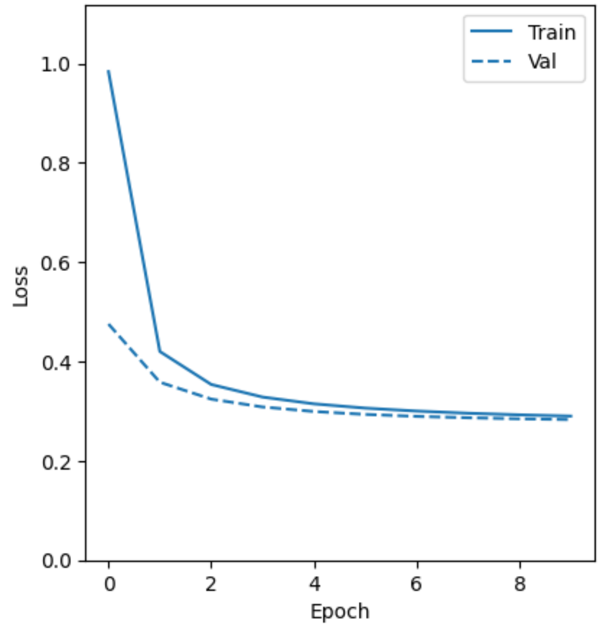
We train the first CNN model.

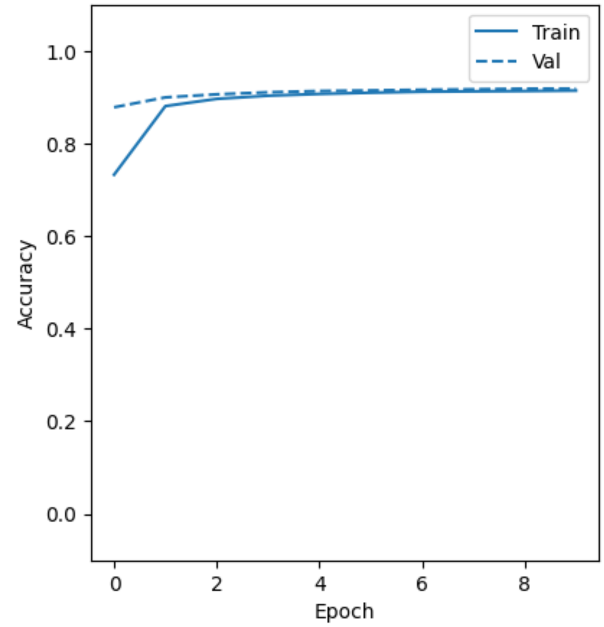
In [ ]:
```
model1_history = model1.fit(
    x_train,
    y_train,
    epochs=EPOCHS,
    callbacks = [early_stopping],
    validation_data=(x_test, y_test),
    batch_size=BATCH_SIZE)
```

```
Epoch 1/10
469/469 ━━━━━━━━━━━━━━━━━━━━ 1s 1ms/step - accuracy: 0.5493 - loss: 1.4949 - val_acc
uracy: 0.8781 - val_loss: 0.4755
Epoch 2/10
469/469 ━━━━━━━━━━━━━━━━━━━━ 1s 1ms/step - accuracy: 0.8745 - loss: 0.4512 - val_acc
uracy: 0.8995 - val_loss: 0.3583
Epoch 3/10
469/469 ━━━━━━━━━━━━━━━━━━━━ 1s 1ms/step - accuracy: 0.8942 - loss: 0.3639 - val_acc
uracy: 0.9061 - val_loss: 0.3242
Epoch 4/10
469/469 ━━━━━━━━━━━━━━━━━━━━ 1s 1ms/step - accuracy: 0.9020 - loss: 0.3342 - val_acc
uracy: 0.9105 - val_loss: 0.3084
Epoch 5/10
469/469 ━━━━━━━━━━━━━━━━━━━━ 1s 1ms/step - accuracy: 0.9061 - loss: 0.3190 - val_acc
uracy: 0.9132 - val_loss: 0.2993
Epoch 6/10
469/469 ━━━━━━━━━━━━━━━━━━━━ 1s 1ms/step - accuracy: 0.9093 - loss: 0.3096 - val_acc
uracy: 0.9145 - val_loss: 0.2935
Epoch 7/10
469/469 ━━━━━━━━━━━━━━━━━━━━ 1s 1ms/step - accuracy: 0.9120 - loss: 0.3032 - val_acc
uracy: 0.9159 - val_loss: 0.2896
Epoch 8/10
469/469 ━━━━━━━━━━━━━━━━━━━━ 1s 1ms/step - accuracy: 0.9129 - loss: 0.2986 - val_acc
uracy: 0.9169 - val_loss: 0.2867
Epoch 9/10
469/469 ━━━━━━━━━━━━━━━━━━━━ 1s 1ms/step - accuracy: 0.9139 - loss: 0.2952 - val_acc
uracy: 0.9182 - val_loss: 0.2847
Epoch 10/10
469/469 ━━━━━━━━━━━━━━━━━━━━ 0s 1ms/step - accuracy: 0.9154 - loss: 0.2925 - val_acc
uracy: 0.9184 - val_loss: 0.2831
Epoch 10: early stopping
Restoring model weights from the end of the best epoch: 1.
```

I don't know why the early stopping callback restored the weights of epoch 1, since the best val_accuracy was found in epoch 10. So I ran the training again. After I plotted the training history.

In [ ]:
```
plot_metrics(model1_history)
```





In [ ]:

```
set_seeds()
model1 = make_model(1,1,2, False)
model1_history = model1.fit(
    x_train,
    y_train,
    epochs=10,
    validation_data=(x_test, y_test),
    batch_size=BATCH_SIZE)
```

```
Epoch 1/10
469/469 ━━━━━━━━━━━━━━━━━━━━ 1s 1ms/step - accuracy: 0.5493 - loss: 1.4949 - val_acc
uracy: 0.8781 - val_loss: 0.4755
Epoch 2/10
469/469 ━━━━━━━━━━━━━━━━━━━━ 1s 1ms/step - accuracy: 0.8745 - loss: 0.4512 - val_acc
uracy: 0.8995 - val_loss: 0.3583
Epoch 3/10
469/469 ━━━━━━━━━━━━━━━━━━━━ 1s 1ms/step - accuracy: 0.8942 - loss: 0.3639 - val_acc
uracy: 0.9061 - val_loss: 0.3242
Epoch 4/10
469/469 ━━━━━━━━━━━━━━━━━━━━ 1s 1ms/step - accuracy: 0.9020 - loss: 0.3342 - val_acc
uracy: 0.9105 - val_loss: 0.3084
Epoch 5/10
469/469 ━━━━━━━━━━━━━━━━━━━━ 1s 1ms/step - accuracy: 0.9061 - loss: 0.3190 - val_acc
uracy: 0.9132 - val_loss: 0.2993
Epoch 6/10
469/469 ━━━━━━━━━━━━━━━━━━━━ 1s 1ms/step - accuracy: 0.9093 - loss: 0.3096 - val_acc
uracy: 0.9145 - val_loss: 0.2935
Epoch 7/10
469/469 ━━━━━━━━━━━━━━━━━━━━ 1s 1ms/step - accuracy: 0.9120 - loss: 0.3032 - val_acc
uracy: 0.9159 - val_loss: 0.2896
Epoch 8/10
469/469 ━━━━━━━━━━━━━━━━━━━━ 0s 1ms/step - accuracy: 0.9129 - loss: 0.2986 - val_acc
uracy: 0.9169 - val_loss: 0.2867
Epoch 9/10
469/469 ━━━━━━━━━━━━━━━━━━━━ 1s 1ms/step - accuracy: 0.9139 - loss: 0.2952 - val_acc
uracy: 0.9182 - val_loss: 0.2847
Epoch 10/10
469/469 ━━━━━━━━━━━━━━━━━━━━ 0s 1ms/step - accuracy: 0.9154 - loss: 0.2925 - val_acc
uracy: 0.9184 - val_loss: 0.2831
```

We evaluate the trained model.

In [ ]:
```
loss, acc = model1.evaluate(x_test, y_test, verbose=0)
print('Accuracy: %.3f' % acc)
```

```
Accuracy: 0.918
```

This cannot be any better. Use the model to make a prediction.

In [ ]:
```
err = print_res(model1)
```

```
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 22ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 22ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 8ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 8ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
9
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 8ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
4
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 8ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 8ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 8ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 8ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 8ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
4
```

```
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 8ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 8ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
5
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 8ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
0
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
7
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
9
--
```

```
7 2 1 0 4 1 4 9 5 9
0 6 9 0 1 5 9 7 3 4
9 6 6 5 4 0 7 4 0 1
3 1 3 4 7 2 7 1 2 1
1 7 4 2 3 5 1 2 4 4
6 3 5 5 6 0 4 1 9 5
7 8 9 3 7 4 6 4 3 0
7 0 2 9 1 7 3 2 9 7
7 6 2 7 8 4 7 3 6 1
3 6 9 3 1 4 1 7 6 9
```

In [ ]:

```
err
```

Out[ ]: 6

The accuracy decreased as well as the number of parameters. However, the decrease in accuracy was about 0.09 and the reduce in the number of parameters was (7582 - 1972 =) 5610, resulting in a much smaller model.

Model 1 Parameters:

- *Model Complexity (Paramereters) = 1972*
- *Accuracy: 0.918*
- *Test error in first 100 images: 6*

Adding a CNN Kernel

Because we might assume, that evaluating more pixels together could compensate for the pool stride that might have caused the decrease of accuracy in the previous model. Additionally, we would decrease the complexity of the model one more time.

In [ ]:
```
set_seeds()
model2 = make_model(1,3,2)
model2.summary()
```

Model: "sequential_61"

Layer (type)	Output Shape	Param #
conv2d_121 (Conv2D)	(None, 26, 26, 1)	10
max_pooling2d_120 (MaxPooling2D)	(None, 13, 13, 1)	0
flatten_60 (Flatten)	(None, 169)	0
dense_84 (Dense)	(None, 10)	1,700

Total params: 1,710 (6.68 KB)

Trainable params: 1,710 (6.68 KB)

Non-trainable params: 0 (0.00 B)

The kernel has still 3 × 3 weights and one bias, i.e., `Param = 10` for the CNN layer. No padding is applied, so the CNN layer 'eats' one pixel at the corners, i.e., its output shape is `(26, 26, 1)`. The output of the pooling layers is one quater (half in x and y dimension) of the CNN output. Therefore, the weight for the dense layer were reduced to 1700.

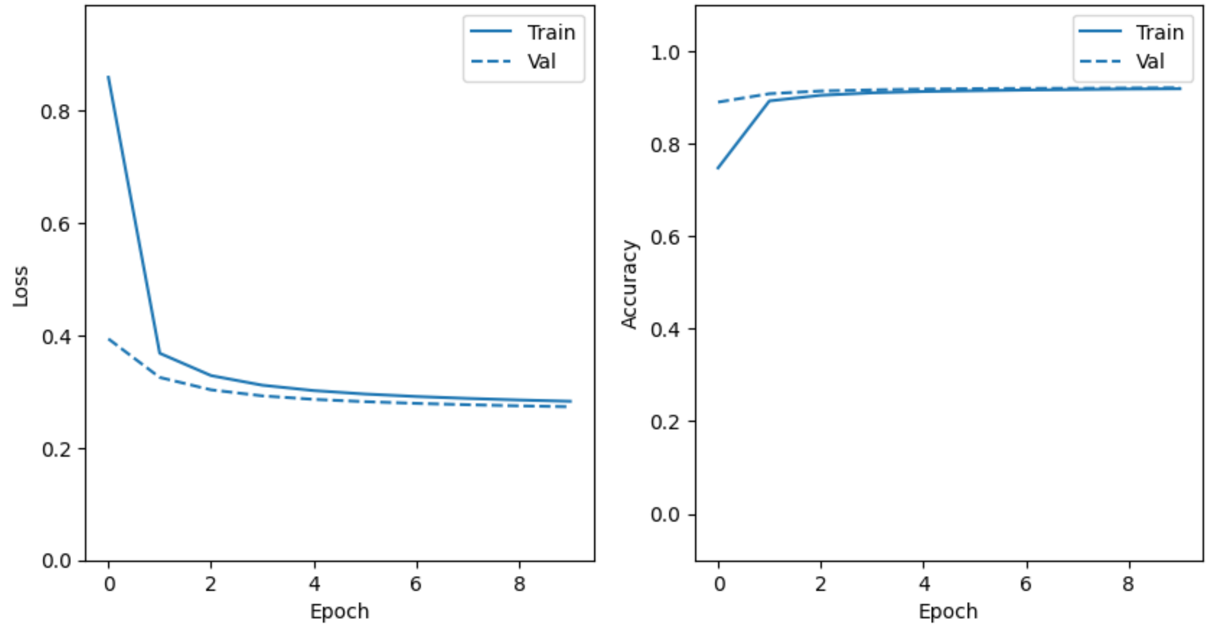
We train the first real CNN model.

In [ ]:
```
model2_history = model2.fit(
    x_train,
    y_train,
    epochs=EPOCHS,
    callbacks = [early_stopping],
    validation_data=(x_test, y_test),
    batch_size=BATCH_SIZE)
```

```
Epoch 1/10
469/469 ━━━━━━━━━━━━━━━━━━━━ 1s 1ms/step - accuracy: 0.5618 - loss: 1.3862 - val_acc
uracy: 0.8892 - val_loss: 0.3933
Epoch 2/10
469/469 ━━━━━━━━━━━━━━━━━━━━ 1s 1ms/step - accuracy: 0.8871 - loss: 0.3864 - val_acc
uracy: 0.9074 - val_loss: 0.3248
Epoch 3/10
469/469 ━━━━━━━━━━━━━━━━━━━━ 1s 1ms/step - accuracy: 0.9027 - loss: 0.3358 - val_acc
uracy: 0.9133 - val_loss: 0.3027
Epoch 4/10
469/469 ━━━━━━━━━━━━━━━━━━━━ 1s 1ms/step - accuracy: 0.9083 - loss: 0.3160 - val_acc
uracy: 0.9156 - val_loss: 0.2920
Epoch 5/10
469/469 ━━━━━━━━━━━━━━━━━━━━ 1s 1ms/step - accuracy: 0.9117 - loss: 0.3053 - val_acc
uracy: 0.9173 - val_loss: 0.2858
Epoch 6/10
469/469 ━━━━━━━━━━━━━━━━━━━━ 1s 1ms/step - accuracy: 0.9137 - loss: 0.2985 - val_acc
uracy: 0.9182 - val_loss: 0.2817
Epoch 7/10
469/469 ━━━━━━━━━━━━━━━━━━━━ 1s 1ms/step - accuracy: 0.9152 - loss: 0.2938 - val_acc
uracy: 0.9189 - val_loss: 0.2787
Epoch 8/10
469/469 ━━━━━━━━━━━━━━━━━━━━ 1s 1ms/step - accuracy: 0.9161 - loss: 0.2902 - val_acc
uracy: 0.9191 - val_loss: 0.2762
Epoch 9/10
469/469 ━━━━━━━━━━━━━━━━━━━━ 1s 1ms/step - accuracy: 0.9174 - loss: 0.2873 - val_acc
uracy: 0.9198 - val_loss: 0.2743
Epoch 10/10
469/469 ━━━━━━━━━━━━━━━━━━━━ 1s 1ms/step - accuracy: 0.9181 - loss: 0.2848 - val_acc
uracy: 0.9204 - val_loss: 0.2726
Epoch 10: early stopping
Restoring model weights from the end of the best epoch: 1.
```

I don't know why the early stopping callback restored the weights of epoch 1, since the best val_accuracy was found in epoch 10. So I ran the training again. After I plotted the training history.

In [ ]:
```
plot_metrics(model2_history)
```



Now we see effective training. We evaluate the trained model.

In my case, I saw it in other settings too

In [ ]:

```
set_seeds()
model2 = make_model(1,3,2)

model2_history = model2.fit(
    x_train,
    y_train,
    epochs=10,
    validation_data=(x_test, y_test),
    batch_size=BATCH_SIZE)
```

```
Epoch 1/10
469/469 ━━━━━━━━━━━━━━━━━━━━ 1s 1ms/step - accuracy: 0.5618 - loss: 1.3862 - val_acc
uracy: 0.8892 - val_loss: 0.3933
Epoch 2/10
469/469 ━━━━━━━━━━━━━━━━━━━━ 1s 1ms/step - accuracy: 0.8871 - loss: 0.3864 - val_acc
uracy: 0.9074 - val_loss: 0.3248
Epoch 3/10
469/469 ━━━━━━━━━━━━━━━━━━━━ 1s 1ms/step - accuracy: 0.9027 - loss: 0.3358 - val_acc
uracy: 0.9133 - val_loss: 0.3027
Epoch 4/10
469/469 ━━━━━━━━━━━━━━━━━━━━ 1s 1ms/step - accuracy: 0.9083 - loss: 0.3160 - val_acc
uracy: 0.9156 - val_loss: 0.2920
Epoch 5/10
469/469 ━━━━━━━━━━━━━━━━━━━━ 1s 1ms/step - accuracy: 0.9117 - loss: 0.3053 - val_acc
uracy: 0.9173 - val_loss: 0.2858
Epoch 6/10
469/469 ━━━━━━━━━━━━━━━━━━━━ 1s 1ms/step - accuracy: 0.9137 - loss: 0.2985 - val_acc
uracy: 0.9182 - val_loss: 0.2817
Epoch 7/10
469/469 ━━━━━━━━━━━━━━━━━━━━ 1s 1ms/step - accuracy: 0.9152 - loss: 0.2938 - val_acc
uracy: 0.9189 - val_loss: 0.2787
Epoch 8/10
469/469 ━━━━━━━━━━━━━━━━━━━━ 1s 1ms/step - accuracy: 0.9161 - loss: 0.2902 - val_acc
uracy: 0.9191 - val_loss: 0.2762
Epoch 9/10
469/469 ━━━━━━━━━━━━━━━━━━━━ 1s 1ms/step - accuracy: 0.9174 - loss: 0.2873 - val_acc
uracy: 0.9198 - val_loss: 0.2743
Epoch 10/10
469/469 ━━━━━━━━━━━━━━━━━━━━ 1s 1ms/step - accuracy: 0.9181 - loss: 0.2848 - val_acc
uracy: 0.9204 - val_loss: 0.2726
```

In [ ]:

```python
loss, acc = model2.evaluate(x_test, y_test, verbose=0)
print('Accuracy: %.3f' % acc)
```

```
Accuracy: 0.920
```

Use the model to make a prediction.

In [ ]:

```python
err = print_res(model2)
```

```
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 22ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 22ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 8ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
9
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 8ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 10ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
4
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 10ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 8ms/step
4
```

```
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
5
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
0
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
7
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 11ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
9
--
```

```
7 2 1 0 4 1 4 9 5 9
0 6 9 0 1 5 9 7 3 4
9 6 6 5 4 0 7 4 0 1
3 1 3 4 7 2 7 1 2 1
1 7 4 2 3 5 1 2 4 4
6 3 5 5 6 0 4 1 9 5
7 8 9 3 7 4 6 4 3 0
7 0 2 9 1 7 3 2 9 7
7 6 2 7 8 4 7 3 6 1
3 6 9 3 1 4 1 7 6 9
```

In [ ]:
```
err
```

Out[ ]: 4

The accuracy increased a little bit about 0.002 while the complexity on the model decreased again by (1972-1710=) 262 compared to model 1.

Model 2 Parameters:

- *Model Complexity (Paramereters) = 1710*
- *Accuracy: 0.92*
- *Test error in first 100 images: 4*

Adding Parallel CNN Kernels

We set `depth=32` , and keep `kernel_width=3x3` , and `pool_stride=2` . *Because we assume that there are different features that are relevant to classify the images. Thus, we need to add more kernels in order to learn filters capable to detect these features. However, we pay for this with much more weights in the dense layer as well as in the convolution layer.*

In [ ]:
```
set_seeds()
model3 = make_model(32,3,2)
model3.summary()
```

Model: "sequential_63"

Layer (type)	Output Shape	Param #
conv2d_123 (Conv2D)	(None, 26, 26, 32)	320
max_pooling2d_122 (MaxPooling2D)	(None, 13, 13, 32)	0
flatten_62 (Flatten)	(None, 5408)	0
dense_86 (Dense)	(None, 10)	54,090

Total params: 54,410 (212.54 KB)

Trainable params: 54,410 (212.54 KB)

Non-trainable params: 0 (0.00 B)

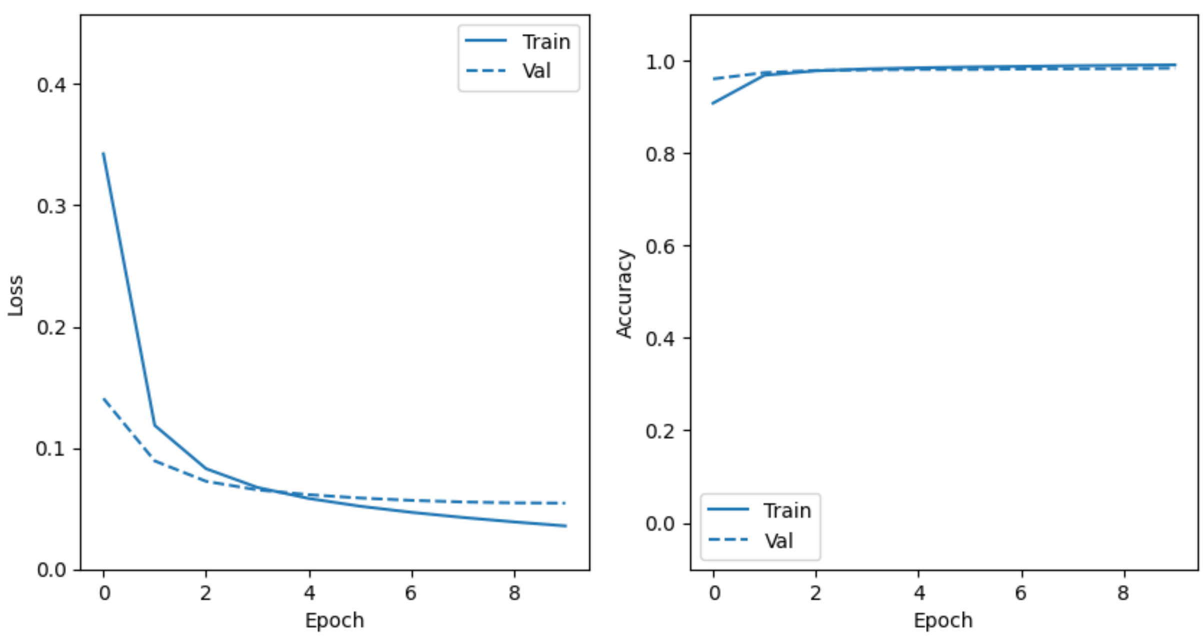
There are 32 kernels has each with 3 × 3 weights and one bias, i.e., `Param = 320` for the CNN layer. As no padding is applied, the CNN layer's output shape is the same for the x and y dimensions , but it adds depth: `(26, 26, 32)` . The output of the pooling layers is one quater (half in x and y dimension) of the CNN output, but it keeps the depth. *Additionally, the dense layers weights grew to 54090.*

We train the first CNN model.

In [ ]:
```
model3_history = model3.fit(
    x_train,
    y_train,
    epochs=EPOCHS,
    callbacks = [early_stopping],
    validation_data=(x_test, y_test),
    batch_size=BATCH_SIZE)
```

```
Epoch 1/10
469/469 ━━━━━━━━━━━━━━━━━━━━ 2s 4ms/step - accuracy: 0.8311 - loss: 0.6482 - val_acc
uracy: 0.9596 - val_loss: 0.1408
Epoch 2/10
469/469 ━━━━━━━━━━━━━━━━━━━━ 2s 3ms/step - accuracy: 0.9636 - loss: 0.1337 - val_acc
uracy: 0.9730 - val_loss: 0.0893
Epoch 3/10
469/469 ━━━━━━━━━━━━━━━━━━━━ 2s 3ms/step - accuracy: 0.9763 - loss: 0.0879 - val_acc
uracy: 0.9778 - val_loss: 0.0725
Epoch 4/10
469/469 ━━━━━━━━━━━━━━━━━━━━ 2s 4ms/step - accuracy: 0.9812 - loss: 0.0696 - val_acc
uracy: 0.9790 - val_loss: 0.0654
Epoch 5/10
469/469 ━━━━━━━━━━━━━━━━━━━━ 2s 3ms/step - accuracy: 0.9836 - loss: 0.0594 - val_acc
uracy: 0.9801 - val_loss: 0.0615
Epoch 6/10
469/469 ━━━━━━━━━━━━━━━━━━━━ 2s 4ms/step - accuracy: 0.9855 - loss: 0.0524 - val_acc
uracy: 0.9800 - val_loss: 0.0587
Epoch 7/10
469/469 ━━━━━━━━━━━━━━━━━━━━ 2s 4ms/step - accuracy: 0.9868 - loss: 0.0471 - val_acc
uracy: 0.9810 - val_loss: 0.0568
Epoch 8/10
469/469 ━━━━━━━━━━━━━━━━━━━━ 2s 3ms/step - accuracy: 0.9881 - loss: 0.0427 - val_acc
uracy: 0.9813 - val_loss: 0.0555
Epoch 9/10
469/469 ━━━━━━━━━━━━━━━━━━━━ 2s 4ms/step - accuracy: 0.9893 - loss: 0.0389 - val_acc
uracy: 0.9818 - val_loss: 0.0547
Epoch 10/10
469/469 ━━━━━━━━━━━━━━━━━━━━ 2s 4ms/step - accuracy: 0.9900 - loss: 0.0355 - val_acc
uracy: 0.9828 - val_loss: 0.0545
Restoring model weights from the end of the best epoch: 10.
```

In [ ]:
```
plot_metrics(model3_history)
```



In [ ]:
```
loss, acc = model3.evaluate(x_test, y_test, verbose=0)
print('Accuracy: %.3f' % acc)
```

```
Accuracy: 0.983
```

In [ ]:
```
err = print_res(model3)
```

```
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 22ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 22ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 8ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 10ms/step
9
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 10ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
4
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 10ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 8ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 10ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 11ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 11ms/step
4
```

```
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
5
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 8ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
0
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 10ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
7
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 10ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
9
--
```

```
7 2 1 0 4 1 4 9 5 9
0 6 9 0 1 5 9 7 3 4
9 6 6 5 4 0 7 4 0 1
3 1 3 4 7 2 7 1 2 1
1 7 4 2 3 5 1 2 4 4
6 3 5 5 6 0 4 1 9 5
7 8 9 3 7 4 6 4 3 0
7 0 2 9 1 7 3 2 9 7
7 6 2 7 8 4 7 3 6 1
3 6 9 3 1 4 1 7 6 9
```

In [ ]:
```
err
```

Out[ ]: 3

Here the accuracy goes up to 0.983 outperform all models trained so far. However, this is also the most complex model with a total parameter size of 54410 parameters.

Model 3 parameters:

- *Model Complexity (Paramereters) = 54410*
- *Accuracy: 0.983*
- *Test error in first 100 images: 3*

Adding another dense (hidden) layer

Because we might assume, that the learning can be improved when applying a more complex network after the convolution layer. However, we pay for this one more time with a more complex model.

In [ ]:
```
set_seeds()
model4 = make_model(32,3,2,True)
model4.summary()
```

Model: "sequential_64"

Layer (type)	Output Shape	Param #
conv2d_124 (Conv2D)	(None, 26, 26, 32)	320
max_pooling2d_123 (MaxPooling2D)	(None, 13, 13, 32)	0
flatten_63 (Flatten)	(None, 5408)	0
dense_87 (Dense)	(None, 100)	540,900
dropout_24 (Dropout)	(None, 100)	0
dense_88 (Dense)	(None, 10)	1,010

Total params: 542,230 (2.07 MB)

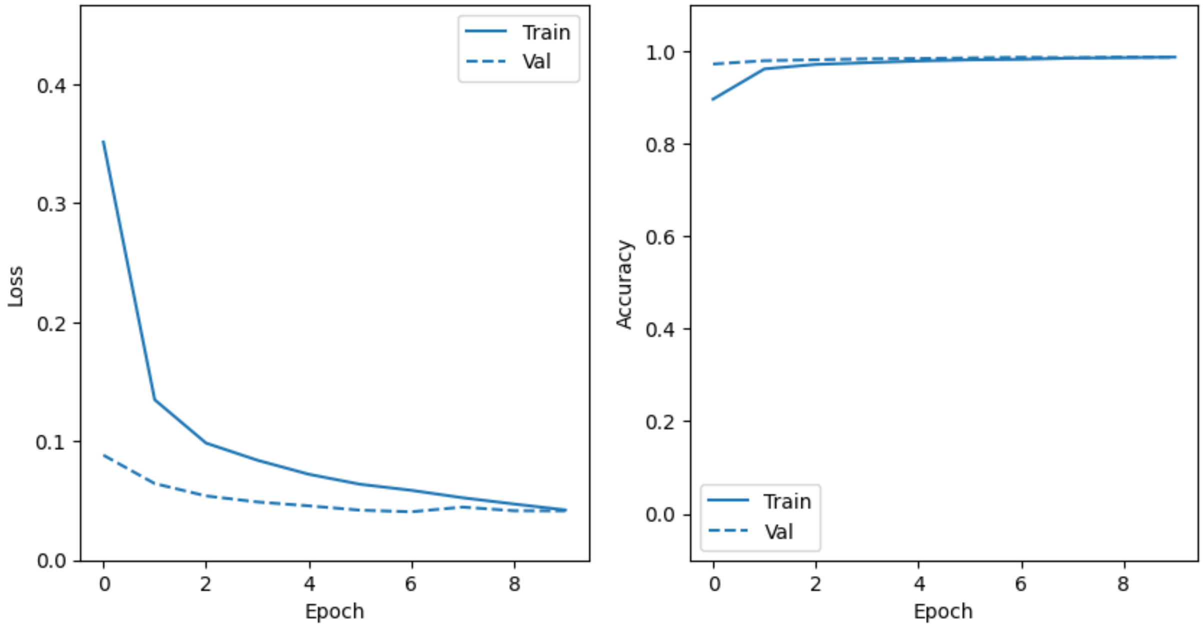
```
Trainable params: 542,230 (2.07 MB)
Non-trainable params: 0 (0.00 B)
```

This lets the number of parameters grow to over half a million! We train this final CNN model.

In [ ]:
```
model4_history = model4.fit(
    x_train,
    y_train,
    epochs=EPOCHS,
    callbacks = [early_stopping],
    validation_data=(x_test, y_test),
    batch_size=BATCH_SIZE)
```

```
Epoch 1/10
469/469 ━━━━━━━━━━━━━━━━━━━━ 4s 7ms/step - accuracy: 0.8085 - loss: 0.6309 - val_acc
uracy: 0.9715 - val_loss: 0.0883
Epoch 2/10
469/469 ━━━━━━━━━━━━━━━━━━━━ 3s 7ms/step - accuracy: 0.9590 - loss: 0.1445 - val_acc
uracy: 0.9787 - val_loss: 0.0644
Epoch 3/10
469/469 ━━━━━━━━━━━━━━━━━━━━ 3s 7ms/step - accuracy: 0.9704 - loss: 0.1011 - val_acc
uracy: 0.9807 - val_loss: 0.0540
Epoch 4/10
469/469 ━━━━━━━━━━━━━━━━━━━━ 3s 7ms/step - accuracy: 0.9749 - loss: 0.0851 - val_acc
uracy: 0.9828 - val_loss: 0.0489
Epoch 5/10
469/469 ━━━━━━━━━━━━━━━━━━━━ 3s 7ms/step - accuracy: 0.9787 - loss: 0.0711 - val_acc
uracy: 0.9834 - val_loss: 0.0457
Epoch 6/10
469/469 ━━━━━━━━━━━━━━━━━━━━ 3s 7ms/step - accuracy: 0.9810 - loss: 0.0629 - val_acc
uracy: 0.9846 - val_loss: 0.0421
Epoch 7/10
469/469 ━━━━━━━━━━━━━━━━━━━━ 3s 7ms/step - accuracy: 0.9824 - loss: 0.0576 - val_acc
uracy: 0.9861 - val_loss: 0.0407
Epoch 8/10
469/469 ━━━━━━━━━━━━━━━━━━━━ 3s 7ms/step - accuracy: 0.9852 - loss: 0.0504 - val_acc
uracy: 0.9850 - val_loss: 0.0447
Epoch 9/10
469/469 ━━━━━━━━━━━━━━━━━━━━ 3s 7ms/step - accuracy: 0.9856 - loss: 0.0468 - val_acc
uracy: 0.9865 - val_loss: 0.0416
Epoch 10/10
469/469 ━━━━━━━━━━━━━━━━━━━━ 3s 7ms/step - accuracy: 0.9872 - loss: 0.0400 - val_acc
uracy: 0.9860 - val_loss: 0.0415
Restoring model weights from the end of the best epoch: 9.
```

In [ ]:
```
plot_metrics(model4_history)
```



In [ ]:
```
loss, acc = model4.evaluate(x_test, y_test, verbose=0)
print('Accuracy: %.3f' % acc)
```

```
Accuracy: 0.987
```

In [ ]:
```
err = print_res(model4)
```

```
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 24ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
9
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 10ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
4
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 10ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 10ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
4
```

```
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
5
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 10ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
0
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
7
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 10ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 10ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
9
--
```

```
7 2 1 0 4 1 4 9 5 9
0 6 9 0 1 5 9 7 3 4
9 6 6 5 4 0 7 4 0 1
3 1 3 4 7 2 7 1 2 1
1 7 4 2 3 5 1 2 4 4
6 3 5 5 6 0 4 1 9 5
7 8 9 3 7 4 6 4 3 0
7 0 2 9 1 7 3 2 9 7
7 6 2 7 8 4 7 3 6 1
3 6 9 3 1 4 1 7 6 9
```

In [ ]:

```
err
```

Out[ ]: 2

Here the accuracy goes up to 0.987 an outperform all models trained so far. However, this is also the most complex model with a total parameter size of 542230 parameters. So we multiplied the number of parameters by 10 while we got an increase in accuracy by only 0.004.

Model 4 parameters:

- *Model Complexity (Paramereters) = 542230*
- *Accuracy: 0.987*
- *Test error in first 100 images: 2*

Maybe we can reach a similar result with a larger kernel, since relevant features could also be detected on a larger part of the image than a 3x3 tile. Additionally, this would reduce the number of trainable parameters.

In [ ]:
```
set_seeds()
model5 = make_model(32,9,2,True)
model5.summary()
```

Model: "sequential_65"

Layer (type)	Output Shape	Param #
conv2d_125 (Conv2D)	(None, 20, 20, 32)	2,624
max_pooling2d_124 (MaxPooling2D)	(None, 10, 10, 32)	0
flatten_64 (Flatten)	(None, 3200)	0
dense_89 (Dense)	(None, 100)	320,100
dropout_25 (Dropout)	(None, 100)	0
dense_90 (Dense)	(None, 10)	1,010

Total params: 323,734 (1.23 MB)

Trainable params: 323,734 (1.23 MB)

Non-trainable params: 0 (0.00 B)

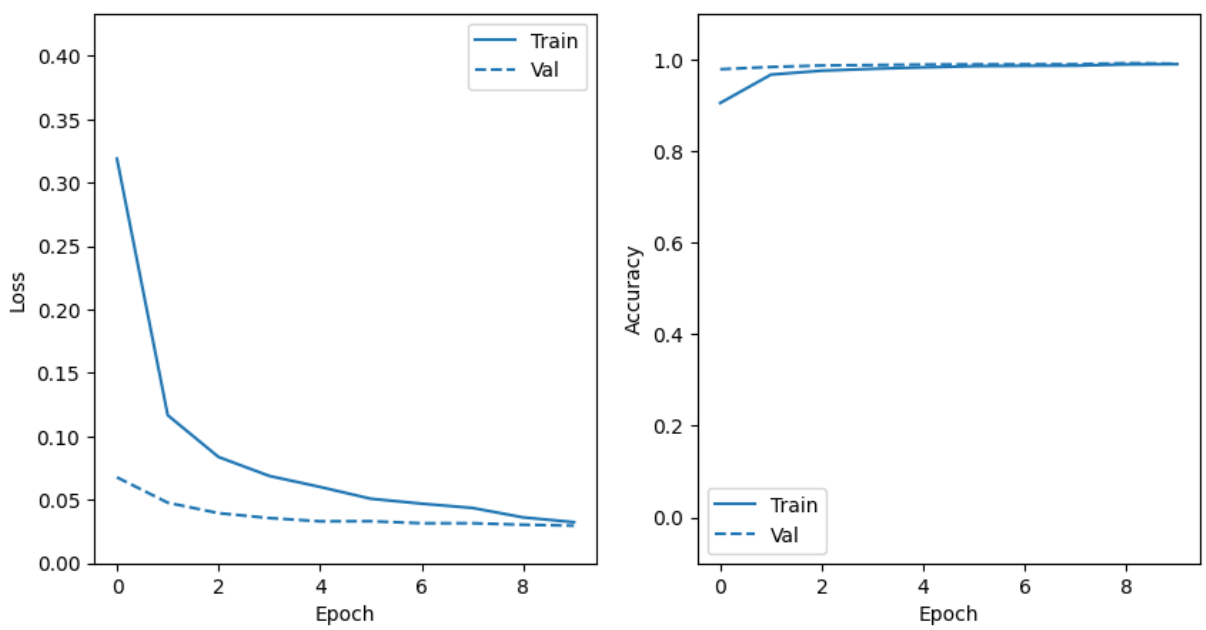
Here, I tested 32 9x9 kernel, which results in a 20x20x32 output shape of the convolution layer. Since I added no padding, the cnn neglect 4 pixels at the corners. Hence, the output shape is (20,20,32). However, I think the relevant features can be found in the middle if the images.

This setting increases the number of parameters in the convolution layer. However, is decrease the number of parameters in the first dense layer on a much larger scale.

In [ ]:
```
model5_history = model5.fit(
    x_train,
    y_train,
    epochs=EPOCHS,
    callbacks = [early_stopping],
    validation_data=(x_test, y_test),
    batch_size=BATCH_SIZE)
```

```
Epoch 1/10
469/469 ━━━━━━━━━━━━━━━━━━━━ 3s 6ms/step - accuracy: 0.8106 - loss: 0.6087 - val_acc
uracy: 0.9783 - val_loss: 0.0678
Epoch 2/10
469/469 ━━━━━━━━━━━━━━━━━━━━ 3s 6ms/step - accuracy: 0.9645 - loss: 0.1269 - val_acc
uracy: 0.9833 - val_loss: 0.0478
Epoch 3/10
469/469 ━━━━━━━━━━━━━━━━━━━━ 3s 6ms/step - accuracy: 0.9752 - loss: 0.0863 - val_acc
uracy: 0.9865 - val_loss: 0.0395
Epoch 4/10
469/469 ━━━━━━━━━━━━━━━━━━━━ 3s 6ms/step - accuracy: 0.9779 - loss: 0.0721 - val_acc
uracy: 0.9874 - val_loss: 0.0357
Epoch 5/10
469/469 ━━━━━━━━━━━━━━━━━━━━ 3s 6ms/step - accuracy: 0.9824 - loss: 0.0605 - val_acc
uracy: 0.9886 - val_loss: 0.0331
Epoch 6/10
469/469 ━━━━━━━━━━━━━━━━━━━━ 3s 6ms/step - accuracy: 0.9856 - loss: 0.0500 - val_acc
uracy: 0.9890 - val_loss: 0.0332
Epoch 7/10
469/469 ━━━━━━━━━━━━━━━━━━━━ 3s 6ms/step - accuracy: 0.9860 - loss: 0.0454 - val_acc
uracy: 0.9892 - val_loss: 0.0316
Epoch 8/10
469/469 ━━━━━━━━━━━━━━━━━━━━ 3s 6ms/step - accuracy: 0.9861 - loss: 0.0426 - val_acc
uracy: 0.9893 - val_loss: 0.0316
Epoch 9/10
469/469 ━━━━━━━━━━━━━━━━━━━━ 3s 5ms/step - accuracy: 0.9885 - loss: 0.0365 - val_acc
uracy: 0.9915 - val_loss: 0.0304
Epoch 10/10
469/469 ━━━━━━━━━━━━━━━━━━━━ 3s 6ms/step - accuracy: 0.9898 - loss: 0.0316 - val_acc
uracy: 0.9903 - val_loss: 0.0297
Restoring model weights from the end of the best epoch: 9.
```

In [ ]:
```
plot_metrics(model5_history)
```



In [ ]:
```
loss, acc = model5.evaluate(x_test, y_test, verbose=0)
print('Accuracy: %.3f' % acc)
```

```
Accuracy: 0.992
```

In [ ]:
```
err = print_res(model5)
```

```
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 23ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
9
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 10ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
4
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 10ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
4
```

```
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 10ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 14ms/step
5
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 10ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
0
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
7
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 10ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
9
--
```

```
7 2 1 0 4 1 4 9 5 9
0 6 9 0 1 5 9 7 3 4
9 6 6 5 4 0 7 4 0 1
3 1 3 4 7 2 7 1 2 1
1 7 4 2 3 5 1 2 4 4
6 3 5 5 6 0 4 1 9 5
7 8 9 3 7 4 6 4 3 0
7 0 2 9 1 7 3 2 9 7
7 6 2 7 8 4 7 3 6 1
3 6 9 3 1 4 1 7 6 9
```

In [ ]:
```
err
```

Out[ ]: 0

Here, we reached both, an increase in accuracy to 0.992 and a decrease in model complexity to 323734 parameters. Hence, this is the best model so far.

Model 5 parameters:

- *Model Complexity (Paramereters) = 323734*
- *Accuracy: 0.992*
- *Test error in first 100 images: 0*

Since this worked great, we should try a larger kernel again, because we might assume that even a 9x9 kernel is to detailed.

In [ ]:
```
set_seeds()
model6 = make_model(32,11,2,True)
model6.summary()
```

Model: "sequential_66"

Layer (type)	Output Shape	Param #
conv2d_126 (Conv2D)	(None, 18, 18, 32)	3,904
max_pooling2d_125 (MaxPooling2D)	(None, 9, 9, 32)	0
flatten_65 (Flatten)	(None, 2592)	0
dense_91 (Dense)	(None, 100)	259,300
dropout_26 (Dropout)	(None, 100)	0
dense_92 (Dense)	(None, 10)	1,010

Total params: 264,214 (1.01 MB)

Trainable params: 264,214 (1.01 MB)

Non-trainable params: 0 (0.00 B)

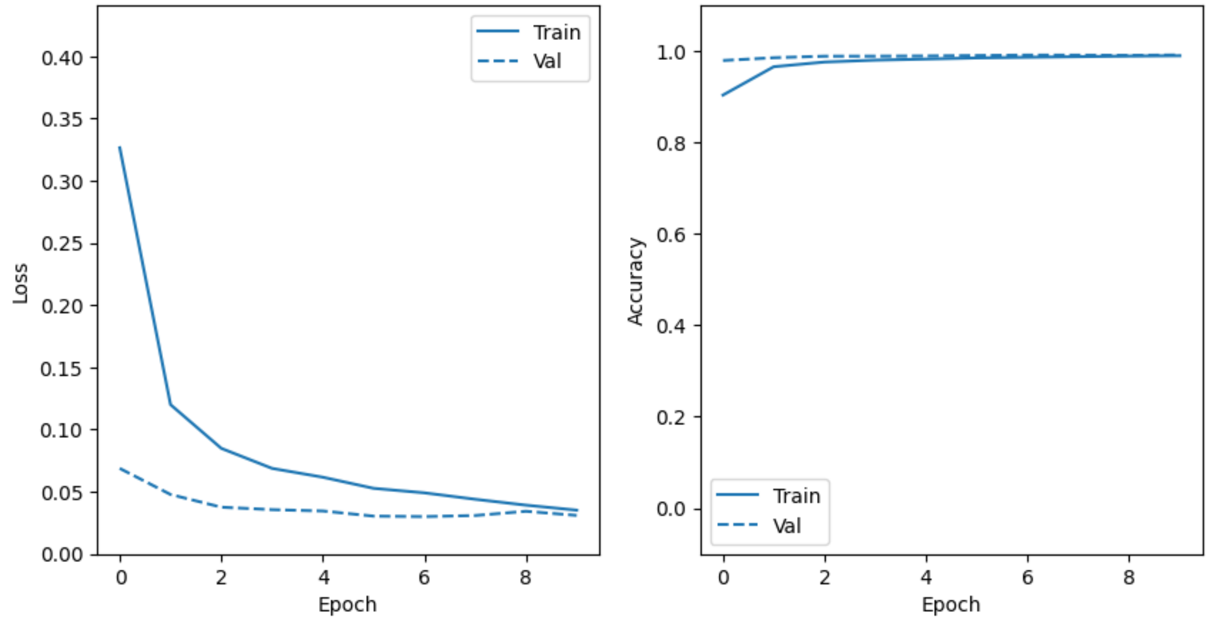
This one more time reduced the model complexity. However, the CNN eats more of the corner pixels, i.e. 5 in each corner, and ,thus, focues more on the center of the image. Let's check the training.

In [ ]:
```
model6_history = model6.fit(
    x_train,
    y_train,
    epochs=EPOCHS,
    callbacks = [early_stopping],
    validation_data=(x_test, y_test),
    batch_size=BATCH_SIZE)
```

```
Epoch 1/10
469/469 ━━━━━━━━━━━━━━━━━━━━ 3s 5ms/step - accuracy: 0.8090 - loss: 0.6164 - val_acc
uracy: 0.9784 - val_loss: 0.0688
Epoch 2/10
469/469 ━━━━━━━━━━━━━━━━━━━━ 2s 5ms/step - accuracy: 0.9626 - loss: 0.1280 - val_acc
uracy: 0.9845 - val_loss: 0.0478
Epoch 3/10
469/469 ━━━━━━━━━━━━━━━━━━━━ 2s 5ms/step - accuracy: 0.9734 - loss: 0.0870 - val_acc
uracy: 0.9878 - val_loss: 0.0376
Epoch 4/10
469/469 ━━━━━━━━━━━━━━━━━━━━ 2s 5ms/step - accuracy: 0.9785 - loss: 0.0709 - val_acc
uracy: 0.9879 - val_loss: 0.0356
Epoch 5/10
469/469 ━━━━━━━━━━━━━━━━━━━━ 2s 5ms/step - accuracy: 0.9808 - loss: 0.0624 - val_acc
uracy: 0.9884 - val_loss: 0.0346
Epoch 6/10
469/469 ━━━━━━━━━━━━━━━━━━━━ 2s 5ms/step - accuracy: 0.9844 - loss: 0.0523 - val_acc
uracy: 0.9894 - val_loss: 0.0304
Epoch 7/10
469/469 ━━━━━━━━━━━━━━━━━━━━ 2s 5ms/step - accuracy: 0.9859 - loss: 0.0491 - val_acc
uracy: 0.9903 - val_loss: 0.0300
Epoch 8/10
469/469 ━━━━━━━━━━━━━━━━━━━━ 2s 5ms/step - accuracy: 0.9867 - loss: 0.0436 - val_acc
uracy: 0.9902 - val_loss: 0.0309
Epoch 9/10
469/469 ━━━━━━━━━━━━━━━━━━━━ 2s 5ms/step - accuracy: 0.9883 - loss: 0.0384 - val_acc
uracy: 0.9897 - val_loss: 0.0343
Epoch 10/10
469/469 ━━━━━━━━━━━━━━━━━━━━ 2s 5ms/step - accuracy: 0.9894 - loss: 0.0345 - val_acc
uracy: 0.9906 - val_loss: 0.0310
Epoch 10: early stopping
Restoring model weights from the end of the best epoch: 1.
```

I don't know why the early stopping callback restored the weights of epoch 1, since the best val_accuracy was found in epoch 10. So I ran the training again. After I plotted the training history.

In [ ]:
```
plot_metrics(model6_history)
```



In [ ]:

```
set_seeds()
model6 = make_model(32,11,2,True)

model6_history = model6.fit(
    x_train,
    y_train,
    epochs=10,
    validation_data=(x_test, y_test),
    batch_size=BATCH_SIZE)
```

```
Epoch 1/10
469/469 ━━━━━━━━━━━━━━━━━━━━ 3s 5ms/step - accuracy: 0.8090 - loss: 0.6164 - val_acc
uracy: 0.9784 - val_loss: 0.0688
Epoch 2/10
469/469 ━━━━━━━━━━━━━━━━━━━━ 2s 5ms/step - accuracy: 0.9626 - loss: 0.1280 - val_acc
uracy: 0.9845 - val_loss: 0.0478
Epoch 3/10
469/469 ━━━━━━━━━━━━━━━━━━━━ 2s 5ms/step - accuracy: 0.9734 - loss: 0.0870 - val_acc
uracy: 0.9878 - val_loss: 0.0376
Epoch 4/10
469/469 ━━━━━━━━━━━━━━━━━━━━ 2s 5ms/step - accuracy: 0.9785 - loss: 0.0709 - val_acc
uracy: 0.9879 - val_loss: 0.0356
Epoch 5/10
469/469 ━━━━━━━━━━━━━━━━━━━━ 2s 5ms/step - accuracy: 0.9808 - loss: 0.0624 - val_acc
uracy: 0.9884 - val_loss: 0.0346
Epoch 6/10
469/469 ━━━━━━━━━━━━━━━━━━━━ 2s 5ms/step - accuracy: 0.9844 - loss: 0.0523 - val_acc
uracy: 0.9894 - val_loss: 0.0304
Epoch 7/10
469/469 ━━━━━━━━━━━━━━━━━━━━ 3s 5ms/step - accuracy: 0.9859 - loss: 0.0491 - val_acc
uracy: 0.9903 - val_loss: 0.0300
Epoch 8/10
469/469 ━━━━━━━━━━━━━━━━━━━━ 2s 5ms/step - accuracy: 0.9867 - loss: 0.0436 - val_acc
uracy: 0.9902 - val_loss: 0.0309
Epoch 9/10
469/469 ━━━━━━━━━━━━━━━━━━━━ 2s 5ms/step - accuracy: 0.9883 - loss: 0.0384 - val_acc
uracy: 0.9897 - val_loss: 0.0343
Epoch 10/10
469/469 ━━━━━━━━━━━━━━━━━━━━ 2s 5ms/step - accuracy: 0.9894 - loss: 0.0345 - val_acc
uracy: 0.9906 - val_loss: 0.0310
```

In [ ]:

```
loss, acc = model6.evaluate(x_test, y_test, verbose=0)
print('Accuracy: %.3f' % acc)
```

```
Accuracy: 0.991
```

In [ ]:

```
err = print_res(model6)
```

```
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 26ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
9
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 12ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 10ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
4
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 10ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 14ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 12ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 11ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
4
```

```
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 11ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
5
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 13ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 10ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
0
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 10ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 10ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
7
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 10ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 10ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 10ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 10ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 10ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 10ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 10ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
9
--
```

```
7 2 1 0 4 1 4 9 5 9
0 6 9 0 1 5 9 7 3 4
9 6 6 5 4 0 7 4 0 1
3 1 3 4 7 2 7 1 2 1
1 7 4 2 3 5 1 2 4 4
6 3 5 5 6 0 4 1 9 5
7 8 9 3 7 4 6 4 3 0
7 0 2 9 1 7 3 2 9 7
7 6 2 7 8 4 7 3 6 1
3 6 9 3 1 4 1 7 6 9
```

In [ ]:
```
err
```

Out[ ]: 0

So, in terms of accuracy, this went into the wrong direction (a little bit). However, we decrease the models complexity. But since there are other approaches to increase model complexity, I will stick to the 9x9 kernel from the previous model.

Model 6 parameters:

- *Model Complexity (Paramereters) = 264214*
- *Accuracy: 0.991*
- *Test error in first 100 images: 0*

Now, let's reduce the number of kernels, in order to decrease the number of parameters a bit more. This is because I assume, that there are less than 32 features necessary to do the classification.

In [ ]:
```
set_seeds()
model7 = make_model(16,9,2,True)
model7.summary()
```

Model: "sequential_68"

Layer (type)	Output Shape	Param #
conv2d_128 (Conv2D)	(None, 20, 20, 16)	1,312
max_pooling2d_127 (MaxPooling2D)	(None, 10, 10, 16)	0
flatten_67 (Flatten)	(None, 1600)	0
dense_95 (Dense)	(None, 100)	160,100
dropout_28 (Dropout)	(None, 100)	0
dense_96 (Dense)	(None, 10)	1,010

Total params: 162,422 (634.46 KB)

Trainable params: 162,422 (634.46 KB)

Non-trainable params: 0 (0.00 B)

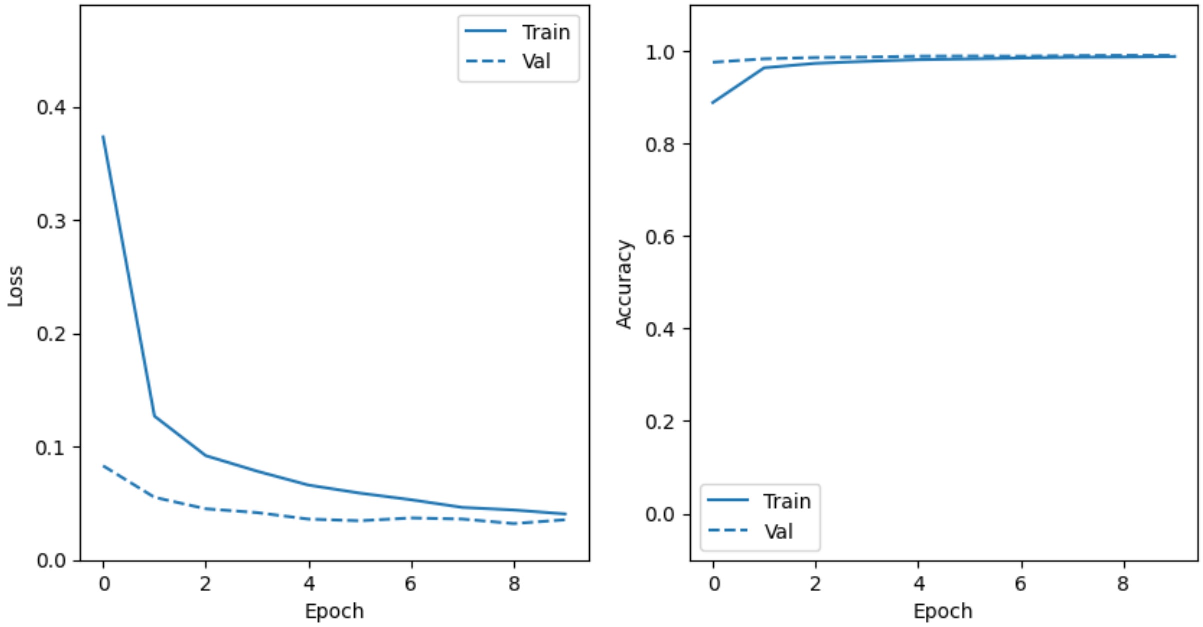
Here, I used 16 kernels instead of 32, which approximately halfes the number of parameters.

In [ ]:
```
model7_history = model7.fit(
    x_train,
    y_train,
    epochs=EPOCHS,
    callbacks = [early_stopping],
    validation_data=(x_test, y_test),
    batch_size=BATCH_SIZE)
```

```
Epoch 1/10
469/469 ━━━━━━━━━━━━━━━━━━━━ 2s 3ms/step - accuracy: 0.7845 - loss: 0.7012 - val_acc
uracy: 0.9748 - val_loss: 0.0831
Epoch 2/10
469/469 ━━━━━━━━━━━━━━━━━━━━ 2s 3ms/step - accuracy: 0.9606 - loss: 0.1369 - val_acc
uracy: 0.9822 - val_loss: 0.0553
Epoch 3/10
469/469 ━━━━━━━━━━━━━━━━━━━━ 2s 3ms/step - accuracy: 0.9718 - loss: 0.0958 - val_acc
uracy: 0.9850 - val_loss: 0.0452
Epoch 4/10
469/469 ━━━━━━━━━━━━━━━━━━━━ 2s 3ms/step - accuracy: 0.9759 - loss: 0.0814 - val_acc
uracy: 0.9859 - val_loss: 0.0418
Epoch 5/10
469/469 ━━━━━━━━━━━━━━━━━━━━ 2s 3ms/step - accuracy: 0.9804 - loss: 0.0665 - val_acc
uracy: 0.9879 - val_loss: 0.0361
Epoch 6/10
469/469 ━━━━━━━━━━━━━━━━━━━━ 2s 3ms/step - accuracy: 0.9829 - loss: 0.0577 - val_acc
uracy: 0.9882 - val_loss: 0.0346
Epoch 7/10
469/469 ━━━━━━━━━━━━━━━━━━━━ 1s 3ms/step - accuracy: 0.9837 - loss: 0.0537 - val_acc
uracy: 0.9878 - val_loss: 0.0371
Epoch 8/10
469/469 ━━━━━━━━━━━━━━━━━━━━ 2s 3ms/step - accuracy: 0.9861 - loss: 0.0461 - val_acc
uracy: 0.9891 - val_loss: 0.0361
Epoch 9/10
469/469 ━━━━━━━━━━━━━━━━━━━━ 2s 3ms/step - accuracy: 0.9861 - loss: 0.0440 - val_acc
uracy: 0.9896 - val_loss: 0.0321
Epoch 10/10
469/469 ━━━━━━━━━━━━━━━━━━━━ 2s 3ms/step - accuracy: 0.9877 - loss: 0.0406 - val_acc
uracy: 0.9895 - val_loss: 0.0355
Epoch 10: early stopping
Restoring model weights from the end of the best epoch: 1.
```

I don't know why the early stopping callback restored the weights of epoch 1, since the best val_accuracy was found in epoch 9. So I ran the training again. After I plotted the training history.

In [ ]:
```
plot_metrics(model7_history)
```



In [ ]:
```
set_seeds()
model7 = make_model(16,9,2,True)

model7_history = model7.fit(
    x_train,
    y_train,
    epochs=9,
    validation_data=(x_test, y_test),
    batch_size=BATCH_SIZE)
```

```
Epoch 1/9
469/469 ━━━━━━━━━━━━━━━━━━━━ 2s 3ms/step - accuracy: 0.7845 - loss: 0.7012 - val_acc
uracy: 0.9748 - val_loss: 0.0831
Epoch 2/9
469/469 ━━━━━━━━━━━━━━━━━━━━ 2s 3ms/step - accuracy: 0.9606 - loss: 0.1369 - val_acc
uracy: 0.9822 - val_loss: 0.0553
Epoch 3/9
469/469 ━━━━━━━━━━━━━━━━━━━━ 1s 3ms/step - accuracy: 0.9718 - loss: 0.0958 - val_acc
uracy: 0.9850 - val_loss: 0.0452
Epoch 4/9
469/469 ━━━━━━━━━━━━━━━━━━━━ 2s 3ms/step - accuracy: 0.9759 - loss: 0.0814 - val_acc
uracy: 0.9859 - val_loss: 0.0418
Epoch 5/9
469/469 ━━━━━━━━━━━━━━━━━━━━ 2s 3ms/step - accuracy: 0.9804 - loss: 0.0665 - val_acc
uracy: 0.9879 - val_loss: 0.0361
Epoch 6/9
469/469 ━━━━━━━━━━━━━━━━━━━━ 1s 3ms/step - accuracy: 0.9829 - loss: 0.0577 - val_acc
uracy: 0.9882 - val_loss: 0.0346
Epoch 7/9
469/469 ━━━━━━━━━━━━━━━━━━━━ 2s 3ms/step - accuracy: 0.9837 - loss: 0.0537 - val_acc
uracy: 0.9878 - val_loss: 0.0371
Epoch 8/9
469/469 ━━━━━━━━━━━━━━━━━━━━ 2s 3ms/step - accuracy: 0.9861 - loss: 0.0461 - val_acc
uracy: 0.9891 - val_loss: 0.0361
Epoch 9/9
469/469 ━━━━━━━━━━━━━━━━━━━━ 1s 3ms/step - accuracy: 0.9861 - loss: 0.0440 - val_acc
uracy: 0.9896 - val_loss: 0.0321
```

In [ ]:
```
loss, acc = model7.evaluate(x_test, y_test, verbose=0)
print('Accuracy: %.3f' % acc)
```

```
Accuracy: 0.990
```

In [ ]:
```
err = print_res(model7)
```

```
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 24ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 24ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
9
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
4
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 12ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
4
```

```
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 10ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 10ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
5
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 12ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
0
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 8ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 10ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
7
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 10ms/step
1
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
9
--
```

```
7 2 1 0 4 1 4 9 5 9
0 6 9 0 1 5 9 7 3 4
9 6 6 5 4 0 7 4 0 1
3 1 3 4 7 2 7 1 2 1
1 7 4 2 3 5 1 2 4 4
6 3 5 5 6 0 4 1 9 5
7 8 9 3 7 4 6 4 3 0
7 0 2 9 1 7 3 2 9 7
7 6 2 7 8 4 7 3 6 1
3 6 9 3 1 4 1 7 6 9
```

In [ ]:
```
err
```

Out[ ]:
```
0
```

So one more time, in terms of accuracy, this went into the wrong direction. However, we decrease the models complexity much more than in the previous model. But since there are other approaches to increase model complexity, I will stick to the 32 kernels from the model 5.

Model 7 parameters:

- *Model Complexity (Paramereters) = 162422*
- *Accuracy: 0.990*
- *Test error in first 100 images: 0*

Let's try to add another convolution layer, because I assume that the features extracted from the first convolution layer can be aggregated one more time to reduce the parameters for the dense layers. However, I will use a smaller kernel, because I have to work the with output of the max pooling layer of 10x10

In [ ]:
```
set_seeds()
model8 = Sequential()
model8.add(Conv2D(32, (9,9), activation='relu',input_shape=in_shape))
model8.add(MaxPool2D((2, 2)))
model8.add(Conv2D(32, (3,3), activation='relu'))
model8.add(MaxPool2D((1, 1)))
model8.add(Flatten())
model8.add(Dense(100, activation='relu'))
model8.add(Dropout(0.5))
model8.add(Dense(n_classes, activation='softmax'))
model8.compile(optimizer='adam', loss='sparse_categorical_crossentropy',metrics=['a

model8.summary()
```

Model: "sequential_70"

Layer (type)	Output Shape	Param #
conv2d_130 (Conv2D)	(None, 20, 20, 32)	2,624
max_pooling2d_129 (MaxPooling2D)	(None, 10, 10, 32)	0
conv2d_131 (Conv2D)	(None, 8, 8, 32)	9,248
max_pooling2d_130 (MaxPooling2D)	(None, 8, 8, 32)	0
flatten_69 (Flatten)	(None, 2048)	0
dense_99 (Dense)	(None, 100)	204,900
dropout_30 (Dropout)	(None, 100)	0
dense_100 (Dense)	(None, 10)	1,010

Total params: 217,782 (850.71 KB)

Trainable params: 217,782 (850.71 KB)

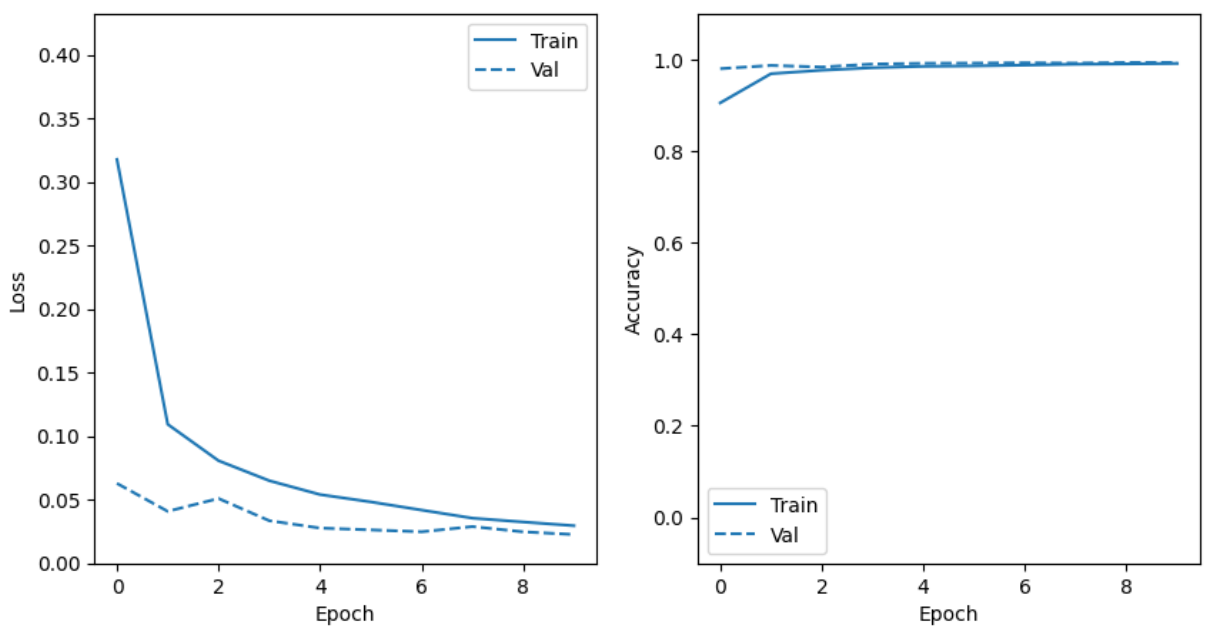
Non-trainable params: 0 (0.00 B)

The second convolution layer eats again 1 corner pixels from the output of the first convolution layer. Because of the stride of 2 in the first pooling layer, the cnn will eat effectively 6 corner pixels of the image. (May a little bit to many.)

In [ ]:
```
model8_history = model8.fit(
    x_train,
    y_train,
    epochs=EPOCHS,
    callbacks = [early_stopping],
    validation_data=(x_test, y_test),
    batch_size=BATCH_SIZE)
```

```
Epoch 1/10
469/469 ━━━━━━━━━━━━━━━━━━━━ 3s 6ms/step - accuracy: 0.8053 - loss: 0.6215 - val_acc
uracy: 0.9795 - val_loss: 0.0628
Epoch 2/10
469/469 ━━━━━━━━━━━━━━━━━━━━ 3s 6ms/step - accuracy: 0.9662 - loss: 0.1177 - val_acc
uracy: 0.9866 - val_loss: 0.0409
Epoch 3/10
469/469 ━━━━━━━━━━━━━━━━━━━━ 3s 6ms/step - accuracy: 0.9768 - loss: 0.0810 - val_acc
uracy: 0.9831 - val_loss: 0.0510
Epoch 4/10
469/469 ━━━━━━━━━━━━━━━━━━━━ 3s 6ms/step - accuracy: 0.9809 - loss: 0.0684 - val_acc
uracy: 0.9896 - val_loss: 0.0335
Epoch 5/10
469/469 ━━━━━━━━━━━━━━━━━━━━ 3s 6ms/step - accuracy: 0.9850 - loss: 0.0555 - val_acc
uracy: 0.9912 - val_loss: 0.0277
Epoch 6/10
469/469 ━━━━━━━━━━━━━━━━━━━━ 3s 6ms/step - accuracy: 0.9858 - loss: 0.0481 - val_acc
uracy: 0.9916 - val_loss: 0.0263
Epoch 7/10
469/469 ━━━━━━━━━━━━━━━━━━━━ 3s 6ms/step - accuracy: 0.9879 - loss: 0.0409 - val_acc
uracy: 0.9924 - val_loss: 0.0248
Epoch 8/10
469/469 ━━━━━━━━━━━━━━━━━━━━ 3s 6ms/step - accuracy: 0.9891 - loss: 0.0359 - val_acc
uracy: 0.9916 - val_loss: 0.0289
Epoch 9/10
469/469 ━━━━━━━━━━━━━━━━━━━━ 3s 6ms/step - accuracy: 0.9904 - loss: 0.0325 - val_acc
uracy: 0.9930 - val_loss: 0.0247
Epoch 10/10
469/469 ━━━━━━━━━━━━━━━━━━━━ 3s 6ms/step - accuracy: 0.9910 - loss: 0.0300 - val_acc
uracy: 0.9930 - val_loss: 0.0226
Restoring model weights from the end of the best epoch: 9.
```

In [ ]:
```
plot_metrics(model8_history)
```



In [ ]:
```
loss, acc = model8.evaluate(x_test, y_test, verbose=0)
print('Accuracy: %.3f' % acc)
```

```
Accuracy: 0.993
```

In [ ]:
```
err = print_res(model8)
```

```
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 29ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
9
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 10ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 10ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 10ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
4
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 10ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 10ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 29ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 11ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 10ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 10ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 10ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
4
```

```
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
5
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
0
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 10ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
7
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 10ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 10ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 10ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 10ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 10ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 10ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
9
--
```

```
7 2 1 0 4 1 4 9 5 9
0 6 9 0 1 5 9 7 3 4
9 6 6 5 4 0 7 4 0 1
3 1 3 4 7 2 7 1 2 1
1 7 4 2 3 5 1 2 4 4
6 3 5 5 6 0 4 1 9 5
7 8 9 3 7 4 6 4 3 0
7 0 2 9 1 7 3 2 9 7
7 6 2 7 8 4 7 3 6 1
3 6 9 3 1 4 1 7 6 9
```

In [ ]:
```
err
```

Out[ ]: 0

This resulted in the best accuracy we have seen so far. Additionally, the models complexity is lower than our current benchmark model 5.

Model 8 parameters:

- *Model Complexity (Paramereters) = 217782*
- *Accuracy: 0.993*
- *Test error in first 100 images: 0*

Now, let's try to add stride to the second convolution layer, because I assume that the output doesn't have to be that detailed in order to do the classification.

In [ ]:
```
set_seeds()
model9 = Sequential()
model9.add(Conv2D(32, (9,9), activation='relu',input_shape=in_shape))
model9.add(MaxPool2D((2, 2)))
model9.add(Conv2D(32, (3,3), activation='relu'))
model9.add(MaxPool2D((2, 2)))
model9.add(Flatten())
model9.add(Dense(100, activation='relu'))
model9.add(Dropout(0.5))
model9.add(Dense(n_classes, activation='softmax'))
model9.compile(optimizer='adam', loss='sparse_categorical_crossentropy',metrics=['a

model9.summary()
```

Model: "sequential_71"

Layer (type)	Output Shape	Param #
conv2d_132 (Conv2D)	(None, 20, 20, 32)	2,624
max_pooling2d_131 (MaxPooling2D)	(None, 10, 10, 32)	0
conv2d_133 (Conv2D)	(None, 8, 8, 32)	9,248
max_pooling2d_132 (MaxPooling2D)	(None, 4, 4, 32)	0
flatten_70 (Flatten)	(None, 512)	0
dense_101 (Dense)	(None, 100)	51,300
dropout_31 (Dropout)	(None, 100)	0
dense_102 (Dense)	(None, 10)	1,010

Total params: 64,182 (250.71 KB)

Trainable params: 64,182 (250.71 KB)

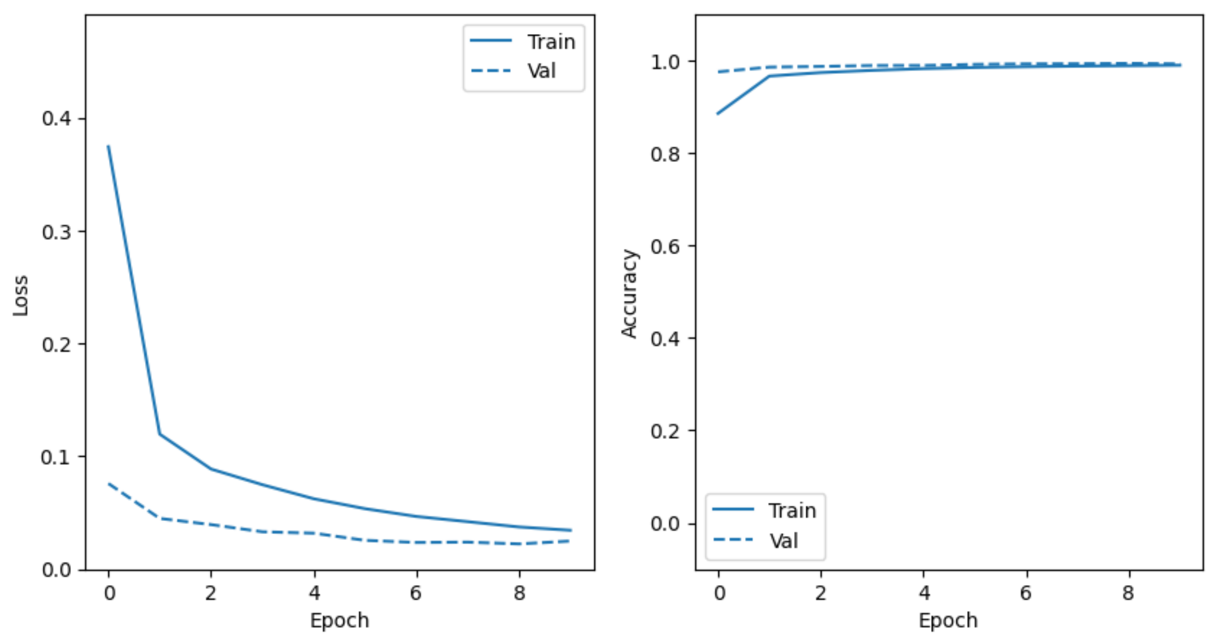
Non-trainable params: 0 (0.00 B)

You see, we have decreased the models complexity enourmos.

In [ ]:
```
model9_history = model9.fit(
    x_train,
    y_train,
    epochs=EPOCHS,
    callbacks = [early_stopping],
    validation_data=(x_test, y_test),
    batch_size=BATCH_SIZE)
```

```
Epoch 1/10
469/469 ━━━━━━━━━━━━━━━━━━━━ 3s 5ms/step - accuracy: 0.7644 - loss: 0.7453 - val_acc
uracy: 0.9748 - val_loss: 0.0759
Epoch 2/10
469/469 ━━━━━━━━━━━━━━━━━━━━ 2s 5ms/step - accuracy: 0.9631 - loss: 0.1295 - val_acc
uracy: 0.9850 - val_loss: 0.0450
Epoch 3/10
469/469 ━━━━━━━━━━━━━━━━━━━━ 2s 5ms/step - accuracy: 0.9729 - loss: 0.0900 - val_acc
uracy: 0.9865 - val_loss: 0.0395
Epoch 4/10
469/469 ━━━━━━━━━━━━━━━━━━━━ 2s 5ms/step - accuracy: 0.9779 - loss: 0.0768 - val_acc
uracy: 0.9885 - val_loss: 0.0332
Epoch 5/10
469/469 ━━━━━━━━━━━━━━━━━━━━ 2s 5ms/step - accuracy: 0.9821 - loss: 0.0630 - val_acc
uracy: 0.9885 - val_loss: 0.0320
Epoch 6/10
469/469 ━━━━━━━━━━━━━━━━━━━━ 2s 5ms/step - accuracy: 0.9846 - loss: 0.0516 - val_acc
uracy: 0.9909 - val_loss: 0.0256
Epoch 7/10
469/469 ━━━━━━━━━━━━━━━━━━━━ 2s 5ms/step - accuracy: 0.9873 - loss: 0.0435 - val_acc
uracy: 0.9921 - val_loss: 0.0237
Epoch 8/10
469/469 ━━━━━━━━━━━━━━━━━━━━ 2s 5ms/step - accuracy: 0.9871 - loss: 0.0423 - val_acc
uracy: 0.9924 - val_loss: 0.0240
Epoch 9/10
469/469 ━━━━━━━━━━━━━━━━━━━━ 2s 5ms/step - accuracy: 0.9883 - loss: 0.0380 - val_acc
uracy: 0.9931 - val_loss: 0.0224
Epoch 10/10
469/469 ━━━━━━━━━━━━━━━━━━━━ 2s 5ms/step - accuracy: 0.9897 - loss: 0.0334 - val_acc
uracy: 0.9921 - val_loss: 0.0250
Restoring model weights from the end of the best epoch: 9.
```

In [ ]:
```
plot_metrics(model9_history)
```



In [ ]:
```
loss, acc = model9.evaluate(x_test, y_test, verbose=0)
print('Accuracy: %.3f' % acc)
```

```
Accuracy: 0.993
```

In [ ]:
```
err = print_res(model9)
```

```
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 29ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
9
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
4
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 10ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 10ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 10ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 10ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
4
```

```
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 10ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
5
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
0
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 10ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 13ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
7
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 10ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 10ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 10ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 10ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 10ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 10ms/step
1
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 10ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 10ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 10ms/step
9
--
```

```
7 2 1 0 4 1 4 9 5 9
0 6 9 0 1 5 9 7 3 4
9 6 6 5 4 0 7 4 0 1
3 1 3 4 7 2 7 1 2 1
1 7 4 2 3 5 1 2 4 4
6 3 5 5 6 0 4 1 9 5
7 8 9 3 7 4 6 4 3 0
7 0 2 9 1 7 3 2 9 7
7 6 2 7 8 4 7 3 6 1
3 6 9 3 1 4 1 7 6 9
```

In [ ]:

```
err
```

Out[ ]: 1

That worked too. We sticked to the level of accuracy, but decreased the models complexity one more time.

Model 9 parameters:

- *Model Complexity (Paramereters) = 64182*
- *Accuracy: 0.993*
- *Test error in first 100 images: 1*

Let's add another convolution layer, because I think that there are some low leverl features that can help to do the classification. This would also result in a further reduction of the number of parameters.

In [ ]:

```
set_seeds()
model10 = Sequential()
model10.add(Conv2D(32, (9,9), activation='relu',input_shape=in_shape))
model10.add(MaxPool2D((2, 2)))
model10.add(Conv2D(32, (3,3), activation='relu'))
model10.add(MaxPool2D((2,2)))
model10.add(Conv2D(32, (3,3), activation='relu'))
model10.add(MaxPool2D((1,1)))
model10.add(Flatten())
model10.add(Dense(100, activation='relu'))
model10.add(Dropout(0.5))
model10.add(Dense(n_classes, activation='softmax'))
model10.compile(optimizer='adam', loss='sparse_categorical_crossentropy',metrics=['
model10.summary()
```

Model: "sequential_72"

Layer (type)	Output Shape	Param #
conv2d_134 (Conv2D)	(None, 20, 20, 32)	2,624
max_pooling2d_133 (MaxPooling2D)	(None, 10, 10, 32)	0
conv2d_135 (Conv2D)	(None, 8, 8, 32)	9,248
max_pooling2d_134 (MaxPooling2D)	(None, 4, 4, 32)	0
conv2d_136 (Conv2D)	(None, 2, 2, 32)	9,248
max_pooling2d_135 (MaxPooling2D)	(None, 2, 2, 32)	0
flatten_71 (Flatten)	(None, 128)	0
dense_103 (Dense)	(None, 100)	12,900
dropout_32 (Dropout)	(None, 100)	0
dense_104 (Dense)	(None, 10)	1,010

Total params: 35,030 (136.84 KB)

Trainable params: 35,030 (136.84 KB)

Non-trainable params: 0 (0.00 B)

I added a third conv layer with a 32 3x3 kernels, which would result in more corner pixels to be eaten by the cnn and, thus, let the cnn focus even more on the center of the image. After the conv layer, I added a pseudo Pooling layer with a stride of 1.

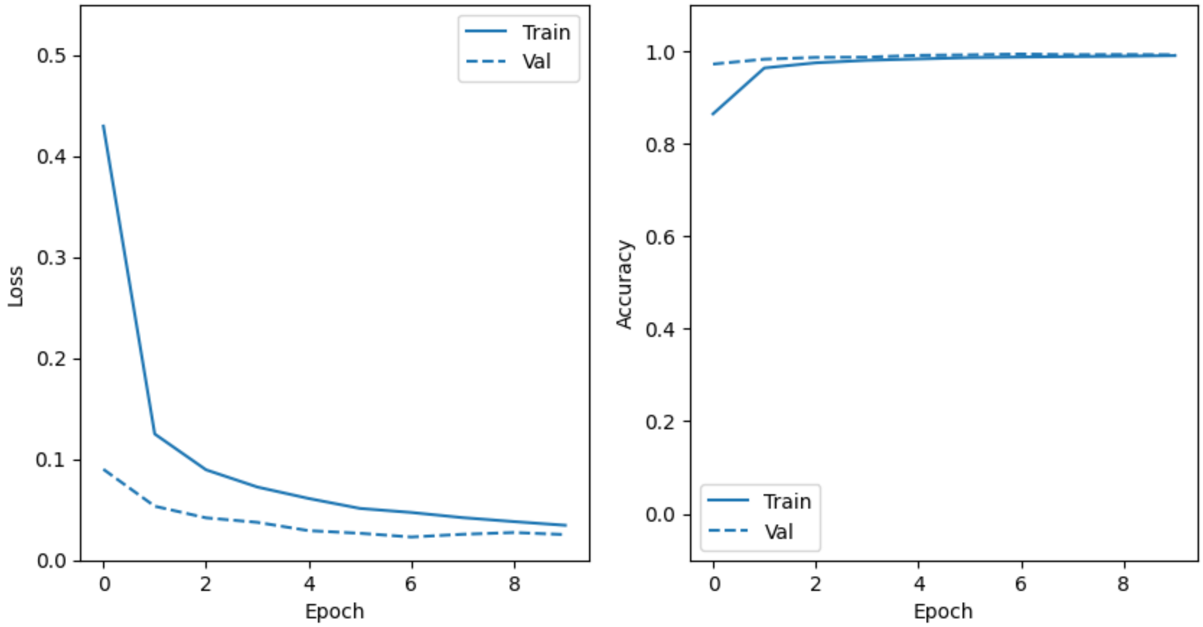
In [ ]:
```
model10_history = model10.fit(
    x_train,
    y_train,
    epochs=EPOCHS,
    callbacks = [early_stopping],
    validation_data=(x_test, y_test),
    batch_size=BATCH_SIZE)
```

```
Epoch 1/10
469/469 ━━━━━━━━━━━━━━━━━━━━ 3s 5ms/step - accuracy: 0.7192 - loss: 0.8423 - val_acc
uracy: 0.9714 - val_loss: 0.0900
Epoch 2/10
469/469 ━━━━━━━━━━━━━━━━━━━━ 2s 5ms/step - accuracy: 0.9603 - loss: 0.1380 - val_acc
uracy: 0.9819 - val_loss: 0.0534
Epoch 3/10
469/469 ━━━━━━━━━━━━━━━━━━━━ 2s 5ms/step - accuracy: 0.9723 - loss: 0.0963 - val_acc
uracy: 0.9858 - val_loss: 0.0419
Epoch 4/10
469/469 ━━━━━━━━━━━━━━━━━━━━ 2s 5ms/step - accuracy: 0.9779 - loss: 0.0777 - val_acc
uracy: 0.9863 - val_loss: 0.0375
Epoch 5/10
469/469 ━━━━━━━━━━━━━━━━━━━━ 2s 5ms/step - accuracy: 0.9818 - loss: 0.0631 - val_acc
uracy: 0.9903 - val_loss: 0.0293
Epoch 6/10
469/469 ━━━━━━━━━━━━━━━━━━━━ 2s 5ms/step - accuracy: 0.9855 - loss: 0.0512 - val_acc
uracy: 0.9913 - val_loss: 0.0267
Epoch 7/10
469/469 ━━━━━━━━━━━━━━━━━━━━ 2s 5ms/step - accuracy: 0.9863 - loss: 0.0478 - val_acc
uracy: 0.9931 - val_loss: 0.0230
Epoch 8/10
469/469 ━━━━━━━━━━━━━━━━━━━━ 2s 5ms/step - accuracy: 0.9878 - loss: 0.0401 - val_acc
uracy: 0.9917 - val_loss: 0.0256
Epoch 9/10
469/469 ━━━━━━━━━━━━━━━━━━━━ 2s 5ms/step - accuracy: 0.9890 - loss: 0.0362 - val_acc
uracy: 0.9921 - val_loss: 0.0274
Epoch 10/10
469/469 ━━━━━━━━━━━━━━━━━━━━ 2s 5ms/step - accuracy: 0.9902 - loss: 0.0332 - val_acc
uracy: 0.9916 - val_loss: 0.0254
Epoch 10: early stopping
Restoring model weights from the end of the best epoch: 1.
```

I don't know why the early stopping callback restored the weights of epoch 1, since the best val_accuracy was found in epoch 7. So I ran the training again. After I plotted the training history.

In [ ]:

```
plot_metrics(model10_history)
```



In [ ]:

```
set_seeds()
model10 = Sequential()
model10.add(Conv2D(32, (9,9), activation='relu',input_shape=in_shape))
model10.add(MaxPool2D((2, 2)))
model10.add(Conv2D(32, (3,3), activation='relu'))
model10.add(MaxPool2D((2,2)))
model10.add(Conv2D(32, (3,3), activation='relu'))
model10.add(MaxPool2D((1,1)))
model10.add(Flatten())
model10.add(Dense(100, activation='relu'))
model10.add(Dropout(0.5))
model10.add(Dense(n_classes, activation='softmax'))
model10.compile(optimizer='adam', loss='sparse_categorical_crossentropy',metrics=['

model10_history = model10.fit(
    x_train,
    y_train,
    epochs=7,
    validation_data=(x_test, y_test),
    batch_size=BATCH_SIZE)
```

```
Epoch 1/7
469/469 ━━━━━━━━━━━━━━━━━━━━ 3s 5ms/step - accuracy: 0.7192 - loss: 0.8423 - val_acc
uracy: 0.9714 - val_loss: 0.0900
Epoch 2/7
469/469 ━━━━━━━━━━━━━━━━━━━━ 2s 5ms/step - accuracy: 0.9603 - loss: 0.1380 - val_acc
uracy: 0.9819 - val_loss: 0.0534
Epoch 3/7
469/469 ━━━━━━━━━━━━━━━━━━━━ 2s 5ms/step - accuracy: 0.9723 - loss: 0.0963 - val_acc
uracy: 0.9858 - val_loss: 0.0419
Epoch 4/7
469/469 ━━━━━━━━━━━━━━━━━━━━ 2s 5ms/step - accuracy: 0.9779 - loss: 0.0777 - val_acc
uracy: 0.9863 - val_loss: 0.0375
Epoch 5/7
469/469 ━━━━━━━━━━━━━━━━━━━━ 2s 5ms/step - accuracy: 0.9818 - loss: 0.0631 - val_acc
uracy: 0.9903 - val_loss: 0.0293
Epoch 6/7
469/469 ━━━━━━━━━━━━━━━━━━━━ 2s 5ms/step - accuracy: 0.9855 - loss: 0.0512 - val_acc
uracy: 0.9913 - val_loss: 0.0267
Epoch 7/7
469/469 ━━━━━━━━━━━━━━━━━━━━ 2s 5ms/step - accuracy: 0.9863 - loss: 0.0478 - val_acc
uracy: 0.9931 - val_loss: 0.0230
```

In [ ]:

```
loss, acc = model10.evaluate(x_test, y_test, verbose=0)
print('Accuracy: %.3f' % acc)
```

```
Accuracy: 0.993
```

In [ ]:

```
err = print_res(model10)
```

```
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 32ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 32ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 10ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 10ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 12ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
9
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
4
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 10ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
4
```

```
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 10ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 10ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 10ms/step
5
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 10ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 10ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 10ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
0
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 10ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 10ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 10ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 10ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
7
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 10ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 10ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 10ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 10ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 10ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 10ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 10ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 10ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
9
--
```

```
7 2 1 0 4 1 4 9 5 9
0 6 9 0 1 5 9 7 3 4
9 6 6 5 4 0 7 4 0 1
3 1 3 4 7 2 7 1 2 1
1 7 4 2 3 5 1 2 4 4
6 3 5 5 6 0 4 1 9 5
7 8 9 3 7 4 6 4 3 0
7 0 2 9 1 7 3 2 9 7
7 6 2 7 8 4 7 3 6 1
3 6 9 3 1 4 1 7 6 9
```

In [ ]:
```
err
```

Out[ ]: 0

Once again, we sticked to the level of accuracy, but decreased the models complexity one more time.

Model 10 parameters:

- *Model Complexity (Paramereters) = 35030*
- *Accuracy: 0.993*
- *Test error in first 100 images: 0*

Next, I will try to remove the first dense layer, because we could assume that the classification can be done by one layer, since we extracted the relevant features by the conv layers.

In [ ]:
```
set_seeds()
model11 = Sequential()
model11.add(Conv2D(32, (9,9), activation='relu',input_shape=in_shape))
model11.add(MaxPool2D((2, 2)))
model11.add(Conv2D(32, (3,3), activation='relu'))
model11.add(MaxPool2D((2,2)))
model11.add(Conv2D(32, (3,3), activation='relu'))
model11.add(MaxPool2D((1,1)))
model11.add(Flatten())
model11.add(Dense(n_classes, activation='softmax'))
model11.compile(optimizer='adam', loss='sparse_categorical_crossentropy',metrics=['
model11.summary()
```

Model: "sequential_74"

Layer (type)	Output Shape	Param #
conv2d_140 (Conv2D)	(None, 20, 20, 32)	2,624
max_pooling2d_139 (MaxPooling2D)	(None, 10, 10, 32)	0
conv2d_141 (Conv2D)	(None, 8, 8, 32)	9,248
max_pooling2d_140 (MaxPooling2D)	(None, 4, 4, 32)	0
conv2d_142 (Conv2D)	(None, 2, 2, 32)	9,248
max_pooling2d_141 (MaxPooling2D)	(None, 2, 2, 32)	0
flatten_73 (Flatten)	(None, 128)	0
dense_107 (Dense)	(None, 10)	1,290

Total params: 22,410 (87.54 KB)

Trainable params: 22,410 (87.54 KB)

Non-trainable params: 0 (0.00 B)

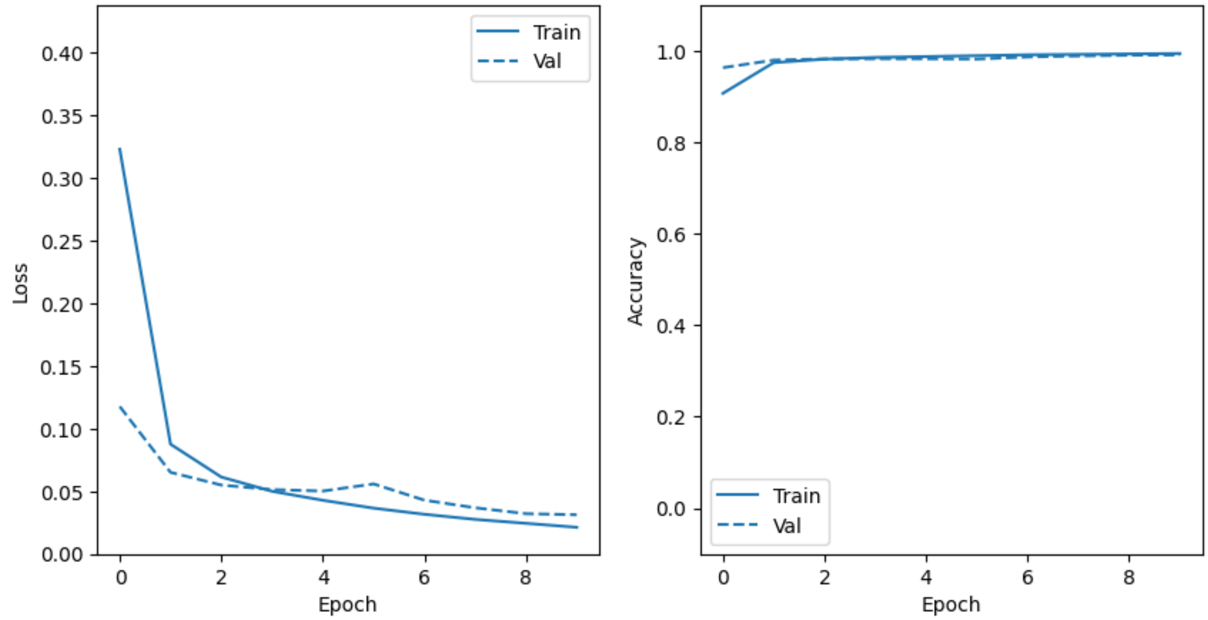
Now, there is only one dense layer, which reduces the complexity of the model one more time.

In [ ]:
```
model11_history = model11.fit(
    x_train,
    y_train,
    epochs=EPOCHS,
    callbacks = [early_stopping],
    validation_data=(x_test, y_test),
    batch_size=BATCH_SIZE)
```

```
Epoch 1/10
469/469 ━━━━━━━━━━━━━━━━━━━━ 3s 5ms/step - accuracy: 0.7976 - loss: 0.7007 - val_acc
uracy: 0.9628 - val_loss: 0.1176
Epoch 2/10
469/469 ━━━━━━━━━━━━━━━━━━━━ 2s 5ms/step - accuracy: 0.9712 - loss: 0.0968 - val_acc
uracy: 0.9792 - val_loss: 0.0651
Epoch 3/10
469/469 ━━━━━━━━━━━━━━━━━━━━ 2s 5ms/step - accuracy: 0.9811 - loss: 0.0639 - val_acc
uracy: 0.9818 - val_loss: 0.0549
Epoch 4/10
469/469 ━━━━━━━━━━━━━━━━━━━━ 2s 5ms/step - accuracy: 0.9850 - loss: 0.0518 - val_acc
uracy: 0.9823 - val_loss: 0.0513
Epoch 5/10
469/469 ━━━━━━━━━━━━━━━━━━━━ 2s 5ms/step - accuracy: 0.9869 - loss: 0.0447 - val_acc
uracy: 0.9822 - val_loss: 0.0502
Epoch 6/10
469/469 ━━━━━━━━━━━━━━━━━━━━ 2s 5ms/step - accuracy: 0.9888 - loss: 0.0382 - val_acc
uracy: 0.9816 - val_loss: 0.0559
Epoch 7/10
469/469 ━━━━━━━━━━━━━━━━━━━━ 2s 5ms/step - accuracy: 0.9910 - loss: 0.0333 - val_acc
uracy: 0.9862 - val_loss: 0.0429
Epoch 8/10
469/469 ━━━━━━━━━━━━━━━━━━━━ 2s 5ms/step - accuracy: 0.9916 - loss: 0.0290 - val_acc
uracy: 0.9884 - val_loss: 0.0368
Epoch 9/10
469/469 ━━━━━━━━━━━━━━━━━━━━ 2s 5ms/step - accuracy: 0.9927 - loss: 0.0255 - val_acc
uracy: 0.9901 - val_loss: 0.0321
Epoch 10/10
469/469 ━━━━━━━━━━━━━━━━━━━━ 2s 5ms/step - accuracy: 0.9938 - loss: 0.0218 - val_acc
uracy: 0.9905 - val_loss: 0.0313
Epoch 10: early stopping
Restoring model weights from the end of the best epoch: 1.
```

I don't know why the early stopping callback restored the weights of epoch 1, since the best val_accuracy was found in epoch 10. So I ran the training again. After I plotted the training history.

In [ ]:
```
plot_metrics(model11_history)
```



In [ ]:

```
set_seeds()
model11 = Sequential()
model11.add(Conv2D(32, (9,9), activation='relu',input_shape=in_shape))
model11.add(MaxPool2D((2, 2)))
model11.add(Conv2D(32, (3,3), activation='relu'))
model11.add(MaxPool2D((2,2)))
model11.add(Conv2D(32, (3,3), activation='relu'))
model11.add(MaxPool2D((1,1)))
model11.add(Flatten())
model11.add(Dense(n_classes, activation='softmax'))
model11.compile(optimizer='adam', loss='sparse_categorical_crossentropy',metrics=['

model11_history = model11.fit(
    x_train,
    y_train,
    epochs=10,
    validation_data=(x_test, y_test),
    batch_size=BATCH_SIZE)
```

```
Epoch 1/10
469/469 ━━━━━━━━━━━━━━━━━━━━ 3s 5ms/step - accuracy: 0.7976 - loss: 0.7007 - val_acc
uracy: 0.9628 - val_loss: 0.1176
Epoch 2/10
469/469 ━━━━━━━━━━━━━━━━━━━━ 2s 5ms/step - accuracy: 0.9712 - loss: 0.0968 - val_acc
uracy: 0.9792 - val_loss: 0.0651
Epoch 3/10
469/469 ━━━━━━━━━━━━━━━━━━━━ 2s 5ms/step - accuracy: 0.9811 - loss: 0.0639 - val_acc
uracy: 0.9818 - val_loss: 0.0549
Epoch 4/10
469/469 ━━━━━━━━━━━━━━━━━━━━ 2s 5ms/step - accuracy: 0.9850 - loss: 0.0518 - val_acc
uracy: 0.9823 - val_loss: 0.0513
Epoch 5/10
469/469 ━━━━━━━━━━━━━━━━━━━━ 2s 5ms/step - accuracy: 0.9869 - loss: 0.0447 - val_acc
uracy: 0.9822 - val_loss: 0.0502
Epoch 6/10
469/469 ━━━━━━━━━━━━━━━━━━━━ 2s 5ms/step - accuracy: 0.9888 - loss: 0.0382 - val_acc
uracy: 0.9816 - val_loss: 0.0559
Epoch 7/10
469/469 ━━━━━━━━━━━━━━━━━━━━ 2s 5ms/step - accuracy: 0.9910 - loss: 0.0333 - val_acc
uracy: 0.9862 - val_loss: 0.0429
Epoch 8/10
469/469 ━━━━━━━━━━━━━━━━━━━━ 2s 5ms/step - accuracy: 0.9916 - loss: 0.0290 - val_acc
uracy: 0.9884 - val_loss: 0.0368
Epoch 9/10
469/469 ━━━━━━━━━━━━━━━━━━━━ 2s 5ms/step - accuracy: 0.9927 - loss: 0.0255 - val_acc
uracy: 0.9901 - val_loss: 0.0321
Epoch 10/10
469/469 ━━━━━━━━━━━━━━━━━━━━ 2s 5ms/step - accuracy: 0.9938 - loss: 0.0218 - val_acc
uracy: 0.9905 - val_loss: 0.0313
```

In [ ]:

```
loss, acc = model11.evaluate(x_test, y_test, verbose=0)
print('Accuracy: %.3f' % acc)
```

```
Accuracy: 0.990
```

In [ ]:

```
err = print_res(model11)
```

```
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 29ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 10ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 10ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
9
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 10ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 10ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
4
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 10ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 10ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 10ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 10ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
4
```

```
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 10ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 11ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
5
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 10ms/step
0
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 10ms/step
7
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 10ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 10ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 10ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 11ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 10ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
9
--
```

```
7 2 1 0 4 1 4 9 5 9
0 6 9 0 1 5 9 7 3 4
9 6 6 5 4 0 7 4 0 1
3 1 3 4 7 2 7 1 2 1
1 7 4 2 3 5 1 2 4 4
6 3 5 5 6 0 4 1 9 5
7 8 9 3 7 4 6 4 3 0
7 0 2 9 1 7 3 2 9 7
7 6 2 7 8 4 7 3 6 1
3 6 9 3 1 4 1 7 6 9
```

In [ ]:
```
err
```

Out[ ]: 1

Here, the accuracy decreased as well as the models complexity.

Model 11 parameters:

- *Model Complexity (Paramereters) = 22410*
- *Accuracy: 0.99*
- *Test error in first 100 images: 1*

In the last experiment I ran, I tried to add same padding. I added it to the second convolution layer, because I assumed previously that the this layer might 'eat' to many of the corner pixels. Therefore, setting the padding, the shape would stay stable and all incomming pixels are taken into account.

In [ ]:
```
set_seeds()
model12 = Sequential()
model12.add(Conv2D(32, (9,9), activation='relu',input_shape=in_shape))
model12.add(MaxPool2D((2, 2)))
model12.add(Conv2D(32, (3,3), activation='relu', padding='same') )
model12.add(MaxPool2D((2,2)))
model12.add(Conv2D(32, (3,3), activation='relu'))
model12.add(MaxPool2D((1,1)))
model12.add(Flatten())
model12.add(Dense(100, activation='relu'))
model12.add(Dropout(0.5))
model12.add(Dense(n_classes, activation='softmax'))
model12.compile(optimizer='adam', loss='sparse_categorical_crossentropy',metrics=['
model12.summary()
```

Model: "sequential_76"

Layer (type)	Output Shape	Param #
conv2d_146 (Conv2D)	(None, 20, 20, 32)	2,624
max_pooling2d_145 (MaxPooling2D)	(None, 10, 10, 32)	0
conv2d_147 (Conv2D)	(None, 10, 10, 32)	9,248
max_pooling2d_146 (MaxPooling2D)	(None, 5, 5, 32)	0
conv2d_148 (Conv2D)	(None, 3, 3, 32)	9,248
max_pooling2d_147 (MaxPooling2D)	(None, 3, 3, 32)	0
flatten_75 (Flatten)	(None, 288)	0
dense_109 (Dense)	(None, 100)	28,900
dropout_34 (Dropout)	(None, 100)	0
dense_110 (Dense)	(None, 10)	1,010

Total params: 51,030 (199.34 KB)

Trainable params: 51,030 (199.34 KB)

Non-trainable params: 0 (0.00 B)

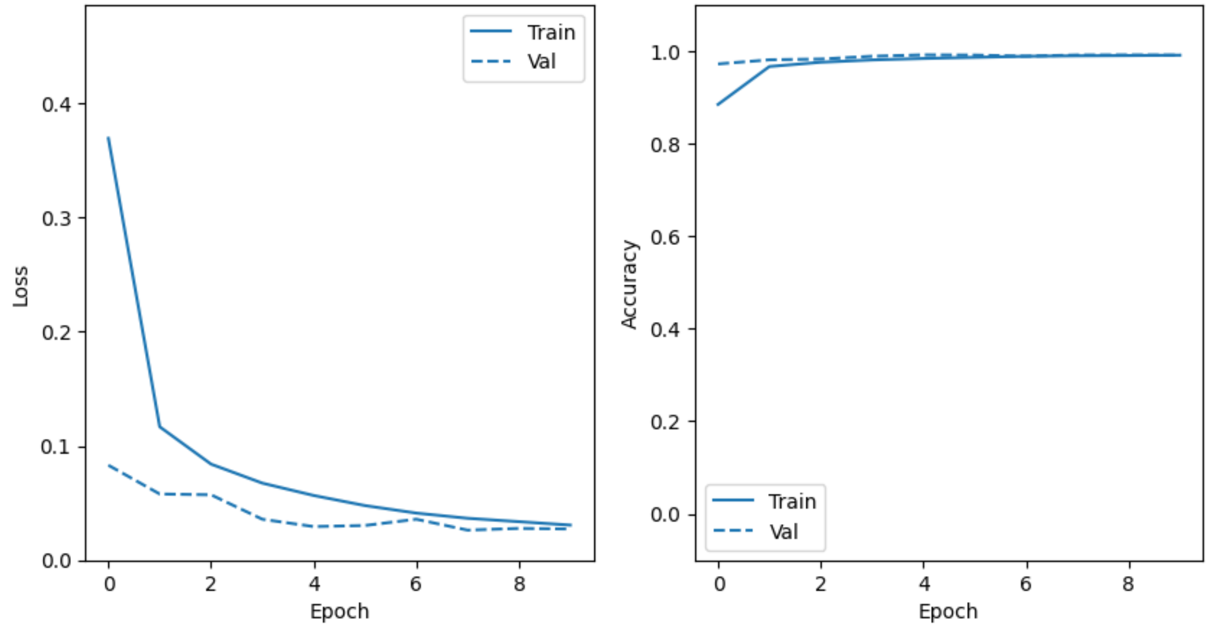
I added padding to the second convolution layer. Hence, the ouput shape is the same like in the first pooling layer. The cnn 'eats' pixels only in the first and third conv layer, resulting in overall 8 'eaten' corner pixels.

In [ ]:
```
model12_history = model12.fit(
    x_train,
    y_train,
    epochs=EPOCHS,
    callbacks = [early_stopping],
    validation_data=(x_test, y_test),
    batch_size=BATCH_SIZE)
```

```
Epoch 1/10
469/469 ━━━━━━━━━━━━━━━━━━━━ 4s 6ms/step - accuracy: 0.7597 - loss: 0.7358 - val_acc
uracy: 0.9717 - val_loss: 0.0832
Epoch 2/10
469/469 ━━━━━━━━━━━━━━━━━━━━ 3s 6ms/step - accuracy: 0.9632 - loss: 0.1292 - val_acc
uracy: 0.9806 - val_loss: 0.0580
Epoch 3/10
469/469 ━━━━━━━━━━━━━━━━━━━━ 3s 6ms/step - accuracy: 0.9747 - loss: 0.0874 - val_acc
uracy: 0.9823 - val_loss: 0.0573
Epoch 4/10
469/469 ━━━━━━━━━━━━━━━━━━━━ 3s 6ms/step - accuracy: 0.9790 - loss: 0.0711 - val_acc
uracy: 0.9883 - val_loss: 0.0359
Epoch 5/10
469/469 ━━━━━━━━━━━━━━━━━━━━ 3s 6ms/step - accuracy: 0.9838 - loss: 0.0568 - val_acc
uracy: 0.9913 - val_loss: 0.0294
Epoch 6/10
469/469 ━━━━━━━━━━━━━━━━━━━━ 3s 6ms/step - accuracy: 0.9859 - loss: 0.0481 - val_acc
uracy: 0.9908 - val_loss: 0.0304
Epoch 7/10
469/469 ━━━━━━━━━━━━━━━━━━━━ 3s 6ms/step - accuracy: 0.9886 - loss: 0.0405 - val_acc
uracy: 0.9885 - val_loss: 0.0359
Epoch 8/10
469/469 ━━━━━━━━━━━━━━━━━━━━ 3s 6ms/step - accuracy: 0.9888 - loss: 0.0387 - val_acc
uracy: 0.9910 - val_loss: 0.0263
Epoch 9/10
469/469 ━━━━━━━━━━━━━━━━━━━━ 3s 6ms/step - accuracy: 0.9895 - loss: 0.0352 - val_acc
uracy: 0.9913 - val_loss: 0.0279
Epoch 10/10
469/469 ━━━━━━━━━━━━━━━━━━━━ 3s 6ms/step - accuracy: 0.9906 - loss: 0.0309 - val_acc
uracy: 0.9908 - val_loss: 0.0274
Epoch 10: early stopping
Restoring model weights from the end of the best epoch: 1.
```

I don't know why the early stopping callback restored the weights of epoch 1, since the best val_accuracy was found in epoch 9. So I ran the training again. After I plotted the training history.

In [ ]:
```
plot_metrics(model12_history)
```



In [ ]:
```
set_seeds()
model12 = Sequential()
model12.add(Conv2D(32, (9,9), activation='relu',input_shape=in_shape))
model12.add(MaxPool2D((2, 2)))
model12.add(Conv2D(32, (3,3), activation='relu', padding='same') )
model12.add(MaxPool2D((2,2)))
model12.add(Conv2D(32, (3,3), activation='relu'))
model12.add(MaxPool2D((1,1)))
model12.add(Flatten())
model12.add(Dense(100, activation='relu'))
model12.add(Dropout(0.5))
model12.add(Dense(n_classes, activation='softmax'))
model12.compile(optimizer='adam', loss='sparse_categorical_crossentropy',metrics=['

model12_history = model12.fit(
    x_train,
    y_train,
    epochs=9,
    validation_data=(x_test, y_test),
    batch_size=BATCH_SIZE)
```

```
Epoch 1/9
469/469 ━━━━━━━━━━━━━━━━━━━━ 3s 6ms/step - accuracy: 0.7597 - loss: 0.7358 - val_acc
uracy: 0.9717 - val_loss: 0.0832
Epoch 2/9
469/469 ━━━━━━━━━━━━━━━━━━━━ 3s 6ms/step - accuracy: 0.9632 - loss: 0.1292 - val_acc
uracy: 0.9806 - val_loss: 0.0580
Epoch 3/9
469/469 ━━━━━━━━━━━━━━━━━━━━ 3s 6ms/step - accuracy: 0.9747 - loss: 0.0874 - val_acc
uracy: 0.9823 - val_loss: 0.0573
Epoch 4/9
469/469 ━━━━━━━━━━━━━━━━━━━━ 3s 6ms/step - accuracy: 0.9790 - loss: 0.0711 - val_acc
uracy: 0.9883 - val_loss: 0.0359
Epoch 5/9
469/469 ━━━━━━━━━━━━━━━━━━━━ 3s 6ms/step - accuracy: 0.9838 - loss: 0.0568 - val_acc
uracy: 0.9913 - val_loss: 0.0294
Epoch 6/9
469/469 ━━━━━━━━━━━━━━━━━━━━ 3s 6ms/step - accuracy: 0.9859 - loss: 0.0481 - val_acc
uracy: 0.9908 - val_loss: 0.0304
Epoch 7/9
469/469 ━━━━━━━━━━━━━━━━━━━━ 3s 6ms/step - accuracy: 0.9886 - loss: 0.0405 - val_acc
uracy: 0.9885 - val_loss: 0.0359
Epoch 8/9
469/469 ━━━━━━━━━━━━━━━━━━━━ 3s 6ms/step - accuracy: 0.9888 - loss: 0.0387 - val_acc
uracy: 0.9910 - val_loss: 0.0263
Epoch 9/9
469/469 ━━━━━━━━━━━━━━━━━━━━ 3s 6ms/step - accuracy: 0.9895 - loss: 0.0352 - val_acc
uracy: 0.9913 - val_loss: 0.0279
```

In [ ]:

```
loss, acc = model12.evaluate(x_test, y_test, verbose=0)
print('Accuracy: %.3f' % acc)
```

```
Accuracy: 0.991
```

In [ ]:

```
err = print_res(model12)
```

```
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 32ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 10ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 10ms/step
9
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 10ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 10ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
4
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 10ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 10ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 10ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 10ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 10ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 10ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 10ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 10ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
4
```

```
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 13ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 10ms/step
5
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 10ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 10ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 10ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
0
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 10ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 10ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 10ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 10ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 10ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 10ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
7
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 10ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 10ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 10ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 10ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 10ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 10ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 10ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 10ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 10ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
1/1 ━━━━━━━━━━━━━━━━━━━━ 0s 9ms/step
9
--
```

```
7 2 1 0 4 1 4 9 5 9
0 6 9 0 1 5 9 7 3 4
9 6 6 5 4 0 7 4 0 1
3 1 3 4 7 2 7 1 2 1
1 7 4 2 3 5 1 2 4 4
6 3 5 5 6 0 4 1 9 5
7 8 9 3 7 4 6 4 3 0
7 0 2 9 1 7 3 2 9 7
7 6 2 7 8 4 7 3 6 1
3 6 9 3 1 4 1 7 6 9
```

In [ ]:
```
err
```

Out[ ]: 0

Unfotunately, this was not good for neither, accuracy and models complexity. The accuracy shrank to 0.988 and the number of parameters rise to 51030

Model 11 parameters:

- *Model Complexity (Paramereters) = 51030*
- *Accuracy: 0.991*
- *Test error in first 100 images: 0*

Some words about learning convergence according to the loss curves: All of them converged very fast after 2 epochs. The first learning step was quite big in each experiments compared to the other learning steps. However, they converged at different losses, since the accuracy was different (, which is not guaranteed, but at least an indicator for a smaller of bigger loss.)

Since the loss curves are quite similar, I neglected the describtion of them for each experiment in detail. Maybe, there is a setting I missed at which I could have seen an interesting difference?

In the end model 10 was the best one I found.

Model 10 parameters:

- *Model Complexity (Paramereters) = 35030*
- *Accuracy: 0.993*
- *Test error in first 100 images: 0*