# ECE 533 Fall 2015 Lab 4
# Image Deblurring

In this lab, we will explore different methods for removing blur from images. Specifically, in past classes we examined smoothing filters for images, where the smoothed image may be expressed as

$$g[m,n] = (f * h)[m,n] = \sum_{s=-S/2}^{S/2} \sum_{t=-S/2}^{S/2} f[m-s, n-t]h[s,t],$$

where $f$ is the original image and $h$ is the filter.

This convolution model can also be used to represent blurred images that we observe directly, where the blur may be caused by atmospheric effects, motion of the object relative to the camera, or an out-of-focus camera. In this case, we observe

$$g[m,n] = (f * h)[m,n] = \sum_{s=-S/2}^{S/2} \sum_{t=-S/2}^{S/2} f[m-s, n-t]h[s,t] + \epsilon[m,n],$$

where $\epsilon$ represents noise and quantization effects induced by the imaging sensor. **In this lab, we will explore some methods for estimating $f$ given $g$ and $h$.**

1. Inverse filters
   (a) Let $h$ correspond to a Gaussian low pass filter and let $\epsilon$ correspond to Gaussian noise with variance $\sigma^2$. Write a script that will compute $g$ from $h$ by performing convolution in the Fourier domain.
   (b) In the Fourier domain, we can write our observed image $g$ as
   $$G[k,\ell] = F[k,\ell]H[k,\ell] + E[k,\ell],$$
   where $E$ corresponds to the Fourier transform of the noise. The inverse filter multiplies each $G[k,\ell]$ by $1/H[k,\ell]$. Write a function to perform this inverse filter operation.
   (c) Apply your inverse filter to your image for a range of (small) values of $\sigma^2$. If your original pixel values range between 0 and 255, for instance, have $\sigma^2$ range from 0 to 2. How does your inverse filter work for $\sigma = 0$? For larger values of $\sigma$? Make a plot of mean squared error vs. $\sigma^2$.
2. Wiener filters
   (a) The Wiener filter assumes prior knowledge of the *power spectral density* of our image,
   $$S[k,\ell] = \mathbb{E}[|F[k,\ell]|^2].$$
   The Fourier coefficients of the Wiener filter can be written as
   $$W[k,\ell] = \frac{H^*[k,\ell]S[k,\ell]}{|H[k,\ell]|^2 S[k,\ell] + V[k,\ell]},$$

where $V[k, \ell]$ is the power spectral density of the noise $\epsilon$ and is $\sigma^2$ for Gaussian noise with variance $\sigma^2$. Write a function to perform the Wiener filter operation given $S$ and $V$.

(b) A common model of the power spectral density of images is called the $1/f$ model; that is,

$$S[k, \ell] \propto 1/(k^2 + \ell^2)^c$$

for some constant $c$. Try your Wiener filter for different values of $c$; what seems to work best?

(c) When does the Wiener filter perform better than the inverse filter?

(d) **(Bonus)** Devise a better power spectral density model than the $1/f$ model above. Explain how you came up with it and demonstrate how it outperforms the $1/f$ model experimentally.

3. The above experiments were conducted with a Gaussian blur. Repeat those experiments with (a) a motion blur and (b) different Gaussian filter bandwidths. What do you expect the inverse and Wiener filters to do as the blurring filter bandwidth approaches zero? Is this what you see experimentally?

4. Sometimes we have to deblur images with imperfect knowledge of $h$. Try to blur an image with a Gaussian with bandwidth $\sigma_{h_1}$ and deblur assuming a Gaussian with bandwidth $\sigma_{h_2}$. What do you observe for different $h_1, h_2$ pairs?

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## A. Deadline

Reports, including images, descriptions, and code, should be turned in via Moodle. Turn in the result of "publish" in Matlab or iPython Notebooks. Written problem solutions should also be submitted digitally. If you photograph or scan hand-written solutions, make sure they are legible. Due by 2:30pm on Nov. 2.