

Economic Growth

Lecture 6: Measuring Productivity

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Introduction

- Some aspects of factor accumulation –
 - investment rate,
 - population growth rate,
 - human capital

– explain some variation in income differences across countries.
- Taken together, do the different aspects of factor accumulation explain all of the variation among countries?
 - Answer: **No**
- Productivity explains the remaining part of the variation among countries

Productivity

- **Productivity:** Effectiveness with which factors of production are converted into output.
- Productivity = (Real) Output / Input
- Output = Nominal GDP / Price Deflator (CPI or PPI)
- Labor productivity = Output / Hours Worked
- Total Factor Productivity (TFP) = Output / Index of all inputs to production
- A in our Cobb-Douglas production functions is an example of TFP

$$y = Ak^\alpha h^{1-\alpha} \Rightarrow A = \frac{y}{k^\alpha h^{1-\alpha}}$$

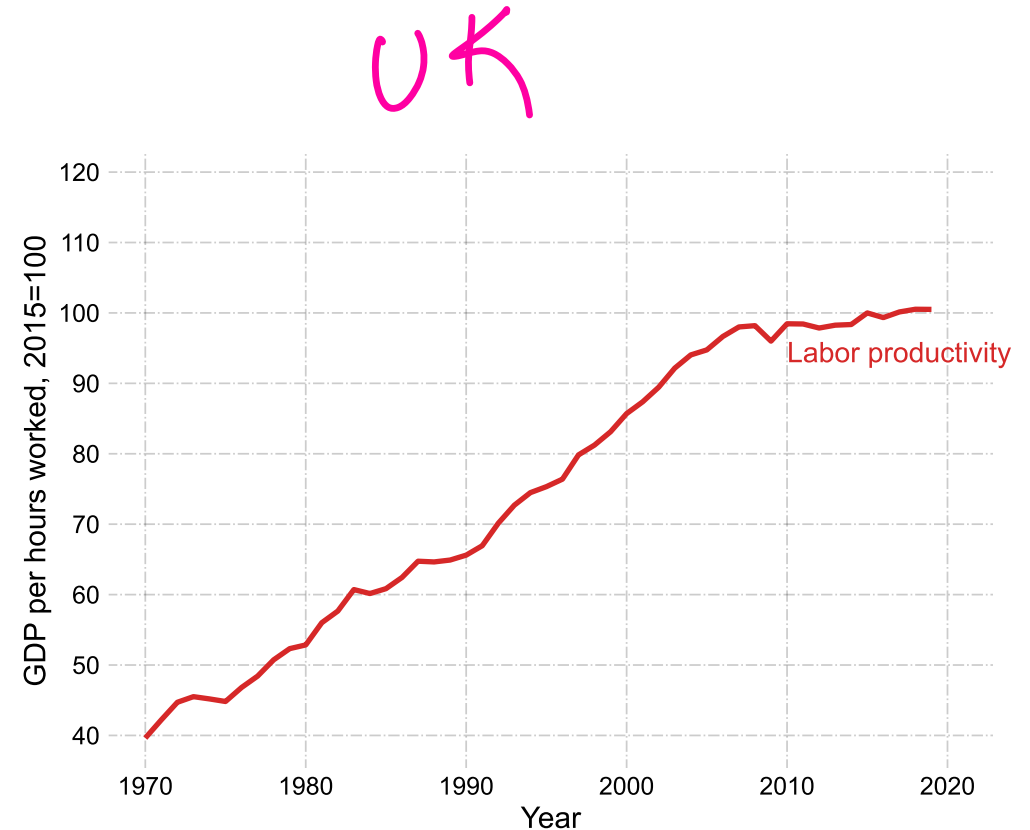
- In this module, main focus is on total factor productivity
 - When we say 'productivity', we mean total factor productivity

Labor productivity

- Labor productivity = $\text{Output} / \text{Hours Worked}$
- Commonly used measure of productivity
- Easy to calculate and interpret
- Capital accumulation leads to labor productivity gains

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- Slowdown in the UK labor productivity growth after 2008
 - Also known as *productivity puzzle*
 - Average annual labor productivity growth from 1970 to 2008: 2.4%
 - Average annual labor productivity growth from 2009 to 2019: 0.5%

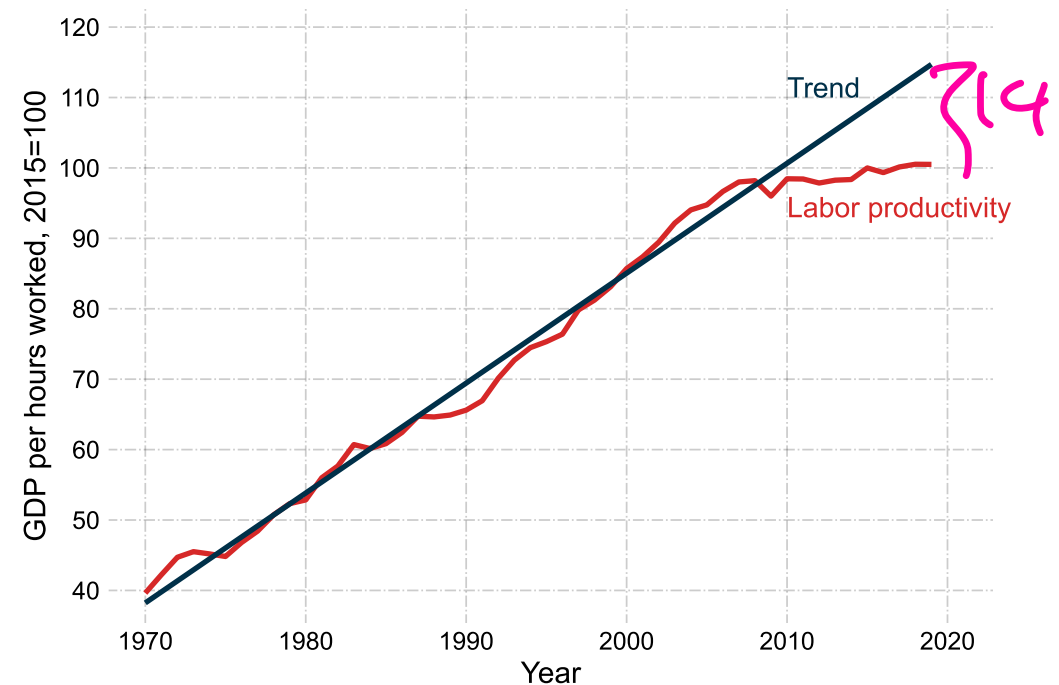


Source: OECD

Data from OECD, GDP per hour worked

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- Had the labor productivity grown at the pre-crisis level, each household would have earned 14% more in 2019.



Source: OECD

Data from OECD, GDP per hour worked

Productivity slowdown

- Many advanced economies experienced slowdown in labor productivity growth in the last decade
- UK experienced one of the severest slowdown.

Labor productivity growth (annualized %)

Country	1970-2008	2009-2019
United States	1.7	0.9
United Kingdom	2.4	0.5
Germany	2.4	1.1
France	2.6	0.9
Japan	3	1.2
New Zealand	1.2	0.5

Data source: OECD, GDP per hour worked

More on (labor) productivity

- You can watch "What is Productivity" by BLS
- You can read Andrew G Haldane's (Chief Economist, Bank of England) speech, Productivity Puzzles
- You can listen to "Tackling the UK's productivity crisis with John Van Reenen".

Accounting exercises

- Countries differ in their output because of

- differences in factor accumulation

- differences in productivity

- In this lecture, we ask:

- How much does productivity differ across countries?

- How much of the variation in income per worker among countries is explained by productivity differences?

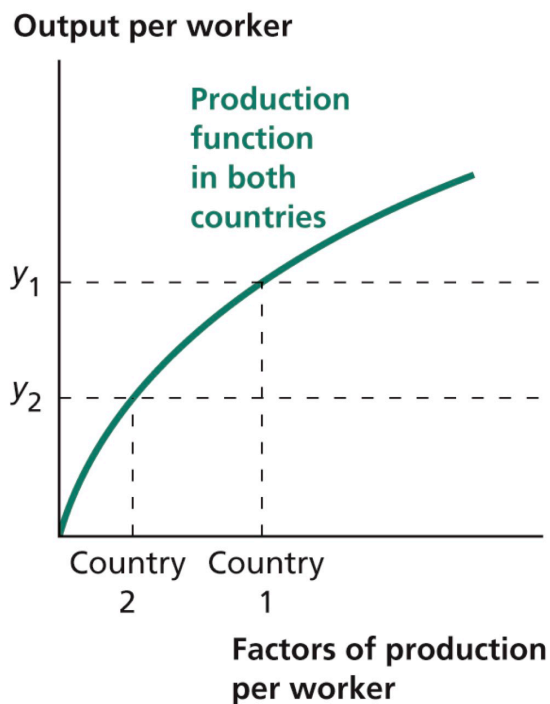
- How much does productivity growth differ among countries?

- How much of the variation in growth rates among countries is explained by variation in productivity growth, and how much by variation in factor accumulation?

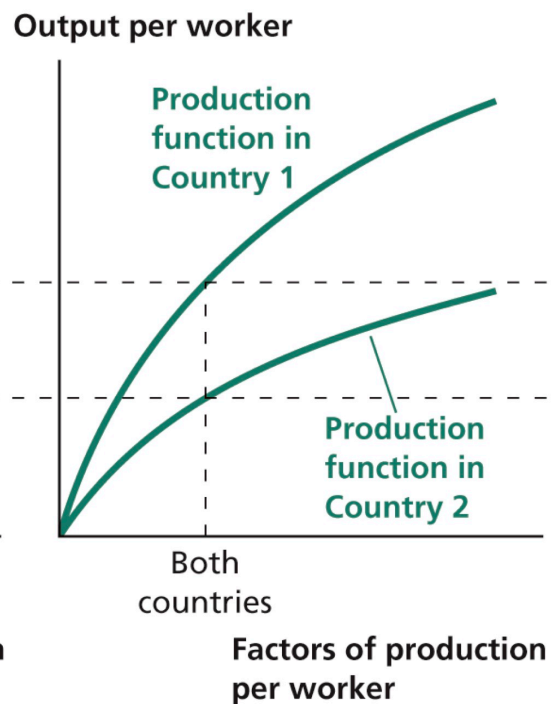
Growth accounting

Possible sources of differences in output per worker

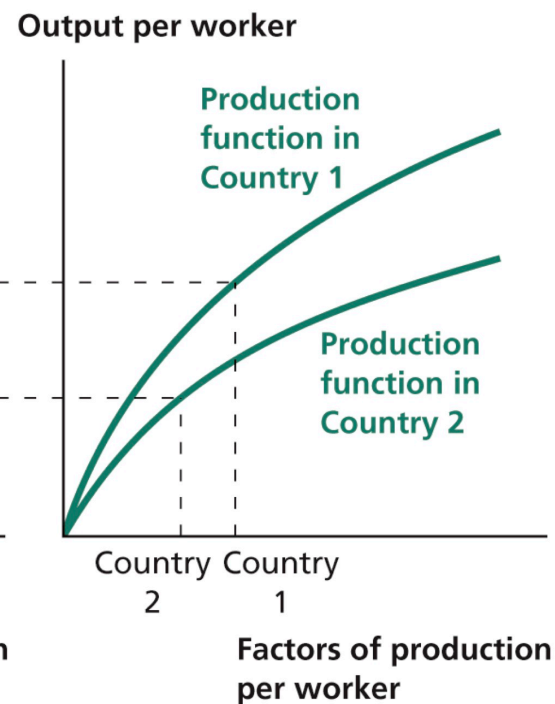
(a) Differences in output due to factor accumulation



(b) Differences in output due to productivity



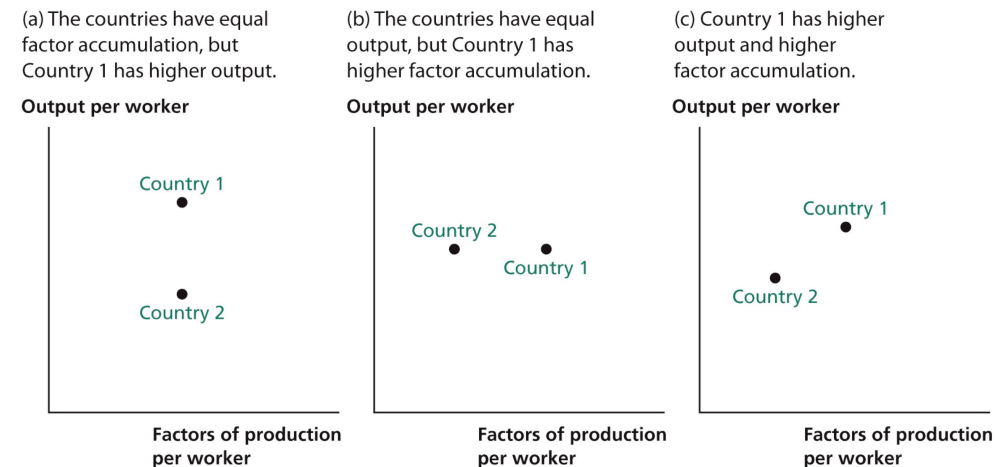
(c) Differences in output due to both productivity and factor accumulation



Graphic from Weil (2013)

Inferring productivity from data on output and factor accumulation

- a. Country 1 is more productive than country 2
- b. Country 2 is more productive than country 1
- c. Need to know the production function to make a comparison



Graphic from Weil (2013)

Measuring productivity differences among countries

- Assume production function is a Cobb-Douglas

$$Y = AK^\alpha(hL)^{1-\alpha}$$

- Output per worker $y = Ak^\alpha h^{1-\alpha}$
- factors of production = $k^\alpha h^{1-\alpha}$
- output = productivity \times factors of production

Productivity differences

- Output per worker of country 1 and country 2

$$y_1 = A_1 k_1^\alpha h_1^{1-\alpha}$$

$$y_2 = A_2 k_2^\alpha h_2^{1-\alpha}$$

- Output per worker ratios:

$$\frac{y_1}{y_2} = \left(\frac{A_1}{A_2} \right) \left(\frac{k_1^\alpha h_1^{1-\alpha}}{k_2^\alpha h_2^{1-\alpha}} \right)$$

ratio of output per worker = ratio of productivity \times ratio of factors of production

- Ratio of productivity

$$\text{ratio of productivity} = \frac{A_1}{A_2} = \frac{\text{ratio of output per worker}}{\text{ratio of factors of production}} = \frac{\left(\frac{y_1}{y_2} \right)}{\left(\frac{k_1^\alpha h_1^{1-\alpha}}{k_2^\alpha h_2^{1-\alpha}} \right)}$$

Numerical example

$$\frac{A_1}{A_2} = \frac{\left(\frac{y_1}{y_2}\right)}{\left(\frac{k_1^\alpha h_1^{1-\alpha}}{k_2^\alpha h_2^{1-\alpha}}\right)}$$

$$\text{ratio of productivity} = \frac{\text{ratio of output}}{\text{ratio of factors of production}}$$

	Output per worker, y	Physical capital per worker, k	Human capital per worker, h
Country 1	24	27	8
Country 2	1	1	1

$$\frac{A_1}{A_2} = \frac{\left(\frac{24}{1}\right)}{\left(\frac{27^{1/3} \times 8^{2/3}}{1^{1/3} \times 1^{2/3}}\right)} = \frac{24}{\left(\frac{3 \times 4}{1}\right)} = 2$$

- Country 1 is twice as productive as country

Development accounting

Development accounting

Development accounting: breaking down income differences into parts accounted for by

- productivity differences
- factor accumulation differences

$$y = A \underbrace{k^\alpha h^{1-\alpha}}_{\text{factors of prod.}}$$

We ask

- How much of the variation in income per worker among countries is explained by productivity differences?
- How much of the variation in income per worker among countries is explained by factors of production differences?

Development accounting, algorithm

Using data in a given year, say t :

1. Pick a base country
2. Find output per worker of country i relative to the base country, $\frac{y_i}{y_{base}} = ?$
3. Find physical capital per worker of country i relative to the base country, $\frac{k_i}{k_{base}} = ?$
4. Find human capital per worker of country i relative to the base country, $\frac{h_i}{h_{base}} = ?$
5. Find factors of production of country i relative to the base country, $\frac{k_i^\alpha h_i^{1-\alpha}}{k_{base}^\alpha h_{base}^{1-\alpha}} = ?$
6. Find productivity of country i relative to the base country, $\frac{A_i}{A_{base}} = ?$
7. Comment on the values you found in above steps

Development accounting relative to the US (2018)

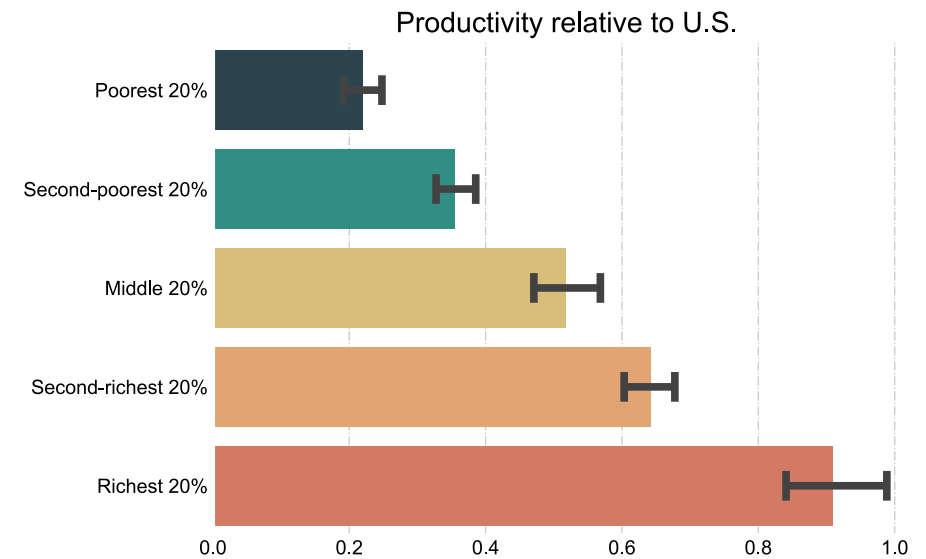
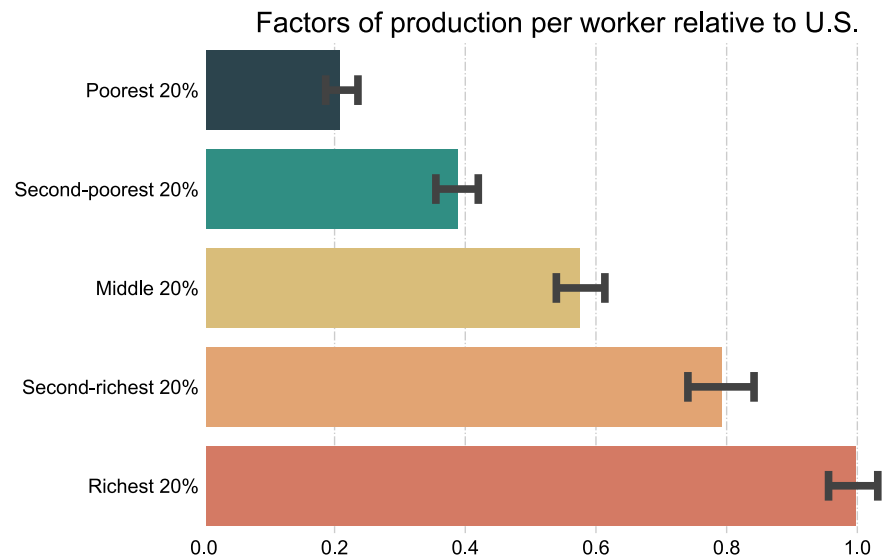
Country	Output per worker, y	Capital per worker, k	Human capital, h	Factors of production, $k^\alpha h^{1-\alpha}$	Productivity, A
India	0.14	0.16	0.57	0.37	0.38
Myanmar	0.09	0.08	0.49	0.26	0.34
Mexico	0.35	0.46	0.74	0.63	0.56
Brazil	0.26	0.34	0.81	0.6	0.43
South Korea	0.62	0.94	1	0.98	0.63
Japan	0.56	0.86	0.96	0.92	0.61
United States	1	1	1	1	1
Canada	0.75	1.04	0.99	1.01	0.75
United Kingdom	0.71	1.07	1.01	1.03	0.69
Norway	1.11	1.38	0.98	1.1	1.02

Data source: Penn World Tables, version 10.0

Development accounting, cont'd

- Large differences in productivity
 - 38% for Myanmar
 - 1.02% for Norway
 - 63% for South Korea
- Canada and UK: similar output per worker hours, Canada is more productive, the UK has more factors of production.
- Japan has factors of production close to the US (92%), but is about 40% less productive than the US.

Roles of factors of production and productivity in determining output per worker, 2018



Data source: Penn World Tables, version 10.0

- Productivity among the poorest one-fifth of countries is 22% of the US level.
- Similar contributions of productivity and factor accumulation to output per worker differences
 - 51% of variation in output per worker is due to factor accumulation
 - 49% of the variation is due to productivity

Growth accounting

Differences in the growth rate of productivity among countries

Growth accounting: breaking a country's growth into parts accounted for by

- growth in productivity
- growth in the quantity of factors of production

We ask

- How much income growth on average is accounted for by the productivity growth?
- How much income growth on average is accounted for by increases in factors of production?

Measuring a country's productivity growth

$$y = A \times K^\alpha h^{1-\alpha}$$

output = productivity \times factors of production

$$C = D \times E$$
$$g_C = g_D + g_E$$

- growth rate of output = growth rate of productivity + growth rate of factors of production
- growth rate of productivity = growth rate of output - growth rate of factors of production

Measuring a country's productivity growth

output = productivity \times factors of production

$$\log(C \times D^E)$$

$$\log C + \log(D^E)$$

$$\log C + E \log D$$

- growth rate of output = growth rate of productivity + growth rate of factors of production
- growth rate of productivity = growth rate of output - growth rate of factors of production
- Assume a Cobb-Douglas production function
- Output per worker, $y = Ak^\alpha h^{1-\alpha}$
- Take natural log of each side of equation

$$\ln(y) = \ln(Ak^\alpha h^{1-\alpha}) = \ln(A) + \alpha \ln(k) + (1 - \alpha) \ln(h)$$

→ chain rule

- Differentiate with respect to time

$$\frac{\frac{dy}{dt}}{y} = \frac{\frac{dA}{dt}}{A} + \alpha \frac{\frac{dk}{dt}}{k} + (1 - \alpha) \frac{\frac{dh}{dt}}{h}$$

$$\hat{y} = \hat{A} + \alpha \hat{k} + (1 - \alpha) \hat{h} \Rightarrow \hat{A} = \hat{y} - (\alpha \hat{k} + (1 - \alpha) \hat{h})$$

growth rate
of factors
of production.

where \hat{x} is the growth rate of x .

Growth accounting, algorithm

For a country, i , from year t to year $t + n$, find

1. the average annual growth rate of income per worker, $\hat{y} = \frac{\ln y_{i,t+n} - \ln y_{i,t}}{n}$

2. the average annual growth rate of physical capital per worker, $\hat{k} = \frac{\ln k_{i,t+n} - \ln k_{i,t}}{n}$

3. the average annual growth rate of human capital per worker, $\hat{h} = \frac{\ln h_{i,t+n} - \ln h_{i,t}}{n}$

4. the average annual growth rate of factors of production per worker,
 $f.\hat{o}.p. = \alpha\hat{k} + (1 - \alpha)\hat{h}$

5. the average annual growth rate of productivity,
 $\hat{A} = \hat{y} - f.\hat{o}.p.$

Comment on the above stats, in particular highlight

1. percentage of output per worker growth accounted for by the productivity growth, $\frac{\hat{A}}{\hat{y}}$

2. percentage of output per worker growth accounted for by the factors of production, $\frac{f.\hat{o}.p.}{\hat{y}}$

- percentage of output per worker growth accounted for by the physical capital accumulation, $\frac{\alpha\hat{k}}{\hat{y}}$

- percentage of output per worker growth accounted for by the human capital accumulation, $\frac{(1-\alpha)\hat{h}}{\hat{y}}$

Numerical example

$$\hat{A} = \hat{y} - \alpha \hat{k} - (1 - \alpha) \hat{h}$$

	Output per worker, y	Physical capital per worker, k	Human capital per worker, h
Erewhon in 1975	1	20	5
Erewhon in 2010	4	40	10
Annual growth rate	4%	2%	2%

$$\text{growth rate of output} = \left(\frac{\ln(\text{output in 2010}) - \ln(\text{output in 1975})}{35} \right) = .04$$

$$\hat{A} = .04 - \frac{1}{3} \times .02 - \frac{2}{3} \times .02 = .02$$

4a → (1-α)

Cont. of prod.

$$\frac{0.02}{0.04} = 50\%$$

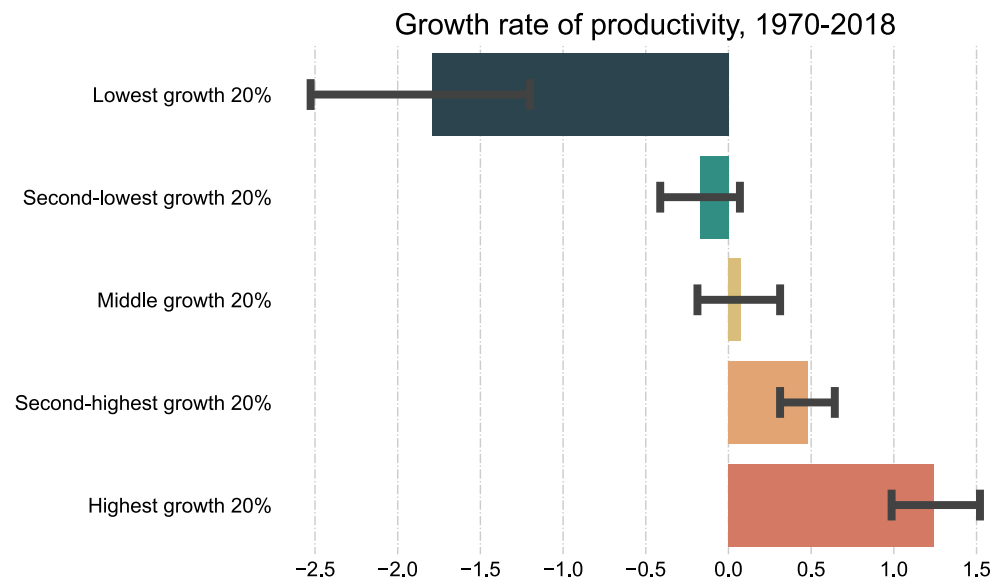
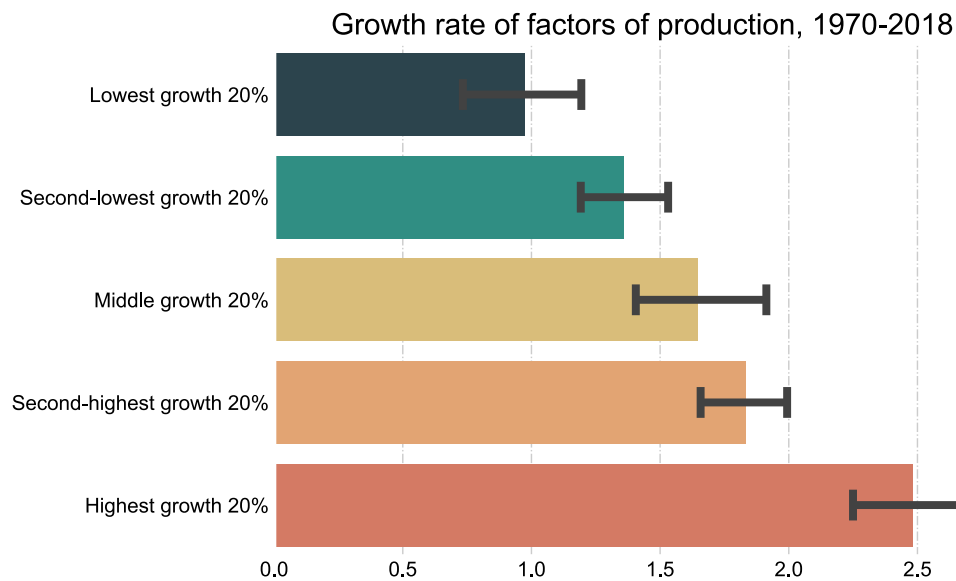
Growth accounting for the UK

- Years 1970 – 2018
- 1.85% output per worker growth annually
- physical capital stock per worker growth rate of 2.47% per year
- human capital growth rate of 0.66% per year
- $\alpha = 1/3$

$$\hat{A} = 0.0185 - \frac{1}{3} \times 0.0247 - \frac{2}{3} \times 0.0066 = 0.0058$$

- Productivity growth explains 32% of income growth ($0.0058/0.0185 = 32\%$)
- 68% of growth is explained by the accumulation of factors of production

Roles of factors of production and productivity in determining growth, 1970 - 2018



Growth accounting, further analysis

- Growth rate of factors of production ranges from 0.97% to 2.48% (1.51 percentage point gap)
- Productivity growth in the fastest growing one-fifth of countries is 1.25% per year
- Productivity growth in the slowest growing one-fifth of countries is -1.79% per year
- Gap between the highest and the lowest productivity growth is 3.04 percentage point
- Productivity growth is much more important source of income growth
- 71% of variation in the growth rates is the result of variation in productivity growth
- 29% of the variation is due to variation in factor accumulation

Problems with measuring productivity

- Quality of schooling
- Misreporting of investment
- Many different types of assets
- Quality differences across different versions of same asset
- Finding appropriate weights to sum up real quantities of different assets
- Production function is not necessarily constant
- Factor shares vary across countries

→ affect human capital measures
→ measuring physical cap.

$$y = A k^{\alpha} h^{1-\alpha}$$

capital's share of income

Summary

- **Development Accounting:** Breaking down income differences into parts accounted for by
 - productivity differences
 - factor accumulation differences
- **Growth accounting:** Breaking a country's growth into parts accounted for by
 - growth in productivity
 - growth in the quantity of factors of production

To review this lecture:

Read [Chapter 7 of Economic Growth](#) by David N. Weil