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EC569 Economic Growth
Problem Set #1
Due Date: Monday, January 28th, 2019
Quiz open: Monday, January 28th 16:00 – Wednesday, January 30th 16:00
Note: Once you begin your quiz, you will have 2 hours to complete the quiz. You can take the quiz only once.

1. Consider the Solow model. Suppose the production function is $Y = AK^{\alpha}(hL)^{1-\alpha}$.

a. Derive change in capital per worker, $\dot{k}$, as a function of capital per worker, k, and other exogenous variables in the model.

b. Suppose that investment rate is 20%, $\gamma = .2$, depreciation rate is 5%, $\delta = .05$, population growth rate is 1%, $n = .01$, $A = 1$, $h = 1$, and capital income share is .33, $\alpha = .33$. Find steady state capital per worker, income per worker, consumption per worker.

In part c. and d. you will simulate the model, i.e. starting from an initial capital per worker level, k_0, you will calculate capital per worker levels for the subsequent 100 periods. If you know capital per worker at time t, k_t, you can find change in capital per worker, $\dot{k}_t$, using the formula you derived in part a. Then you can approximate capital per worker at time $t+1$ as $k_{t+1} \approx k_t + \dot{k}_t$.[1]

c. Suppose that initial capital stock per worker is equal to 5.8, $k_0 = 5.8$. Simulate the economy for 100 periods, and calculate capital per worker, output per worker, consumption per worker, change in capital per worker for each time period. Plot your results. Does the economy converge to the steady state? Use the excel template posted on Moodle.

d. Repeat part c with initial capital stock per worker equal to 6.2, $k_0 = 6.2$. Compare your results with part c. Use the excel template posted on Moodle.

2. Thanos is in a hunt to collect all six Infinity Stones in order to wipe out half of the humanity.[2] Avengers, while fighting hard to prevent Thanos from collecting the Infinity Stones, want to know the economic implications of a potential halving of human population on Earth. Using the Solow model, starting from the steady state you found in question 1.b, calculate the capital per worker, output per worker, consumption per worker for the 100 years after the potential wipeout. Use parameter values of question 1. Plot your results [3].

[1] This formula is not exact as $\dot{k}$ is the instantaneous change in k, not change over a time period. However, the above approximation is good enough for us.

[2] Check out Avengers: Infinity War

[3] Essentially, this is an exercise similar to 1.c and 1.d. All you need to do is to figure out what happens at the initial period, and simulate the model for 100 years like you did in 1.c and 1.d.

Briefly comment. Considering the assumptions of the Solow model, do you think the potential impacts of such wipeout would be more severe? If so, how? Retaining the simplicity of the Solow model, how can you reflect such destruction in the Solow model?

3. Go to the following website:
https://www.rug.nl/ggdc/historicaldevelopment/maddison/releases/maddison-project-database-2018

(If necessary, copy the link above and paste in your browser.)
Download Maddison Project Database 2018 either in Excel or Stata format (you are free to use whichever software you are comfortable with). Pick the United Kingdom, Germany, Senegal, Indonesia, Mexico, Portugal, China, and Singapore.

a. Plot real income per capita of the countries over time from 1900 onwards.[4] The variable you need to plot is cgdppc[5]. Briefly comment on the graph.

b. Fill in Table 1. In case data is missing for some countries for a particular year in the table, you can leave that cell empty.

Table 1: Real GDP per capita of countries relative to UK GDP per capita

Country	Income relative to the UK				
	1900	1950	1980	2000	2010
Senegal					
Indonesia					
Mexico					
Portugal					
Germany					
Singapore					
China					

[4]Some countries may have missing data. In that case, plot only available years.
[5]Real GDP per capita in 2011US$, multiple benchmarks (suitable for cross-country income comparisons)

Extra Questions

Note: The following questions will NOT be on the quiz and you don't have to answer them. But, in case you want to learn more about data and enhance your knowledge of the Solow model, please give your best effort to do them.

4. This question is a continuation of question 3. Now, calculate the annual growth rate of these countries using variable rgdpnapc [6]. For each country, you need to calculate a growth rate at each year. Plot growth rates over time. Briefly comment on the graph. Fill in Table 2.

Table 2: GDP per worker growth rate

Country	Average Growth Rate			Std. deviation of Growth Rates		
	1918 - 1939	1945 - 1980	1980 - 2007	1918 - 1939	1945 - 1980	1980 - 2007
Senegal						
Indonesia						
Mexico						
Portugal						
Germany						
Singapore						

5. Exercise #1 from Chapter 2 of Jones and Vollrath (2013): *A decrease in the investment rate.* Suppose the U.S. congress enacts legislation that discourages saving and investment, such as the elimination of the investment tax credit that occurred in 1990. As a result, suppose the investment rate falls permanently from γ to γ'. Examine the policy change in the Solow model, assuming the economy begins in steady state. Sketch a graph of how (the natural log of) output per worker evolves over time with and without the policy change. Does the policy change permanently reduce the level or the growth rate of output per worker?

6. Exercise #5 from Chapter 2 of Jones and Vollrath (2013): *Can we save too much?* Consumption is equal to output minus investment: $c = (1-\gamma)y$. In the context of the Solow model, what is the savings rate that maximizes steady-state consumption per worker? What is the marginal product of capital in this steady state? Show this point in a Solow diagram. Be sure to draw the production function on the diagram, and show consumption and saving and a line indicating the marginal product of capital. Can we save too much?

[6] Real GDP per capita in 2011US$, 2011 benchmark (suitable for cross-country growth comparisons)