The Solow Model

Lecture 2

The Solow Model

$Y = F(K, ehL)$ → production function exhibit constant returns to scale.

$zY = F(zK, zehL) \Rightarrow F$ is CRS

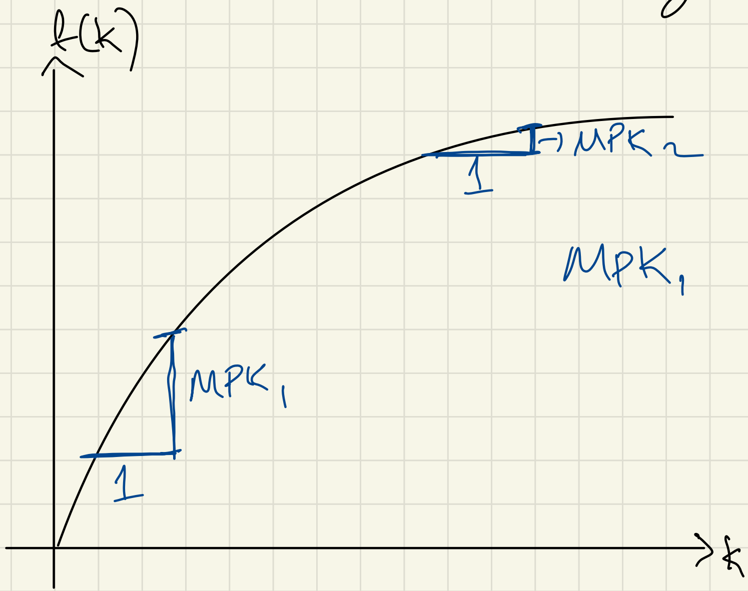
$$\frac{1}{L} Y = \frac{1}{L} F(K, ehL) \underset{\text{CRS}}{=} F\left(\frac{K}{L}, \frac{eh\cancel{L}}{\cancel{L}}\right) = F\left(\frac{K}{L}, eh\right)$$

$k \equiv \frac{K}{L}$, capital per worker, $y \equiv \frac{Y}{L}$, output per worker

$y = F(k, eh) \equiv f(k)$

$k(t)$, $y(t)$ Drop t index for not cluttering the notes

$f(k)$ exhibits diminishing marginal product of capital



$MPK_1 > MPK_2$

Change in capital stock = Investment − Depreciation

$$\dot{K} = I - D \qquad \dot{K} = \frac{dK(t)}{dt}$$

$$\dot{K} = \underline{\gamma Y} - \underline{\delta K}$$

Assumption: a constant fraction of capital depreciates

Assumption: a constant fraction of output is invested into capital

$$\frac{\dot{K}}{K} = \frac{\gamma Y - \delta K}{K} = \frac{\gamma Y}{K} - \frac{\delta \cancel{K}}{\cancel{K}} \Rightarrow \boxed{\frac{\dot{K}}{K} = \frac{\gamma Y}{K} - \delta} \quad 1)$$

$k \equiv \frac{K}{L}$, Goal express $\frac{\dot{k}}{k}$ as a function of $\frac{\dot{K}}{K}$

$$\ln k^{(t)} = \ln(K^{t}/L_{t}) = \ln K^{(t)} - \ln L^{(t)}$$

Chain rule

$$\frac{d\, f(g(x))}{dx} = \frac{df}{dg} \times \frac{dg}{dx}$$

Assumption: L grows at a rate n.

Differentiate w.r.t. t

$$\frac{d \ln k(t)}{dt} = \frac{d\ln k}{dk}\frac{dk}{dt} = \frac{1}{k}\frac{dk(t)}{dt} = \frac{\dot{k}}{k}$$

2) $$\boxed{\frac{\dot{k}}{k} = \frac{\dot{K}}{K} - \frac{\dot{L}}{L} = \frac{\dot{K}}{K} - n}$$

$$\frac{\dot{K}}{K} = \frac{\gamma Y}{K} - \delta$$

$$\frac{\dot{k}}{k} = \frac{\dot{K}}{K} - n \Rightarrow \frac{\dot{K}}{K} = \underbrace{\frac{\dot{k}}{k} + n}$$

$$\frac{M}{N} = \frac{M/c}{N/c}$$

$$\frac{\dot{k}}{k} + n = \frac{\gamma Y}{K} - \delta \Rightarrow \frac{\dot{k}}{k} = \frac{\gamma Y}{K} - \delta - n = \frac{\gamma Y/L}{K/L} - (\delta + n) = \frac{\dot{k}}{k}$$

$$= k\left(\frac{\gamma y}{k} - (\delta+n)\right) = \left(\frac{\dot{k}}{k}\right) k$$

$$\boxed{\text{Law of motion for capital per worker} \quad \dot{k} = \gamma y - (\delta+n)k}$$

$$\Leftarrow \frac{\gamma y \cancel{k}}{\cancel{k}} - (\delta+n)k = \frac{\dot{k}}{\cancel{k}} \cancel{k}$$

Change in physical capital per worker = Investment per worker − depreciation (due to) − population growth (due to)