ECE 533 FALL 2015 LAB 5
IMAGE RECONSTRUCTION

In this lab, we will explore different methods for reconstructing imaging from distorted observations. We consider observations of the form

$$y = Hf + \varepsilon,$$

where ε is noise, $f \in \mathbb{R}^N$ is a vectorized version of the N-pixel image of interest, $g \in \mathbb{R}^M$ is a vector of the M observations we record, and $H \in \mathbb{R}^{M \times N}$ is a matrix representing distortions to our image. For instance, H could correspond to a Gaussian blur.

In this lab, we will consider estimating f from y using Tikhinov and ℓ_1 regularization.

1. **Tikhinov regularization:** Our strategy here is to choose $\hat{f}$ to minimize

$$\phi(f) := \|y - Hf\|_2^2 + \tau \|Gf\|_2^2$$

for some $\tau > 0$.

   (a) Let h correspond to a Gaussian low pass filter and let ϵ correspond to Gaussian noise with variance σ^2. Write an expression for the filter corresponding to Tikhinov regularization.

   (b) Let $G = I$ be the identity operator. Write a Matlab function or script to deblur y. Try your estimator for different values of τ – how does τ impact the accuracy of your reconstruction?

   (c) Let G correspond to a Laplacian filter. Write a Matlab function or script to deblur y. Try your estimator for different values of τ – how does τ impact the accuracy of your reconstruction?

2. ℓ_1 **regularization:** Our strategy here is to choose $\hat{f}$ to minimize

$$\phi(f) = \|y - Hf\|_2^2 + \tau \|Gf\|_1$$

for some $\tau > 0$.

   (a) Let h correspond to a Gaussian low pass filter and let ϵ correspond to Gaussian noise with variance σ^2. Write an expression for the iterative updates associated with a projected gradient algorithm.

   (b) Let $G = I$ be the identity operator. Write a Matlab function or script to deblur y. Try your estimator for different values of τ – how does τ impact the accuracy of your reconstruction?

   (c) Let G correspond to a Haar wavelet decomposition. Write a Matlab function or script to deblur y. Try your estimator for different values of τ – how does τ impact the accuracy of your reconstruction?

A. Deadline

Reports, including images, descriptions, and code, should be turned in via Moodle. Turn in the result of "publish" in Matlab or iPython Notebooks. Written problem solutions should also be submitted digitally. If you photograph or scan hand-written solutions, make sure they are legible. Due by 2:30pm on Nov. 9.