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Rumours and markets

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Abstract

The paper presents a simple model to study the effects of rumours on markets. Agents in our economy communicate with their local neighbours which gives rise to the possible spread of a rumour. As the rumour affects beliefs of the agents the evolution of the rumour has a direct impact on market outcomes. Our results show that if the rumour dies out long-run equilibrium prices correspond to pre-rumour values. However, if the rumour stays present it produces a *price run-up* for the good that is positively targeted by the rumour. Price run-ups related to rumours have been observed in empirical studies by Rose [Rose, A.M., 1951. Rumor in the stock market. Public Opinion Quarterly 15, 461–486], Pound and Zeckhauser [Pound, J., Zeckhauser, R., 1990. Clearly heard on the street: the effect of takeover rumors on stock prices. Journal of Business 63, 291–308] and Zivney et al. [Zivney, T., Bertin, W.J., Torabzadeh, K.M., 1996. Overreaction to take-over speculation. Quarterly Review of Economics and Finance 36, 89–115]. The present model provides an analytical foundation for this finding.

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1. Introduction

> "Rumour, . . . speed lends her strength, and she wins vigour as she goes, . . . clinging to the false and wrong, yet heralding truth."
> — Vergil, Aeneis, Book IV.

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Rumours are part of our everyday life. Sometimes they contain confidential information about public figures, in other cases they have hot news concerning important social or economic issues. Rumours can shape the public opinion of a society or a market by affecting and coordinating the individual beliefs of its members.

Although rumours are well-known in real life they are almost absent in economic theory. Only in recent years economists have started to look at rumours, both from a theoretical and an empirical point of view (cf. Koenig, 1985; Kapferer, 1989; Pound and Zeckhauser, 1990; Banerjee, 1993; Zivney et al., 1996; an early exemption is Rose, 1951). The reason seems intuitive. While economic theory focuses mainly on rational behaviour, rumours were often thought to be something rather irrational. In some sense, they did not fit into the model.

The aim of this paper is to propose an analytical framework for studying the effects of rumours on markets. Our model combines standard microeconomic theory and particle system theory in a way that we can analyze (i) the spread of a rumour through word-of-mouth communication and (ii) the rumour's impact on demand and prices of goods in a competitive market. Roughly said, our main results show that if word-of-mouth communication is strong enough the rumour produces a significant *price run-up* for the good that is positively targeted by the rumour. Price run-ups related to rumours have been observed in an early study by Rose (1951) and more recently by Pound and Zeckhauser (1990) and Zivney et al. (1996). To our knowledge the present model provides the first analytical foundation for this finding.

Rose (1951) considers the short-run movement of a sample of US stock prices during periods between 1937 and 1938 and between 1948 and 1949. The author calculates a so-called "factor of stickiness" to measure the effect of rumour. His findings support the hypothesis that if rumour affects stock prices, it will do so by "creating a unidirectional trend" (Rose, 1951, p. 468), i.e., it will cause prices to move in a single direction over some short period of time.

Pound and Zeckhauser (1990) have examined the effects of takeover rumours on stock prices. They study a sample of rumours published in the "Heard on the Street" (HOTS) column that appears daily in the *Wall Street Journal*. Using a sample from 1983 until 1985, their main findings are as follows. (1) The market reacts efficiently to published rumours, i.e., no excess returns could be realized by buying (or selling) rumoured takeover targets at the time the rumour appeared in the press. (2) In the period immediately before publication of the rumour, a sizeable and consistent price run-up for the takeover target occurs, while on the day of publication itself no significant reaction of the market can be observed. (3) Rumours correctly predicted imminent takeover bids less than half of the time.

Zivney et al. (1996) take a similar approach but consider both the HOTS and the "Abreast of the Market" (AOTM) column in the *Wall Street Journal* to identify the correct initial rumour publication date.[1] They focus on the period from 1985 until 1988 since articles dealing with rumour information were most numerous in 1988. Their main findings are that similarly to Pound and Zeckhauser (1990) the market generally reacts efficiently to rumour publication. However, rumours published in the AOTM column lead to a clear short-term overreaction, giving rise for profitable investment opportunities. Moreover, like Pound and

[1] The reason is that, usually, some days or weeks before a rumour is published in the HOTS column it appears in the AOTM column.

Zeckhauser (1990) they also observe a significant price run-up during the 20-day period before publication of the rumour.

The present paper focuses on the observed price run-up as one of the main empirical findings in the studies above. Our model can explain the run-up as a direct consequence of communication between traders in a competitive market. Due to a rumour that is transmitted from one trader to another, individual beliefs and trading strategies are affected, thereby leading to a price run-up for the good that is positively targeted by the rumour. The transmission of the rumour relies on local communication between traders, which follows an infection dynamic as a typical mechanism for the spread of a rumour. Our model thus endogenizes the evolution of market prices as a result of rumour diffusion among traders in the economy.

The idea of using infection dynamics for the transmission of a rumour is also part of the model discussed in Banerjee (1993). Banerjee uses an individual Bayesian updating approach to obtain a stochastic process that describes the evolution of a rumour in time. The dynamics are approximated through a system of differential equations that are known from the epidemiological literature. Results are then obtained from an analysis of the approximating deterministic system.

Contrary to Banerjee's model we take a simplifying step and model the rumour transmission as a purely mechanical act. The gain is that we can analyze the stochastic process directly. In particular, we do not have to approximate the process in order to obtain results about the evolution of the rumour.[2] The second and main difference of our model is that communication between agents is restricted to local neighbourhoods. While Banerjee's approach assumes agents to meet all other agents in the population, i.e., transmission of the rumour is global, we believe that the assumption of local interaction is more appropriate for studying the dynamics of a rumour. Mostly, rumours reach us through people that are closely connected to us, through friends, people living next door, and colleagues for example. Moreover, since rumours often arise in case of uncertainty, we also discuss rumours mostly with our immediate environment in order to get a clearer picture of the situation. Other models that are similar in spirit focussing on interaction through word-of-mouth communication or recruitment mechanisms include Banerjee (1992), Bikhchandani et al. (1992), Kirman (1993), Ellison and Fudenberg (1995) and Vettas (1997, 1998). The main difference with respect to these models lies in the underlying question that is posed. While the above literature concentrates mainly on the problem of social learning and efficiency, our study focuses on the effects of word-of-mouth communication, as a particular kind of social interaction, on market outcomes, and in particular prices. With this respect, the work of Kirman (1993) is, perhaps, most closely related.

Both methodologically and analytically, our model is close to early work of Föllmer (1974) on random economies. Föllmer studies the existence and properties of equilibria in an economy where individual preferences of agents are random and depend on the local environment of agents. Our approach is similar to Föllmer's analysis in the sense that we also consider agents' characteristics as being dependent on their local environment. However, the

[2] This avoids also some unpleasant problem: it is known that the long-run behaviour of the approximating deterministic system can be very different from the one of the original stochastic process. See, for example, Blume (1997) on this issue.

model differs in two main respects. Firstly, we model agents' beliefs rather than preferences. Secondly, our model explicitly considers a dynamic framework that captures the evolution of beliefs over time. Equilibria with respect to the interaction between agents are stationary (or, invariant) distributions of the underlying stochastic process.

To the extent that we model the evolution of agents' beliefs in a market economy our approach is also close to Blume and Easley (1992) and Sandroni (2000). However, while this literature is mostly interested in the idea of market selection and convergence to rational expectations, our focus here is somewhat different. For example, we do not distinguish between beliefs that are rational, i.e., correct, and beliefs that are not. In fact, we do not even assume that correct beliefs exist at all. Instead our main aim is to show how different beliefs are able to diffuse within in a market economy that is structured by local communication and how the diffusion of these beliefs affects market outcomes.

The paper is organized as follows. Section 2 presents the basic set-up to analyze the spread of a rumour. We define individual beliefs and describe how the rumour affects the beliefs of agents in the economy. The transmission dynamic of the rumour is based on word-of-mouth communication between neighbouring agents, leading to a stochastic process that models the evolution of the rumour in time. The set of invariant distributions of this process is given and convergence to invariant distributions is characterized. Section 3 applies the set-up to a simple market situation, in which agents can trade Arrow securities. We calculate so-called long-run equilibrium prices, where markets clear given that beliefs form an invariant distribution. Our results support the empirical finding of a price run-up for the security that is positively targeted by the rumour. Section 4 concludes with possible generalizations of the model.

2. Rumours

2.1. Structure of the economy

Consider the following economy. There is a countable infinite set of agents located on the one-dimensional integer lattice $\mathbb{Z}$.[3] Identify each agent with his or her location, so $\mathbb{Z}$ represents the set of agents. Typically, agents are denoted as $x, y, z \in \mathbb{Z}$. Assume the economy to be endowed with a neighbourhood structure of local interaction. For every agent $x \in \mathbb{Z}$ let $N(x) = \{y \in \mathbb{Z} | \, |y - x| = 1\}$ be the set of neighbours, where $|\cdot|$ denotes the Euclidean distance in $\mathbb{Z}$.

In our model neighbourhoods specify the range of social interaction of agents, where social interaction is assumed to mean communication of a rumour. An agent $x \in \mathbb{Z}$ directly communicates with his adjacent neighbours $y \in N(x)$. However, since neighbourhoods are overlapping he also communicates indirectly with all other agents in the economy. This seems a natural assumption, in particular for situations of word-of-mouth communication that we have in mind.

[3] The assumption of dimension one is not critical. Results similarly hold for any higher dimensional lattice. This follows from the fact that the relevant behaviour of the underlying stochastic process (the contact process) is independent of the dimension of the lattice (cf. Liggett, 1985).

We assume that there are two possible states of the world $s \in \{1, 2\}$. At any time an agent in the economy is characterized by his beliefs about the states of the world. Time is continuous. Let $\theta_t(x) \in [0, 1]$ be the belief of agent x at time $t \geq 0$ that $s = 1$ is the true state of the world. Consequently, $1 - \theta_t(x)$ is the belief of agent x that $s = 2$ is the true state of the world. To give an example with regard to rumours, suppose, e.g., that $s = 1$ models the state of the world where eventually, a given firm is the object of a takeover and $s = 2$ where it is not. In our model, a rumour will affect the beliefs of agents about the different states of the world.[4]

We assume that there exists a pre-rumour belief $b \in [0, 1]$ that models the beliefs in the economy before the rumour appears. Once the rumour is introduced at say, $t = t_0$, it affects beliefs of those agents that communicate the rumour. In particular, we assume that the rumour induces a shift in beliefs from b to some other belief $\tilde{b} \not\equiv b$. Thus, after the rumour has appeared beliefs in the economy can be described as follows. For any agent x at time $t \geq t_0$,

$$\theta_t(x) = \begin{cases} b & \text{if} \quad x \text{ is not affected by rumour} \\ \tilde{b} & \text{if} \quad x \text{ is affected by rumour,} \end{cases} \tag{1}$$

with $b, \tilde{b} \in [0, 1]$.

2.2. Communication of the rumour

Having defined the general structure of the economy and the relation between rumour and beliefs, the next step is to define the transmission dynamic of the rumour. The main idea is that agents who are affected by the rumour are also the ones that communicate the rumour to their neighbours.

Formally, we model the rumour dynamic as an interacting particle system, the latter being a continuous-time Markov process on the state space of all configurations of beliefs in the economy $\Theta = \{\theta | \theta : \mathbb{Z} \to \{b, \tilde{b}\}\}$. (See Liggett, 1985 for a good introduction into the theory of interacting particle systems.) The main objective in order to define an interacting particle system is to specify so-called flip rates $r(x, \theta_t) \in [0, \infty)$ for $x \in \mathbb{Z}, \theta_t \in \Theta$. In our model, these rates determine the probability that given configuration θ_t at time $t \geq 0$, agent x changes his belief $\theta_t(x)$ in such a way that for $\delta \downarrow 0$

$$\text{Prob}[\theta_{t+\delta}(x) \neq \theta_t(x)] = r(x, \theta_t)\delta + o(\delta). \tag{2}$$

We make the following assumptions.

Assumption 1. *If no neighbouring agent is affected by the rumour, the probability for an agent, who is not yet affected by the rumour, to become affected is zero. On the other hand, if at least one neighbour is affected by the rumour, the probability to become affected by the rumour oneself is strictly positive. Moreover, this probability is strictly increasing in*

[4] In the market economy that is considered in the next section the two states will represent the fact that one out of two Arrow securities pays out rather than the other.

the number of neighbours that are affected by the rumour. *Formally*, *if* $\theta(x) = b$,

$$r(x, \theta) = \lambda \sum_{y \in N(x)} 1_{\{\theta(y)=\tilde{b}\}}, \tag{3}$$

with $0 < \lambda < \infty$.

Due to the lack of evidence that is typically involved in the transmission of a rumour (agents do not know whether the rumour is true or not), the probability with which an agent believes whether a rumour is true depends on the number of neighbours that communicate the rumour. Intuitively, if I hear a story once, I may believe it or not, but if I hear it also from another person there must at least be something in it that is true. Therefore, Assumption 1 says that every neighbour who communicates the rumour increases the probability for an agent to become affected by the rumour and in consequence also to start spreading the rumour himself. The result is that the rumour follows a dynamic which is similar to an infection process.

Assumption 2. *Agents can forget the rumour*. *The probability to forget the rumour is constant and independent of neighbours*. *Formally*, *if* $\theta(x) = \tilde{b}$, $r(x, \theta) = 1$.

Due to other objects that may appear in everyday communication the importance or relevance of a rumour may decline. In consequence, agents that have heard the rumour may stop thinking of it and forget about it after a while. This assumption should not be confused with the main idea that rumours are reinforced by frequent hearing. Since the latter effect is already captured by Assumption 1, the possibility to forget the rumour is simply introduced as a counter-dynamic to the spread of the rumour. Not having such an assumption would imply that every agent, who is infected by rumour once, would continue spreading it forever. The constant probability assumption is made to keep the model simple. It can be generalized — without changing the results — to non-constant probabilities, which are decreasing with the number of neighbours that communicate the rumour. In that case, the fact that neighbours confirm the rumour does have an influence on forgetting. (See the discussion in Section 4.) In our model, the forget rate is normalized to 1. Since any multiplication of flip rates by some constant leads to the same type of dynamics, such normalization allows us to control the relationship between forgetting and being infected by the rumour via the single parameter λ. Definition 1 summarizes our two assumptions.

Definition 1. *For every* $x \in \mathbb{Z}$ *and* $\theta \in \Theta$,

$$r(x, \theta) = \begin{cases} \lambda \sum_{y \in N(x)} 1_{\{\theta(y)=\tilde{b}\}} & if \quad \theta(x) = b \\ 1 & if \quad \theta(x) = \tilde{b}, \end{cases} \tag{4}$$

with $0 < \lambda < \infty$.

In our model the parameter λ is constant and exogenous. It determines the degree of social interaction and communication between neighbours. In this sense it also captures the willingness of agents to talk about the rumour. If λ was equal to zero there would be no communication between neighbours, a situation we shall not be interested in during the following. If $\lambda > 0$, the value directly determines the probability for an agent to become infected by the rumour by talking to one of his neighbours. As it can be seen from Eqs. (4) and (2), the higher the value of λ the higher this probability is.

By Definition 1 the dynamics of the rumour follow those of the so-called contact process. This process was first studied by Harris (1974), see also Chapter VI of Liggett (1985). All results on the spread of the rumour within the economy are based on the evolutionary behaviour of the contact process.

2.3. Evolution of the rumour

We now analyze the evolution of the rumour, i.e., the process $\{\theta_t\}_{t\geq 0}$. Suppose that the rumour appears at time $t_0 = 0$. We assume that the initial configuration of beliefs is randomly determined by some probability distribution μ_0 on Θ.

There exists a canonical one-to-one correspondence between the space Θ and the space $\Delta = \{A |\, A \subset \mathbb{Z}\}$. The function $\chi : \Theta \to \Delta$ maps each configuration of beliefs θ into $\chi(\theta) = \{x |\, \theta(x) = \tilde{b}\}$, the set of agents that are infected by rumour. It is often convenient to think of $\{\theta_t\}_{t\geq 0}$ as a process on Δ. Using this correspondence we denote by δ_A the Dirac measure that puts probability one on the configuration where exactly all agents $x \in A \subset \mathbb{Z}$ are infected by the rumour.

The first observation is the following. For any parameter value λ, the Dirac measure $\delta_\emptyset$, putting full mass on the configuration $\theta = \emptyset$, where nobody in the economy is affected by the rumour, is an invariant distribution of the process $\{\theta_t\}_{t\geq 0}$. Once nobody in the economy is affected by the rumour the rumour will be gone forever. There is no spontaneous source for the rumour except at time zero.

The second and crucial observation is that — in spite of its simplicity — the rumour exhibits a *phase transition*.[5] This means that there exists a critical value λ^* such that for any $\lambda < \lambda^*$, $\delta_\emptyset$ is the unique invariant distribution and therefore also the unique limiting distribution independent of initial conditions. In this case the evolution of the rumour is ergodic. However, for any $\lambda > \lambda^*$, the set of invariant measures is equal to the convex set spanned by the extremal measures $\delta_\emptyset$ and ν_λ, where the latter is the limiting distribution obtained from starting with the initial configuration $\theta = \mathbb{Z}$, i.e., the configuration where everybody is affected by the rumour. Hence, λ^* is the critical value where the ergodicity of the rumour breaks down. Obviously, the interesting regime — also from an economic point of view — is the non-ergodic one, since only then the rumour has a chance to affect market behaviour at all. The following proposition is a well-known result for the contact process. For a proof see, e.g., Liggett (1985).

Proposition 1. *There exists a critical value $\lambda^* < \infty$, such that $\forall\lambda < \lambda^*$, $\delta_\emptyset$ is the unique invariant and limiting distribution of the rumour. If $\lambda > \lambda^*$, the set of invariant distributions*

[5] The phenomenon of phase transitions has been studied in other economic applications before. See, e.g., Föllmer (1974) for an early economic analysis of phase transitions in the situation of a random exchange economy. Other examples include Allen (1982) and Durlauf (1993). For a short and rather informal introduction into the economic sense of phase transitions see also Hors (1995) and Hors and Lordon (1997).

is equal to the non-degenerate convex set $\mathcal{I} = \{\nu \mid \nu = \sigma\delta_\emptyset + (1-\sigma)\nu_\lambda,\ 0 \le \sigma \le 1\}$. *The measure* ν_λ *is non-trivial in the sense that* $\nu_\lambda(\theta = \emptyset) = 0$. *It is obtained as the limit* $\nu_\lambda = \lim_{t\to\infty} \mu_t$, *when* μ_0 *is the initial distribution of the rumour putting probability one on configuration* $\theta = \mathbb{Z}$.

Since $\nu_\lambda(\theta = \emptyset) = 0$, the measure ν_λ ensures that with probability one at least some agent in the economy has belief $\tilde{b}$, i.e., is infected by rumour. Hence, convergence to the distribution ν_λ corresponds to saying that the rumour will be persistently present in the economy, while convergence to the distribution $\delta_\emptyset$ corresponds to saying that the rumour disappears. This motivates the following definition.

Definition 2. *For given* λ *let* μ_t *be the distribution of the rumour at time t. The rumour* ***disappears*** *if* $\lim_{t\to\infty} \mu_t = \delta_\emptyset$, *or equivalently*, $\lim_{t\to\infty} \mu_t(\theta = \emptyset) = 1$. *If* $\lambda > \lambda^*$, *the rumour is* ***persistently present*** *if* $\lim_{t\to\infty} \mu_t = \nu_\lambda$. *In this case* $\lim_{t\to\infty} \mu_t(\theta(x) = \tilde{b}) > 0$ *for every* $x \in \mathbb{Z}$.

Passing from the ergodic to the non-ergodic regime the behaviour of the process undergoes an abrupt change. While in the first regime it is sure that the rumour will disappear for every initial distribution, in the second regime the evolution of the process is much more ambiguous. Since the set of invariant distributions $\mathcal{I}$ is no longer a singleton but the whole "interval" between $\delta_\emptyset$ and ν_λ, in principle any measure $\nu \in \mathcal{I}$ is a candidate for the limiting distribution of the process starting with some initial distribution μ_0. The next proposition clarifies the evolution of the rumour for arbitrary initial distributions. For a proof see again Liggett (1985). A special class of initial distributions, for which the evolution can be determined very easily, is the class of translation invariant distributions.[6]

Let τ be the stopping time for the rumour to enter the state $\theta = \emptyset$. Denote τ^{μ_0} the stopping time when starting with initial distribution μ_0.

Proposition 2. *Consider the case where* $\lambda > \lambda^*$. *Let* μ_0 *be any arbitrary initial distribution. Then*

$$\nu = \lim_{t\to\infty} \mu_t = \sigma\delta_\emptyset + (1-\sigma)\nu_\lambda, \tag{5}$$

where $\sigma = \mathrm{Prob}[\tau^{\mu_0} < \infty]$, i.e., σ *equals the probability for the rumour to disappear in finite time. If* μ_0 *is translation invariant*, $\sigma = \mu_0(\theta = \emptyset)$.

For translation invariant distributions the value of σ determining the mixture between $\delta_\emptyset$ and ν_λ is explicitly given by the probability for the initial configuration to be $\theta = \emptyset$, signifying that nobody in the economy hears the rumour at the beginning. Once we can

[6] A probability measure μ on Θ is *translation invariant* if for any finite collection of sites $(x_1, \dots, x_k)$, with $x_j \in \mathbb{Z}$, any profile $(i_1, \dots, i_k)$, with $i_j \in \{b, \tilde{b}\}$, $k \ge 1$, and $z \in \mathbb{Z}$

$$\mu(\theta(z + x_1) = i_1, \dots, \theta(z + x_k) = i_k) = \mu(\theta(x_1) = i_1, \dots, \theta(x_k) = i_k),$$

i.e., probabilities do not depend on z.

ensure this probability to be zero we obtain weak convergence to distribution ν_λ, thus we know that the rumour will be persistently present in the economy for any time in the future. At the same time, we see that the rumour dies out if and only if the initial state equals $\theta = \emptyset$, μ_0-almost surely, i.e., at the beginning there is simply nobody who is infected by the rumour.

This result has a nice consequence. Assume for example that at the beginning everybody in the economy has the same chance to hear the rumour. In order to model this, consider the initial distribution to be determined as follows. A random process independently assigns to each agent belief b or $\tilde{b}$. Assume this process to be binomially distributed with parameter ϵ, i.e., we get the Bernoulli product measure μ^ϵ with $\mu^\epsilon(\theta(x) = \tilde{b}) = \epsilon$, for every agent $x \in \mathbb{Z}$. In this situation $\sigma = \mu^\epsilon(\theta = \emptyset) = 0$ if and only if $\epsilon > 0$. Thus, if we know that the probability for every agent in the economy to hear the rumour is strictly positive, we can conclude that in consequence the rumour will never die out but will be persistently present, even if ϵ is arbitrarily small. Only if $\epsilon = 0$ the rumour will (trivially) die out since it will not even be known at the beginning.

The assumption of μ_0 being translation invariant can be very restrictive. As we have just seen, this means that everybody has in fact the same access to relevant information, an assumption that is often too strong for many economic situations. We may therefore also be interested in initial distributions where the rumour starts spreading from an arbitrary given set of agents $A \in \Delta$. In this case the situation looks as follows (cf. Liggett, 1985). If A is finite, we loose weak convergence, i.e., $\sigma > 0$. However, as long as A is nonempty it holds that $\sigma < 1$, i.e., the probability for the rumour to survive is strictly positive. Moreover, σ monotonically decreases as A grows. In particular, if A is infinite, $\sigma = 0$, i.e., we again obtain weak convergence to the distribution ν_λ, independent on how dense the initial set A of infected agents is. Thus, in our model the rumour is persistently present with probability one whenever the initial distribution contains an infinite number of agents that are infected. If this number is finite (but larger than zero), the rumour is persistently present with probability that is strictly positive and increases with the number of agents that are initially infected. For the following analysis, when it comes to persistence of the rumour we are therefore mostly interested in initial distributions where A is either infinite or where it is finite but large.

Finally, Griffeath (1981) and Durrett and Griffeath (1983) have shown that for both $\lambda < \lambda^*$ and $\lambda > \lambda^*$, convergence to invariant distributions is exponentially rapid. In consequence, both measures $\delta_\emptyset$ and ν_λ may serve as good approximations for the distribution of the rumour as time has sufficiently passed.

3. Markets

3.1. Equilibrium prices of arrow securities

In this section we apply our model of a rumour economy to a simple market situation. Corresponding to our two states of the world, we assume that there are two Arrow securities, each paying a return of 1 in case the corresponding state of the world occurs, and zero otherwise. Let $q_s(x)$ be agent x's holding of Arrow security $s \in \{1, 2\}$. We assume that each agent has an initial endowment of securities $\omega = (\omega_1, \omega_2)$ and a utility function u depending on income, that is sufficiently well-behaved (i.e., continuously differentiable, strictly increasing and strictly quasiconcave on $\mathbb{R}$).[7]

Given belief $\theta(x)$, agent $x \in \mathbb{Z}$ maximizes his expected utility choosing the optimal portfolio (q_1, q_2), i.e.,

$$\max_{q_1, q_2} \theta(x)u(q_1) + (1 - \theta(x))u(q_2), \tag{6}$$

satisfying his budget constraint $p_1(q_1 - \omega_1) + p_2(q_2 - \omega_2) = 0$ and taking security prices $p = (p_1, p_2)$ as given. Due to our assumptions on utility the solution to (6) defines a single-valued, continuous demand function of agent x for security s that is strictly decreasing in p_s and strictly increasing in the belief agent x holds for state s. We denote this function by $q_s(x, \theta(x), \omega, p)$ with $s \in \{0, 1\}$.

Example. Assume agent's utility function to be logarithmic in income: $u(w) = \log(w)$. In this case demand equals

$$q_1(x, \theta(x), \omega, p) = \frac{\theta(x)}{p_1}(p_1\omega_1 + p_2\omega_2), \tag{7}$$

$$q_2(x, \theta(x), \omega, p) = \frac{1 - \theta(x)}{p_2}(p_1\omega_1 + p_2\omega_2). \tag{8}$$

Methodologically, our set-up is related to the model of Föllmer (1974), who studies an exchange economy on a random field. In fact, the rumour economy described in the previous section is a dynamic analogue to the random field approach. We can therefore use Föllmer's definition of an equilibrium price. Let μ be a probability measure on the space of belief configurations Θ.

Definition 3. *Given a distribution μ on the space of belief configurations Θ, a price vector $p = (p_1, p_2)$ is an* ***equilibrium price vector*** *if*

$$\lim_{n\to\infty} \frac{1}{|B_n|} \sum_{x\in B_n} q_s(x, \theta(x), \omega, p) - \omega_s = 0, \qquad \mu - a.s. \tag{9}$$

for $s \in \{1, 2\}$, where B_n is the box of size n centered around the origin, i.e., $B_n = \{-n/2, \dots, n/2\}$ for n even.

Equilibrium prices are prices where per capita expected excess demand equals zero, μ-a.s. Note that this requires the equality in (9) to hold *for all* configurations that have positive probability given the measure μ. In other words, for equilibrium prices p markets clear whatever configuration of the economy is actually realized.

[7] It is possible to allow individual endowments to be drawn from a family of i.i.d. random variables that operate on a given set of endowments. However, as such a setting does not change our results substantially (in equilibrium the role of fixed endowments is taken by expected endowments), we stick to the simpler framework and assume that endowments are identical.

Let μ_t be the distribution of the rumour at time t. From the analysis in the preceding section we know that, taking the limit $t \to \infty$, the distribution μ_t converges to an invariant distribution of the rumour. If $\lambda < \lambda^*$, the unique invariant distribution is $\delta_\emptyset$. The rumour always disappears. If $\lambda > \lambda^*$, the rumour stays persistently present for a large class of initial distributions, i.e., we obtain $\lim_{t\to\infty} \mu_t = \nu_\lambda$. Moreover, convergence to equilibrium $\delta_\emptyset$, if $\lambda < \lambda^*$, and to ν_λ, if $\lambda > \lambda^*$, is exponentially rapid. Hence, both limiting distributions may serve as an approximation for the distribution of the rumour when t is large, that is when the rumour has been around in the population for a sufficient amount of time. We use this approximation in order to calculate so-called *long-run equilibrium prices*.

Before doing so, however, we prove that finite time equilibrium prices converge to long-run equilibrium prices of the economy. This guarantees that our model forms indeed a coherent set-up.

Proposition 3. *Let μ_t be the distribution of beliefs in the economy at time t. Assume $\lim_{t\to\infty} \mu_t = \nu$ with $\nu \in \{\delta_\emptyset, \nu_\lambda\}$. For any time t, let p_t be an equilibrium price at time t. Denote $p^* = \lim_{t\to\infty} p^t$. Then p^* is an equilibrium price for the limiting distribution ν.*

Proof. The claim is proven by contradiction. By Definition 3 in order for p^* to form an equilibrium price for the distribution ν, per capita expected excess demand must be equal to zero, ν-a.s., i.e.,

$$\zeta_s(\theta, \omega, p^*) = \lim_{n\to\infty} \frac{1}{|B_n|} \sum_{x\in B_n} q_s(x, \theta(x), \omega, p^*) - \omega_s = 0, \qquad \nu - a.s.$$

for $s \in \{1, 2\}$. Suppose that $\nu(\zeta_s(\theta, \omega, p^*) \neq 0) > 0$ for some $s \in \{0, 1\}$. Since ν is the weak limit of the sequence of distributions $\{\mu_t\}_{t\geq 0}$, this implies that

$$\mu_t(\zeta_s(\theta, \omega, p^*) \neq 0) > 0,$$

for t sufficiently large. By continuity of ζ with respect to p (which follows from the continuity of the individual demand function) we get that

$$\mu_t(\lim_{r\to\infty} \zeta_s(\theta, \omega, p^r) \neq 0) > 0,$$

for t sufficiently large. But this implies that eventually there is some $r > 0$ for which

$$\mu_r(\zeta_s(\theta, \omega, p^r) \neq 0) > 0.$$

Since p^r is an equilibrium price for the distribution at time r, this gives the contradiction. □

3.2. Long-run equilibrium prices

With regard to long-run equilibrium prices, the crucial fact we can exploit is the observation that for distribution $\nu \in \{\delta_\emptyset, \nu_\lambda\}$ the following holds for any price vector p, ν-a.s.:

$$\lim_{n\to\infty} \frac{1}{|B_n|} \sum_{x\in B_n} q_s(x, \theta(x), \omega, p) - \omega_s = E^\nu[q_s(0, \theta(0), \omega, p) - \omega_s] \tag{10}$$

with $s \in \{1, 2\}$ and 0 denoting the agent located at the origin. This follows from ergodic theory since both measures $\delta_\emptyset$ and ν_λ are translation invariant and extremal within the set of translation invariant measures (cf. Liggett, 1985). Intuitively, Eq. (10) shows that we can aggregate the information in the economy with regard to per capita excess demand by looking at the expected excess demand of the agent at site zero, who thus acts as a true "representative agent" for the economy. Note, however, that this relation breaks down as soon as we consider any other invariant distribution for the rumour, i.e., a real mixture of both extremal measures $\delta_\emptyset$ and ν_λ. In that case we are left with Eq. (9) again.

Applying Definition 3 to distributions $\nu \in \{\delta_\emptyset, \nu_\lambda\}$, the condition for long-run equilibrium prices thus reduces to a much simpler one: a price vector p is a long-run equilibrium price vector for the distribution $\nu \in \{\delta_\emptyset, \nu_\lambda\}$ if

$$E^\nu[q_s(0, \theta(0), \omega, p) - \omega_s] = 0 \tag{11}$$

for $s \in \{1, 2\}$. For notational convenience, let us denote by $\alpha_\nu = \nu(\theta(0) = \tilde{b})$ the probability that agent 0 is infected by rumour given distribution $\nu \in \{\delta_\emptyset, \nu_\lambda\}$.

Our first result shows that assuming that endowments are strictly positive and beliefs are not extreme long-run equilibrium prices exist.

Proposition 4. *Suppose that* $\omega > 0$ *and* $b, \tilde{b} \notin \{0, 1\}$. *Then for* $\nu \in \{\delta_\emptyset, \nu_\lambda\}$ *a long-run equilibrium price exists.*

Proof. Given distribution $\nu \in \{\delta_\emptyset, \nu_\lambda\}$, expected excess demand of agent 0 for security $s \in \{0, 1\}$ equals

$$\begin{aligned} &E^\nu[q_s(0, \theta(0), \omega, p) - \omega_s] \\ &\quad = (1 - \alpha_\nu)q_s(0, b, \omega, p) + \alpha_\nu q_s(0, \tilde{b}, \omega, p) - \omega_s \\ &\quad = (1 - \alpha_\nu)(q_s(0, b, \omega, p) - \omega_s) + \alpha_\nu(q_s(0, \tilde{b}, \omega, p) - \omega_s). \end{aligned} \tag{12}$$

Because the right hand side of (12) is a continuous function that is positive for p_s sufficiently small and negative for p_s sufficiently large, there exists a price vector $p = (p_1, p_2)$ such that the above function equals zero for $s \in \{0, 1\}$. □

Example (continued). If $u(w) = \log(w)$, equilibrium prices can be calculated directly. In this case, the excess demand function is linear in beliefs, which implies that

$$E^\nu[q_1(0, \theta(0), \omega, p) - \omega_1] = \frac{\gamma}{p_1}(p_1\omega_1 + p_2\omega_2) - \omega_1, \tag{13}$$

$$E^{\nu}[q_2(0, \theta(0), \omega, p) - \omega_2] = \frac{1-\gamma}{p_2}(p_1\omega_1 + p_2\omega_2) - \omega_2, \tag{14}$$

where $\gamma = (1-\alpha_\nu)b + \alpha_\nu \tilde{b}$ is the expected belief of agent 0. Combining both equations, implies that

$$\frac{p_1}{p_2} = \frac{\gamma}{1-\gamma}\frac{\omega_2}{\omega_1}. \tag{15}$$

Thus, if utility is logarithmic in income, relative equilibrium prices are determined by the ratio of the expected belief of agent 0 times the inverse ratio of endowments.

3.3. Rumour effects

The following proposition contains our main result showing the effect that rumours can have on prices in the economy.

Proposition 5. *If the rumour disappears, long-run equilibrium prices reflect pre-rumour beliefs. If, however, the rumour stays persistently present, long-run equilibrium prices deviate from pre-rumour values. The rumour generates a price run-up for the security that is positively targeted by the rumour.*

Proof. Suppose first that the rumour disappears, i.e., $\nu = \delta_\emptyset$ and hence $\alpha_\nu = \alpha_{\delta_\emptyset} = 0$. In this case, the equilibrium equation simplifies to

$$q_s(0, b, \omega, p) - \omega_s = 0,$$

for $s \in \{0, 1\}$. Prices are determined by the pre-rumour belief b only.

Suppose next that the rumour stays persistently present, i.e., the process converges to ν_λ, in which case $\alpha_\nu = \alpha_{\nu_\lambda} > 0$. Suppose the rumour is positive about state 1, i.e., $\tilde{b} > b$. The equilibrium price for security 1 has to meet the condition

$$(1-\alpha_{\nu_\lambda})(q_1(0, b, \omega, p) - \omega_1) + \alpha_{\nu_\lambda}\left(q_1(0, \tilde{b}, \omega, p) - \omega_1\right) = 0. \tag{16}$$

Let p_1^b be the equilibrium price reflecting pre-rumour beliefs. Clearly, p_1^b sets the first term on the left hand side equal to zero. The second term, however, is strictly positive, as demand is strictly increasing in the belief and $\tilde{b} > b$. Now, since demand is strictly decreasing in the price p_1, the long-run equilibrium price — if the rumour stays persistently present — must be strictly larger than p_1^b. Thus, the rumour generates a price run-up for security 1. $\square$

The driving force behind the rumour effect in Proposition 5 is intuitive. Since the rumour contains positive information about the likelihood of state 1, it induces those agents, who are affected by the rumour, to shift part of their demand to the corresponding security. This eventually leads to an increase of the equilibrium price p_1. The particular feature of our model is that the shift in demand is an endogenous result of the spread and survival of the rumour in the economy. The observation of a price run-up has been made in an early study of Rose (1951) and more recently also by Pound and Zeckhauser (1990) and Zivney et al. (1996). To our knowledge the present model provides the first analytical foundation for this finding.

The solution to Eq. (16) depends on $\alpha_{\nu_\lambda} = \nu_\lambda(\theta(0) = \tilde{b})$, i.e., the probability that agent 0 is infected by rumour given distribution ν_λ, which again depends on λ capturing the intensity of communication between agents. Using a result from Liggett (1985, Theorem 1.33, Chapter VI), who shows that for $\lambda \geq 2$, $\nu_\lambda(\theta(0) = \tilde{b}) \geq (1/2) + \sqrt{(1/4) - (1/2\lambda)}$, we see that α_{ν_λ} monotonically increases with λ and converges to 1 as λ goes to infinity. In consequence, in the limit the influence of pre-rumour belief b vanishes completely, and long-run equilibrium prices are uniquely determined by rumour beliefs $\tilde{b}$.

Proposition 6. *As communication becomes stronger, long-run equilibrium prices are more strongly affected by rumour*. *In the limit* ($\lambda = \infty$), *prices deviate completely from pre-rumour beliefs, i.e., are determined by rumour beliefs only*.

Intensifying the communication increases the probability for agents to eventually believe in the rumour, thereby shifting beliefs and demand towards the security that is positively targeted by the rumour. Our model predicts that the price run-up for the security is the stronger the hotter the communication is.

Example (continued). Consider the logarithmic-utility case again. From Eq. (15) it can immediately be seen that long-run equilibrium prices equal

$$\frac{p_1}{p_2} = \frac{b}{1-b}\frac{\omega_2}{\omega_1} \tag{17}$$

if the rumour disappears. If the rumour stays present,

$$\frac{p_1}{p_2} = \frac{\gamma}{1-\gamma}\frac{\omega_2}{\omega_1}, \tag{18}$$

with $\alpha_\nu > 0$. Assuming $\tilde{b} > b$, it follows that $\gamma > b$, and hence equilibrium price p_1 is higher if the rumour stays around than if it disappears. How large can this effect be? Consider, e.g., the following situation. Suppose that $0 < b < \frac{1}{2}$ and $\tilde{b} \approx 1$. Thus, before appearance of the rumour the general opinion in the market is slightly in favour of state 2, while agents that are affected by rumour believe that state 1 is the true state with very high probability.[8] From Eq. (15) and Proposition 6 it follows that, if we let λ go to infinity, i.e., intensify the communication between agents, and consider the limit of long-run prices as $\tilde{b}$ approaches 1, i.e., as beliefs become more extreme, the ratio of relative prices converges to infinity: $\lim_{\tilde{b}\to 1} \lim_{\lambda\to\infty} \frac{p_1}{p_2} = \infty$. That is, there exists no finite upper bound for the relative price run-up for the security that is positively targeted by the rumour if communication and rumour-effects become extreme.

[8] The assumption that $0 < b < 1/2$ is not necessary, any $b \notin \{0, 1\}$ will do it.

4. Discussion

The model in this paper focuses on a rumour as a particular example of information transmission, affecting beliefs of agents who trade within a two-good exchange economy. The aim has been to look for reasonable dynamics that model the evolution of a rumour and that can be used to answer questions concerning the impact of rumours on economic variables, such as market demand and, especially, equilibrium prices. Still, there are a lot of issues that have to be addressed in order to obtain a true understanding of the relation between rumours and markets. We conclude the paper by discussing some of them.

4.1. Endogenous infection rate

One of the most important elements is certainly the parameter λ that determines the probability for uninformed agents to be infected by the rumour from neighbours. While in the present model λ was modelled exogenously, an obvious extension is to endogenize that parameter and derive it from other variables. With this respect, a crucial variable will then be an agent's subjective value of the information that is transmitted. This again could be linked to prices of some commodities or expected returns of an investment opportunity, depending on the situation one wants to analyze. In any case, the next step is to connect the transmission of the rumour in some way or other with agents' behavior in the market and derive it from agents' decision-making procedure. Here, Banerjee (1993) represents a promising attempt using a Bayesian updating approach. Other ideas may come from alternative models focusing on individual and social learning.

4.2. Heterogeneity

Analytical results for a true endogenization of λ seem technically very demanding. A first step is to incorporate some heterogeneity into the system. Suppose, for instance, that agents have different forget-rates. Each agent $x \in \mathbb{Z}$ is infected by the rumour with equal rate λ (times the number of neighbours that communicate the rumour) but has an individual forget-rate $c(x)$. To keep things simple suppose that any agent x within a subset $X \subset \mathbb{Z}$ has a forget-rate equal to $c(x) = c < 1$, while all other agents $x \notin X$ have rate $c(x) = 1$. Thus, agents within X are less forgetful or perhaps more enthusiastic in communicating a rumour.[9] If the set X is sufficiently dense it will affect the evolution of the rumour in the sense that the critical value λ^* is strictly smaller than before. In consequence, the rumour has a greater chance to survive. Precisely, if X is such that there exists a length l so that each interval of length l in $\mathbb{Z}$ contains at least one agent $x \in X$, then $\lambda^*(X) < \lambda^*$, where $\lambda^*(X)$ denotes the critical value of the system governed with lower forget-rates in X and λ^* is the critical value obtained in the present model (Madras et al., 1994). On the other hand, if X is sufficiently thin the behaviour of the system stays the same. Madras et al. give a condition where it is sufficient that for every distance d there are only finitely many pairs of agents in X, who are located not further than d apart from each other.

[9] I believe everybody would know at least one person of this type.

Bramson et al. (1991) have analyzed the system when the distribution of different forget rates within the population is determined by some i.i.d. random process. Suppose that initially, before the rumour spreads, each agent is told his personal forget-rate. With probability p he has a forget rate equal to $c < 1$ and with probability $1 - p$ he has a rate equal to 1, where $p \in (0, 1)$. It is easy to imagine that the resulting set of agents, to which the rate c is assigned, is sufficiently dense within the population. Therefore, the critical value leading to survival of the rumour is strictly smaller than before.

4.3. Non-constant forget-rates

As mentioned above the part of Assumption 2 that assumes forget-rates to be constant and independent of the number of neighbours, who confirm the rumour, can be relaxed without changing the main results of the model. Suppose, for example, that flip rates are defined as follows. For $x \in \mathbb{Z}$, $\theta \in \Theta$ set

$$r(x,\theta) = \begin{cases} \lambda \sum_{y \in N(x)} 1_{\{\theta(y)=\tilde{b}\}} & \text{if} \quad \theta(x) = b \\ \dfrac{1}{3} & \text{if} \quad \theta(x) = \tilde{b} \quad \text{and} \quad \theta(x-1) = \theta(x+1) = \tilde{b} \\ \dfrac{1}{2} & \text{if} \quad \theta(x) = \tilde{b} \quad \text{and} \quad \theta(x-1) \neq \theta(x+1) \\ 1 & \text{if} \quad \theta(x) = \tilde{b} \quad \text{and} \quad \theta(x-1) = \theta(x+1) = b. \end{cases} \tag{19}$$

If agent x is affected by the rumour and both neighbours confirm the rumour, the forget-rate is equal to (1/3) and in this case it is also the smallest. It increases as the number of confirming neighbours decreases. When no agent in the neighbourhood communicates the rumour the probability to forget the rumour is at its highest value, which is again normalized to one. Using a standard dominance argument the following holds: every rumour that stays persistently present under the original dynamic stays persistently present under the dynamic in (19) as well. The intuition lies at hand. Since agents are only less likely to forget when confirmation of the rumour has an effect, the possibility for the rumour to spread around and stay present becomes only larger. Therefore, results hold the same as before.

4.4. Rumours and trade

In the present model we have not calculated equilibrium prices for the market described by a distribution different than $\delta_\emptyset$ and ν_λ. Although the generality of the model would allow for a definition of such a price the relevant equations are rather difficult to be solved. The model is consistent in the sense that any sequence of finite time equilibrium prices converges to a long-run equilibrium price. While the long-run approach serves as an approximation for the general situation, a richer model will connect the rumour with trading behaviour more directly. Then, it will be interesting not only to look for impacts of the rumour on trade, but also for the possible effects trading may have with regard to the evolution of the rumour.

In order to capture such effects, however, the behaviour of an agent must depend not only on the behaviour of his neighbours but also on the behaviour of the whole economy aggregated in current prices of goods. The difficulty of such a model is less on the behavioural than on the mathematical side. For example, it is rather reasonable to assume that an increase in the price of a good, e.g., shares of some well-known company, together with a rumour on that good, e.g., a takeover rumour, will make an agent believe the rumour even more to be true. In consequence, the probability to get infected by the rumour will be higher as well. Mathematically, however, this leads to a major conceptual change of the model. While in the present model, the state of an agent x depends only on the beliefs of his neighbours $y \in N(x)$, the modified model incorporates a dependence of x on the whole configuration θ. Since this produces a feedback of interaction, the first question is whether such a system can be well-defined at all. If so, the next problem is how do equilibria look like. The difference to the present model is that now equilibria for the stochastic process, in the sense of invariant distributions, and equilibria for the market, in the sense of clearing prices, are highly connected with each other. It appears an open question in which manner such a system can be analyzed and how interesting results can be obtained. Nevertheless, the project is of particular interest since the influence of macrodata, such as market prices, inflation rates, growth rates, share indices, and many more, on the decision making of individuals lies at hand. There are first attempts in that direction that build on work of Föllmer (1979, 1980). See, e.g., Türnich (1995) for more.

4.5. *Counter-rumours and new information*

Finally, an interesting project is to study the possible rise of counter-rumours and the effect of different refutation strategies as special examples of new information in the economy. While the explicit dynamics of a counter-rumour that contains information contrary to the original rumour may be difficult to be analyzed, effects of refutation strategies can be examined already in the present model.[10] Note that the current set-up allows for two possibilities to change the evolution of the rumour. The first is on the set of agents being affected by the rumour, the second on the degree of communication between agents. A firm may want to influence the number of people infected by the rumour, and it can do so, for instance, by circulating specific information through the national press or other official media. Depending on the information that is published the current configuration of beliefs θ will be affected by manipulating agents' individual beliefs $\theta(x)$, $x \in \mathbb{Z}$. For example, information that strongly confirms the rumour is expected to increase the number of agents that have belief $\tilde{b}$, while information that refutes the rumour aims at strongly decreasing that number. Similarly, new information can also influence the degree of communication of the rumour between agents. It can rise general interest into the topic, which in the model would lead to an increase of λ, as well as it can make the rumour less important to the public thereby decreasing λ. However, official refutation or denial of a rumour can well turn into a counter-productive strategy. When the refutation does not lower the probability to believe

[10] While the object of a takeover rumour may, perhaps, be interested in a price run-up for its shares, the situation looks much different in case the rumour contains negative information about a company. In that case our model predicts a fall in the respective price.

in the rumour but increases only the number of agents that hear the rumour and perhaps also the interest of people to talk about the rumour it may be better to keep quiet and wait until the rumour (hopefully) disappears. In particular, the model suggests that whenever $\lambda < \lambda^*$ an official refutation may do more harm than good to the situation, since it may well increase λ making the rumour possibly even persistent, while otherwise it dies out for sure. Similarly, if initially the rumour is known only to a small number of agents it may be a risky strategy for a firm to combat the rumour in public, since it may attract only more publicity. By enlarging the set of rumour-infected agents the rumour becomes only more likely to survive.

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