EC569 Economic Growth
Extended Solow Model
Lecture 5

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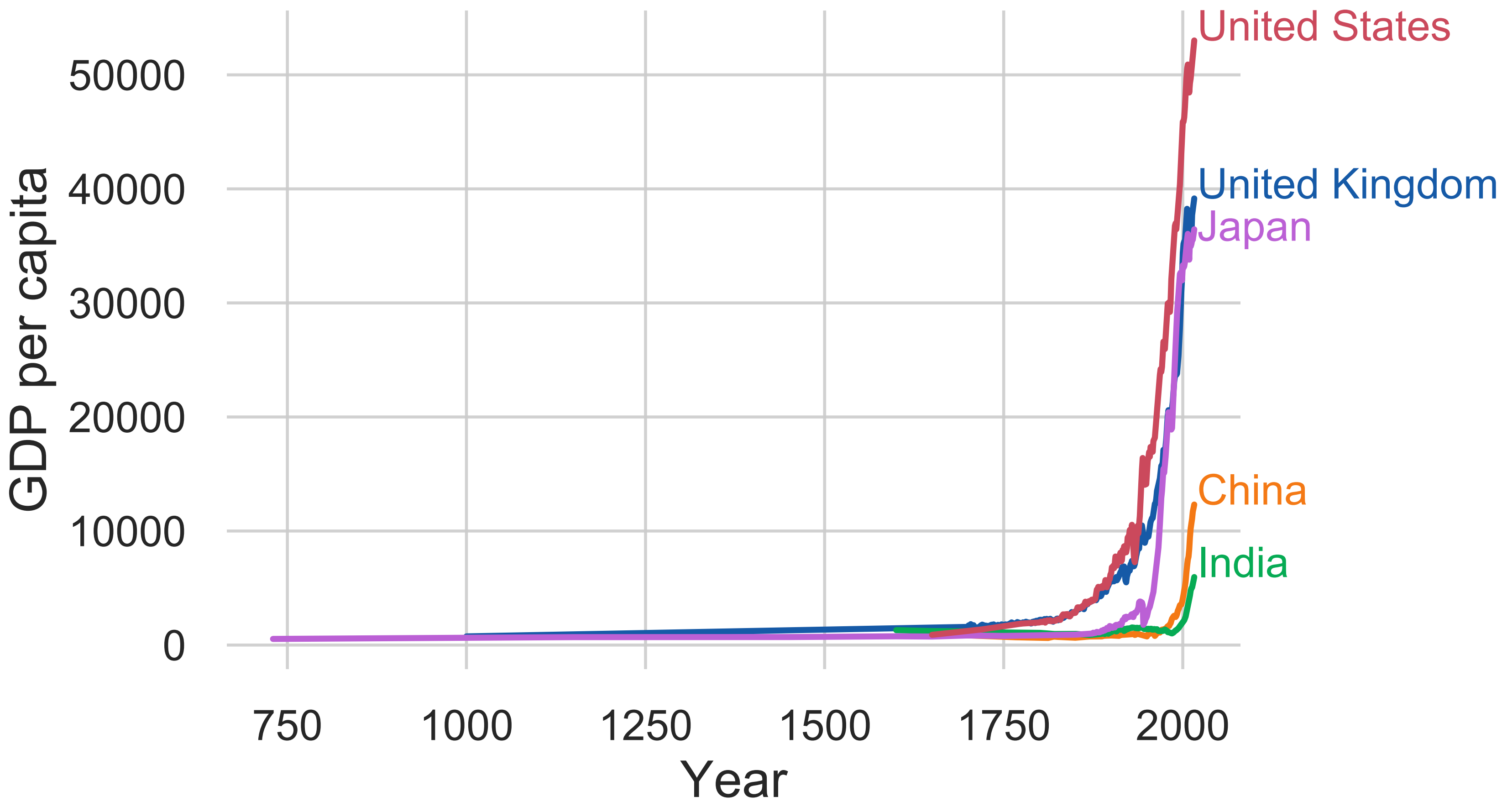
Questions

- What's the role of tehnological progress on economic growth?
- Do relative low income countries grow faster than high income countries?

What's the long-run growth rate of income per capita in the Solow Model?

0

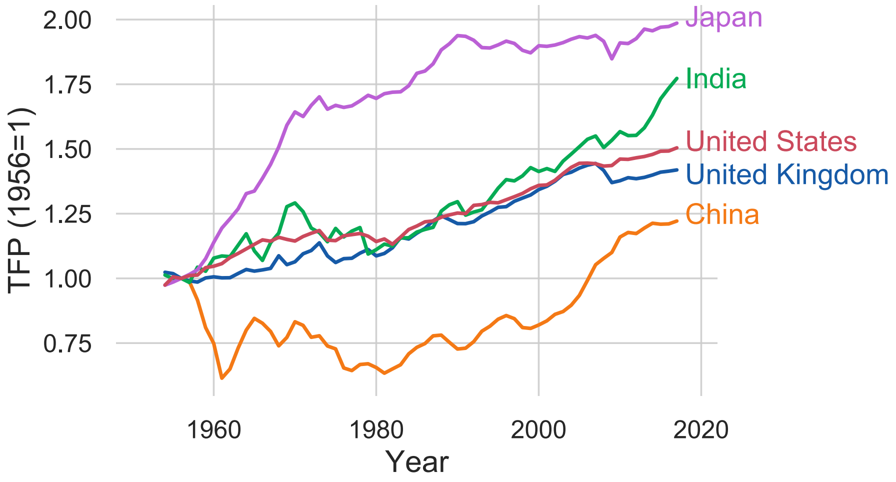
Economies do grow over time



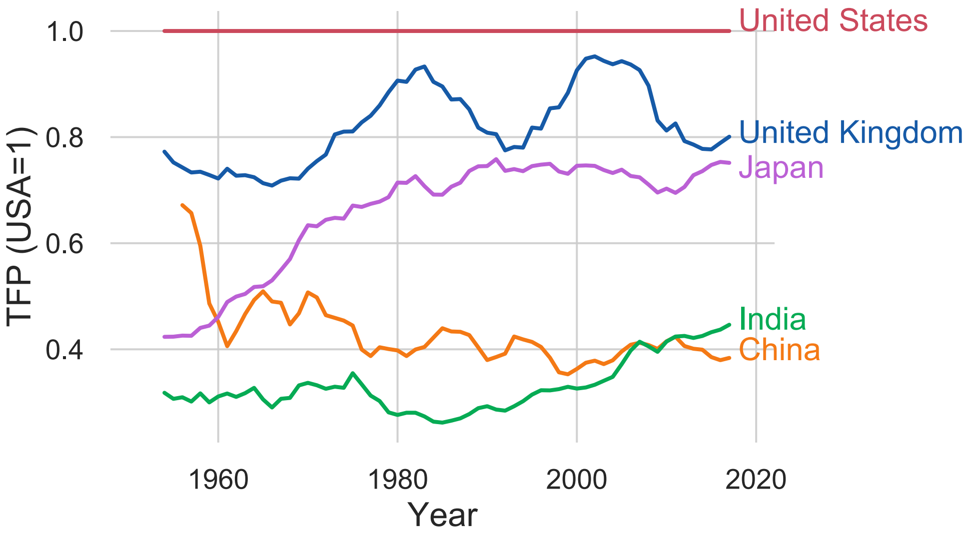
Source: Maddison Project Database (MPD) 2018

Countries get more productive

- TFP stands for total factor productivity
- TFP = total real output / total inputs into production



- It does not mean that India is more productive than the US
- It means the TFP of India has grown more than the US TFP since 1954



Data source: Penn World Tables, version 9.1

Extended Solow Model

- Retain assumptions:
  - Constant returns to scale production function: $F(K, ehL)$
  - A constant fraction, γ, of output is invested.
  - A constant fraction, δ, of physical capital stock depreciates.
  - Labor force participation rate is constant
  - Population grows at a constant rate, n.
  - Human capital, h, is constant.
- Differently, assume
  - There is labor augmenting productivity growth, $F(K, \color{red}{e}hL)$
  - Labor augmenting productivity, e, grows at a constant rate, g.

$$\frac{\dot{e}}{e} = g \Leftrightarrow e(t) = e_0 e^{gt}$$

Clarification...

- Previously, we assumed production function is $AF(K, hL)$
  - Here, A is Hicks-neutral technology
- In the Extended Solow model, technology needs to be labor-augmenting: $F(K, ehL)$
  - Here, e is labor-augmenting or Harrod-neutral technology
- In Cobb-Douglas production, this distinction is not important.
  - $K^{\alpha}(ehL)^{1-\alpha} = AK^{\alpha}(hL)^{1-\alpha}$, where $A \equiv e^{1-\alpha}$

Accumulation of physical capital

- Change in capital stock = Investment - Depreciation

$$\dot{K} = I - D$$

- A constant fraction, γ, of output is invested

$$I = \gamma Y$$

- A constant fraction, δ, of capital depreciates

$$D = \delta K$$

- Change in capital stock:

$$\dot{K} = \gamma Y - \delta K$$

Accumulation of capital-technology ratio

- Goal: derive a formula for the accumulation of capital-technology ratio, $\tilde{k} \equiv \frac{K}{eL}$
- Capital-technology ratio, $\tilde{k}$, sometimes referred as capital per effective labor.
- Why do we need to convert physical capital accumulation equation into capital-technology units?
    - Because capital stock will grow as a result of increasing population and higher productivity.

Accumulation of capital-technology ratio, cont'd

- Accumulation of capital

$$\dot{K} = \gamma Y - \delta K$$

- How do we transform $\dot{K}$ into $\dot{\tilde{k}}$?
  - make use of $\tilde{k} \equiv \frac{K}{eL}$
  - take log of $\tilde{k}(t) \equiv \frac{K(t)}{e(t)L(t)}$:

$$\ln(\tilde{k}(t)) = \ln\left(\frac{K(t)}{e(t)L(t)}\right) = \ln(K(t)) - \ln(e(t)) - \ln(L(t))$$

- Then differentiate with respect to time, t,

$$\frac{\dot{\tilde{k}}(t)}{\tilde{k}(t)} = \frac{\dot{K}(t)}{K(t)} - \frac{\dot{e}(t)}{e(t)} - \frac{\dot{L}(t)}{L(t)}, \quad \frac{\dot{\tilde{k}}}{\tilde{k}} = \frac{\dot{K}}{K} - g - n \Rightarrow \frac{\dot{K}}{K} = \frac{\dot{\tilde{k}}}{\tilde{k}} + g + n$$

- Remember chain-rule: $\frac{df(g(x))}{dx} = \frac{df}{dg}\frac{dg}{dx}$

Accumulation of capital-technology ratio, cont'd (2)

- Divide each side of $\dot{K} = \gamma Y - \delta K$ by K:

$$\frac{\dot{K}}{K} = \frac{\gamma Y}{K} - \delta$$

- Then,

$$\frac{\dot{\tilde{k}}}{\tilde{k}} + g + n = \frac{\dot{K}}{K} = \frac{\gamma Y}{K} - \delta$$

$$\frac{\dot{\tilde{k}}}{\tilde{k}} + g + n = \frac{\gamma Y/(eL)}{K/(eL)} - \delta$$

$$\frac{\dot{\tilde{k}}}{\tilde{k}} + g + n = \frac{\gamma \tilde{y}}{\tilde{k}} - \delta, \text{ where } \tilde{y} \equiv \frac{Y}{eL}$$

$$\frac{\dot{\tilde{k}}}{\tilde{k}} = \frac{\gamma \tilde{y}}{\tilde{k}} - (\delta + g + n) \;\;\Rightarrow\;\; \dot{\tilde{k}} = \gamma \tilde{y} - (\delta + g + n)\tilde{k}$$

Steady State

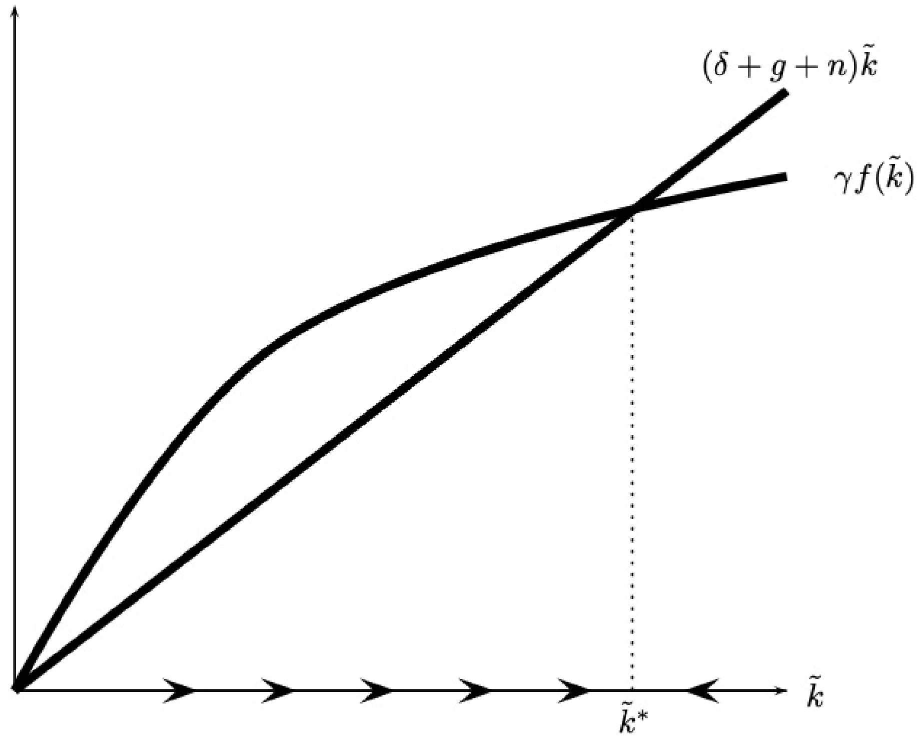
$$\dot{\tilde{k}} = \gamma\tilde{y} - (\delta + g + n)\tilde{k}, \text{ or}$$

$$\dot{\tilde{k}} = \gamma f(\tilde{k}) - (\delta + g + n)\tilde{k}$$

where $\tilde{y} = f(\tilde{k}) = F(K, ehL)/(eL)$

Capital-technology ratio is constant at the steady state.

- if $\gamma f(\tilde{k}) > (\delta + g + n)\tilde{k}$, then $\dot{\tilde{k}} > 0$
- if $\gamma f(\tilde{k}) < (\delta + g + n)\tilde{k}$, then $\dot{\tilde{k}} < 0$
- if $\gamma f(\tilde{k}) = (\delta + g + n)\tilde{k}$, then $\dot{\tilde{k}} = 0$: **steady state**
- If $\tilde{k} < \tilde{k}^*$, capital-technology ratio will increase.



Graphic from Jones and Vollrath (2013)

Steady State, cont'd

- Cobb-Douglas production function

$$Y = K^\alpha (ehL)^{1-\alpha}$$

$$\frac{Y}{eL} = \frac{K^\alpha (ehL)^{1-\alpha}}{(eL)^\alpha (eL)^{1-\alpha}}$$

$$\tilde{y} = \tilde{k}^\alpha h^{1-\alpha}$$

- Law of motion of capital-technology ratio

$$\dot{\tilde{k}} = \gamma \tilde{k}^\alpha h^{1-\alpha} - (\delta + g + n)\tilde{k}$$

- No change of capital-technology ratio at the steady state

$$0 = \gamma (\tilde{k}^*)^\alpha h^{1-\alpha} - (\delta + g + n)\tilde{k}^*$$

$$\Downarrow$$

$$\gamma (\tilde{k}^*)^\alpha h^{1-\alpha} = (\delta + g + n)\tilde{k}^*$$

- Steady state capital-technology ratio
- ◦ (capital-technology ratio = capital per effective labor)

$$\tilde{k}^* = \left(\frac{\gamma}{\delta + g + n}\right)^{1/(1-\alpha)} h$$

- Steady steady output-technology ration:
- ◦ (output-technology ratio = output per effective labor)

$$\tilde{y}^* = (\tilde{k}^*)^\alpha h^{1-\alpha} = \left(\frac{\gamma}{\delta + g + n}\right)^{\alpha/(1-\alpha)} h$$

Per worker values

Output per worker:

$$y(t) = e(t)\tilde{y}(t)$$

- Recall that

$$y(t) = \frac{Y(t)}{L(t)} = e(t)\frac{Y(t)}{e(t)L(t)} = e(t)\tilde{y}(t)$$

Capital per worker:

$$k(t) = e(t)\tilde{k}(t)$$

- Recall that

$$k(t) = \frac{K(t)}{L(t)} = e(t)\frac{K(t)}{e(t)L(t)} = e(t)\tilde{k}(t)$$

Output per worker at the steady state:

$$y(t) = e(t)\tilde{y}^* = e(t)\left(\frac{\gamma}{\delta + g + n}\right)^{\alpha/(1-\alpha)} h$$

- Since $\tilde{y}^*$ is constant at the steady state, y at the steady state grows at the same rate as e: g.

Capital per worker:

$$k(t) = e(t)\tilde{k}^* = e(t)\left(\frac{\gamma}{\delta + g + n}\right)^{1/(1-\alpha)} h$$

- Since $\tilde{k}^*$ is constant at the steady state, k at the steady state grows at the same rate as e: g.

Comparative Statics

Capital-technology ratio at the steady state:

$$\tilde{k}^* = \left(\frac{\gamma}{\delta + g + n} \right)^{1/(1-\alpha)} h$$

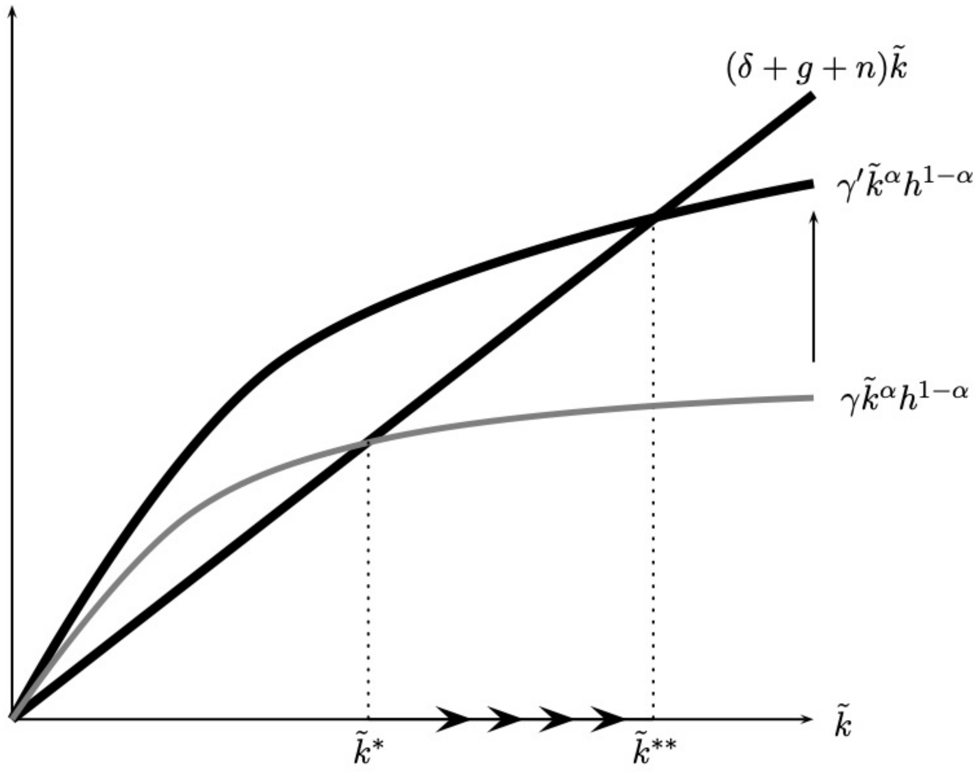
Output-technology ratio at the steady state:

$$\tilde{y}^* = \left(\frac{\gamma}{\delta + g + n} \right)^{\alpha/(1-\alpha)} h$$

- $\tilde{k}^*$ and $\tilde{y}^*$ are rising with investment rate γ, and human capital h,
- $\tilde{k}^*$ and $\tilde{y}^*$ are declining with depreciation rate, δ, population growth rate, n, and rate of technological progress, g.

Increasing the investment rate

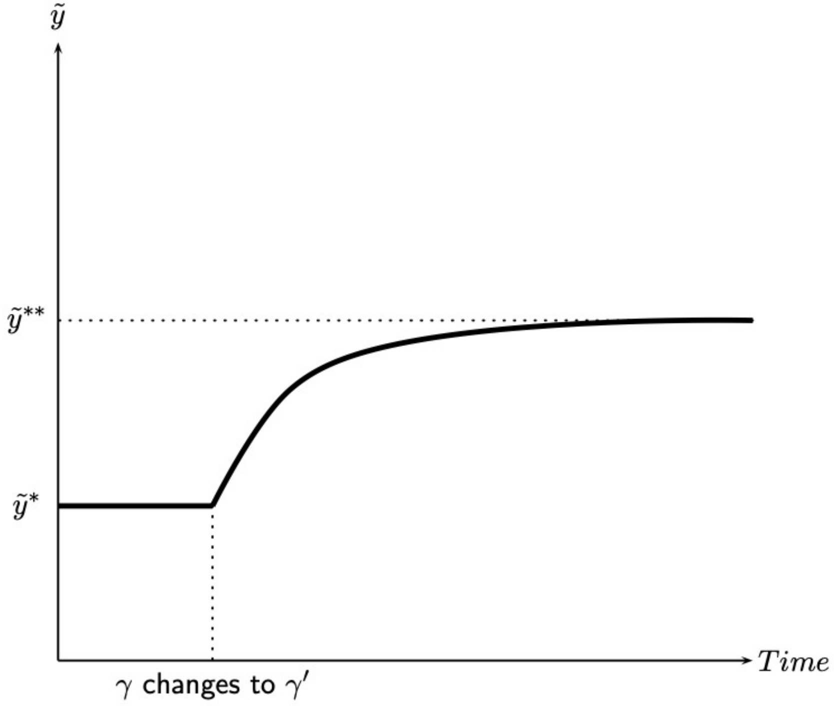
As $\gamma \uparrow$, $\tilde{k}^* = \left(\frac{\gamma}{\delta+g+n}\right)^{1/(1-\alpha)} h \uparrow$, $\tilde{y}^* = \left(\frac{\gamma}{\delta+g+n}\right)^{\alpha/(1-\alpha)} h \uparrow$



Graphic from Jones and Vollrath (2013)

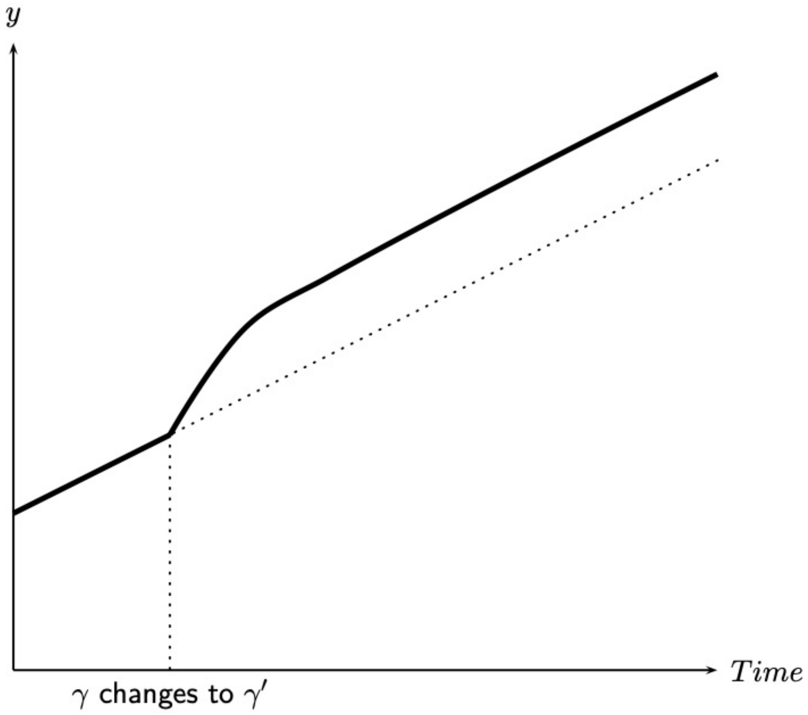
Increasing the investment rate, cont'd

Output-technology ratio over time



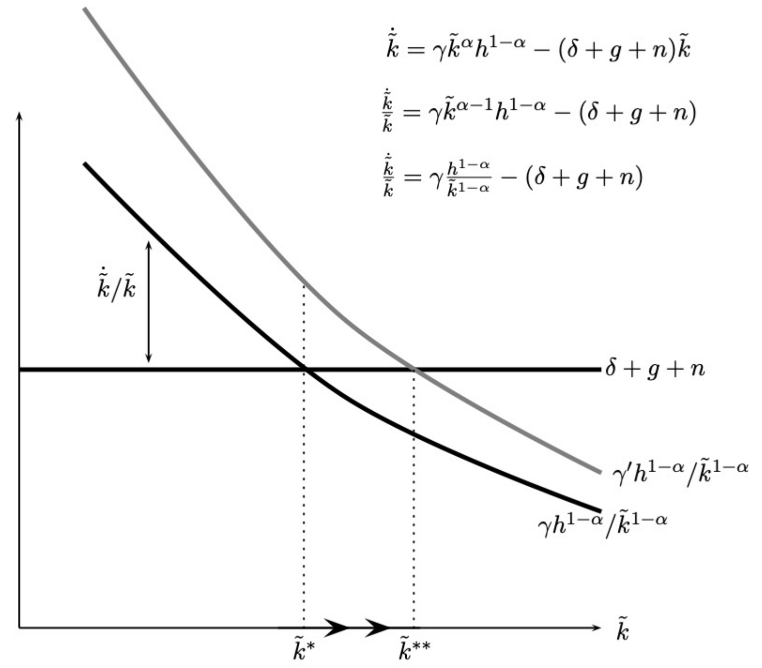
- Notice that $\tilde{k}$ has a similar trajectory as $\tilde{y}$

Output per worker over time

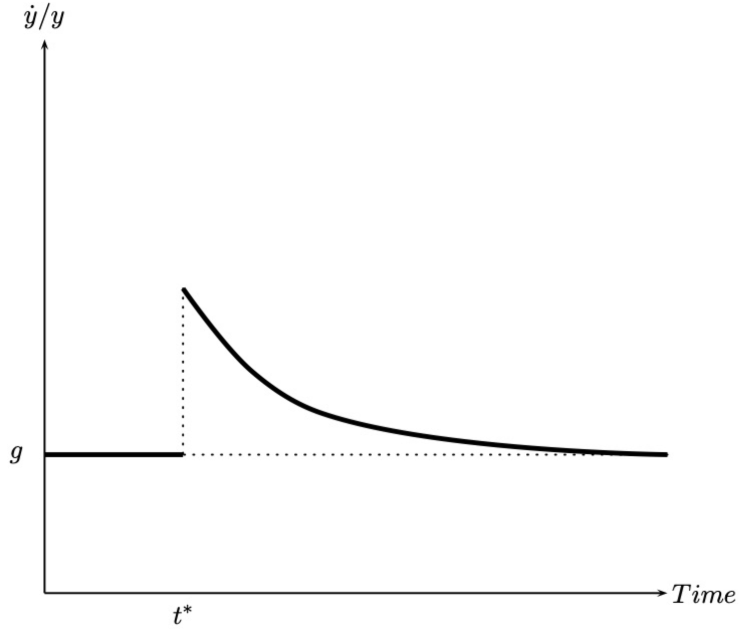


- Notice that k has a similar trajectory as y

Growth rate of capital-technology ratio after an increase in investment rate



- An increase in the investment rate leads to an increase in the growth rate of capital per worker in the short-run.



- Long-run growth rates of output per worker and capital per worker are not affected.

Exercise

Conduct comparative statics for changes in n, δ, g, and h.

Effects of an increase in investment rate

Short-run:

- Growth rates of capital per worker and income per worker increase.
- Capital per worker and income per worker increases.

Long-run (steady-state):

- Capital per worker and income per worker increase.
- Growth rate of capital per worker and income per worker do not change.

Steady state growth rates

Variable	Notation	Definition	Growth rate as the s.s.
Capital-technology ratio	$\tilde{k}$	$\frac{K}{eL}$	?
Output-technology ratio	$\tilde{y}$	$\frac{Y}{eL}$	?
Consumption-technology ratio	$\tilde{c}$	$\frac{C}{eL}$	?
Capital per worker	k	$\frac{K}{L}$	?
Output per worker	y	$\frac{Y}{L}$	?
Consumption per worker	c	$\frac{C}{L}$	?
Capital	K	K	?
Output	Y	Y	?
Consumption	C	C	?

Exogenous Growth Model

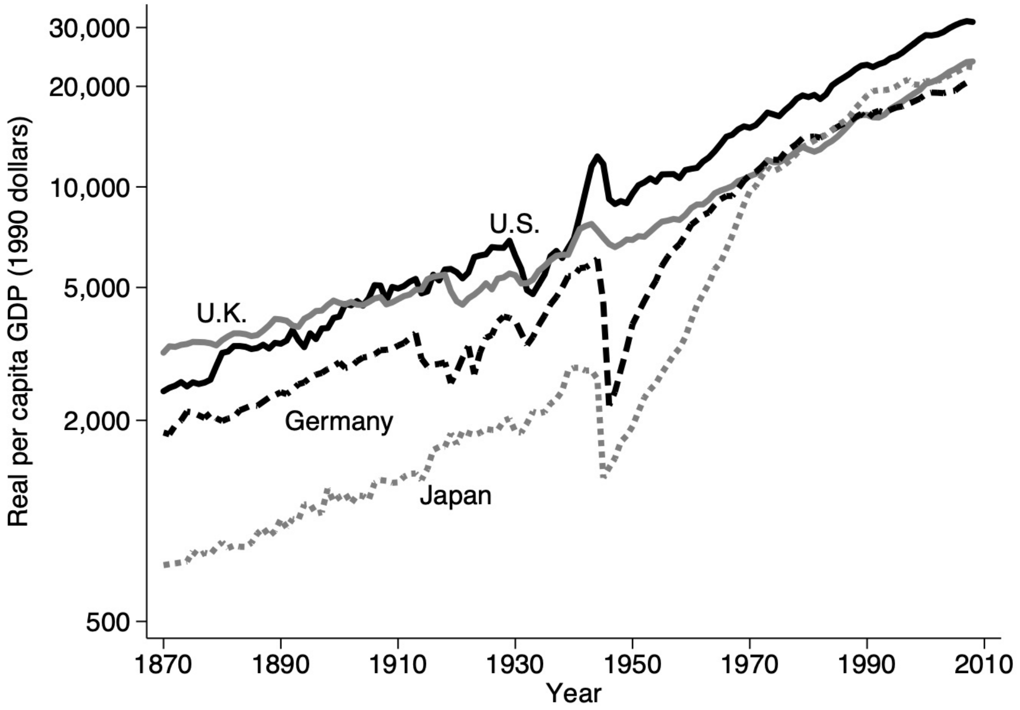
- Technology is the only source of long-run growth.
- Technology is exogenous: not a results of interactions of agents in the model
- Hence, the extended Solow model is an example of *exogenous growth models.*
- It is also referred as *Neo-classical growth model*
- In upcoming lectures, we will analyze *endogenous growth models*
    - Technological progress as a result of actions of model agents.

Convergence

Convergence

- Are *poor* countries growing faster than *rich* countries?
- Are *poor* countries *closing the gap*?
- **Converge:** The phenomenon of *poor* countries catching up with the *rich* countries.

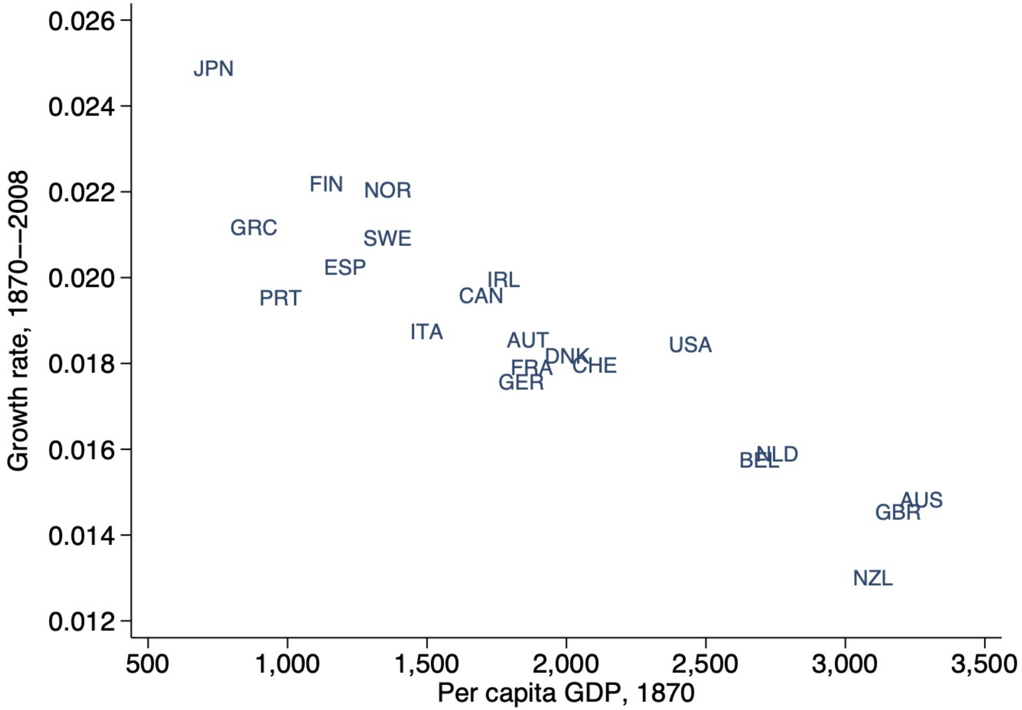
Convergence in a sample of industrialized countries, 1870-2008



Data source: Maddison (2010)

Graph from: Jones and Vollrath (2013)

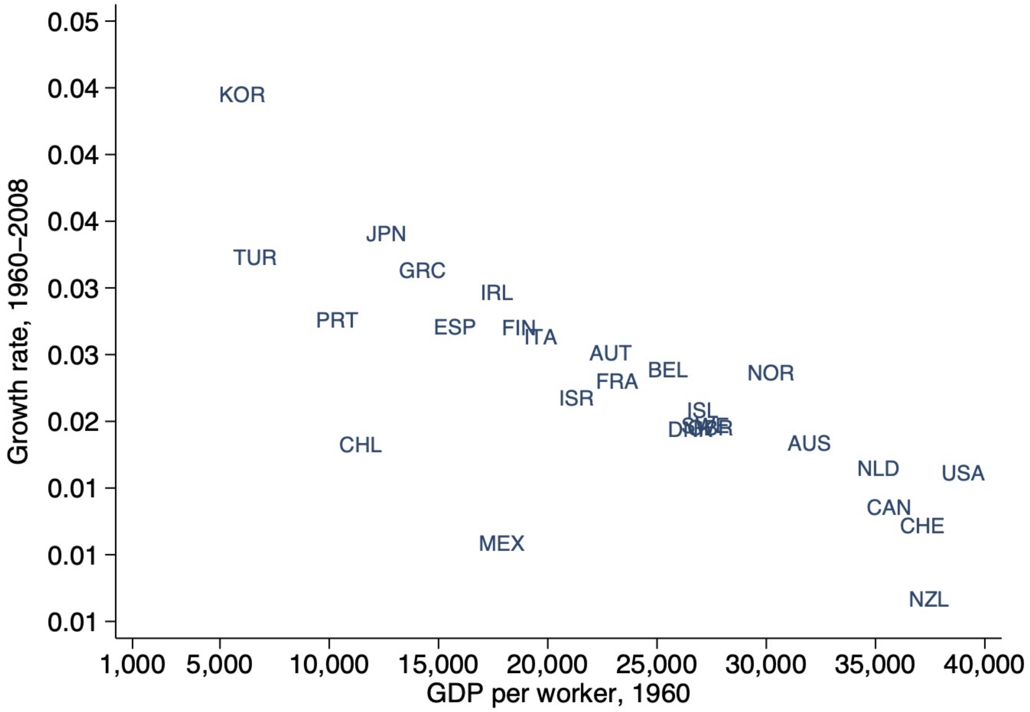
Convergence in a sample of industrialized countries, cont'd, 1870-2008



Data source: Maddison (2010)

Graph from: Jones and Vollrath (2013)

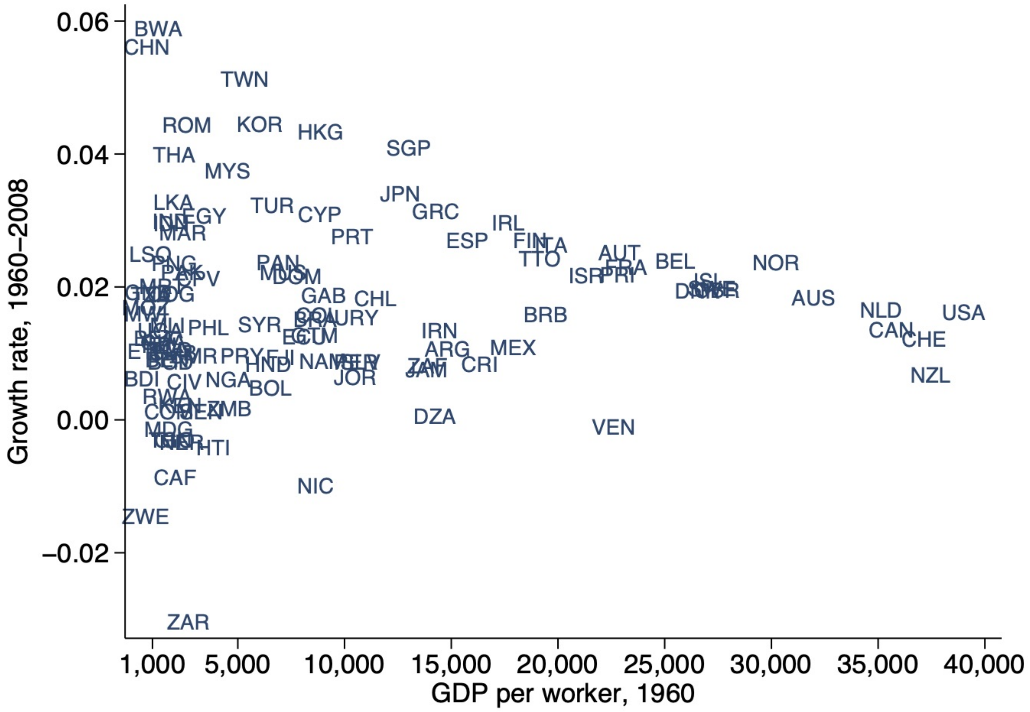
Convergence in OECD countries, 1960-2008



Data source: Penn World Tables Mark 7.0 and Summers and Heston (1991)

Graph from: Jones and Vollrath (2013)

The lack of convergence for the World, 1960-2008



Data source: Penn World Tables Mark 7.0 and Summers and Heston (1991) Graph from: Jones and Vollrath (2013)

Notice that, in our last seminar, we looked at the convergence of the countries with the most recent data, and there was convergence of countries after 1995.

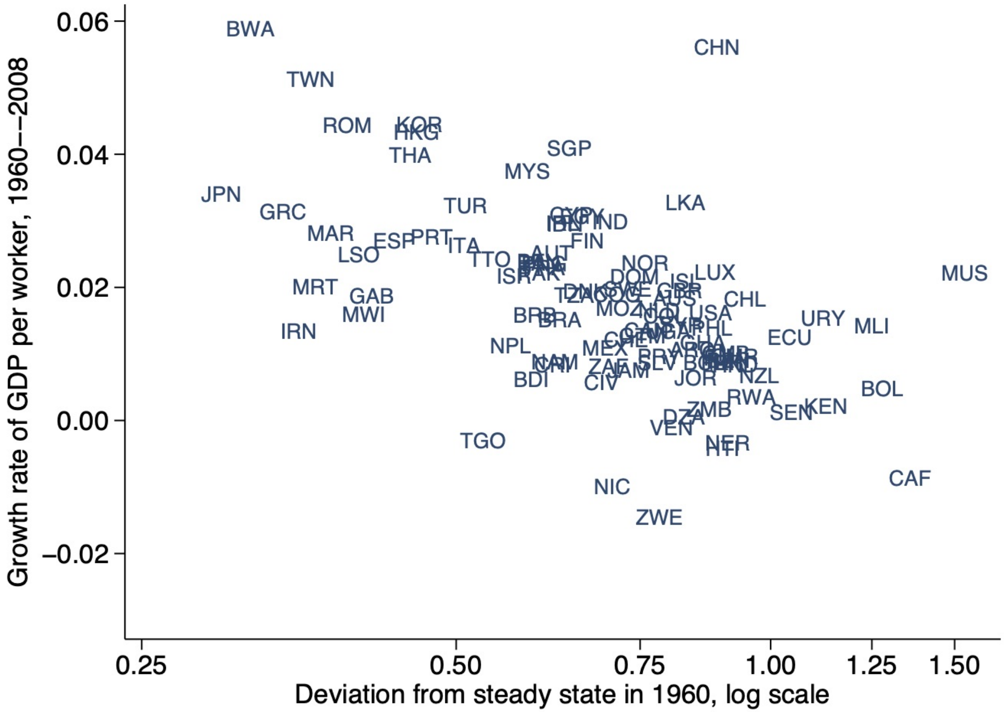
Convergence (?)

- How do we reconcile the converge in OECD but lack of convergence for the world?
- Prediction of the Solow model:
  - Among the countries with the same steady state, poor countries should grow faster than rich countries.
- Steady state depends on investment rate, population growth rate, technological progress rate
- OECD countries show similarities in investment rate, population growth rate, rate of technological progress.
- More variation in the World in these statistics.

Conditional convergence

- Mankiw, Romer, and Weil (1992), and Barro and Sala-i-Martin (1992):
- Convergence of countries "conditional on" their steady states
- Countries that are poor relative to their steady states tend to grow faster.

Conditional convergence for the World, 1960-2008



Data source: Author's calculations using Penn World Tables 7.0, update of Summers and Heston (1991).

Note: The variable on the x-axis is $\hat{y}_{60}/\hat{y}^*$. Estimates of A for 1970 are used to compute the steady state.

Graph from: Jones and Vollrath (2013)

Summary

- We developed a model in which technological progress is the only source of long-run growth
- We looked at the convergence of countries: countries with similar steady states converge but not all countries.
- Most recent data, as discussed in our last seminar, actually show convergence.