EC569 Economic Growth
Physical Capital
Lecture 2

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Physical Capital

Capital: Tools – the physical objects that extend our ability or do work for us:

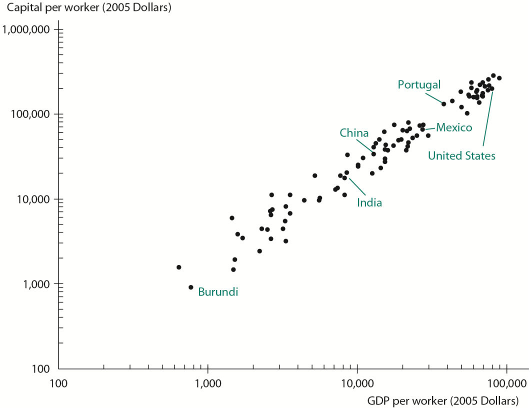
- machines
- buildings
- infrastructure
- vehicles
- computers

Key Characteristics of Capital

- Productive
- It is produced (not like piece of land) (investment)
- Its use is limited: limited number of people can use a given piece of capital at one time (not like ideas)
- It can earn a return (incentive for its creation)
- It wears out (depreciation)

Capital vs GDP

More capital per worker, more output per worker



Source: Calculations based on Heston et al. (2010).

Source: Weil (2013)
[US is richer than Mexico or India because it has more capital]?

The Solow model

- Build around two equations:
  - Production function
    - Constant returns to scale production function
  - Capital accumulation equation
    - Household invest a constant share of income in capital stock

Note: There are many varieties of the Solow model.

Production Function

$Y = AF(K, hL)$, where Y is output, K is physical capital, L is labor, A is productivity, and h is human capital.
Assume:

- $F(\cdot)$ has **constant returns to scale:**
  $F(zK, zhL) = zF(K, hL)$, where z is a constant.

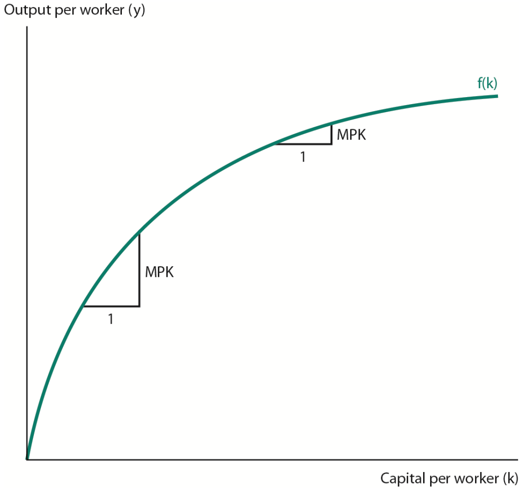
  $$\left(\frac{1}{L}\right) Y = \left(\frac{1}{L}\right) F(K, hL) = F\left(\frac{K}{L}, \frac{hL}{L}\right) = F\left(\frac{K}{L}, h\right)$$

  Define $k \equiv K/L$, capital per labor, $y \equiv Y/L$, output per labor

  $$y = F(k, h) \equiv f(k)$$

- $F(\cdot)$ displays **diminishing marginal product:**
  MPK $= f(k+1) - f(k) \downarrow$ as $k \uparrow$

Production function, cont'd



Source: Weil (2013)

Cobb-Douglas Production Function

$$Y = AK^{\alpha}(hL)^{1-\alpha},$$

where A is productivity, K is capital, h is human capital, and L is labor, $0 < \alpha < 1$.

$$\frac{Y}{L} = \frac{AK^{\alpha}(hL)^{1-\alpha}}{L} = A\frac{K^{\alpha}}{L^{\alpha}}\frac{(hL)^{1-\alpha}}{L^{1-\alpha}} = A\left(\frac{K}{L}\right)^{\alpha} h^{1-\alpha}$$

$$y = Ak^{\alpha}h^{1-\alpha}$$

- Marginal product of physical capital is positive

$$\text{mpk} = \frac{\partial y}{\partial k} = \alpha Ak^{\alpha-1}h^{1-\alpha} > 0$$

- Diminishing marginal product of capital

$$\frac{\partial^2 y}{\partial k^2} = (\alpha - 1)\alpha Ak^{\alpha-2}h^{1-\alpha} < 0$$

Capital's share of national income

- Factors of production will be paid their marginal product

$$\begin{aligned} r &= \alpha A K^{\alpha-1}(hL)^{1-\alpha} \\ w &= (1-\alpha) A K^{\alpha}(hL)^{-\alpha} \quad \text{(in efficiency units)} \end{aligned}$$

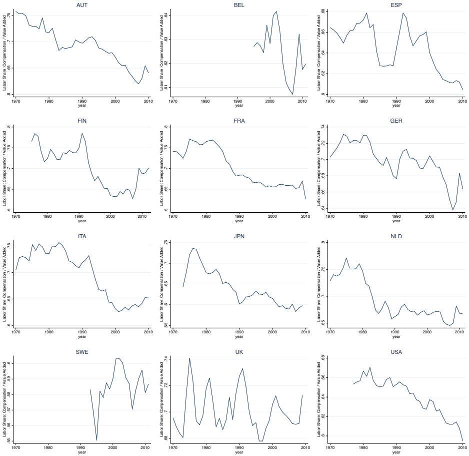
- Capital's share of income=Fraction of national income (Y) that is paid out as rent on capital

$$\text{Capital's share of income} = \frac{MPK \times K}{Y} = \frac{\alpha A K^{\alpha-1}(hL)^{1-\alpha} K}{Y} = \alpha$$

$$\text{Labor's share of income} = \frac{MPL \times hL}{Y} = \frac{(1-\alpha) A K^{\alpha}(hL)^{-\alpha} hL}{Y} = 1-\alpha$$

- Capital's share of income is generally estimated as 1/3.

Figure 1: International Comparison: Labor Share by Country



Notes: Each panel plots the ratio of aggregate compensation over value-added for all industries in a country based on KLEMS data.

Source: Autor, D., Dorn, D., Katz, L. F., Patterson, C., & Van Reenen, J. (2017). The Fall of the Labor Share and the Rise of Superstar Firms (No. dp1482). Centre for Economic Performance, LSE.

Solow model, production function, assumptions, cnt'd

- Productivity, A, is constant. Will be relaxed later
- Human capital, h, is constant. Will be relaxed later
- Labor force participation rate is constant.
- Population grows at a rate n, and so do labor force, L.

$$L(t) = L_0 e^{nt}$$

- Trick: to find the growth rate of a variable, take log and differentiate w.r.t. time, t.

$$\ln L = \ln L_0 + nt, \qquad \frac{\dot{L}}{L} = n.$$

- E.g. population grows are a rate, $n = .01$ or 1% per year.
- $\dot{L} \approx L(t+1) - L(t)$

Accumulation of physical capital

- Investment: a constant fraction of output is invested
- Consumption: output minus investment.
- Depreciation: a constant fraction of capital wears out
- Change in physical capital stock = Investment - Depreciation

Accumulation of physical capital, cont'd

- Change in capital stock = Investment - Depreciation

$$\dot{K} = I - D$$

- A constant fraction, γ, of output is invested

$$I = \gamma Y$$

- A constant fraction, δ, of capital depreciates

$$D = \delta K$$

-

$$\dot{K} = \gamma Y - \delta K$$

Accumulation of capital per worker

- Goal: write physical capital accumulation equation with per worker variables
$$\dot{K} = \gamma Y - \delta K$$
- Divide each side by K:
$$\frac{\dot{K}}{K} = \frac{\gamma Y}{K} - \delta$$
- take log and differentiate $k \equiv K/L$:
$$\frac{\dot{k}}{k} = \frac{\dot{K}}{K} - \frac{\dot{L}}{L}, \qquad \frac{\dot{k}}{k} = \frac{\dot{K}}{K} - n$$
- Substitute above and get
$$\frac{\dot{k}}{k} = \frac{\gamma Y/L}{K/L} - \delta - n \quad \Rightarrow \quad \dot{k} = \gamma y - (\delta + n)k$$

Accumulation of capital per worker, cont'd

Change in capital stock per worker =

$$\dot{k} = \gamma y - (\delta + n)k$$

$$\dot{k} = \gamma f(k) - (\delta + n)k$$

Example: $f(k) = Ak^{\alpha}h^{1-\alpha}$, $\alpha = 1/3$, $A = 1$, $k_0 = 1$, $h = 1$, $n = .01$, $\gamma = .2$, and $\delta = .05$. Calculate $\dot{k}$, k_1, y_1.

$$\dot{k} = \gamma f(k) - (\delta + n)k = .2 \times 1^{1/3} - (.05 + .01) \times 1 = .2 - .06 = .14$$

$$k_1 \approx k_0 + \dot{k} = 1 + .14 = 1.14$$

$$y_1 = 1.14^{1/3} \approx 1.04$$

What about k_2 and y_2?

Consumption

- Income-expenditure identity in a closed economy (without the government)
$$y = c + i,$$
where c is consumption per worker.
- $c = (1 - \gamma)y$
- investment-savings equality
$$\begin{aligned} s &= y - c \\ s &= i = \gamma y, \end{aligned}$$
where s is savings per worker. Then γ is also saving rate.

Steady State

Capital per worker and output per worker are constant at the steady state.

- if $\gamma f(k) > (\delta + n)k$, then $\dot{k} > 0$: capital stock growing
- if $\gamma f(k) < (\delta + n)k$, then $\dot{k} < 0$: capital stock shrinking
- if $\gamma f(k) = (\delta + n)k$, then $\dot{k} = 0$: capital stock consant **(steady state)**

The Solow Diagram

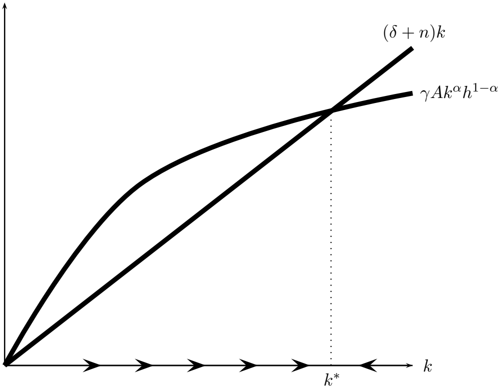


Diagram from Jones and Vollrath (2013)

If $k < k^*$, capital per worker and output per worker will increase.
If $k > k^*$, capital per worker and output per worker will decrease.

Steady State, cont'd

Cobb-Douglas production function

$$y = Ak^{\alpha}h^{1-\alpha}$$

No change of capital stock per worker at the steady state

$$\dot{k} = \gamma A k^{\alpha} h^{1-\alpha} - (\delta + n)k$$

$$0 = \gamma A (k^*)^{\alpha} h^{1-\alpha} - (\delta + n)k^*$$

$$\gamma A (k^*)^{\alpha} h^{1-\alpha} = (\delta + n)k^*$$

$$k^* = h\left(\frac{\gamma A}{\delta + n}\right)^{1/(1-\alpha)}$$

Steady steady output per worker:

$$y^* = A(k^*)^{\alpha}h^{1-\alpha} = A^{1/(1-\alpha)}\left(\frac{\gamma}{\delta + n}\right)^{\alpha/(1-\alpha)} h$$

Comparative Statics

$$k^* = h\left(\frac{\gamma A}{\delta + n}\right)^{1/(1-\alpha)}$$

$$y^* = A^{1/(1-\alpha)}\left(\frac{\gamma}{\delta + n}\right)^{\alpha/(1-\alpha)} h$$

- k^* and y^* are rising with investment rate γ, technology A, human capital h,
- k^* and y^* are declining with depreciation rate, δ, and population growth rate, n.

Increasing the investment rate

As $\gamma \uparrow$, $k^* = h\left(\frac{\gamma A}{\delta+n}\right)^{1/(1-\alpha)} \uparrow$, $y^* = A^{1/(1-\alpha)}\left(\frac{\gamma}{\delta+n}\right)^{\alpha/(1-\alpha)} h \uparrow$

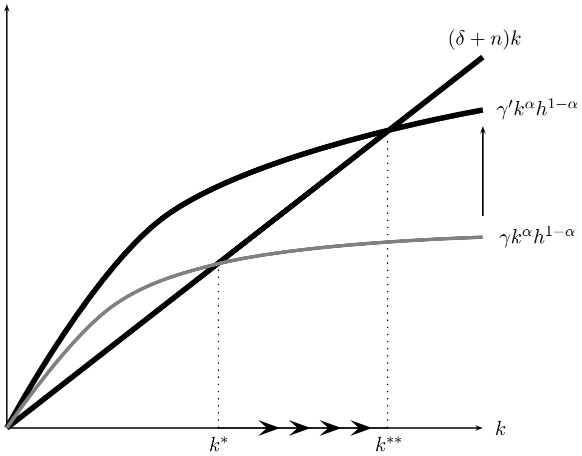


Diagram from Jones and Vollrath (2013)

Income after an increase in investment rate

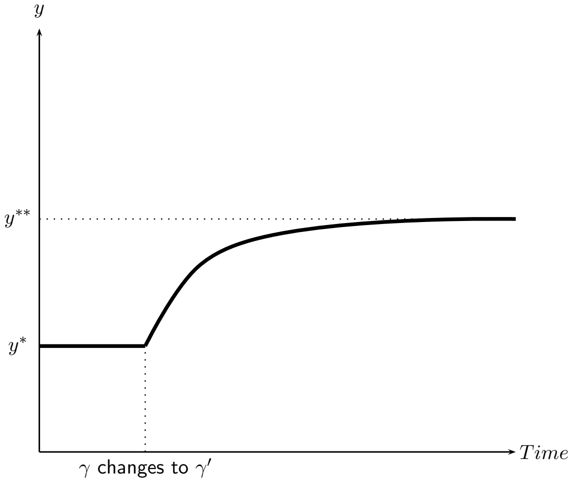


Diagram from Jones and Vollrath (2013)

Income Differences

Consider 2 countries, i and j, that differ only in investment rates, γ_i and γ_j.

$$y_i^* = A^{1/(1-\alpha)} \left(\frac{\gamma_i}{\delta + n} \right)^{\alpha/(1-\alpha)} h$$

$$y_j^* = A^{1/(1-\alpha)} \left(\frac{\gamma_j}{\delta + n} \right)^{\alpha/(1-\alpha)} h$$

$$\frac{y_i^*}{y_j^*} = \left(\frac{\gamma_i}{\gamma_j} \right)^{\alpha/(1-\alpha)}$$

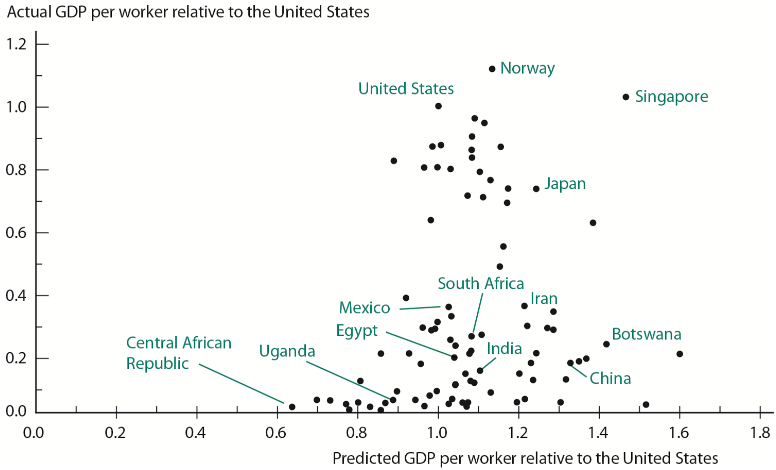
Quantitative predictions:

- Suppose country i has an investment rate of 20%
- country j has an investment rate of 5%
- $\alpha = 1/3$, hence $\alpha/(1-\alpha) = 1/2$

$$\frac{y_i^*}{y_j^*} = \left(\frac{.2}{.05} \right)^{1/2} = 4^{1/2} = 2$$

Income per capita in country i would be twice the level of country j

Predicted versus Actual GDP per Worker



Source: Author's calculations using data from Heston, Summers, and Aten (2011).

Source: Weil(2013)

Predicted vs Actual GDP per Worker

- Correlation of actual and predicted values is .17
- China and Botswana are predicted to have higher GDP per worker than the US.
- CAR GDP per worker predicted to be 63% of US, but only 1.9% in actual data
- Later chapters: quantity of capital is determined by other factors as well
  - rate of population growth (Chapter 4)
  - other factors of production in addition to physical capital (Chapter 6)
  - differences in productivity (Chapter 7)
- Countries may not be in their steady state.

Growth Rates

The Solow model predicts that growth is faster, the farther away from steady state is an economy. Look at the growth rate of k

$$\frac{\dot{k}}{k} = \frac{\gamma A h^{1-\alpha}}{k^{1-\alpha}} - (\delta + n).$$

As k rises, the growth rate of k falls.

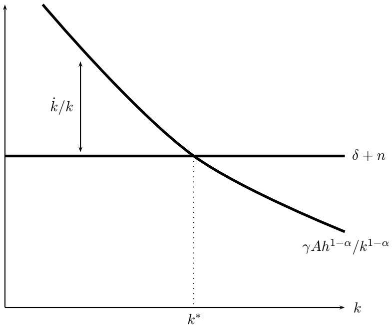


Diagram from Jones and Vollrath (2013)

Growth Rates, cont'd

In the Solow model (current version)

- No long-run growth of output per worker:

$$y^* = A(k^*)^\alpha h^{1-\alpha}, \quad \frac{\dot{y}^*}{y^*} = \alpha \frac{\dot{k}^*}{k^*} = 0$$

- Transitional growth
  - Higher growth rate for lower capital stock countries
  - **Convergence toward the steady state:** A country's per-worker output will grow or shrink from some initial position toward the steady-state level determined by the investment rate.

Economics of Solow model

- What is the main source of economic growth in the Solow model?

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  - Diminishing marginal product of physical capital.
- What is the growth rate of output at the steady state?
  - Population growth rate, n

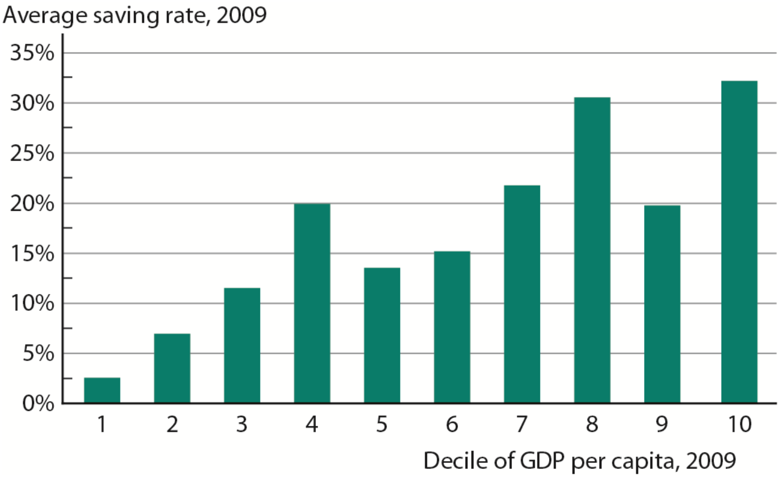
Relative Growth Rates, Predictions

- If two countries have the same rate of investment but different levels of income, the country with lower income will have higher growth.
- If two countries have the same level of income but different rates of investment, then the country with a higher rate of investment will have higher growth.
- A country that raises its level of investment will experience an increase in its rate of income growth.

Saving rate vs Investment Rate

- Why do investment rates differ across countries?
- Every act of investment corresponds to an act of saving

Saving Rate by Decile of Income per Capita

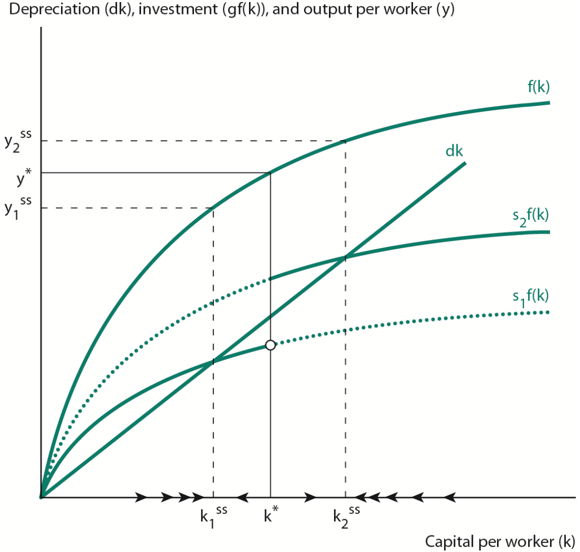


Source: Weil (2013)

What determines saving rates?

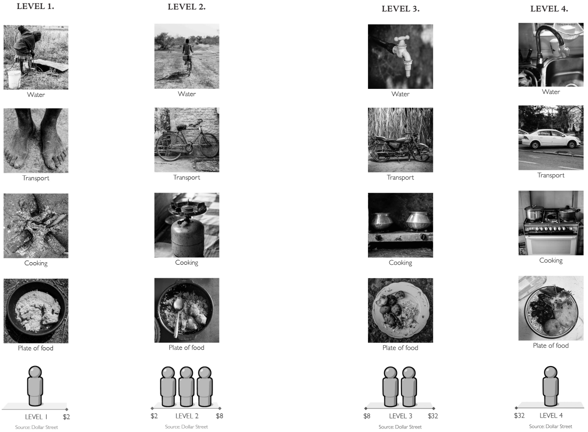
- Exogenous (determined outside of the model): government policy, income inequality, culture, geography
- Endogenous (determined within the model): countries that are rich save more
- Poor countries living on the verge of subsistence

Saving Dependent on Income Level



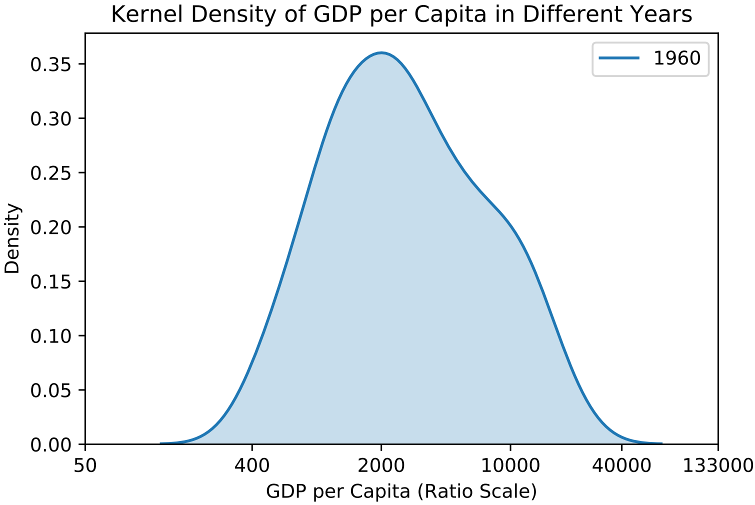
Source: Weil (2013)

Thank you!



Source: Rosling, H., Rosling, O., & Rönnlund, A. R. (2018). Factfulness: Ten Reasons We're Wrong about the World–and why Things are Better Than You Think. St Martin's Press.

Income distrubution over time



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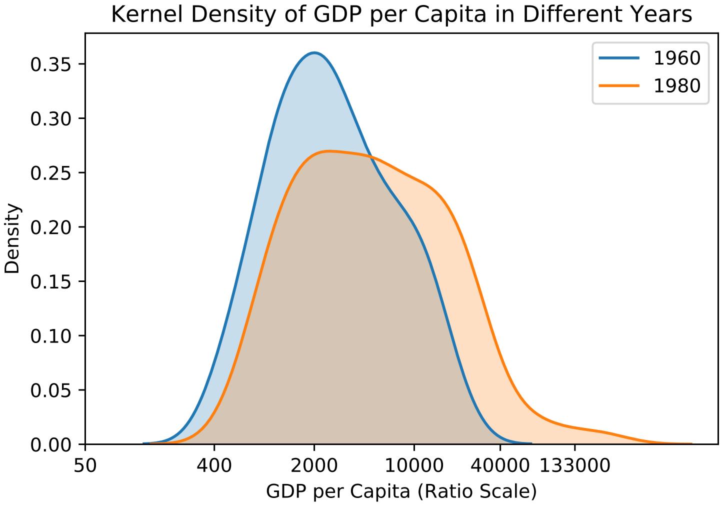

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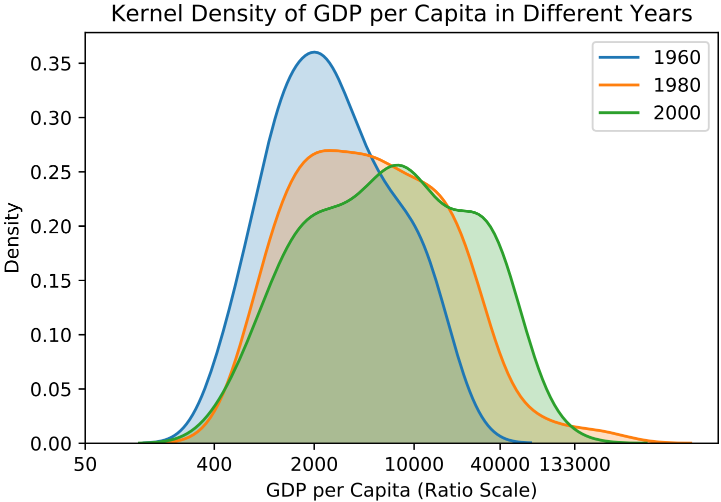

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