



Programming Assignment - Part 5

Submission Deadline:

5. Jun 2018

5 Confidence (90 Points)

In the lecture, the concept of confidence has been introduced. In this section, you will implement the calculation of confidence intervals and do some simulations to get to know this concept.

5.1 Implementation

Your TIC should be able to create a confidence interval based on the samples in its internal array and a confidence level α , given as an input parameter.

Task 5.1.1: Calculating Confidence Intervals

10 Points

Implement the two methods *report_confidence_interval()* and *is_in_confidence_interval()*. The first method should return the half width of the symmetric confidence interval. The second method should calculate, whether a given sample x is in the confidence interval. Use the first function in the implementation of the second one.

You can test the basic functionality of the confidence intervals by running the test in *part5_tests.py*.

Task 5.1.2: Calculating Bootstrap Confidence Intervals

10 Points

Recall the method to create bootstrap confidence intervals. Implement the two methods *report_bootstrap_confidence_interval()* and *is_in_bootstrap_confidence_interval()*. The first method should return the lower and upper bound of the bootstrap confidence interval. The second method should calculate and decide whether a given sample x is in the bootstrap confidence interval. Use the first function in the implementation of the second one.

Hint: the length of each resampled array must be equal to the length of the original data. However, you should repeat this resampling a high number of times. We use 5000 as default value.

You can test the basic functionality of the confidence intervals by running the test in *part5_tests.py*.

5.2 Simulation Study IV

In the following, you will implement your simulation in a way that it can be interrupted based on the width of the confidence interval. The width of the confidence interval should be two times $\epsilon = 0.0015$.

You will perform the simulation in two ways: The first way is to make multiple runs with a fixed time and to stop your simulation after a specific amount of runs when a given width of the confidence interval is reached. The second way is to make one run and take batches of a fixed time. You calculate your confidence interval based on the values you get from each batch. Once the width is reached, you stop your simulation.

Do the coding for this section in file *part5_simstudy.py*.

Task 5.2.1: Multiple Runs Confidence

10 Points

Implement simulation routine, that runs fixed time simulations until the width of the confidence interval is small enough. For this, call your simulation like in the previous tasks with a simulation time of 100 s and 1000 s respectively. After each run, extract the blocking probability and add it to a new TIC. Calculate the width of the confidence interval and stop your simulations when it is below two times ϵ . Use $\rho = 0.9$ and $S=4$ and calculate the number of runs for a confidence level of 0.9 and 0.95.

Task 5.2.2: Batch Confidence

15 Points

Now there should be no event *SimulationTermination* and the simulation keeps running. For each $N = 100$ ($N = 1000$) packets, the blocking probability is read and is added to a new TIC. Then the statistics collected in *simresult* should be reset manually. Stop the simulation when the width of the confidence interval is below two times ϵ . Calculate the total simulation time for a confidence level of 0.9 and 0.95 ($\rho = 0.9$, $S=4$). Hint: You have to implement the function *do_simulation_n_limit()* in class *Simulation*, such that the simulation stops after a total packet count of N . However, you can take most of the code from your regular simulation routine.

Task 5.2.3: Confidence Plots I

5 Points

Implement a plotting function that takes confidence intervals, or their boundaries, as input. On the x-axis you should have one id per sample (confidence interval). On the y-axis, you should plot the mean value with the corresponding confidence interval.

Task 5.2.4: Confidence Plots II

10 Points

Make the following simulation study. Use an $M/M/1/\infty$ system and perform multiple runs simulation. Make your study for system offered traffic 0.5 and 0.9. Use confidence levels of

0.9 and 0.95. Use 100 s and 1000 s as simulation time. Calculate the confidence interval for the system throughput of 30 runs. Repeat this 100 times. Take each confidence interval and plot it with your implemented function (see task 5.2.3). On the x-axis you should have values from 1 to 100 (for every repetition, each of 30 runs) and on the y-axis the given calculated mean value with the error bars. Also plot the theoretical value as a dashed line.

5.3 Analysis and General Questions

Answer the following questions separately and in full sentences. Explain your answers.

Task 5.3.1: Confidence Interval Width

5 Points

How many runs do you need in each setup of tasks 5.2.1 and 5.2.2? Compare all eight values and explain, why they differ and which are the advantages of the setups.

Task 5.3.2: Confidence Interval Width

5 Points

Compared to the actual blocking probability calculated by an analytic formula

$$P(S) = \frac{(1 - \rho)\rho^{S+1}}{1 - \rho^{S+2}} \quad (1)$$

the blocking probability in task 5.2.1 always differs. Why is this the case and does it differ in task 5.2.2 as well?

Task 5.3.3: Confidence Interval Width

10 Points

What can you observe in the plots of task 5.2.4? What are the differences between the plots? How many intervals are covering the true value and how many not? How does the skewness affect the calculation of confidence intervals?

Task 5.3.4: Variable Simulation Time

5 Points

Would it also be an option to do a simulation with an infinite simulation time (and stop as soon as a given confidence level is reached) and take the samples for the blocking probability each time a packet is dropped? Justify your answer!

Task 5.3.5: Bootstrapping

5 Points

What's the advantage of bootstrap confidence interval over normal confidence interval? Under which case would the two types of confidence interval become the same?

Total: 90 Points