Gross Worker Flows over the Life Cycle*

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Abstract

This paper analyzes the gross worker flows over the life cycle. We first document the life-cycle patterns of flows across different labor market states (employment, unemployment, and not in the labor force) as well as job-to-job transitions in the US. Next, we build a model of the aggregate labor market that incorporates the life cycle of workers, consumption-saving decisions, and labor market frictions. Finally, we estimate the model with the US data, and use the estimated model to investigate the effect of taxes and transfers on aggregate labor market outcomes, such as unemployment rate and the labor-force participation rate, through the lens of gross labor market flows.

Keywords: worker flows, life cycle, taxes and transfers

JEL Classification: E24, J21, J22, J64.

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1 Introduction

Over the past several decades, the study of the aggregate labor market has made significant progress by analyzing gross flows in the labor market. By investigating beyond the net changes in labor market stocks, such as the unemployment rate and employment-population ratio, our understanding of the dynamics of the labor market and the effect of labor market policies has deepened substantially.

This paper contributes to this strand of literature. We analyze worker flows across three different labor market states: *employment* (E), *unemployment* (U), and *not in the labor force* (or *nonparticipation*) (N). These gross flows influence the policy-relevant labor market stocks, such as the employment-population ratio, unemployment rate, and labor-force participation rate. In addition, we consider an important worker flow: the flow of employed workers across different jobs. Recent studies have found such *job-to-job transitions* play an important role in macroeconomic outcome by reallocating workers to appropriate jobs. For example, Engbom (2017) argues the patterns of job-to-job transitions, combined with human capital accumulation, can explain a large part of the differences in life-cycle wage growth patterns across OECD countries.[1]

Our focus is on the life cycle of workers. Various studies have documented the flows and stocks in the labor market vary substantially with age. For example, the unemployment rate for young workers is known to be higher than for prime-age workers, and young workers tend to experience more frequent job-to-job transitions than older workers. All gross flows, including the ones involving the participation margin, are important in shaping the heterogeneous outcomes in the labor market across different age groups. A recent accounting exercise by Choi et al. (2015) reveals, for example, a low movement from the N state to the E state and from the N state to the U state (we will call them NE flow and NU flow) accounts for a large part of the low participation and unemployment rates for old workers.

In this paper, we ask how labor market policies affect the flows and stocks in the labor market for different age groups of workers. We approach this question from a quantitative-theoretic perspective. We build a dynamic general equilibrium model that replicates the behavior of individuals we focus on, run policy experiments using the model, and interpret the mechanism. We particularly focus on the effects of taxes and transfers, which have been studied in the macroeconomic literature especially since the influential work of Prescott (2004).

Our model features frictional labor market with an operative labor-supply margin, based on Krusell et al. (2010, 2011, 2017). Krusell et al. (2010), in particular, studied the effects of taxes and transfers using an infinite-horizon model, and found important interactions between frictions and the labor-supply margin in this class of models. Our departure from their analysis is that we explicitly consider the worker life cycle. This departure is important

[1] Barlevy (2002) and Mukoyama (2014) analyze the effect of job-to-job transitions on aggregate productivity. Their model analyses imply the effect of job-to-job transitions on the aggregate productivity can be sizable.

because (i) the heterogeneity in worker flows across different age groups is so significant that analyzing the policy effects with explicit treatment of this heterogeneity is itself very important, (ii) this framework is the first that features labor market frictions and the operative labor-supply margin in a life-cycle context, and this framework can be applied to many other policy experiments, and (iii) quantitatively matching the model to data is quite challenging because seven *life-cycle profiles* (six flow rates between three labor market states and job-to-job flow rate) is significantly more complex than seven *numbers* (corresponding flow rates averaged over the life cycle). As can be seen below, various changes in the model, compared to Krusell et al. (2010, 2011, 2017), are necessary for the model to replicate salient life-cycle patterns of worker flows in the data.

We find the increase in labor tax (and transfers) indeed has heterogeneous effects on different age groups of workers. The pattern of the changes in employment largely reflects the pattern of the changes in labor-force participation, underscoring the importance of the endogenous participation margin. The unemployment stock only changes for the young workers. By explicitly analyzing the gross flows, we can pin down which flows are responsible for these changes in stocks.

The main contribution of this paper is theoretical; we provide a framework that can be used for various policy analyses, and we conduct a policy exercise that has been analyzed extensively in the macroeconomic literature. Our model features (i) the worker life cycle, (ii) the frictional labor market with heterogeneous jobs, and (iii) the operative labor-supply margin with concave utility and self-insurance. The model is able to fit the quantitative features of the life-cycle patterns of labor market flows and stocks, allowing us to analyze the effect of tax policies on the labor market outcomes of different age groups of workers. The policy experiment uncovers the impact of the tax-and-transfer policy on the life-cycle patterns of labor market flows and stocks. We intentionally keep the model parsimonious so that the main mechanisms remain transparent despite the quantitative nature of the policy experiment. In particular, as in Krusell et al. (2010, 2011, 2017), the labor-market frictions are modeled using a simple "island" structure, because the most important channel for our experiment is operative labor supply.

The paper is related to several strands of literature. First, the particular policy we consider, taxes-and-transfers, has been analyzed by various authors in the macroeconomic literature. Examples include Prescott (2004), Ohanian et al. (2008), Alonso-Ortiz and Rogerson (2010), and Krusell et al. (2010). Compared to the previous studies, this paper is novel in that we explicitly consider life-cycle elements in a framework that features incomplete asset markets and labor market frictions. Incorporating life-cycle elements is important because patterns of transitions across labor-market stocks are markedly heterogeneous over the life cycle. Incorporating frictions enables us to talk about the effect of taxes and transfers on unemployment. The structure of incomplete asset markets with concave utility allows us to consider each consumer's asset-accumulation behavior and life-cycle behavior, particularly how the wealth effect operates, in a natural manner. An important interaction also exists between self-insurance and precautionary saving in that transfers can act as insurance against employment shocks.

Second, several recent papers have analyzed life-cycle worker flows in a frictional labor market. The contributions include Chéron et al. (2013), Esteban-Pretel and Fujimoto (2014), Menzio et al. (2016), and Jung and Kuhn (2019). None of these papers, however, explicitly model the endogenous participation margin, through which the labor-supply channel works. As we see later in detail, the operative labor-supply channel is essential for the policy experiment in this paper.

Third, an extensive macroeconomic literature that considers life-cycle labor supply exists. Examples include Rogerson and Wallenius (2009), Low et al. (2010), and Erosa et al. (2016). These studies do not explicitly match the patterns of gross worker flows observed in the data. The explicit analysis of gross worker flows allows us to relate the effect of the policy on stocks to the patterns of reallocation in an economy with heterogeneous agents.

Finally, from a modeling perspective, our model has the Bewley-Huggett-Aiyagari (BHA) structure with a frictional labor market and operative labor supply with indivisible labor. Thus, our model shares many features with Chang and Kim (2006). Compared to Chang and Kim (2006), our model incorporates worker life cycle and frictional labor market.

The paper is organized as follows. The next section summarizes the empirical pattens of the gross worker flows over the life cycle. Section 3 sets up the model, and we calibrate the model in Section 4. Section 5 describes the results. Section 6 concludes.

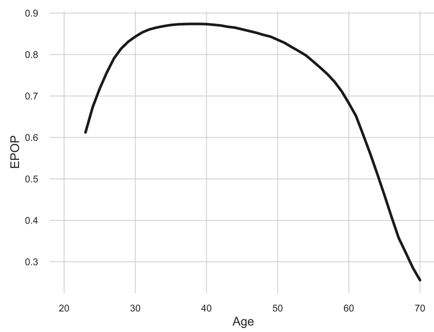
2 Empirical observations

In this section, we summarize the life-cycle patterns of worker flows and stocks in the US data.

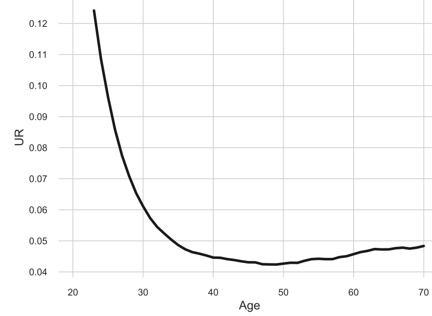
2.1 Data

We use the monthly files of the Current Population Survey (CPS) from 1994 to 2017. Because our preliminary analysis found notable differences in labor market flows between men and women (likely related to decisions to stay at home and take care of children, which is more common for women and is beyond the scope of our model analysis), we decided to limit the sample to the population of men. This sample selection, of course, doesn't mean incorporating women's labor-supply behavior is not important—the analysis of this paper should be viewed as merely a first step.

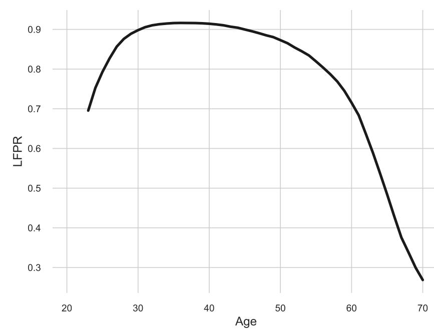
To calculate transition rates between different labor market states, we longitudinally match observations over two consecutive months by using data on household and person identification variables, as well as data on sex, race, and age, as is standard in the literature. Additionally, we correct for transitions that are plausibly spurious by using the deNUNifying procedure (purging the temporary appearance of U state by, for example, replacing N-U-



(a) Employment-to-population ratio



(b) Unemployment rate



(c) Labor-force participation rate

Figure 1: Labor market ratios in the data

N with N-N-N) as described in Elsby et al. (2015). Life-cycle profiles are obtained by estimating weighted OLS regressions of each labor market stock and flow on a set of age dummies.

2.2 Labor market stocks

First, we describe the life-cycle patterns of stocks in the labor market. In this study, we focus on male workers ages 23 to 70. All the data figures are means of six-year-moving windows, and the horizontal-axis labels are upper bounds of the windows.[2] Figure 1 plots the age profile of employment. The employment-to-population ratio exhibits an inverted-U shape: smaller fractions of young and old workers are employed than middle-aged workers. As we can see from the comparison between panel (a) and panel (c), this pattern of employment mostly mimics the pattern of labor-force participation. The unemployment rate also exhibits a strong life-cycle pattern, although the pattern is markedly different from the one associated with labor-force participation. Young workers below 30 years old experience significantly

[2] We calculate our data moments from age 16 onward, take the rolling means, but report only age 23 and above, which is the age group we focus on.

higher unemployment rate than other age groups, while the unemployment rate is almost flat past the age 40. This pattern of unemployment also contributes nontrivially to the low employment-to-population ratio, especially for young workers.

2.3 Labor market flows

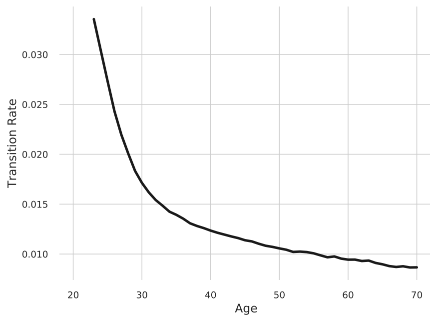
The main innovation of this paper is to provide a model analysis for gross worker flows. The patterns in the data have previously been described by Choi et al. (2015), for example; thus, our summary here will be brief. Figure 2 plots the monthly gross worker flows over the life cycle. The notations are conventional: with E for employment, U for unemployment, and N for nonparticipation, the flow rate ij represents the movement of the worker from state i to state j. The EE flow rate represents the job-to-job transition rate. The flow rate ij is computed by dividing the number of workers who moved from state i to state j between time t to time $t+1$, divided by the stock of the state i at time t.

All gross flow rates have strong life-cycle patterns. Overall, young workers tend to have higher mobility across states (and across jobs) compared to other age groups, although the very old workers have a strong tendency to move into the N state, likely because of their retirement.

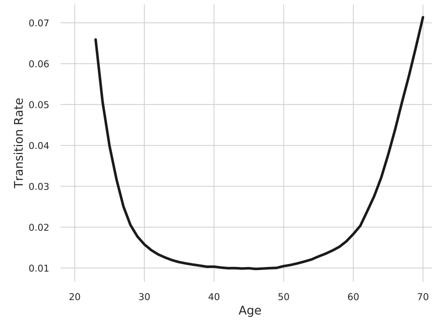
By comparing the patterns of gross flows with the stocks in the previous section, Figure 2 shows the large inflows into N (panels (b) and (d)) in the young and very old contribute to the inverted-U pattern of the labor-force participation rate, although the outflow rates (panels (e) and (f)) have offsetting effects for young workers. For the unemployment stock, the high inflow rates from E and N (panels (a) and (f)) contribute to high unemployment rates of young workers, although the outflow rates (panels (c) and (d)) have offsetting effects. Thus, overall, to explain the patterns of labor-force participation, accounting for the particularly strong life-cycle pattern of the inflow into N is important. For the unemployment rate, the large flow into U is the key to understanding the high unemployment rate of young workers. Therefore, analyzing the life-cycle behavior of employment requires explicit analysis of gross flows involving both U and N.

The behavior of flows and stocks in the steady state does not necessarily directly speak to their reactions to the policies. However, they provide an important guideline to construct and quantify the relevant model. In the next section, we construct a model that contains all relevant elements and is sufficiently flexible to match the data patterns. The results of the policy experiments that come out of the model analysis are credible in the sense that the model itself is consistent with the life-cycle patterns we observe in the data.

A separate, yet interesting question is to investigate the reasons that young workers' flow rates are so high. The model in the next section, which incorporates both individuals' voluntary movements across states (as a reaction to changes in productivity and wealth) and labor market frictions, has a potential to provide insights for the origins of young workers' high mobility.

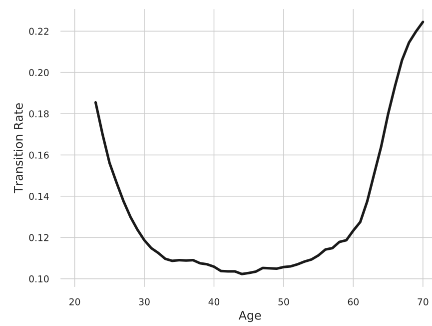


(a) EU

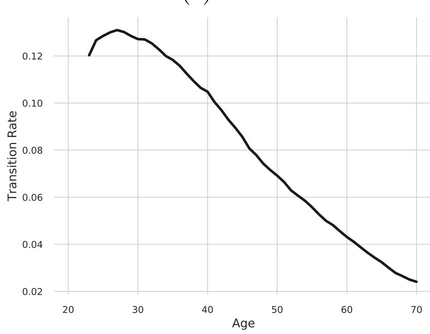


(b) EN

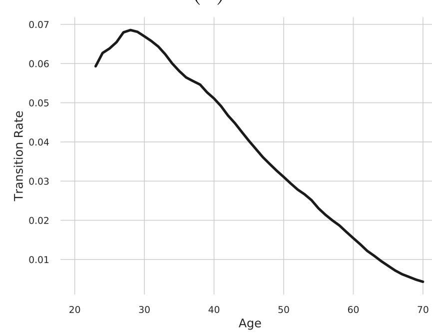
(c) UE



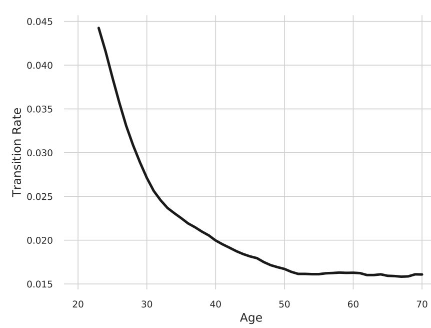
(d) UN



(e) NE



(f) NU



(g) EE

Figure 2: Gross flows in the data

3 Model

The model extends Krusell et al. (2010, 2011, 2017) to a life-cycle setting. In addition to worker life cycle, the model features a frictional labor market with heterogeneous jobs and operative labor-supply margin with concave utility and self-insurance. Thus, the model has the BHA structure with labor market frictions and operative labor supply. An attractive feature of this type of model is that the individuals in the model behave consistently with the permanent-income hypothesis, which has been extensively studied in the consumption-saving literature. Krusell et al. (2011, 2017) have already shown (the infinite-horizon version of) the model is consistent with overall behavior of the gross flows in the economy, including the duration of each state, flow rates for wealth quintiles, and business-cycle properties.

Similar to Krusell et al. (2010), the model features a general equilibrium in that the prices depend on the aggregate capital (which the workers accumulate) and the aggregate labor. One important caveat (shared by Krusell et al. (2010, 2011, 2017)) is that the labor market frictions are exogenous and assumed to be policy invariant. This modeling decision reflects our focus on the labor-supply margin in the analysis of taxes and transfers.

3.1 Overall model structure

Three types of agents—workers, firms, and the government—exist in the economy. The workers supply labor and rent capital out to the firms. The total worker population is normalized to 1. Using capital and labor, firms produce the final good that can be used for consumption and investment. The government taxes labor and transfers it back to all workers in a lump-sum manner. All markets are perfectly competitive. Both the rental market for capital and the final-good market are frictionless as in the standard BHA model. As in the BHA model, the financial market is incomplete. The workers can self-insure by accumulating capital stock.

In the labor market, the worker's labor supply is indivisible in the sense that she can supply either zero or one unit of labor each period. The labor market is frictional. For the frictional labor market to be compatible with perfect competition, we consider the following arrangement, similar to Krusell et al. (2017).

The economy has two islands, *work island* and *leisure island*. All firms are located in the work island. All workers in the work island are employed by firms and receive wages. The work island is divided into many (continuum of) *districts*, and each worker who lives in the district works for one of the firms that are located in that district. The total measure of districts is normalized to 1.

Each worker's productivity has three components: the age component, the general productivity, and the match-specific productivity. The general productivity applies to the worker when working with any firm, whereas the match-specific productivity applies when working in the firm that is located in that district. In other words, the match-specific productivity is specific to the district-worker match. Because many firms exist in the district, the wages are still determined competitively even though the match-specific component exists. All workers in the leisure island do not work.

The mobility of workers across islands is limited, and this lack of mobility is a source of the labor market frictions. Workers in the leisure island receive an opportunity to move to a randomly drawn district every period. The frequency of this job opportunity depends on the search effort of the worker; if the worker searches, in which case she is categorized as *unemployed*, she receives job opportunities more frequently than when she does not search, in which case she is categorized as *not in the labor force*. Within the work island, moving across different districts is limited; every period, an employed worker may receive an opportunity to move to a different district (an "outside job offer") with some probability. We assume that the worker doesn't move across firms within a district (therefore no job-to-job transitions occur within a district), given that, in equilibrium, the worker would receive an identical wage from any firm within the same district. With some probability, an employed worker receives a separation shock and is forced to move to the leisure island. Employed workers can voluntarily move to the leisure island anytime they want to.

3.2 Workers

A worker is characterized by (i) her labor market state: employed (has a job), unemployed (not employed but actively searching for a job), not in the labor force (not employed and not searching for a job), (ii) her wealth (in capital stock), a, (iii) her idiosyncratic general productivity, z, (iv) her match-specific productivity (if employed), μ, and (v) her age, j. Let s_j be the survival probability of a worker from age j to $j+1$. Then each worker maximizes:

$$\mathbf{U}_w = \sum_{j=1}^{J} \left(\beta^j \prod_{t=1}^{j} s_t \right) E_0[\log(c_j) - d_j],$$

where c_j is the consumption at age $j \in \{1, ..., J\}$ and d_j is the disutility of working or searching, which are detailed below. $E_0[\cdot]$ represents the expected value taken at age 0.

Idiosyncratic general productivity, z, is stochastic and follows an AR(1) process. The job-offer probabilities, which are age dependent, are denoted as $\lambda_u(j)$, $\lambda_n(j)$, and $\lambda_e(j)$ for unemployed, not in the labor force, and employed workers at age j. An unemployed worker incurs a search cost of ψ for active searching. An employed worker with general productivity z, match-specific productivity μ, and age j receives wage

$$\omega_j(\mu, z) \equiv g(j)\mu z\tilde{\omega},$$

where the function $g(j)$ is the deterministic age component of market productivity and $\tilde{\omega}$ is the wage per efficiency unit of labor. While working in a firm, μ follows an AR(1) process. At the end of a period, a match is destroyed with a probability, σ_j, depending on age of the worker. A worker in the leisure island receives b units of the final goods from home production.

Upon being matched, the worker draws the match-specific component of productivity μ. We assume the true quality of the match may not be immediately revealed with a probability ζ. In each period, if the match quality is unknown, it remains unknown with probability ζ. In that case, the value of μ is assumed to be $\bar{\mu}$. With probability $1-\zeta$, the true quality is revealed. This gradual learning of match quality is necessary to make the job-to-job transition process in the model match the data. Without such a mechanism, young workers learn their match quality too quickly, and the job-to-job transition rate declines too rapidly with age. Similar formulations are used by Esteban-Pretel and Fujimoto (2014) and Menzio et al. (2016).

We assume the true match-quality shocks for the newly matched are drawn independently from a Pareto distribution with parameters (μ_1, α), where μ_1 is the lower bound of the support of the match-quality distribution, and α determines the rate at which the density of the distribution decreases (note M denotes the random variable and μ denotes its realization):

$$\Pr[M > \mu] = \begin{cases} \left(\dfrac{\mu_1}{\mu}\right)^{\alpha} & \text{for } \mu \geq \mu_1, \\ 1 & \text{for } \mu < \mu_1. \end{cases}$$

The new match quality for an employed worker who obtains an outside job offer is drawn from the same distribution.

The timing within a period is the following. First, idiosyncratic general productivity shocks and match-specific productivity shocks for already-employed workers realize. Second, some nonemployed workers find jobs, and the initial match-specific shocks for new jobs are drawn. Some employed workers receive an opportunity to move to another district, with a new match-specific shock realization. Third, nonemployed workers with job opportunities decide whether to accept the match, and employed workers with moving opportunities decide whether to move. Then, production and consumption take place. At the end of the period, a possible death and the separation shock occur.

Let the value function of an employed worker at age j be $W_j(a, z, \mu)$, the value function of an unemployed worker be $U_j(a, z)$, and the value function of a worker who is not in the labor force be $N_j(a, z)$.

The Bellman equation for the employed is:

$$\begin{aligned} W_j(a, z, \mu) = \max_{c_j, a'} \Big\{ u(c_j) - \psi\gamma + \beta s_j E_{\mu', z'} [\, & (1-\sigma_j)(1-\lambda_e(j)) T_{j+1}(a', z', \mu') \\ & + (1-\sigma_j)\lambda_e(j) S_{j+1}(a', z', \mu') \\ & + \sigma_j (1-\lambda_e(j)) O_{j+1}(a', z') \\ & + \sigma_j \lambda_e(j) F_{j+1}(a', z')] \Big\}, \end{aligned}$$

subject to

$$c_j + a' = (1+r)a + (1-\tau)\omega_j(\mu, z) + \mathbf{T}$$

and

$$a' \geq 0,$$

where

$$T_{j+1}(a', z', \mu') = \max\{W_{j+1}(a', z', \mu'), O_{j+1}(a', z')\},$$

$$S_{j+1}(a', z', \mu') = \int_{\underline{\mu}}^{\bar{\mu}} \max\{T_{j+1}(a', z', \mu'), W_{j+1}(a', z', \mu)\} dG(\mu),$$

$$O_{j+1}(a', z') = \max\{U_{j+1}(z', a'), N_{t+1}(a', z')\},$$

and

$$F_{j+1}(a', z') = \int_{\underline{\mu}}^{\bar{\mu}} \max\{W_{j+1}(a', z', \mu), O_{j+1}(a', z')\} dG(\mu).$$

Here, r is the real interest rate, τ is the labor income tax rate, and $\mathbf{T}$ is the lump-sum government transfer. Each employed worker faces four possible scenarios in the next period: (i) not receiving a separation shock (σ_j) or an outside job offer, in which case she needs to decide between continuing with employment or becoming nonemployed (the value function T), (ii) not receiving a separation shock, but receiving an outside job offer, in which case she additionally needs to decide whether to switch jobs (the value function S), where $G(\cdot)$ is the outside wage-offer distribution, (iii) receiving a separation shock and no outside offer, in which case she becomes nonemployed and needs to decide whether to search (the value function O), or (iv) receiving a separation shock and an outside job offer, in which case she can move directly to another firm (the value function F). While employed, a worker faces disutility of work equal to ψ times γ.

The Bellman equation for the unemployed is:

$$U_j(a, z) = \max_{a', c_j} \left\{ u(c_j) - \psi + \beta s_j E_{z'}[\lambda_u(j) F_{j+1}(a', z') + (1 - \lambda_u(j)) O_{j+1}(a', z')] \right\},$$

subject to

$$c_j + a' = (1 + r)a + b + \mathbf{T}$$

and

$$a' \geq 0,$$

where b is home production and ψ is the disutility of active search effort.

Those not in the labor force are not subject to the disutility of active search, but their job-offer probability will be different (lower), as explained later:

$$N_j(a, z) = \max_{a', c_j} \left\{ u(c_j) + \beta s_j E_{z'}[\lambda_n(j) F_{j+1}(a', z') + (1 - \lambda_n(j)) O_{j+1}(a', z')] \right\}.$$

subject to

$$c_j + a' = (1 + r)a + b + \mathbf{T}$$

and

$$a' \geq 0.$$

3.3 Firms

In each district k of the work island, competitive firms with a constant-returns-to-scale production function operate. The production function for the representative firm in district k takes the Cobb-Douglas form,

$$Y_k = AK_k^{\theta} L_k^{1-\theta},$$

where $\theta \in (0, 1)$, and A is productivity. The inputs K_k and L_k are the capital and labor (in efficiency units) demands. Capital is freely mobile across districts, although labor mobility is restricted. Total capital and labor demand in the economy are

$$K = \int_0^1 K_k dk$$

and

$$L = \int_0^1 L_k dk.$$

We assume firms within a district are homogeneous and the allocation of workers to the districts is entirely random. With the law of large numbers, each district's factor demand becomes the same in a stationary equilibrium. Therefore,

$$K = K_k$$

and

$$L = L_k$$

hold in a stationary equilibrium. Because r and $\tilde{\omega}$ depend only on K_k/L_k, these equalities imply r and $\tilde{\omega}$ are common across districts.[3] Capital stock depreciates at a rate δ.

3.4 Government

The government collects tax on labor income and also confiscates assets of the deceased individuals in the economy. It redistributes all revenue to individuals in the economy uniformly while running a balanced budget. Thus, the government budget constraint is

$$\mathbf{T} = \tau \int e(i)\omega_{j(i)}(i)(\mu(i), z(i))di + \int a(i)(1 - s(i))di, \tag{1}$$

where $e(i)$ is the employment status of agent i with 1 for employed and 0 for not employed, and $s(i)$ is the survival status of agent i, 1 for surviving agents, and 0 for deceased agents.

[3] With common technology, free capital mobility, and a constant-returns-to-scale production function, K_k/L_k is equalized across districts even when L_k is unevenly distributed. See Appendix A.9 of Krusell et al. (2017).

3.5 Equilibrium

We solve for a stationary equilibrium in which the real interest rate and wage profile are constant over time. After all new matching opportunities realize (with new idiosyncratic productivity and match-specific shocks), workers make the following decisions.

(i) A nonemployed worker at age j, wealth a, idiosyncratic productivity z, and who has an offer of match-specific productivity μ accepts the offer and becomes employed if and only if

$$W_j(a, z, \mu) \geq O_j(a, z).$$

(ii) A nonemployed worker at age j, wealth a, and idiosyncratic productivity z who rejected a job offer or did not receive a job offer decides to be in the labor force if and only if

$$U_j(a, z) \geq N_j(a, z).$$

(iii) An employed worker at age j, wealth a, idiosyncratic productvity z, and current match-specific productivity μ, who does not have an outside job offer stays in her job if only if

$$W_j(a, z, \mu) \geq O_j(a, z).$$

(iv) An employed worker at age j, wealth a, idiosyncratic productvity z, current match-specific productivity μ, and outside offer μ' switches jobs if and only if

$$W_j(a, z, \mu') > T_j(a, z, \mu).$$

(v) Each worker makes optimal consumption and investment decisions according to the Bellman equations described in Section 3.2.

Capital and labor markets clear.

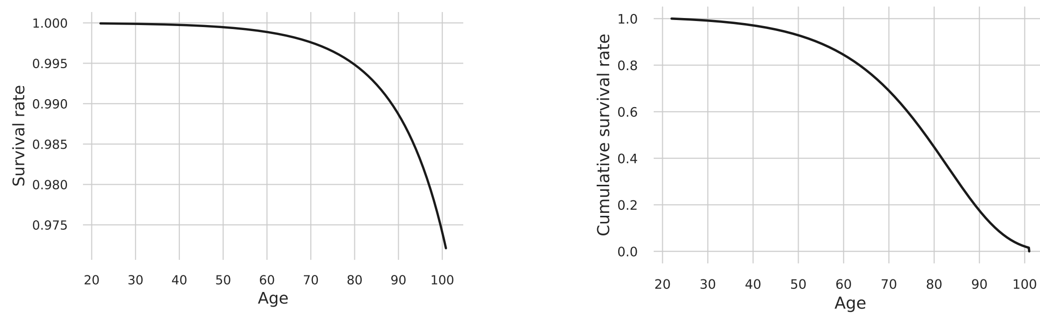
(i) Total assets supplied are equal to total capital demand,

$$\int a_i di = K.$$

(ii) Labor supply in efficiency units is equal to labor demand,

$$\int e(i) z_i \mu_i g_i di = L.$$

As described in Section 3.4, the government runs a balanced budget, represented by the constraint (1): the total lump-sum transfer is equal to the sum of labor income tax revenue and wealth of the deceased agents.



(a) Survival rate

(b) Cumulative survival rate

Figure 3: Survival rate

4 Calibration

In quantifying the model, first, a subset of parameters are calibrated using external information. Then the remaining parameter values are estimated so that the distance between the model outcome and the data are minimized.

Each period corresponds to one month.[4] Following Krusell et al. (2010), we consider $\tau = 0.30$ as the benchmark. On the production side, θ is set at 0.3, and the depreciation rate, δ, is set at 0.0088, which corresponds to a 10.5% annual depreciation rate[5]. For the consumers, $\beta = 0.996$ and the death probabilities at each age are taken from life tables at the Social Security Administration.[6] The calibrated survival rates are plotted in Figure 3. The relative disutility of working compared to search, γ, is set to $\gamma = 40/3.5$, which corresponds to the ratio of the average hours worked by the workers to the average hours the unemployed actively search for a job, taken from Mukoyama et al. (2018). The persistence parameter of the monthly AR(1) idiosyncratic productivity (z) process is set to $\rho_z = 0.97$ and the persistence parameter of the monthly AR(1) match-specific productivity (μ) process is set to $\rho_\mu = 0.98$. The interest rate, r, is targeted to be equal to 0.0043, which corresponds to a 5.26% annual compound interest rate. A is set to 0.501 to normalize $\tilde{\omega}$ to 1 in equilibrium. We assume match-specific productivity of matches with unrevealed quality is equal to median productivity, $\bar{\mu} = 1.0$.

For age-dependent parameters, we allow them to be a simple function of age. Specifically, let the age component of market productivity, $g(j)$, the logarithms[7] of job-offer arrival rates, $\log \lambda_e(j)$, $\log \lambda_u(j)$, $\log \lambda_n(j)$, and the logarithm of exogenous job separation rate be

[4]We assume model age $j = 1$ corresponds to an annual age of 22. The monthly age after which everyone dies for sure is $J = 948$, which corresponds to an annual age of 101.

[5]A 10.5% annual depreciation rate is calculated by multiplying δ with 12, $12\delta = 0.105$. Compounded depreciation rate is about 10%, $1 - (1 - \delta)^{12} \approx 0.10$.

[6]Our calibrated survival rate is given by the following function: $s_j = (1 - (0.000149 \exp(0.0751((j-1)/12 + 22)))^{1/12}$ for j in $1, 2, ...946$, and $s_j = 0$ for $j > 946$. Essentially, workers at age 947 and 948 die for sure.

[7]Throughout this text, when we write "logarithm" we mean natural logarithm.

characterized as second-degree polynomials of age, j:

$$\lambda_e(j) = \exp(\lambda_{e,2}j^2 + \lambda_{e,1}j + \lambda_{e,0}),$$

$$\lambda_u(j) = \exp(\lambda_{u,2}j^2 + \lambda_{u,1}j + \lambda_{u,0}),$$

$$\lambda_n(j) = \exp(\lambda_{n,2}j^2 + \lambda_{n,1}j + \lambda_{n,0}),$$

$$\sigma(j) = \exp(\sigma_2 j^2 + \sigma_1 j + \sigma_0),$$

and

$$g(j) = g_2 j^2 + g_1 j + g_0.$$

The remaining parameters that need to be calibrated are

$$\boldsymbol{\xi} \equiv \{\lambda_{e,2}, \lambda_{e,1}, \lambda_{e,0}, \lambda_{u,2}, \lambda_{u,1}, \lambda_{u,0}, \lambda_{n,2}, \lambda_{n,1}, \lambda_{n,0}, \sigma_2, \sigma_1, \sigma_0, g_2, g_1, g_0, \psi, \sigma_\mu, \sigma_z, b, \zeta, \alpha\},$$

where σ_z and σ_μ are the standard deviations of AR(1) shocks of idiosyncratic productivity and match-specific productivity. To estimate these parameters, we minimize the sum of the squared log distance between (i) gross worker flows and average market wage and (ii) the corresponding moments from the model simulations. More precisely, for a given $\boldsymbol{\xi}$, we solve for the value functions and decision rules recursively and simulate the model according to the decision rules. To simulate the model, we need to make assumptions about the initial distribution of workers' state variables. We assume each worker begins life at the leisure island with no assets. Then, we calculate the monthly transition rates from one state to another state as follows, using the employment-to-unemployment transition (EU) as an example:

$$EU^{model}(\boldsymbol{\xi}, j) = \frac{\text{Measure of the employed at age } j \text{ moving to unemployment the next period}}{\text{Measure of the employed at age } j}.$$

Because our model is stationary and no aggregate shocks occur, we drop the time index. We calculate age-specific transition rates between labor market states by taking the mean of the monthly transition rates. Continuing with the EU transition as an example,

$$EU^{model}(\boldsymbol{\xi}, a) = \frac{1}{12} \sum_{j=12(a-22)+1}^{12(a-21)} EU^{model}(\boldsymbol{\xi}, j),$$

where a is the (annual) age. We assume model-age $j = 1$ corresponds to age 22 in the data. We calculate the monthly wage rate by taking the average of wages of same-aged workers. Then, we convert the monthly wage rate to the age-specific wage as above. However, we normalize the average wage at age 42 to 1:

$$\bar{\omega}(\boldsymbol{\xi}, a) = \frac{1}{12} \sum_{j=12(a-22)+1}^{12(a-21)} \omega^{model}(\boldsymbol{\xi}, j),$$

$$\omega^{model}(\boldsymbol{\xi}, a) = \frac{\bar{\omega}(\boldsymbol{\xi}, a)}{\bar{\omega}(\boldsymbol{\xi}, 42)}.$$

Let $X^{model}(\boldsymbol{\xi})$ be the collection of transition rates among worker states and normalized average wages,

$$\begin{aligned} X^{model}(\boldsymbol{\xi}) = \quad & \text{vec}\Bigg(\Bigg\{ EU^{model}(\boldsymbol{\xi}, a), EN^{model}(\boldsymbol{\xi}, a), EE^{model}(\boldsymbol{\xi}, a), \\ & NU^{model}(\boldsymbol{\xi}, a), NE^{model}(\boldsymbol{\xi}, a), UN^{model}(\boldsymbol{\xi}, a), \\ & UE^{model}(\boldsymbol{\xi}, a), \omega^{model}(\boldsymbol{\xi}, a) \Bigg\}_{a=23}^{70} \Bigg), \end{aligned}$$

and let X^{data} be the collection of the six-year rolling average of transition rates and normalized average wages[8] of males observed in the data,

$$\begin{aligned} X^{data} = \quad & \text{vec}\Bigg(\Bigg\{ EU^{data}(\boldsymbol{\xi}, a), EN^{data}(\boldsymbol{\xi}, a), EE^{data}(\boldsymbol{\xi}, a), \\ & NU^{data}(\boldsymbol{\xi}, a), NE^{data}(\boldsymbol{\xi}, a), UN^{data}(\boldsymbol{\xi}, a), \\ & UE^{data}(\boldsymbol{\xi}, a), \omega^{data}(\boldsymbol{\xi}, a) \Bigg\}_{a=23}^{70} \Bigg). \end{aligned}$$

In the numerical solution of the model, we discretize idiosyncratic and match-quality AR(1) processes using the Tauchen method. In our calibration exercise, we minimize the sum of the squared log distance between $X^{model}(\boldsymbol{\xi})$ and X^{data} by choosing (ξ) :

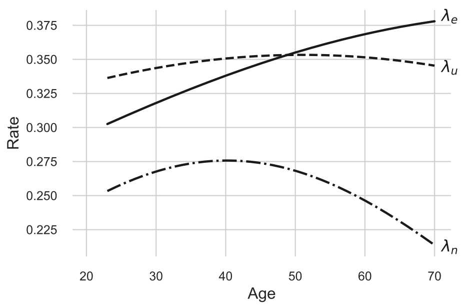
$$\min_{\boldsymbol{\xi}} |(\log X^{model}(\boldsymbol{\xi}) - \log X^{data})|'|(\log X^{model}(\boldsymbol{\xi}) - \log X^{data})|.$$

The calibrated parameters that do not have an age component are shown in Table 1. The estimated coefficients are in Appendix A. Figure 4 visualizes the calibrated outcome in graphs. As we discussed in the introduction, these results are of independent interest. These results reveal the nature of frictions for different age groups of individuals, which we cannot observe directly in the data.

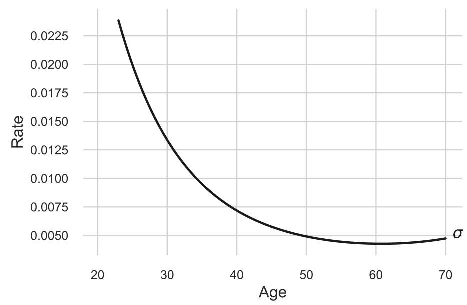
Panel (a) of Figure 4 shows job-offer arrival rates over the life cycle for the unemployed (dashed line), the employed (solid line), and the nonparticipant (dot-dash line). For nonemployed workers, job-offer arrival rates increase until they reach prime age, and then decreases. The decrease in the job-offer arrival rate is sharper for the non-participant. As expected, the job-offer arrival rate for the unemployed is greater than that of the non-participant, highlighting the active-job-search trade-off: active job search is costly but results in a higher probability of receiving an offer.

The job-offer arrival rate for the employed has two notable features. First, the overall level of λ_e is similar to λ_u. This result is reminiscent of Tobin's (1972) argument that no evidence

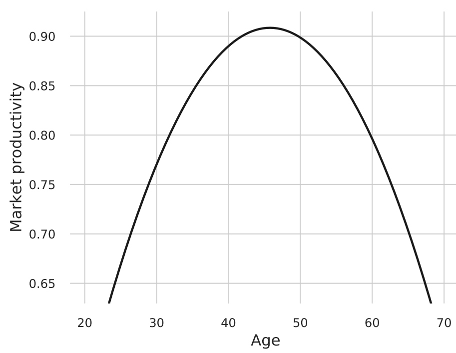
[8] Average wages at each age is expressed relative to average wage of 42-year-old male workers.



(a) Job offer arrival rate



(b) Exogenous job separation rate



(c) Age component of market productivity

Figure 4: Age dependent parameters

Table 1: Age independent parameters

Parameter	**Definition**	**Value**
β	Discount factor	0.996
θ	Elasticity of output w.r.t. capital	0.3
δ	Depreciation rate	0.0088
ψ	Disutility of active job search	0.04
A	Total factor productivity	0.501
ρ_μ	Persistence parameter of monthly AR(1) match-specific productivity	0.98
σ_μ	Std. dev. of innovations in match specific productivity	0.11
ρ_z	Persistence parameter of monthly AR(1) idiosyncratic productivity	0.97
σ_z	Std. dev. of innovations in idiosyncratic productivity	0.09
b	Home productivity	0.14
ζ	Unknown match quality probability	0.28
α	Shape parameter of Pareto distribution (new job wage offers)	7.49
$\bar{\mu}$	Match quality for unrevealed matches	1.0
γ	Disutility of work over disutility of active job search	11.4
J	Monthly age at which everyone dies	948

exists that employed workers are less efficient in job search than nonemployed workers.[9] In fact, employed workers appear to be more efficient in search than nonemployed workers when they are old. This finding, of course, is consistent with the fact that job-to-job transitions are less frequent than UE transitions, because the employed workers tend to be choosier because of their outside options. Second, unlike λ_u and λ_n, λ_e increases monotonically with age. This pattern could, for example, reflect that employed workers can build a better network as they become older. Although our model is too stylized to investigate this point further, it seems to be an interesting hypothesis for future inquiry.

Panel (b) of Figure 4 shows that exogenous job separation decreases over the life cycle of an individual with a slight increase after age 60. The age component of market productivity in Panel (c) displays an inverse-U shape. Market productivity increases until middle age and then decrease toward the end of an agent's working life. This pattern is largely consistent with the results from direct measurements from microeconomic data, widely used in quantitative public finance literature.[10]

Several results from Figure 4 are surprising when compared with the actual worker flows. First, despite the strong life-cycle pattern of UE flows, the job-offer probability of unemployed

[9] Mukoyama (2014) reports a similar outcome with a simple job-ladder model when the separation rate strongly depends on match quality.

[10] For example, Conesa et al. (2009) use the measurement from Hansen (1993).

Table 2: Aggregate statistics from the experiment

Tax	N	E	U	u	$lfpr$	Labor	Efficiency
0.30	0.343	0.620	0.037	0.057	0.657	0.571	0.921
0.45	0.520	0.453	0.027	0.056	0.480	0.449	0.990

individuals, λ_u, is almost flat over the life cycle. Second, although the EE flow rates decline over the life cycle, the offer probability, λ_e, increases over the age. These results, once again, caution against identifying the patterns of actual worker flows with the patterns of the opportunities that workers face.

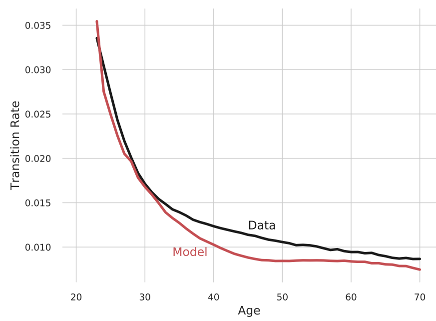
Comparing panels (a) and (b), one can conclude the large U stock for young workers is mostly the result of a large separation shock σ. Investigating why σ exhibits such a pattern is beyond the scope of this paper, but it is an important future research topic. In the context of Mortensen and Pissarides's (1994) model, one can interpret the σ shock as an event where the job-worker match suffers from a large negative productivity shock. The matches involving young workers, not having as much information on the strengths and weaknesses of the individuals, may be subject to these shocks more frequently.

Our model outcomes against the targeted data moments are plotted on Figure 5. We are able to match qualitative features of the flows rates by age quite well. For some flow rates, EU, EN, and EE, we are able to match the entire life-cycle dynamics almost perfectly.

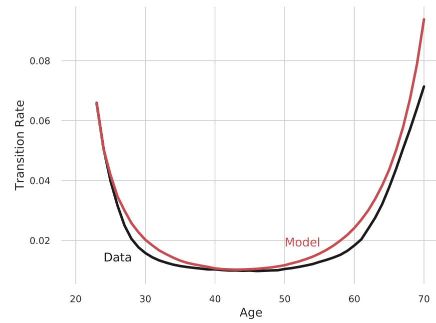
5 Results

Our primary interest is the effect of an increase in labor tax. In an influential work, Prescott (2004) argue the difference in total hours between the US and the continental Europe can largely be explained by the difference in tax system. Although various studies have followed up on Prescott's (2004) study, none has explicitly analyzed a model with gross worker flows in a life-cycle economy. Our model reveals two novel effects of the tax and transfer: the effects on reallocation (worker flows) over the life cycle and the decomposition of effects on nonemployment into the one on unemployment and the one on nonparticipation.

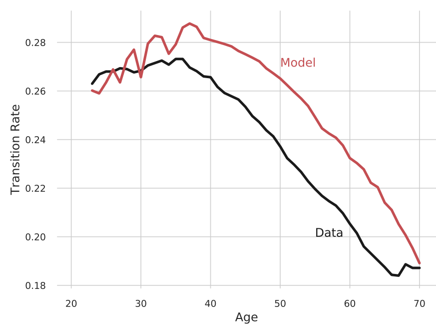
Following the experiment by Krusell et al. (2010), we consider an experiment of raising the labor tax rate τ from 0.30 to 0.45. Table 2 summarizes the results at the aggregate level. The magnitude of decline in aggregate employment is somewhat stronger than the infinite-horizon economy in Krusell et al. (2010); here, E declines by $0.453/0.620 = 0.73$, whereas in Krusell et al. (2010), the corresponding value is $0.488/0.633 = 0.77$. One factor that increases the impact of tax in the life-cycle economy is the heterogeneity of responses across different age groups. Figure 6 draws the composition of the labor market states at each age, for both the benchmark (30% tax) and the experiment (45% tax). Although employment



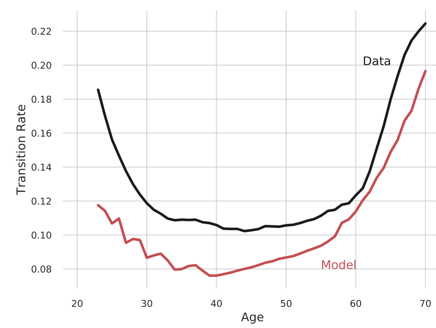
(a) EU



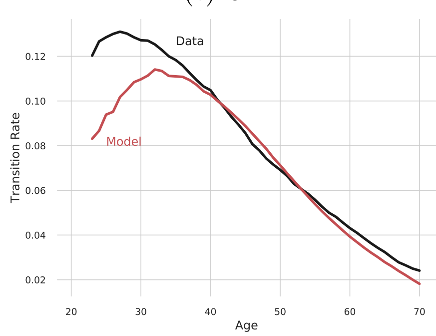
(b) EN



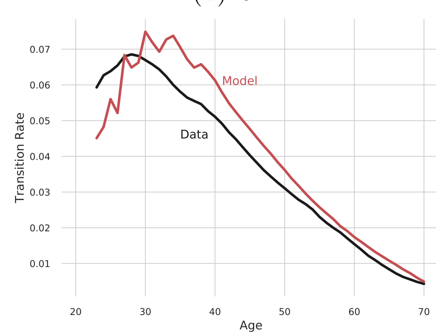
(c) UE



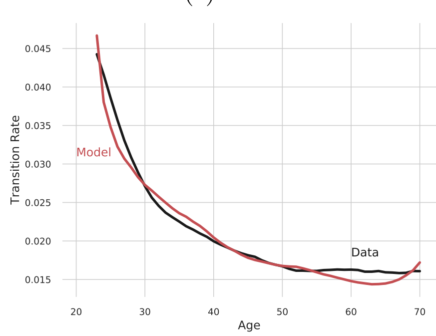
(d) UN



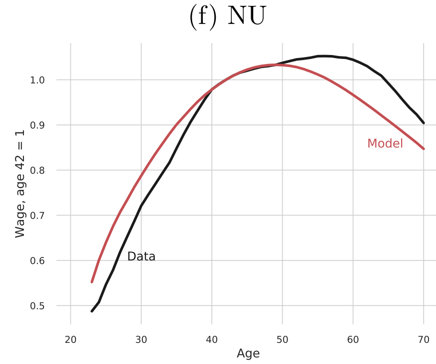
(e) NE



(f) NU

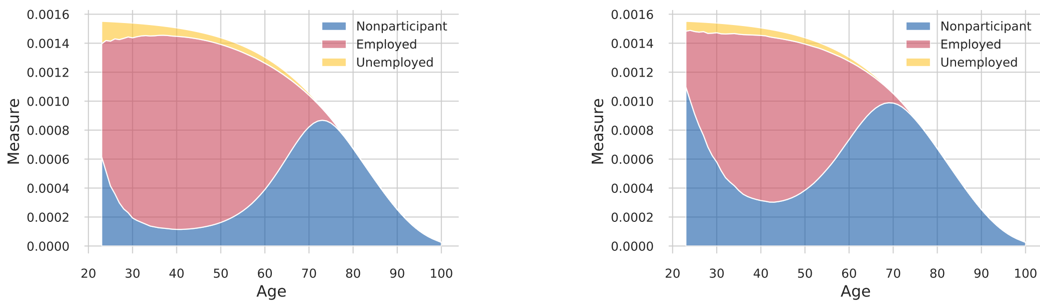


(g) EE



(h) Wage

Figure 5: Model moments and calibration targets



(a) Benchmark

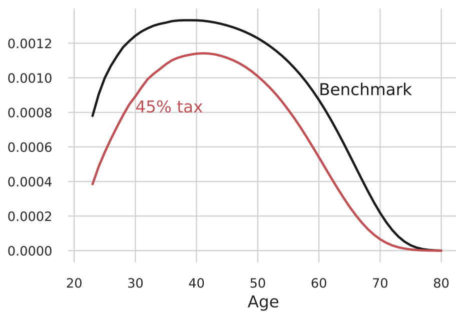
(b) 45% tax

Figure 6: Composition of the labor market states over the lifecycle

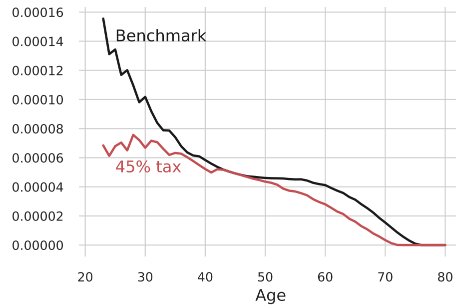
clearly decreases and the nonparticipation increases in all ages, the decline in participation is particularly strong in young workers. Because young workers tend to be less productive than the prime-aged workers (see panel (c) of Figure 4), the impact on employment tends to be larger for a given change in the efficiency units of labor. Because the total labor in efficiency units is the quantity that is pinned down in the general equilibrium, the employment response is determined so that the efficiency units becomes the value that clears the market. In this life-cycle economy, because of the low productivity of young workers, the employment has to respond more, so that the total labor in efficiency units changes by a sufficient amount.

The unemployment rate in Table 2 slightly declines with the tax increase. This finding is in contrast to the baseline case in Krusell et al. (2010), where the unemployment rate increases. Once again, the key is in the behavior of young workers. Figure 7 compares the stocks in Figure 6 one by one. A striking difference can be seen in panel (b): the unemployment stock for young workers declines dramatically. The rates comparable to Figure 1 for the data are plotted in Figure 8. Somewhat surprisingly, the life-cycle profile of the unemployment rate changes very little for all ages. Even for the very young workers, where the total U stock changes significantly in Figure 7, the change in the unemployment rate is relatively small because E also falls significantly. Overall, therefore, the change in employment is mostly driven by the labor force participation. The response of the labor-force participation rate is the largest for the young workers.

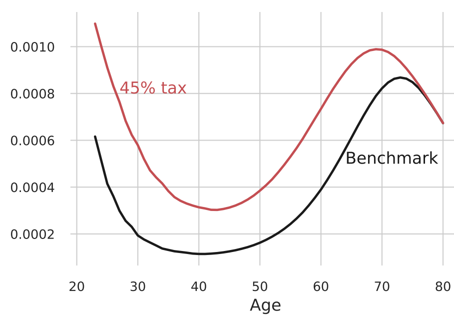
Now we investigate the gross worker flows. Figure 9 draws each labor market transition rate for the benchmark and 45% tax case. First, we investigate the cause of the decrease in U stocks in Figure 7. Among the flows that involve the U state, two flows show a strong impact on young workers. The first is the EU flow. Because only high-productivity workers participate when the tax is high, the likelihood of moving from E to U when the match quality becomes worse is lower in a high-tax situation. The second is NU flow. Two (potential) reasons exist for moving from N to U: (i) running down assets (the wealth effect), and (ii) improvement of the idiosyncratic productivity. The reduction in labor income and the increase in the lump-sum transfer implies the individuals in the N state do not (have to)



(a) Employed

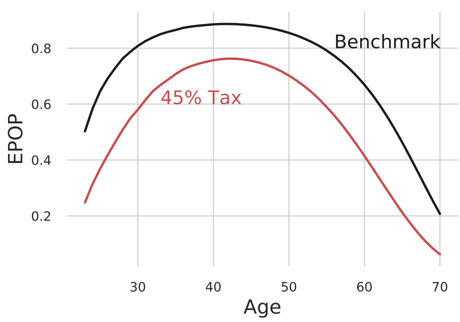


(b) Unemployed

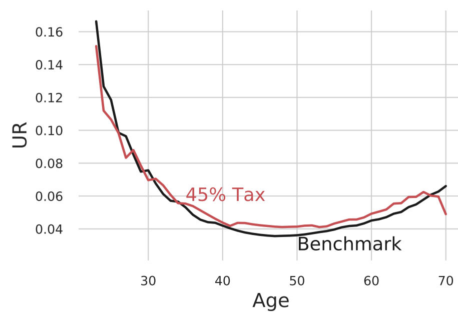


(c) Not in the labor force

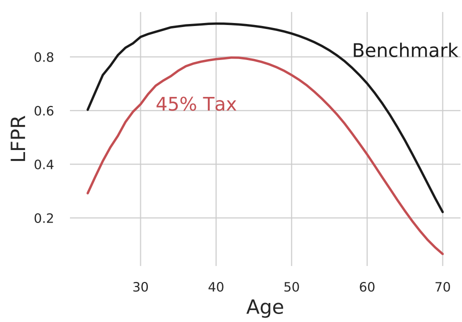
Figure 7: Labor market stocks after a tax hike



(a) Employment to population ratio

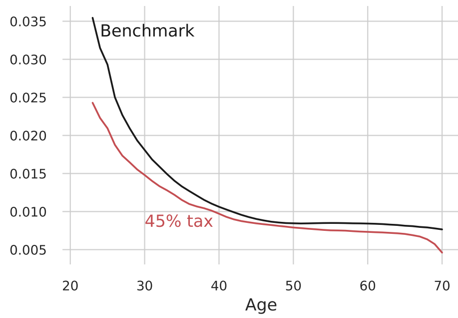


(b) Unemployment rate

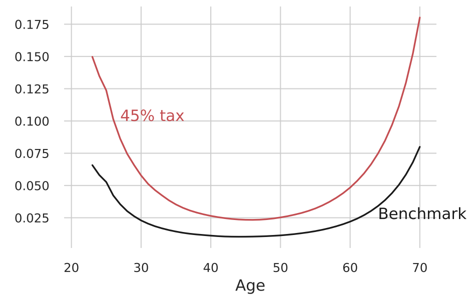


(c) Labor force participation rate

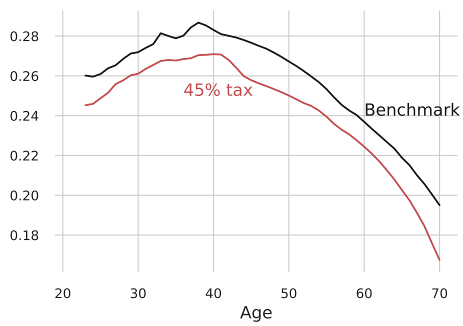
Figure 8: Labor market ratios after a tax hike



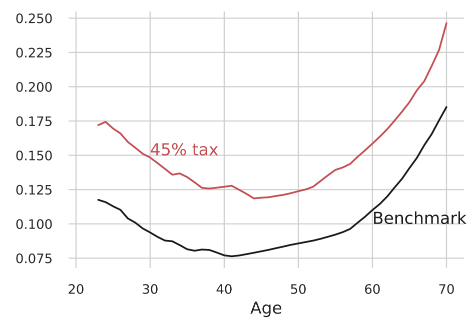
(a) EU



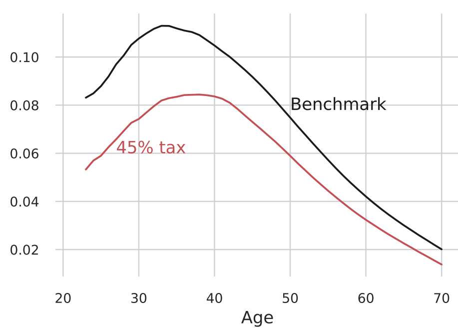
(b) EN



(c) UE

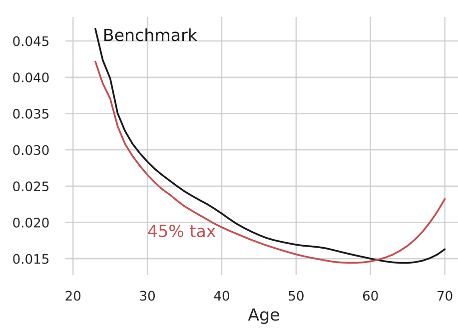


(d) UN



(e) NE

(f) NU



(g) EE

Figure 9: Gross worker flow rates after a tax hike

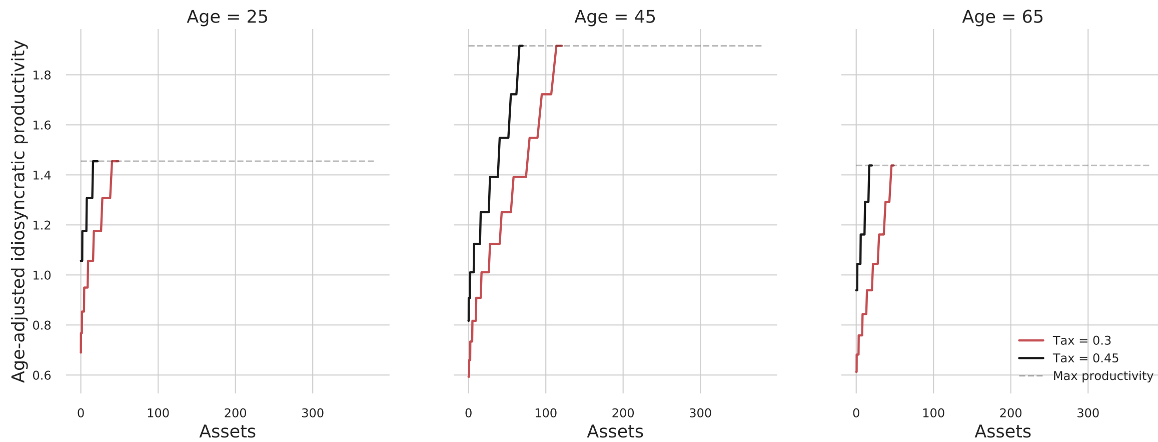


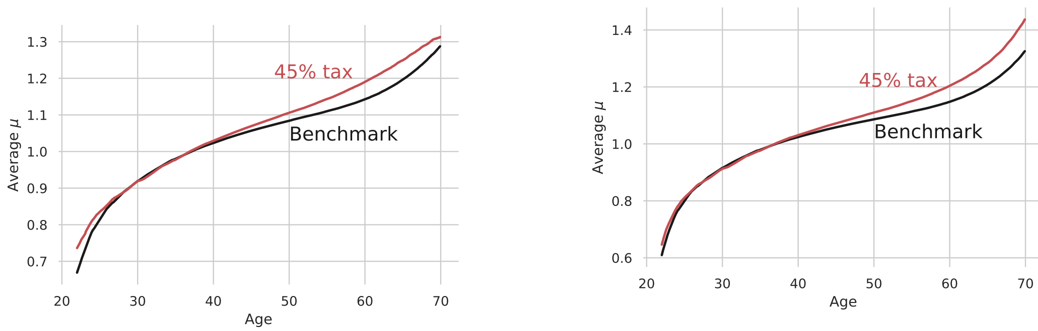
Figure 10: Age-adjusted idiosyncratic productivity cutoffs for labor force participation

run down assets while nonemployed as quickly when the labor tax is high. In other words, the income is smoother across states, and thus reducing the precautionary saving (and precautionary work) motive for the individuals. The impact of a lump-sum transfer is larger for a young worker, who tends to have lower labor income and a lower level of assets. Thus, in explaining the decrease in U for young workers, (i) the selection of employed workers and (ii) the improved opportunities for consumption smoothing play important roles.

Second, in relation to labor-force participation rates, both NE and NU flow shift substantially more for young workers. This finding is in contrast to the shifts of the opposite-direction flows, EN and UN, which are fairly uniform across all ages. Combined with the fact that the employment response is largely coming from the participation margin, we conclude that the outflow from non-participation is the key to generating the life-cycle pattern of the employment response to the taxes.

Analyzing more deeply at the micro level, Figure 10 plots the cutoff levels of the age-adjusted idiosyncratic productivity ($g(j) \times z$) for given assets. Above the cutoff level, a non-employed worker decides to participate in the labor market. Three panels for different ages (25, 45, and 65 years old) compare the cutoffs for the baseline ($\tau = 0.3$) and the experiment ($\tau = 0.45$). The amount of the shift of the cutoffs turns out to be not too different across different ages. Note that the aggregate responses are affected by the combinations of the change in the cutoffs and the location of the distributions of the state variables (in particular, the joint distributions of asset and productivity), as well as the change in distributions by the policy. Overall, younger workers exhibit more action in the aggregate participation margin, largely because they tend to have a lower level of wealth (where taxes have a larger impact) and more workers tend to be in the neighborhood of the cutoff lines.

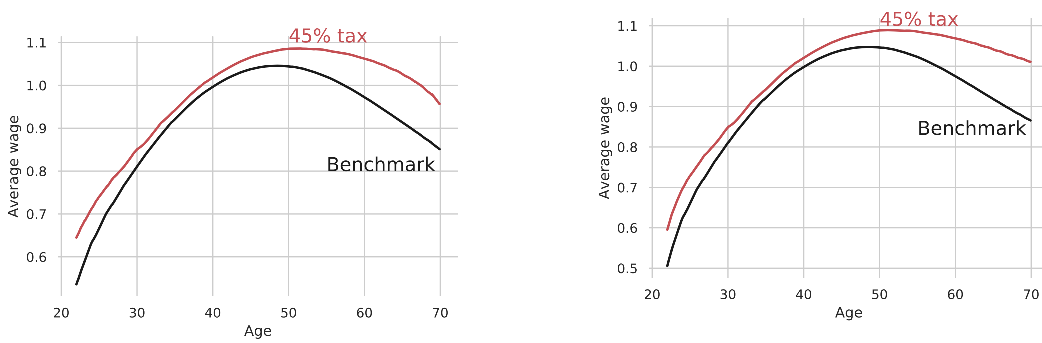
In concluding the experiment, we show two more consequences of the labor tax. First, Figure 11 plots the average values of μ for each age (including (a) and excluding (b) the "unrevealed" matches). As one would expect, the case with a higher tax exhibits a higher



(a) All employed

(b) Employed with revealed match-specific productivity

Figure 11: Mean match-specific productivity



(a) All employed

(b) Employed with revealed match-specific productivity

Figure 12: Mean wages

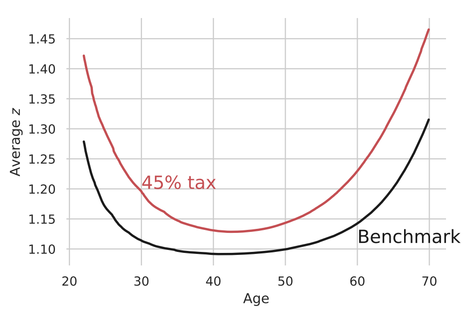


Figure 13: Mean idiosyncratic productivity

level of μ, because the workers are choosier. However, the difference becomes only visible after 40 years of age. One reason is that, with a higher tax, workers have less opportunity to climb the job ladder, as they stay employed for a shorter duration. Young workers spend more time in U and N states, and therefore, does not experience as many job-to-job transitions when the taxes are high. Even conditional on employment, the EE transitions are less frequent—see panel (g) of Figure 9. This type of reallocation effect is absent in past analyses, such as Prescott (2004), and highlights the importance of explicitly incorporating worker flows in the analysis of taxes and transfers.

Second, the wages before tax are plotted in Figure 12. Wages have three components (aside from the age component); μ, z, and $\tilde{\omega}$. The first two components increase as seen in Figure 11 and in Figure 13 with the increase in labor tax. Both μ and z are higher on average, because of the selection. The base wage $\tilde{\omega}$ decreases because K/L is lower, which is expected. Workers save less because of a reduction in the precautionary saving motive and a reduction in the incentives for the low-productivity workers to participate in the labor force. The overall impact of the tax hike on mean wages is positive. The overall effect is fairly strong for old workers—the wage increase almost offsets the increase in τ. Younger workers (below 40 years old) do not experience a significant increase in pre-tax wages. Therefore, from the (static) welfare standpoint, individuals in different age groups experience the effect of the tax increase very differently, even if these individuals are employed in both regimes.

6 Conclusion

This paper analyzes the responses of gross worker flows to the increase in labor tax for different stages of the worker life cycle. Our model features life-cycle permanent-income consumers who can self-insure from various shocks by accumulating assets. In the labor market, the individuals can make the labor market participation decision under labor-market frictions.

The calibrated model can match the salient features of the life-cycle patterns of the gross worker flows in the data. The estimated parameter values reveal how the nature of frictions vary across the worker's life cycle. The pattern of the job-separation shock has an important impact on the life-cycle behavior of unemployment stocks.

With the calibrated model, we run an experiment of increasing the labor tax. An increase in labor tax decreases employment and labor-force participation for all age groups, although the changes are larger for younger workers. Unemployment stock decreases significantly only for young workers. The analysis of gross worker flows finds the changes in EU flow and NU flow (inflow into unemployment) are the main causes of the age heterogeneity in the unemployment response. For the N state, the outflow from N is of prominent importance. Overall, young workers move less into the U state and also leave less from the N state when the labor tax is high. The channels through the changes in the gross flows also have an impact on productivity and wages, highlighting the importance of explicitly considering effects on reallocation in the analysis of taxes and transfers. The reallocation effects are heterogeneous across age groups.

Although we view our study as progress compared to the existing literature, much room remains for future research. The model of this paper, as in the case with Krusell et al. (2017), does not address the endogenous response of the frictions to the change in taxes. The Diamond-Mortensen-Pissarides framework, for example, would suggest a change in the labor tax can affect the firms' vacancy-posting behavior and eventually alter the frequency of workers receiving job offers. Two reasons for this particular omission exist. First, our focus is the labor-supply response, which has been the focus of the literature starting from Prescott (2004). Second, incorporating such a mechanism into a BHA-style model is challenging (see, e.g., Krusell et al. (2010) and Mukoyama (2013)). Incorporating such an effect is outside the scope of this paper, but it is an important future research agenda.

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Appendix

A Age coefficients on frictions and productivity

The age component of market productivity, $g(j)$, the logarithms of job-offer arrival rates, $\log\lambda_e(j), \log\lambda_u(j), \log\lambda_n(j)$, and the logarithm of exogenous job-separation rate are characterized as second degree polynomials of age, j:

$$\begin{aligned}
\lambda_e(j) &= \exp(\lambda_{e,2}j^2 + \lambda_{e,1}j + \lambda_{e,0}), \\
\lambda_u(j) &= \exp(\lambda_{u,2}j^2 + \lambda_{u,1}j + \lambda_{u,0}), \\
\lambda_n(j) &= \exp(\lambda_{n,2}j^2 + \lambda_{n,1}j + \lambda_{n,0}), \\
\sigma(j) &= \exp(\sigma_2 j^2 + \sigma_1 j + \sigma_0), \\
g(j) &= g_2 j^2 + g_1 j + g_0.
\end{aligned}$$

The calibrated coefficients are shown in Table 3.

Table 3: Coefficients of polynomials

Parameter	**Definition**	**Value**
$\lambda_{u,2}$	Log of the job offer rate of the unemployed	-4.377929E-07
$\lambda_{u,1}$		2.9457896E-04
$\lambda_{u,0}$		-1.089935
$\lambda_{e,2}$	Log of the job offer rate of the employed	-4.146005E-07
$\lambda_{e,1}$		6.295554E-04
$\lambda_{e,0}$		-1.196155
$\lambda_{n,2}$	Log of the job offer rate of the non-participant	-1.998678E-06
$\lambda_{n,1}$		8.269823E-04
$\lambda_{n,0}$		-1.37384389
σ_2	Log of the job separation rate	8.414869E-06
σ_1		-7.62585E-03
σ_0		-3.728505
g_2	Age component of market productivity	-3.86754339E-06
g_1		2.11836052E-03
g_0		6.18438302E-01

B Details of computation

In the model, a worker is characterized by (i) her labor market state: employed (has a job), unemployed (not employed but actively searching for a job), not in the labor force (not employed and not searching for a job), (ii) her wealth, a, (iii) her idiosyncratic general productivity, z, (iv) her match-specific productivity (if employed), μ, and (v) her age, j. Age in the model is monthly and ranges from 1 to 948. Model age $j = 1$ corresponds to annual age 22 in the data. At each age j, an s_j fraction of workers survive to age $j+1$. At age 948, which corresponds to age 101 in the data, everyone dies for sure.

To solve the model numerically, we discretize the state space. For assets, a, we create a log-spaced grid of 100 points between 0 and 380. The discretization of asset space is independent of other model parameters. We also discretize the state space of z and μ. However, discretization of $z-$space and $\mu-$space depend on other model parameters and are explained in the later paragraphs.

Our model is in a general equilibrium. However, we first solve and calibrate our model focusing on the household side, and later we ensure general equilibrium conditions hold. First, given the interest rate, r, and wage rate in efficiency units, $\tilde{\omega}$, we solve workers' problems. We then ensure firms demand capital and labor to the extent that marginal product of capital is equal to the interest rate plus the depreciation rate, and the marginal product of labor is equal to the wage rate per efficiency unit of labor.

Recall that $\boldsymbol{\xi} \equiv \{\lambda_{e,2}, \lambda_{e,1}, \lambda_{e,0}, \lambda_{u,2}, \lambda_{u,1}, \lambda_{u,0}, \lambda_{n,2}, \lambda_{n,1}, \lambda_{n,0}, \sigma_2, \sigma_1, \sigma_0, g_2, g_1, g_0, \psi, \sigma_\mu, \sigma_z, b, \zeta, \alpha\}$ is the set of parameters to be calibrated within the model. Let $\boldsymbol{\xi^o} \equiv \{\beta, \{s_j\}_{j=1}^{948}, \rho_\mu, \rho_z\}$ be the set of parameters calibrated externally.

We solve our model as follows:

1. Given parameter values $\boldsymbol{\xi}$ and $\boldsymbol{\xi^o}$, we first create the discrete state space to solve the model numerically.

   (a) We discretize the support of the AR(1) idiosyncratic productivity process using the Tauchen method. We create a grid of idisyncratic productivities, $z-$grid, consisting of 15 points. The Tauchen method also generates transition probabilities from current idiosyncratic productivity, z, to the next period's idiosyncratic productivity, z': $P_{z,z'}$ for z and z' in $z-$grid.

   (b) Similarly, we discretize the support of the AR(1) match-specific productivity process of the worker-firm pair using the Tauchen method. $\mu-$grid consists of 15 points. The probability of a match-specific productivity, μ, becoming μ' in the next period if the worker-firm pair survives is denoted as $P_{\mu,\mu'}$ for μ and μ' in $\mu-$grid.

   (c) Match-specific productivity for a new job (for the workers in the leisure islands, and for the workers who have a job but receive an outside offer) is drawn from a

Pareto distribution:

$$\Pr[M > \mu] = \begin{cases} \left(\frac{\mu_1}{\mu}\right)^{\alpha} & \text{for } \mu \geq \mu_1, \\ 1 & \text{for } \mu < \mu_1, \end{cases}$$

where μ_1 is the lowest point in the $\mu-$grid. Notice M is the random variable, μ is its realization. Let μ_k be the $k-$th lowest point in the $\mu-$grid. Then, probability of receiving an outside offer with match-specific productivity of μ_k is equal to:

$$S(\mu_k) = \begin{cases} \left(\frac{\mu_1}{\mu_{k-1}}\right)^{\alpha} - \left(\frac{\mu_1}{\mu_k}\right)^{\alpha} & \text{for } k > 1, \\ 0 & \text{for } k = 1. \end{cases}$$

(d) Recall that with a probability, ζ, match-specific productivity is unrevealed (or unkown). To account for the unknown state, we add one more grid point to $\mu-$grid, which now consists of 16 points. We assume that if the match-specific productivity is not known, workers are paid as if they have the median match-specific productivity. Let $P^{ext}_{\mu,\mu'}$ represent transition probabilities in the extended μ grid:

$$P^{ext}_{\mu,\mu'} = \begin{cases} P_{\mu,\mu'} & \text{if both } \mu \text{ and } \mu' \text{ are known,} \\ 0 & \text{if } \mu \text{ is known but } \mu' \text{ is unknown,} \\ (1-\zeta)S(\mu') & \text{if } \mu \text{ is unknown but } \mu' \text{ is known,} \\ \zeta & \text{if both } \mu \text{ and } \mu' \text{ are unknown.} \end{cases} \tag{2}$$

Similarly, let $S^{ext}(\mu_k)$ be the probability of receiving an outside job offer with match-specific productivity, μ_k, while taking into account that match quality might be unknown.

$$S^{ext}(\mu_k) = \begin{cases} (1-\zeta)S(\mu_k) & \text{if } \mu_k \text{ is known,} \\ \zeta & \text{if } \mu_k \text{ is unknown.} \end{cases}$$

2. Given parameter values $\boldsymbol{\xi}$ and $\boldsymbol{\xi^o}$, target interest rate r, wage rate in efficiency units, $\tilde{\omega}$, and government transfer to households, $\mathbf{T}$, we recursively solve for the value functions of the workers: $W_j(a,z,\mu)$, $U_j(a,z)$, $N_j(a,z)$, $T_j(a,z,\mu)$, $S_j(a,z,\mu)$, $O_j(a,z)$, and $F_j(a,z)$. Starting from age $j = 948$, when the continuation value is equal to 0, we solve for the consumption/saving decision of age $j = 948$ workers and calculate the value function as described in section 3.2. We iterate this process until we reach age $j = 1$. We linearly interpolate the continuation value and solve for the optimal saving decision using the golden-section method.

3. Using the value functions from step 2, we generate the decision rules for labor force participation, accepting/rejecting an offer, and switching jobs as described in section 3.5.

4. Starting from age $j = 1$, we simulate the model using the decision rules from 3. We assume all the workers are born at the leisure island with no assets and no wage offer at hand.

5. After observing gross worker flows between three labor market states and job-to-job flows, we calculate the transition rates as described in section 4.

6. We solve for a stationary equilibrium in which the interest rate and wage rate are constant and the distribution of agents over the state space is stationary. Hence, given the model parameters, interest rate, and wage rate, total capital supply in the economy is equal to $K = \int a_i di$ and total labor supply in efficiency units is equal to $\int e(i) z_i \mu_i g_i di = L$, where i represents a worker and $e(i)$ is the employment status of the workers with $e(i) = 1$ if i is employed and 0 otherwise. The integration is over all the workers (both in the work island and leisure island) in the model.

7. The next step is to ensure general equilibrium market-clearing conditions. Given the parameters the firm faces, $\boldsymbol{\xi^f} = \{A, \theta, \delta\}$, we solve for the firm's problem and also ensure the government budget is balanced. We repeat steps 1 to 6 until the following conditions hold:

   i) Capital market clears: $\theta A (K/L)^{\theta-1} = r + \delta$

   ii) Labor market clears: $(1-\theta) A (K/L)^{\theta} = \tilde{\omega}$

   iii) Government budget constraint holds: $\mathbf{T} = \tau \int e(i) \omega_{j(i)}(i)(\mu(i), z(i)) di + \int a(i)(1 - s(i)) di$,

B.1 Calibration

To calibrate the model, we use the following algorithm. In calibration of many parameters of the model, we do not need to solve for the general equilibrium. Therefore, we first calibrate the model parameters using partial equilibrium, and then we calibrate the remaining parameters to ensure market-clearing conditions in the general equilibrium hold.

1. Set $\boldsymbol{\xi^o}$ to their respected values as described in section 4.

2. Interest rate, r, is targeted to be equal to 0.043, and $\tilde{\omega}$ is normalized to 1. The one-time normalization of $\tilde{\omega}$ is achieved by setting A such that the marginal product of labor is equal to 1, which is explained in step 6b.

3. For an initial $\boldsymbol{\xi}$ and $\mathbf{T}$, we solve and simulate the model as described above and calculate the gross worker flows from the simulated data. We also calculate average wage for each age and normalize average wage at (annual) age 42 to 1. Hence, average wages in both data and simulated data are expressed relative to average wage at age 42.

4. We calculate the sum of the log distance between the gross worker flows and normalized wage from data and from simulated data. In this step, we also calculate the government budget balance and add the log distance between revenues and expenditures to the sum of the log of the distance between the model and data moments.

5. We repeat steps 3 and 4 for different $\boldsymbol{\xi}$ and $\mathbf{T}$ values until the distance is minimized. We use the Powell method from Scipy minimization sub-package as our minimizer.

6. Once the minimization is done in partial equilibrium, we calibrate the parameters the firm faces, $\boldsymbol{\xi^f}$, in order to ensure the general equilibrium conditions hold:

   (a) θ is set to be equal to 0.3.

   (b) We calculate the aggregate capital stock, K, and aggregate labor in efficiency units, L, from the household side. We chose A such that marginal product of labor is equal to 1, $(1-\theta)A(K/L)^{\theta} = 1$.

   (c) Given K/L from the household size, we choose δ such that marginal product of capital minus depreciation is equal to the interest rate: $\theta A(K/L)^{\theta-1} - \delta = r$.