Growth and Welfare Implications of Sector-Specific Innovations*

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Abstract

I examine the optimal government subsidy of R&D activities when sectors are heterogeneous. To this end, I build an endogenous growth model where R&D drives macroeconomic growth and firm dynamics in two sectors with different characteristics: consumption-goods sector and investment-goods sector. I highlight how various externalities in the innovation process affect the allocation of innovative resources across industries. I calibrate the model to U.S. data and study the quantitative properties of the model. By explicitly examining the transition path after the change in subsidy, I highlight the tradeoff between the consumption level in the short run and the long-run growth. I find that the optimal combination of the subsidy rates as a fraction of firm R&D expenditures is 83 percent in consumption sector and 88 percent in investment sector. By moving from the baseline subsidy rates (10 percent in both sectors), the society can achieve 21 percent welfare gain in consumption equivalent terms. The annual GDP growth rate increases from 1.6 percent to 3.3 percent by this change in subsidy. I also analyze the optimal combination of R&D subsidies when the government budget is limited.

Keywords: Endogenous Growth, Innovation, Research and Development, Investment Specific Technological Change

JEL classification: O31, O38, O41, L16

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1 Introduction

Governments support business research and development (R&D) in varying amounts. In 2011, the United States federal government's total support to business R&D was 0.26% of its GDP. The fraction was 0.43% in Korea and .01% in Mexico [OECD (2015)]. Inter-country variation in the government support of R&D suggests that setting the optimal amount of support is not straightforward. In this paper, I characterize the optimal amount of government subsidy to business R&D in a quantitative environment.

I build an endogenous growth model with firm dynamics to analyze the optimal R&D subsidy. I use the model and firm dynamics data to identify inefficiencies in the R&D expenditures of two sectors that have different characteristics: the consumption-goods sector and the investment-goods sector. Next, I characterize the subsidy rates in these sectors that are needed to correct inefficiencies in innovation. I base my model on the seminal work of Klette and Kortum (2004), in which innovation by incumbent and entering firms generates firm dynamics and derives macroeconomic growth. Klette and Kortum show that their model can qualitatively account for various stylized facts about firm dynamics. Hence, I identify the inefficiencies in the firm R&D expenditures with a framework that has a good fit on the firm dynamics data. I extend the Klette and Kortum model by introducing capital stock and by having not only consumption-goods sector but also investment-goods sector. Firm entry and expansion rate differences across these sectors and the sustained decline in the prices of investment goods relative to consumption goods, observed in the data (developed by Gordon (1990), and extended by Cummins and Violante (2002)), imply that in these sectors there are different magnitudes of inefficiency and rates of technological change. A model that takes the heterogeneity of innovative activity across sectors into account will provide more accurate information about the growth and welfare implications of R&D subsidies.

In this paper, an innovation is modeled as an increase in the quality of an existing good in the market along a quality ladder. My two sectors consist of many differentiated products, each of which is produced by a production line. In this setting, firms are simply collections of the production lines they possess. Innovating firms capture the market of the innovated product from the existing producer and earn monopoly rents, which last as long as the innovators hold the blueprints of the highest quality versions of the goods they produce. Firms lose these rents following innovation on the same goods by other firms. Firms, therefore, expand and shrink according to this creative-destruction process, entering the market when they successfully innovate and exiting if they lose the blueprints of all the goods they produce.

In a market economy the creative-destruction process contains inefficiencies, which lead to differences in market economy and social planner innovation rates in each sector. Innovation rate in a sector is defined as the measure of differentiated products innovated at a time over the measure of total products in that sector. An innovator reaps monopoly rents as long as it holds the blueprints of the highest-quality version of the product she innovated. However, the benefit of innovation to the society is the extra production from the innovation, which may be different from the monopoly rents. Also, unlike a limited lifetime of monopoly rents that accrue to the innovator, the social benefit of the innovation lasts forever. On the one hand, an entrepreneur's inability to appropriate all of the consumer surplus it created causes market economy innovation rates to fall below those of social planner levels (the appropriability effect). Also, the limited time of monopoly rents that accrue to the innovator contributes to under investment in innovation (the inter-temporal spillover effect). On the other hand, an entrepreneur does not take into account the profit loss that is imposed on the current producer of the product that they have taken over, and this moves market innovation rates above the socially efficient level (the business-stealing effect). Depending on the sizes of these distortions, market economy innovation rates can be below or above the socially efficient levels. Therefore, a government may be able to employ tax/subsidy systems to correct the distortions in the economy, which increases the welfare of households.

The above stated distortions lead not only to deviations from the socially optimal levels of industry innovation rates but also to the misallocation of innovative resources across sectors. We observe such misallocations even when the industries have identical innovation functions. Holding the total R&D labor constant at the competitive equilibrium level, the social planner can generate welfare gains by reallocating research labor across industries. Moreover, the above stated externalities distort the allocation of research labor in (potentially) different directions. For example, the monopoly power effect tilts innovation rates toward the consumption sector, while the inter-temporal spillover effect pushes innovation rates toward the industry with the lower socially optimal innovation rate. The former is a result of the relative profits of the consumption sector exceeding the relative influence of average quality in the consumption sector on final consumption. The later is a result of the inter-temporal spillover effect being larger in the industry with the higher innovation rate. In an environment with identical innovation functions across industries, the inter-temporal spillover effect distorts allocation of research labor toward the investment sector.

Setting the optimal amount of subsidy for each sector requires knowledge of the elasticity of R&D with respect to the subsidy and the magnitudes of externalities in each sector. Various studies have estimated the former using data on firm level R&D expenditures and changes in government subsidy rates [Bloom et al. (2002), Hall et al. (2010), CBO (2005)]. The magnitudes of externalities are not observable. To infer the sizes of externalities in each sector and devise the optimal R&D subsidy system, I identify model parameters related to innovation that are obtained from the model's implications on firm dynamics. Recent literature emphasizes that the firm dynamics data contains important information about the innovation process. Klette and Kortum (2004) show that their model qualitatively generates many empirical facts on firm size distribution and firm growth rates. Lentz and Mortensen (2008), whose model is based on Klette and Kortum, estimate model parameters from Danish data, and the model quantitatively fits related firm dynamics moments.

Differences in the firm dynamics of the two sectors, as observed in the US data, imply that in the two sectors there are different magnitudes of externalities. In the model, the expected lifetime of monopoly rents that accrue to an innovator after successful innovation is linked to the entry rate in the firm's sector (the inter-temporal spillover). Hence, a higher entry rate in the consumption sector suggests a larger inter-temporal spillover effect. The business-stealing effect is about the difference between the profit an innovating firm captures and the net benefit of innovation to the society, including the profit loss the incumbent producer faces after an innovation. The net benefit of innovation to society is the extra production it enables, which is summarized by the size of the quality improvement (the quality ladder step size). The lower the quality ladder step size, the lower the benefit of innovation to society, and the larger the difference between the private and social benefits of an innovation. Therefore, the business-stealing effect is inversely related to the quality ladder step size. The GDP growth rate and the growth rate of the investment goods price relative to the consumption goods price identifies the quality ladder step size in each sector. The calibration exercise shows that the consumption-goods sector has lower quality ladder step size than the investment-goods sector. Because of the inverse relationship between the quality ladder step size and the business stealing effect, the consumption-goods sector experiences a larger business-stealing effect.

As explained above, market innovation rates are inefficient. To gauge whether there is under or over investment in innovation, I solve for the social planner problem that is subject to the innovation functions of the firms but can dictate to firms what amount of R&D they should conduct and how much they should produce. Over the long-run, the social planner sets the innovation rates that are substantially higher than the market rates in each sector: 12 percentage points higher in the consumption industry and 17 percentage points higher in the investment industry. Over the long term, the increased innovation rates correspond to a 1.7 percentage point increase in the GDP growth rate. Starting from the balanced growth path of the market economy with 1.6% GDP growth rate, the social planner immediately decreases the GDP growth rate to about 1 percent, and then the growth rate gradually increases to its new balanced growth path value of 3.3 percent. However, consumption and GDP follow different trajectories. Like the consumption growth rate, consumption decreases initially as more labor is employed in research. Although the consumption growth rate increases gradually, it remains below its market economy balanced growth path level for some time. Eventually, the consumption growth rate converges with its balanced growth path level of 3.3 percent. Long-run consumption growth outweighs the short-run consumption loss, and the transition from the market economy balanced growth path to the social planner balanced path leads to an almost 22 percent welfare gain to households, as measured in consumption-equivalent terms. Thus, the market is under-investing in innovation, and a benevolent government can increase the welfare of the household by subsidizing R&D.

The amount of resources allocated to R&D at the social planner's balanced growth path is about six times the market economy resource allocation to innovation. This is larger than what is reported in recent literature that employs related models but different methods. Using Danish data, Lentz and Mortensen (2008) build a model in which firms have persistently different abilities to create higher quality products, and they estimate their model. By using their 2008 paper estimates, Lentz and Mortensen (2015) show that the social planner increases resource allocation to innovation three-fold compared to market outcome, which would generate a 21 percent welfare gain, as measured in the tax to social planner consumption. To calculate the welfare gain Lentz and Mortensen compare only the steady states of the market and the social planner economy. The 15 percent welfare gain that I estimate takes into account an additional factor: the transition path.

In my model, in a decentralized environment, the government can increase the welfare of society by employing R&D subsidies and a capital investment subsidy. In my benchmark calibration, subsidizing 83 percent of consumption sector incumbents' R&D expenditures and 88 percent of investment sector incumbents' R&D expenditures generates a welfare gain close to that of the social planner. The government also employs an entry subsidy such that the marginal social cost of entrant innovation equals the marginal social cost of incumbent innovation. This result suggests that government can substantially increase the welfare of a society by heavily subsidizing innovation with constant rates. The government finances these subsidies with lump-sum taxation of households. Similarly, Grossmann et al. (2013) calculate socially optimum time-dependent R&D subsidy rate and find that the welfare loss of setting R&D subsidy rate to its long-run value immediately instead of employing a time-varying R&D subsidy rate is quite low. They also show that the optimal R&D subsidy is approximately 81.5%. Both results are in line with my findings. Akcigit et al. (2016) address optimal R&D policy within a mechanism design framework. They show that when firms are heterogenous in research productivity and there is asymmetric information about research productivity of firms, the optimal subsidy system depends many factors including age of the innovating firms, current and lagged quality of the products of the firms, current and lagged R&D expenditures of the firms. Since there is no asymmetric information in my model and per-good research productivity of firms are constant within a sector, the optimal R&D subsidy is same for all the firms in a sector.

Comparing my model with Atkeson and Burstein (2018) (AB hereafter) model highlights the main contribution of this research: the analysis of optimal innovation subsidies in an environment with heterogenous sectors. AB build a model that nests many endogenous and semi-endogenous growth models. They develop a method to linearly approximate output and productivity trajectories after a policy-induced change in the innovation intensity of the economy. Their methodology relies on a few critical assumptions, one of which is conditional efficiency: any given level of innovative resources in the economy is efficiently allocated across sectors. Whenever this assumption does not hold, their analysis applies only to proportional changes in innovation subsidies to different agents in the economy. As explained above, this key assumption does not hold in my model. Further, my methodology applies to non-proportional changes in subsidies to firms in different industries. To be clear, under the assumptions of the AB model, the AB method approximates output and productivity trajectories quite well with more politically feasible innovation policies (a 10% increase in policy-induced changes in R&D labor). AB also have a richer model in other aspects; to name a few, different levels of inter-temporal spillovers, technological progress with quality ladders as well as technological progress with expanding varieties, and so on.

Subsidizing the investment sector produces a larger welfare gain than subsidizing the consumption sector. Two factors contribute to the difference in welfare gains. First, each sector has a similar elasticity of innovation with respect to the user cost of R&D, but the investment sector has a higher innovative step. Hence, any decrease in the user cost would lead to similar changes in innovation rates, but a given change in investment sector innovation leads to a higher consumption growth rate and, hence, a larger welfare gain than would the same amount of change in consumption sector innovation. Second, an increase in the investment sector innovation rate leads to a larger reduction in the price of investment goods. A larger rate of decline in the price of investment goods increases the user cost of capital, which results in a lower accumulation of capital. Consequently, consumption production grows more slowly than it would otherwise. During earlier periods, the investment-sector-subsidized economy has a lower consumption than the consumption sector subsidized economy. In other words, the first factor dominates and the welfare gain of subsidizing investment sector R&D is higher.

To achieve welfare-maximizing innovation rates, the government needs to subsidize innovation at roughly 85 percent. This large subsidy rate is mostly the result of two factors. First, there are significant distortions in the economy. As explained above, using related models, Atkeson and Burstein (2018) and Lentz and Mortensen (2015) find substantial under-investment in innovation in the market economy. Similarly, Jones and Williams (2000) find that the market economy typically under-invests in innovation. Second, R&D subsidies encourage innovation by decreasing the cost of innovation, but they also discourage incumbent innovation by reducing the expected lifetime of an innovation. A higher subsidy leads to a higher firm value, which increases the entry rate. When the entry rate increases, an incumbent firm is more likely to lose its monopoly rents by successfully innovating, and this reduces the expected time period of monopoly rents and the value of innovation (inter-temporal substitution effect increases). Thus, innovation will be discouraged. To compensate for the shortened expected lifetime of innovation, firms need to be subsidized even more.

This paper is organized as follows. Section (2) describes the model while Section (3) theoretically characterizes distortions in the economy and analyzes impacts of the distortions on the allocation of innovative resources across sectors. Section (4) calibrates the model. Section (5) numerically compares market outcome to the social planner's equilibrium. Section (6) characterizes the subsidy system that would maximize household welfare. Section (7) compares my results with the results of Atkeson and Burstein (2018) and highlights my contributions. Section (8) concludes.

2 Model

Time is continuous. There are two sectors in the economy: consumption goods and investment goods. Each sector consists of a unit measure of differentiated goods. In turn, each differentiated good has possibly countably many quality levels. Households rent capital to firms, which are owned by the households. Differentiated goods producers engage in research and development (R&D), which results in higher quality levels of existing products in the market.

2.1 Households

An infinitely-lived representative household chooses time paths of consumption, capital holding, investment in capital, and firm holdings to maximize the discounted sum of utility from consumption, $C(t)$,

$$\max \int_0^\infty \exp(-\rho t) \ln C_t dt,$$

subject to the law of motion for capital stock and a budget constraint:

$$\begin{aligned} \dot{K} &= X - \delta K, \\ P_c C + (1 - s_{in}) P_x X + \dot{A} &= RA + wL + rK - T, \end{aligned}$$

where K is the capital stock, X is investment, P_c is the consumption goods price index and normalized to 1, P_x is the investment goods price index, s_{in} is the capital investment subsidy rate, A is total value of the firms, R is the interest rate, w is the wage rate, L is labor supply, r is the rental rate of capital, and T is the lump-sum tax. Henceforth, I will drop time subscripts for notational ease.

Consumption is a CES aggregate of differentiated consumption goods:

$$C = \exp\left(\int_0^1 \ln\left(\sum_{j=0}^{J(\omega)} q^j(\omega) c^j(\omega)\right) d\omega\right), \tag{1}$$

where $q^j(\omega)$ is the quality of version j of product ω, $c^j(\omega)$ is the quantity consumed of version j of product ω, and $J(w)$ is the highest quality version of ω. As seen in Equation (1), households have perfectly substitutable preferences over the different quality adjusted versions of each product. In equilibrium, this formulation leads to the following demand function:

$$c^j(\omega) = \begin{cases} \frac{Z}{p^j(\omega)} & \text{if } \frac{q^j(\omega)}{p^j(\omega)} \geq \frac{q^{j'}(\omega)}{p^{j'}(\omega)} \text{ for all } j' \\ 0 & \text{otherwise,} \end{cases} \tag{2}$$

where $p^j(\omega)$ is the price of version j of product ω, $Z = P_c C$ is the total consumption expenditure, and $P_c = \exp\left(\int_0^1 \ln\left(\frac{p(\omega)}{q(\omega)}\right) d\omega\right) = 1$.

Investment, X, is also a CES aggregate of differentiated investment goods, which are located in a different interval than consumption goods.

$$X = \exp\left(\int_0^1 \ln\left(\sum_{j=0}^{J(\omega)} q^j(\omega) x^j(\omega)\right) d\omega\right), \tag{3}$$

where $x^j(\omega)$ is the quantity invested in version j of product ω. The corresponding demand function is

$$x^j(\omega) = \begin{cases} \frac{I}{p^j(\omega)} & \text{if } \frac{q^j(\omega)}{p^j(\omega)} \geq \frac{q^{j'}(\omega)}{p^{j'}(\omega)} \text{ for all } j' \\ 0 & \text{otherwise,} \end{cases} \tag{4}$$

where $I = P_x X$ is the total investment expenditure, and $P_x = \exp\left(\int_0^1 \ln \frac{p(\omega)}{q(\omega)} d\omega\right)$ is the investment price index.

2.2 Firms

A firm is defined by the set of differentiated goods it produces. Each good is produced by a unique production unit. A firm can own countably many production units. It can expand the set of production units by innovating on other goods it currently does not produce. Similarly, it can lose its existing goods to other innovating firms. Furthermore, if a single good producer loses its only production unit, it exits the market. Lastly, entrepreneurs can enter the market by innovating on a good located on the unit interval.

2.3 Innovation by Incumbents

The amount of research labor a firm hires and the number of goods it produces jointly determine its Poisson innovation arrival rate, β. Innovation is not directed. The good that is innovated is randomly drawn from a uniform distribution on the unit interval of goods in the market. A firm innovates only in the sector that it currently operates in. An innovation increases the quality of the good by an exogenous factor of $\lambda > 1$. This factor is the quality ladder step size and represents the innovativeness of a firm. Throughout the paper, the term *innovativeness* will be used to signify the factor by which the quality of a product increases after a successful innovation. More innovative firms can increase the quality of a good by a larger factor. The level of innovativeness varies by sector but is invariant across firms within each sector.

After a successful innovation, the innovator and the firm which has the blueprints of producing the second highest quality version (runner-up) of the good engage in Bertrand competition. The innovator charges a price equal to λ times marginal cost of production of second highest quality version of the good. Since consumers have infinitely elastic preferences over the quality adjusted varieties of a good, the innovator takes over the market. The innovator expands by one good, the runner-up shrinks by one good.

A firm currently producing m goods and hiring l_R units of labor for the purpose of research innovates at a rate $\varphi(m, l_R) = \beta$, where $\varphi(\cdot)$ is a constant returns to scale production function, increasing in both arguments, and strictly concave in l_R. Firms with experience in innovation, particularly those that retain their products despite innovation by other firms, are better at producing ideas. The number of goods in the production function is a proxy for a firm's experience in innovation.

2.4 Consumption Goods Producers

I turn to the firm problem: how much to invest in R&D to maximize the value of the firm. For more detailed description of the firm problem see Appendix (B.1). Each production unit of a firm has a Cobb-Douglas production function with capital elasticity α. In equilibrium gross profit (before R&D expense) of each production unit is equal to

$$\pi = \left(1 - \frac{1}{\lambda_c}\right) Z. \tag{5}$$

I use the profit function in (5) to derive the firm's value function, which can be expressed either as a function of the level of research labor or, more conveniently, the level of innovation arrival rate per good. Let $\phi(\beta, m)$ denote the level of l_R implicitly defined by $\varphi(m, l_R) = \beta$. Since $\varphi(\cdot)$ is strictly increasing in l_R, and homogenous of degree one, $\phi(\cdot)$ is well-defined and homogenous of degree one and convex in β. Set $\phi_c(b_c, m) = m\chi_c b_c^{\gamma}$, where $b_c = \beta/m$, and $\chi_c > 0$ is a scale parameter.

Since value function of the firm is linear in the number of goods it produces, the value of a production unit, $\nu_c Z$, is described as follows:

$$(R - g_Z + \tau_c - b_c)\nu_c = \left(1 - \frac{1}{\lambda_c}\right) - (1 - s_c^i)\chi_c b_c^{\gamma} \frac{w}{Z}. \tag{6}$$

The first order condition for innovation arrival rate per product is:

$$(1 - s_c^i) w \gamma \chi_c b_c^{\gamma - 1} = \nu_c Z. \tag{7}$$

In my solution of the stationary equilibrium, the growth rate of consumption expenditures, g_Z, is constant and equal to the growth rate of wages, g_w. Appendix B.2 solves the problem and shows the equality of g_Z and g_w.

Entrepreneurs can enter the market by innovating on a product. Like incumbents, they hire research labor to develop better qualities of products. An entrepreneur must hire $\xi_c(z_c, \bar{z}_c) \equiv \psi_c \chi_c z_c \bar{z}_c^{\gamma-1}$ units of labor to secure a z_c Poisson innovation rate, where, $\psi_c > 0$ is a parameter to differentiate the cost of incumbent and entrant innovation, and $\bar{z}_c$ is the entry rate in the market. As more entrants try to enter to the market, it requires more effort to develop a successful product. This formulation is a reduced form of the limited availability of venture capital to entrepreneurs. The value of entry is therefore

$$RV_E = \max_{z_c \geq 0} \{-(1 - s_c^e) w \xi_c(z_c, \bar{z}_c) + z_c[\nu_c Z - V_E]\},$$

where s_c^e is entry subsidy rate (i.e. entrant innovation subsidy) in the consumption sector. Free entry drives down the value of entry to zero. Hence, in equilibrium

$$(1 - s_c^e) w \psi_c \chi_n \bar{z}_c^{\gamma-1} = \nu_c Z. \tag{8}$$

2.5 Investment Goods Producers

Firms in this sector share the same Cobb-Douglas production function with capital elasticity α. The profit of a production unit, $\pi = \left(1 - \frac{1}{\lambda_x}\right) I$, is derived in a manner analogous to that of the consumption goods producers. The value of a production unit, $\nu_x I$, is described as follows:

$$(R - g_I + \tau_x - b_x)\nu_x = \left(1 - \frac{1}{\lambda_x}\right) - (1 - s_x^i)\chi_x b_x^{\gamma} \frac{w}{I}, \tag{9}$$

$$(1 - s_x^i) w \gamma \chi_x b_x^{\gamma-1} = \nu_x I, . \tag{10}$$

where equation (10)is the first order condition of innovation arrival rate per product.

Entrants, on the other hand, have the following problem:

$$RV_E = \max_{z_x \geq 0} \{-(1 - s_x^e) w \xi_x(z_x, \bar{z}_x) + z_x[\nu_x I - V_E]\},$$

where s_x^e is the entry subsidy in investment sector. Free entry implies

$$(1 - s_x^e) w \psi_x \chi_x \bar{z}_x^{\gamma-1} = \nu_x I. \tag{11}$$

2.6 Equilibrium

A symmetric balanced growth path competitive equilibrium is defined by a tuple of firm decisions $\{k_{i,t}, l_{i,t}, l_{R,i,t}, b_{i,t}, z_{i,t}, \tau_{i,t}, c_t, x_t\}$, where $i = c, x$ represents consumption and investment sectors, a tuple of household decisions $\{c_t, x_t, C_t, X_t, K_t\}$, a tuple of prices $\{w_t, r_t, R_t, p_{c,t}, p_{x,t}, P_{c,t}, P_{x,t}\}$, aggregate expenditures $\{Z_t, I_t\}$, average quality levels in each sector, $\{Q_{c,t}, Q_{x,t}\}$, and value of production units per aggregate expenditure in a firm's sector, $\{\nu_c, \nu_x\}$. In equilibrium the following conditions hold.

- $\{p_c, p_x\}$ are the Bertrand equilibrium prices of highest quality products.

- Given prices of differentiated goods and household demand functions (2) and (4) k_c, l_c and k_x, l_x solve the firm cost-minimization problems in the consumption and investment sectors.
- Given prices and nominal aggregate expenditures, $\{\nu_c, b_c, z_c\}$ solve equations (6), (7), (8) for $i = c$, and $\{\nu_x, b_x, z_x\}$ solve equations (9), (10), (11) for $i = x$.
- Innovation rate in a sector is equal to sum of incumbent and entrant innovation rates: $\tau_{i,t} = z_{i,t} + b_{i,t}$, where $i = c, x$.
- Given prices, $\{c_t, x_t, C_t, X_t, K_t\}$ are the balanced growth path values of the household optimization problem.
- The labor market clears: $l_c + l_x, +\chi_c b_c^\gamma + \chi_x b_x^\gamma + \psi_c \chi_c z_c^\gamma + \psi_x \chi_x z_x^\gamma = L$,
- The capital market clears: $K_t = k_{c,t} + k_{x,t}$,
- Nominal expenditures, $\{Z, I\}$, grow at the same rate.
- Average quality levels of industries are $Q_c = \exp\left(\int_0^1 \ln\left(q(\omega)\right) d\omega\right)$, $Q_x = \exp\left(\int_0^1 \ln\left(q(\omega)\right) d\omega\right)$.

This equilibrium is discussed in detail in Appendix B.2.

3 Optimality of Innovation

As a characteristic of Schumpeterian creative–destruction type models, the competitive equilibrium innovation rate may not be socially optimal. An innovating firm improves the quality of an existing good, destroys the profit accrued by the incumbent producer and gains monopoly power on production of the product that it innovated. While deciding the amount of R&D to conduct, it considers the monopoly profits that it will accrue until another firm innovates on that good and captures the product. However, the social benefit of an innovation goes on forever since every innovator improves the quality upon the existing quality level. Also, innovators ignore the profit loss of the existing producer of the good. Therefore, the competitive equilibrium innovation rate is generically inefficient. After defining the social planner problem, I will discuss each externality further in Section 3.2.

In order to identify how the externalities affect the economy, I define and solve the social planner's problem. Then, I compare the competitive equilibrium first order conditions with the social planner first order conditions, and discuss the differences caused by externalities.

3.1 Social Optimum

The social planner maximizes the discounted sum of utility from consumption:

$$\max \int_0^\infty e^{-\rho t} \ln(K_{c,t}^\alpha L_{c,t}^{1-\alpha} Q_{c,t}) dt \quad \text{subject to}$$

the resource constraints of capital, $K_{c,t} + K_{x,t} = K_t$, and labor, $L_{c,t} + L_{x,t} + \psi_c \chi_c z_{c,t}^{\gamma} + \chi_c b_{c,t}^{\gamma} + \psi_x \chi_x z_{x,t}^{\gamma} + \chi_x b_{x,t}^{\gamma} \leq 1$, total innovation rates in each sector, $z_{c,t} + b_{c,t} = \tau_{c,t}$ and $z_{x,t} + b_{x,t} = \tau_{x,t}$, the law of motion for capital stock, $\dot{K}_t = K_{x,t}^{\alpha} L_{x,t}^{1-\alpha} Q_{x,t} - \delta K_t$, the average quality levels in each sector, $Q_{c,t} = \exp\left(\int_0^1 \ln\left(q_t(\omega)\right) d\omega\right)$, $Q_{x,t} = \exp\left(\int_0^1 \ln\left(q_t(\omega)\right) d\omega\right)$, and laws of motion for the technology index of the consumption sector, $\frac{\dot{Q}_{c,t}}{Q_{c,t}} = \tau_{c,t} \log \lambda_c$, and the investment sector, $\frac{\dot{Q}_{x,t}}{Q_{x,t}} = \tau_{x,t} \log \lambda_x$.

The social planner problem (SP) can be divided into two parts: 1) a static problem where a given level of total innovation in a sector is allocated to entrants and incumbents, and 2) a dynamic problem where the time paths of labor, capital and innovation are determined.

In the static problem, the social planner minimizes the research cost of a fixed aggregate innovation in a sector by choosing incumbent entry and innovation rates[1]:

$$\min_{z_i, b_i} \psi_i \chi_i z_i^{\gamma} + \chi_i b_i^{\gamma} \quad \text{subject to} \quad z_i + b_i = \tau_i,$$

where z_i is the entry rate, b_i is the innovation rate by incumbents, τ_i is the aggregate innovation rate, and $i = c, x$ represents sectors. Note that the social planner takes into account the externality created by entrants on each other. The resulting cost function (in labor units) for a sector is

$$C_i(\tau_i) = \frac{\psi_i \chi_i \tau_i^{\gamma}}{\left(1 + \psi_i^{1/(\gamma-1)}\right)^{\gamma-1}}. \tag{12}$$

The economy-wide research cost function is the sum of innovation costs across sectors,

$$C(\tau_c, \tau_x) = \frac{\psi_c \chi_c \tau_c^{\gamma}}{\left(1 + \psi_c^{1/(\gamma-1)}\right)^{\gamma-1}} + \frac{\psi_x \chi_x \tau_x^{\gamma}}{\left(1 + \psi_x^{1/(\gamma-1)}\right)^{\gamma-1}}.$$

Using the research labor cost function found in the static problem, the social planner then maximizes the discounted sum of utility from consumption:

$$\max \int_0^{\infty} e^{-\rho t} \ln(K_{c,t}^{\alpha} L_{c,t}^{1-\alpha} Q_{c,t}) dt,$$

subject to constraints stated above, whilst labor constraint is written as $L_{c,t} + L_{x,t} + C(\tau_{c,t}, \tau_{x,t}) \leq 1$.

3.2 Distortions

Innovative activity leads to various distortions in the market equilibrium conditions relative to the social optimum. First, an improvement in the quality level of a good gives market power to the innovator, i.e. she can charge a markup over the marginal

[1] Note that time index is dropped for the sake of simplicity.

cost of production. Second, quality improvements occur over existing innovations ('standing on the shoulders of giants'). Hence, an innovation increases the quality level of a good forever, but the innovator gets the benefit for a limited time, until the next innovation on the good. Third, innovation destroys the profit accruing to the incumbent ('business stealing'). Fourth, the cost of entry into the market by an entrepreneur increases with the measure of total innovative activity by entrants. The first order condition for innovation rates (denoted with by ' ^ ') in competitive equilibrium, and first order condition for social planner innovation (denoted with ' * ') are as follows:

$$c'(\hat{b})w = \frac{1(\pi - c(\hat{b})w)}{\rho + \tau - b}, \tag{13}$$

$$c'(b^*)F_L(K, L, Q) = \frac{\ln(\lambda)F(K, L, Q)}{\rho}, \tag{14}$$

where $c(\cdot)$ is the R&D cost function with in the competitive equilibrium[2] and $F(\cdot)$ is sector-level production function. In equation (14), I rely on the envelope condition: $C'(\tau_i) = c'(b_i)$ for $i = c, x$ in the social planner's problem. For the sake of simplicity of notation, I dropped sector subscripts. Equations (13) and (14) hold for each sector. Following Aghion and Howitt (1992), I compare Equations (13) (competitive equilibrium first order condition) and (14) (social planner F.O.C.) to understand the effect of these distortions on innovation level. These F.O.C.s equate the marginal cost of innovation to the discounted benefit of innovation. The marginal cost of the innovation in the competitive equilibrium is $c'(b)w$, while the marginal cost in the SP problem is $c'(b)F_L(K, L, Q)$. Since firms have monopoly power, the marginal product of labor may differ from the wage rate. This *monopoly-distortion effect* causes the competitive equilibrium innovation level to exceed the SP level (Aghion and Howitt (1992)).

Second, the private flow benefit of innovation is the monopoly profit minus research cost, $\pi - c(b)w$, whereas the social benefit is total output $F(K, L, Q)$, so that b^* exceeds $\hat{b}$ [*Appropriability*]. Third, as a result of innovation, the monopolist takes over the market for the good, and it does not consider the loss the incumbent incurs. Hence, we have '1' in front of π. However, the social planner considers the change in utility as a result of collective innovation. Hence it has $\log \lambda$ in front of $F(\cdot)$. This *business-stealing effect* leads to a higher level of private innovation. Fourth, the private innovator accrues the benefits as long as she has the monopoly power over the good. Therefore, she discounts the profits at a rate $\rho + \tau - b$. However, the benefits of innovation accrue to society forever, since the quality increase lasts eternally. This *inter-temporal spillover effect* yields higher social planner innovation levels.

[2] R&D cost function is $c_i(b_i) = \chi_i b_i^{\gamma}$, $i = c, x$.

The Euler equations in the market economy and social planner are

$$\frac{1}{\lambda_x}\alpha\hat{K}_x^{\alpha-1}\hat{L}_x^{1-\alpha}Q_x - \delta - \rho = \frac{1}{1-\alpha}\hat{\tau}_x \ln\lambda_x, \tag{15}$$

$$\alpha {K^*}_x^{\alpha-1}{L^*}_x^{1-\alpha}Q_x - \delta - \rho = \frac{1}{1-\alpha}\tau_x^* \ln\lambda_x. \tag{16}$$

Monopoly pricing distorts the price of the investment good, leading to differences in the Euler Equations (15) and (16): $1/\lambda_x$ appears in front of the marginal product of capital in the investment goods sector in competitive equilibrium, but not in the social planner equation. This distortion leads to less private capital than the social optimum. However, innovation in investment goods also affects the change in relative price of investment goods. The higher the innovation, the greater the decline in price of investment goods. The greater pace of decline in the price of investment goods makes acquiring capital in initial periods costlier. Hence, regimes that have a higher innovation in investment goods have lower level of capital.

Though it is not the focus of this paper, there is another externality created by the entry process, namely entrants do not internalize the extra entry cost they impose on other entrants. Equation (17) shows the competitive equilibrium first order condition for allocation of innovation between entrants and incumbents whereas Equation (18) shows the social planner allocation of innovation. In the social planner allocation marginal costs of entry and incumbent innovations are equated, but not in the competitive equilibrium. This leads to a more than optimal entry rate.

$$\psi\chi\hat{z}^{\gamma-1} = \gamma\psi\chi\hat{b}^{\gamma-1} \tag{17}$$

$$\gamma\psi\chi {z^*}^{\gamma-1} = \gamma\psi\chi {b^*}^{\gamma-1} \tag{18}$$

Lastly, since capital and labor markets are competitive, the only distortion in the factor demand equations comes from monopoly pricing of the goods. Equating relative factor prices across sectors, I get the undistorted capital labor ratios. Equation (19) is identical in market equilibrium and in the social planner allocation:

$$\frac{1-\alpha}{\alpha}\frac{K_x}{L_x} = \frac{1-\alpha}{\alpha}\frac{K_c}{L_c}. \tag{19}$$

3.3 Allocation of Innovative Resources Across Sectors

To understand the social planner's allocation of innovative resources across sectors, consider an economy where investment goods producers use only labor, that is, $X_t = L_{x,t}Q_{x,t}$. Substituting the production functions into equation 14 at the balanced growth path yields

$$C_c'(\tau_c) = \frac{\ln\lambda_c L_c/(1-\alpha)}{\rho}, \quad \text{and} \quad C_x'(\tau_x) = \frac{\ln\lambda_x L_x}{\rho}.$$

Similarly, equation 16 is rewritten as

$$-\delta + \frac{\alpha}{1-\alpha}\frac{L_c}{K/Q_x} = \rho + \tau_x \ln \lambda_x.$$

The equality of the growth rates of investment and capital stock along the balanced growth path (BGP) implies

$$\frac{L_x}{K/Q_x} - \delta = \tau_x \ln \lambda_x.$$

Rearranging these first order conditions at the BGP leads to the optimality condition,

$$\frac{C_c'(\tau_c)}{C_x'(\tau_x)} = \frac{1}{\alpha}\frac{\ln \lambda_c}{\ln \lambda_x}\frac{\rho + \delta + \tau_x \ln \lambda_x}{\delta + \tau_x \ln \lambda_x}, \tag{20}$$

and the labor resource constraint,

$$1 = \frac{\rho C_c'(\tau_c)(1-\alpha)}{\ln \lambda_c} + \frac{\rho C_x'(\tau_x)}{\ln \lambda_x} + C_c(\tau_c) + C_x(\tau_x). \tag{21}$$

Equation (20) shows that the allocation of innovative resources across sectors depends on the relative *influence* of sectors, $1/\alpha$. Here, *influence* of a sector is defined as the total impact of an increase in the sector's productivity on output (Acemoglu et al. (2012), Bigio and La'o (2016)). As α increases, so that the investment sector becomes more influential, the social planner raises the innovation rate in this sector. Figure 1 depicts equations (20) and (21) for two different values of α. For a given innovation rate in the consumption sector, an increase in α tilts the optimality condition to the right, and the social planner assigns a higher innovation rate to the investment sector, the latter given by the intersection of the solid black optimality curve and the broken red labor constraint curve.

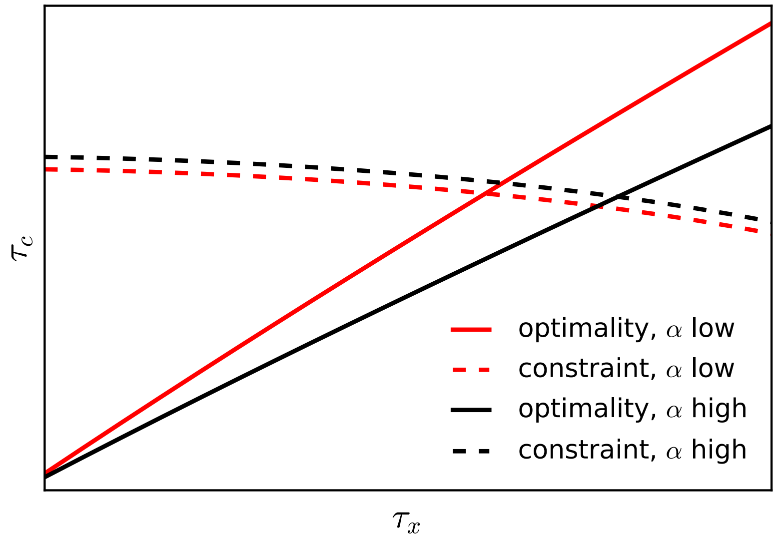
Further, equation (21) shows that an increase in the investment sector's influence frees up labor from production, and the social planner can allocate more labor to innovation. Combining these effects, the social planner allocates more innovative resources to the investment sector as the investment sector becomes more influential. However, the effect on the socially optimum consumption-sector innovation rate is ambiguous.

3.4 Misallocation of Innovative Resources Across Sectors

Turning to the competitive equilibrium, relying on equation (13) and assuming entry is subsidized such that a given level of innovation in an industry is optimally allocated to entrants and incumbents, I characterize the ratio of innovation rates across industries as follows:

$$\frac{C_c'(\tau_c)(\rho + \tau_c - b_c) + c_c(b_c)}{C_x'(\tau_x)(\rho + \tau_x - b_x) + c_x(b_x)} = \frac{\pi_x}{\pi_c} = \frac{1}{\alpha}\frac{\lambda_c - 1}{\lambda_x - 1}\lambda_x(1 - s_{in})\frac{\rho + \delta + \tau_x \ln \lambda_x}{\delta + \tau_x \ln \lambda_x}, \tag{22}$$

Figure 1: Social planner allocation of innovative resources across sectors



where the second equality follows from the F.O.C.s at the BGP.[3] Assuming for simplicity that sectors have identical innovative steps, $\lambda_x = \lambda_c$, equations (22) and (20) illustrate three sources of misallocation of innovative resources across sectors. First, λ_x, which appears in the right hand side of equation (22), shows that market power in the investment sector distorts the equilibrium allocation of production, so that the profit ratio differs from the marginal benefits of sectoral innovation ratio. Second, note that the flow return of innovation is production profit, less future R&D flow costs ($\pi_i - c_i(b_i)$, $i = c, x$). These future R&D flow costs, which appear in the left hand side of equation (22)) drive a wedge between the ratio of the flow return of innovation and the ratio of marginal social benefit of innovation. Finally, the inter-temporal spillover effect (z_c and z_x in the left hand side of equation (22)) leads to non-optimal allocation of innovative resources across sectors. Note that monopoly distortion on wages and the business stealing effect do not lead to misallocation of innovative resources across sectors as both industries have the same wage and the business stealing effect is equal across industries when innovative steps do not differ across industries ($\lambda_c = \lambda_x$).

Suppose further that the industries have identical innovation cost functions, that is, $\psi_x = \psi_c = \psi$, $\chi_x = \chi_c = \chi$.[4] I will add the above stated distortions to the social planner allocation one by one. The first distortion, monopoly power, distorts innovation rates toward consumption sector. To see this, suppose that influence ratio is equal to gross profit ratios as in the competitive equilibrium:

$$\frac{C'(\tau_c)}{C'(\tau_x)} = \frac{\lambda}{\alpha}\frac{\rho + \delta + \tau_x \ln \lambda}{\delta + \tau_x \ln \lambda}.$$

Since $\lambda > 1$ and $C'(\cdot)$ is increasing, introducing monopoly power increases the ratio of innovation rate in the consumption sector relative to the investment sector, $\tau_c/\tau_x >$

[3]Entry subsidy equal $1 - (1 - s_i)\gamma$ achieves conditional optimality, where s_i is the incumbent R&D subsidy.
[4]I still assume $\lambda_x = \lambda_c = \lambda$.

τ_c^*/τ_x^*, where τ_c^*/τ_x^* is the socially optimal industry innovation ratio. This distortion stems from the fact that monopoly pricing increases gross profits in the downstream consumption sector, relative to the upstream investment sector.

Incorporating the inter-temporal spillover effect (the third distortion) in the social planner allocation distorts relative innovation rates towards the industry with lower innovation rates. When innovation functions are identical across sectors, the investment sector has a lower innovation rate owing to its reduced influence on the growth rate of final consumption. Embedding inter-temporal spillovers effect into social planner problem alters equation 20 into:

$$\frac{C'(\tau_c)(\rho + z_c)}{C'(\tau_x)(\rho + z_x)} = \frac{1}{\alpha}\frac{\rho + \delta + \tau_x \ln \lambda_x}{\delta + \tau_x \ln \lambda_x}.$$

Given investment-sector innovation rate $\tilde{\tau}_x$, let $\tilde{\tau}_c(\tilde{\tau}_x)$ represent the consumption-sector innovation rate such that equation 20 holds. Without loss of generality, let the equilibrium investment sector innovation rate be τ_x. Suppose, towards a contradiction, that $\tau_c/\tau_x \geq \tilde{\tau}_c(\tau_x)/\tau_x$ in equilibrium. Then

$$1 < \frac{1}{\alpha}\frac{\rho + \delta + \tau_x \ln \lambda_x}{\delta + \tau_x \ln \lambda_x} = \frac{C'(\tilde{\tau}_c(\tau_x))}{C'(\tau_x)} \leq \frac{C'(\tau_c)}{C'(\tau_x)} < \frac{C'(\tau_c)}{C'(\tau_x)},$$

where the last inequality follows from the fact that $\tau_c/\tau_x > 1$, and entrant innovation rates are proportional to total innovation rates. Note that the strict inequality contradicts the fact that τ_c is the equilibrium consumption sector innovation rate. Hence $\tau_c/\tau_x < \tau_c^*/\tau_x^*$ in equilibrium. Intuitively, the greater the probability of an incumbent firm losing its future profit stream, the greater the inter-temporal spillover effect. With identical innovation functions, the socially optimal entry rate, a sufficient statistic for the threat to incumbents, is higher in the consumption sector, which implies that inter-temporal spillover effect disproportionately lowers the innovation rate in the consumption sector.

The impact of net profit considerations (the second distortion) on the relative innovation rates depends on the curvature of the cost function, and the innovation ratios at the social planner problem. Adding this motive to the social planner problem transforms equation 20 into

$$\frac{C'(\tau_c)\rho + c(b_c)}{C'(\tau_x)\rho + c(b_x)} = \frac{1}{\alpha}\frac{\rho + \delta + \tau_x \ln \lambda_x}{\delta + \tau_x \ln \lambda_x}.$$

In the parameterization of the model, $C_i(\tau_i) = A_i \tau_i^\gamma$, where A_i is a reduced form parameter. Similarly, $c_i(b_i) = \chi_i b_i^\gamma$. Maintaining the assumption of identical innovation functions, and noting that $b_i = a\tau_i$ for some reduced form parameter a, yields

$$\frac{\tau_c^{\gamma-1}(\gamma A \rho + \chi a^\gamma \tau_c)}{\tau_x^{\gamma-1}(\gamma A \rho + \chi a^\gamma \tau_x)} = \frac{1}{\alpha}\frac{\rho + \delta + \tau_x \ln \lambda_x}{\delta + \tau_x \ln \lambda_x}.$$

Applying a similar argument as in the analysis of the inter-temporal effect, and using $\gamma - 1 > 1$, it follows that $\tau_c/\tau_x < \tau_c^*/\tau_x^*$. An industry with a higher total innovation rate has higher incumbent innovation rate. Incorporating the net-profit consideration depresses the innovation rate in the downstream industry more than upstream industry, and hence distorts relative innovation rates towards the investment sector.

4 Calibration

I calibrate balanced growth path (BGP) of my model to averages of data on non-financial corporate sector from 1987 to 2017, subject to the availability of data. A unit length of time in the model is considered as a year in the data. Growth rate of output is targeted to match average growth rate of real gross value added of non-financial corporate sector less net taxes on production and imports, less investment in intellectual property products per worker, $g_Y = 0.016$. Similarly, labor's share of income is calculated as compensation of employees over gross value added less net taxes on production and imports, less investment in intellectual property products per worker. Components of value added data is from Bureau of Economic Analysis (BEA), whereas intellectual property data is from Integrated Macroeconomic Accounts for the United States, published by the Board of Governors of the Federal Reserve System (US). The discount rate is targeted to have a 0.97 annual discount factor, which implies a BGP interest rate, $R = 4.6\%$ slightly higher than estimates of Hall (2003) and McGrattan and Prescott (2005). The depreciation rate, δ, is calibrated to have a 5% annual deprecation rate (KLEMS data on U.S.).

To calibrate the curvature of the R&D cost function, γ, I target price elasticity of R&D with respect to its user cost estimated by Bloom et al. (2002). They estimate both short-run and long-run elasticity of R&D with respect to its user cost. Short–run elasticity, the immediate effect of user cost changes, is estimated as 0.35. This value corresponds to a γ value of 3.85 in my model. However, firms' R&D expenditures are highly persistent. A change in user cost at the current period affects R&D in all subsequent periods. Bloom et al. (2002) estimate a long-run elasticity, sum of R&D changes in all subsequent periods, as approximately 1. This elasticity corresponds to a γ value of 2 in my model. However, neither of these estimates correspond exactly to my model. In the model, firms make R&D decision for each period and get the benefit of R&D immediately. In reality, firms commit to R&D for certain periods of time, but not indefinitely. Therefore, I choose $\gamma = 2.5$, a number that corresponds approximately to midway between the short-run and long-run elasticities. R&D subsidy rates for incumbent firms in the two sectors are chosen as 0.1 to match percentage of business R&D financed by government using OECD data on gross domestic expenditure on R&D by sector of performance and source of funds. Since the focus of this paper is on the inefficiencies of total innovation rate in a sector, I remove the inefficiency in entry by subsidizing/taxing entry accordingly, $s_{e,j} = 1 - (1 - s_{i,j})\gamma$, for $j = c, x$.

Table 1 reports the parameters that are calibrated independently from the data or taken from other papers.

Other parameters of the model are calibrated by using the implications of the model

Table 1: Externally calibrated parameters

	Parameter	Value
Depreciation Rate	δ	.05
Discount Rate	ρ	.03
Curvature of R&D cost function	γ	2.5
R&D subsidy, consumption incumbents	s_c^i	.1
R&D subsidy, investment incumbents	s_x^i	.1
R&D subsidy, consumption entrants	s_c^e	-1.25
R&D subsidy, investment entrants	s_x^e	-1.25

on firm entry and expansion rates. The innovation rate by entrants corresponds to the firm entry rate — the measure of entering production units over the total measure of production units in the sector. The innovation rate by incumbents corresponds to firm expansion rates — the measure of production units captured by incumbent firms over the total measure of production units in the sector. Further, since each production unit in a sector employs the same amount of labor, the innovation rate by entrants is equal to the number of jobs created by entering firms over the total employment in that industry (job creation rate by birth). Similarly, the innovation rate by incumbents is equal to the number of jobs created by expanding firms over the total employment in that industry, which is called job creation rate by expansion. Business Dynamics Statistics (BDS) provides job creation rates by entering establishments and job creation rates by expansion of establishments for major SIC industries. To match the data with my model, I make an identifying assumption that an establishment in the data corresponds to a firm in the model.

I then link observed industries to final goods industries in my model. Each industry in the data produces output that is used as a final good consumption, final good investment, and intermediate input to other industries. Hence, there is no one-to-one link between industries in the data and final good industries in my model. To establish such a link, I first calculate – using the BEA Input-Output tables – the amount of labor required from each industry to produce one unit of each final consumption and investment good. Second, using these industry labor contents of final goods production, I construct entry and expansion rates in the final goods industries as weighted averages of industry entry and expansion rates in the data. Appendix A describes the procedure behind the construction of targets for final goods industries in detail. The first four rows of Table 2 show the targeted innovation rates among entrants and incumbents in each sector.

The model also had implications on the growth rate of the the relative price of investment goods. The growth rate of the price of quality adjusted investment goods is approximately -2.7% (Gordon (1990), Cummins and Violante (2002) and DiCecio (2009)). Technological progress in each industry contributes to the growth rate of consumption (equal to the growth rate of GDP, g_Y, equation 23), whereas the growth rate of the relative price of investment goods depends on the difference of technological

Table 2: Targets

	Variable	Data	Model
Entrant innovation rate, consumption	z_c	.05	.05
Entrant innovation rate, investment	z_x	.047	.047
Incumbent innovation rate, consumption	b_c	.093	.093
Incumbent innovation rate, investment	b_x	.097	.097
GDP per capita growth rate	g_Y	.016	.016
Growth rate of investment good prices relative to consumption good prices	g_{P_x}	-.027	-.027
Labor's share of income		.714	.714

progress in each sector (equation 24):

$$g_C = \tau_c \ln \lambda_c + \frac{\alpha}{1-\alpha}\tau_x \ln \lambda_x, \tag{23}$$

$$g_{P_x} = \tau_c \ln \lambda_c - \tau_x \ln \lambda_x. \tag{24}$$

Therefore, the innovativeness of sectors (λ_c and λ_x) is identified using Equations (23) and (24) and target rates for the consumption growth rate, the change in the relative price of investment goods, and the innovation rates in each industry.

The other parameters of the model seen in Table 3 are calibrated to make the model moments match with the target moments in the data. In the model, labor's share of income is equal to payments to production labor and R&D labor of incumbents over GDP. Note that intellectual property production (R&D) is not included in GDP in the model. The relative costs of entry ψ_x, ψ_c, in each sector are identified by job creation by birth over job creation by expansion rate in these industries. Overall, the results of the calibration exercise indicate that: 1) the quality ladder step size of investment goods is higher than that of consumption goods ($\lambda_x > \lambda_c$), 2) innovation in the investment goods sector is more costly ($\chi_x > \chi_c$). 3) innovation is costlier for entrants ($\psi_x, \psi_c > 1$), 4) entry is more costly in the investment sector ($\psi_x > \psi_c$).

Table 3: Internally calibrated parameters

	Parameter	Value
Quality ladder step size, investment	λ_x	1.25
Quality ladder step size, consumption	λ_c	1.03
R&D cost function parameter, investment	χ_x	6.49
R&D cost function parameter, consumption	χ_c	4.51
Entry cost function parameter, investment	ψ_x	3.00
Entry cost function parameter, consumption	ψ_c	2.52
Elasticity of output w.r.t capital	α	0.26

5 Numerical Analysis

Of the distortions identified in Section 3.2, appropriability and inter–temporal spillover effects cause the economy to under-invest in innovation whereas business stealing and monopoly distortion cause the economy to over-invest in innovation. Whether the economy under-or over-invest in innovation depends on the parameters of the model. In this economy, it is under-investment as shown in Table 4. Column 1 shows the competitive equilibrium consumption growth rate, innovation rates in sectors and discounted capital stock, $\tilde{K} = \frac{K}{Q_x^{1/1-\alpha}}$, which is the capital stock level at the steady state of the economy where variables are discounted accordingly with the technology indices. Column 2 shows the social planner values. Socially optimal innovation rate is 12 percentage points higher than the competitive equilibrium rate in the consumption sector and 17 percentage higher in the investment sector. These higher innovation rates make the economy grow 1.7 percentage point faster under the social planner. In a similar exercise, using a Schumpeterian creative destruction model whose parameters are estimated using Danish firm level data, Lentz and Mortensen (2015) find that the optimal growth rate is twice as much as the competitive equilibrium growth rate.[5]

Later, I discuss the welfare implications of R&D subsidies that push the economy towards social planner allocations. The change in consumption growth rate will play an important role in generating welfare gain. The other important factor that needs to be analyzed is capital stock. Removing the monopoly distortion in the capital Euler equation (15) leads to higher capital accumulation, whereas higher user cost of capital, resulting from higher innovation in investment goods, would result in lower accumulation of capital. Here, the latter force dominates and steady state capital stock of the social planner is lower than capital stock in the market economy. A transition of the economy from competitive equilibrium innovation levels to social planner innovation levels would cause labor allocated to consumption good production to decrease. On the other hand, if the amount of capital invested also decreases then some of the labor allocated to investment good production can be used in consumption good production or research. This will create extra welfare gain in the economy.

To gauge the relative importance of innovations in the two sectors, I conduct the following exercises which are shown in columns 3 (CE τ_c) and 4 (CE τ_x) of Table 4. In the exercise depicted in column 3, I solve the social planner problem while constraining the innovation rate in the consumption sector to the competitive equilibrium value. Now, the social planner allocates more labor to innovative activity in the investment sector and reaches a consumption growth rate close to the unconstrained problem rate. However, in a similar exercise where the investment sector innovation rate is constrained to its competitive equilibrium level instead of the consumption sector, the consumption growth rate is 1.3 percentage points lower than the social planner rate (reported in column 4). Though the social planner increases the innovation rate

[5] They consider a model where firms in a sector have different innovativeness which evolve according to a Markov Process.

Table 4: Competitive Equilibrium vs Social Planner

	CE	SP		
			CE τ_c	CE τ_x
g_C	0.016	0.033	0.030	0.020
τ_c	0.143	0.260	0.143	0.296
τ_x	0.144	0.312	0.325	0.144
$\tilde{K}$	1.995	1.280	1.315	2.421

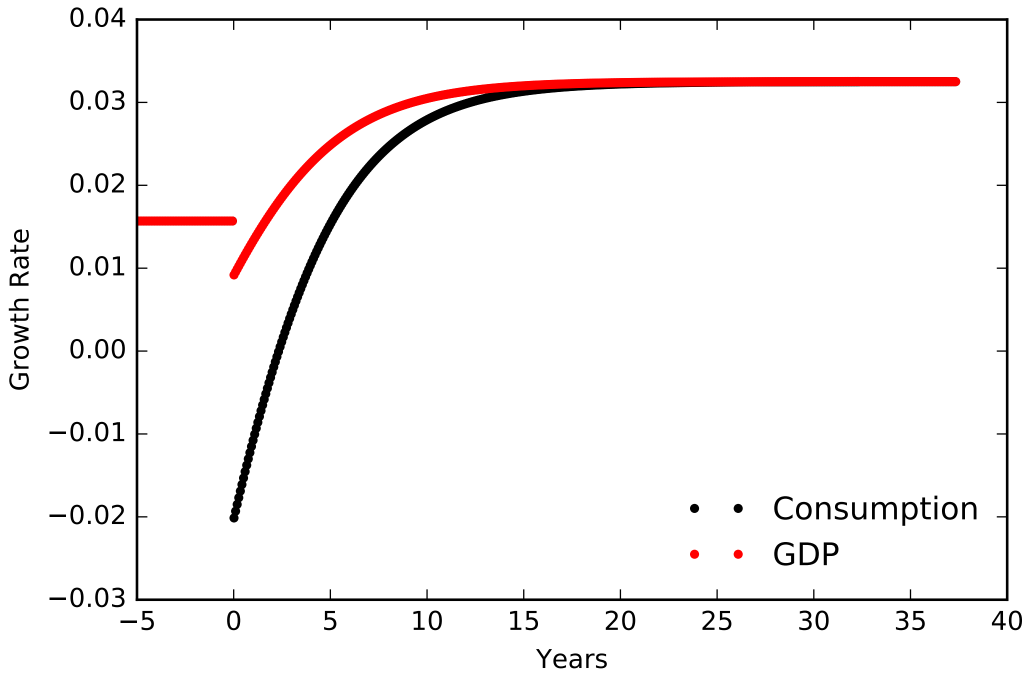
Notes: Column 1 (CE) shows the competitive equilibrium values, column 2 (SP) the social planner values, column 3 (CE τ_c) the social planner values when she is restricted to have competitive equilibrium innovation rate in consumption sector, and column 4 (CE τ_x) when the social planner is restricted to have competitive equilibrium innovation rate in investment sector.

in the consumption sector, it is not enough to compensate purge in the investment sector innovation. These exercises point out that the socially optimum growth rate is significantly higher than the growth rate in the market economy. Also, it is the under-innovation in the investment sector that leads to a large gap between competitive equilibrium and the socially optimum growth rates.

The change in steady state capital stock level is also in line with the analysis comparing the competitive equilibrium with the social planner. When restricting the investment sector innovation to CE levels, steady state capital stock increases. Holding the innovation rate fixed in this sector maintains the user cost of capital at the market economy level. However, the social planner still corrects the monopoly pricing of the investment good. As a result, steady state capital stock increases relative to the competitive equilibrium. However, when the consumption sector innovation is restricted, and the social planner is free to the choose investment sector innovation, capital stock is less than the competitive equilibrium level but higher than the social planner level. The former is expected. The second one may seem contrary to my arguments above. The investment sector innovation rate in the constrained social planner's solution is higher than the unconstrained social planner, and hence the user cost of capital is higher. We would expect a lower accumulation of capital, but we observe a higher accumulation of capital. The reason is: a reduction in the consumption sector innovation frees up some labor which can be allocated to investment good production. This increased labor in the sector leads to higher marginal product of capital. That is why we see an increase in capital accumulation relative to the unconstrained social planner problem.

Analyzing just the balanced growth path does not give the whole picture of welfare and growth implications of the socially optimal innovation. Figure 2 depicts GDP and the consumption growth rate of the social planner equilibrium starting from the balanced growth path of the market economy. Section 6 explains the solution method in detail. Since this is a two sector economy, GDP of the social planner is calculated as if the relative price of the two sectors follows the market economy pricing. The social planner allocates more labor to research and consumption decreases immediately. As the technological progress rate increases, so does the consumption growth rate. However, it takes years for the economy to have a higher consumption growth rate than the market economy balanced growth path. After five years, the consumption growth rate surpasses 1.6 percent and eventually reaches the long-run rate of 3.3 percent. The GDP growth rate also decreases initially, not as much as the consumption growth rate, and then increases gradually to its long-run value of 3.3 percent. Initially, the increase in the growth rate of investment makes the GDP growth rate higher than the consumption growth rate. There are two countering forces that affect the investment growth rate. Discounted capital stock goes down under the social planner, which leads to a reduction in investment. However, because of an increase in the growth rate of the quality of investment goods, investment growth rate goes up. The second force dominates and we observe an increase in the growth rate of investment and hence GDP goes down less than the consumption. Later on, as consumption growth rate keeps increasing and the investment growth rate keeps decreasing, the GDP growth rate converges to 3.3 percent. The transition from the market economy balanced growth path to the social planner equilibrium generates almost 22 percent welfare gain, measured in consumption equivalent terms.

Figure 2: GDP and Consumption Growth Rates, Social Planner



To understand the relative importance of the distortions described in Section 3.4, I conduct the following exercise which is reported in Table 5 . I will add the distortions to the social planner allocation one by one. In line with the theoretical analysis of Section 3.4, monopoly power distorts innovation rates toward consumption sector, and net profit consideration distorts innovation rates toward the investment sector. However, the inter-temporal spillover effect increases the ratio of consumption sector innovation rate over the investment sector innovation rate. As predicted by my theoretical analysis, the inter-temporal spillover distorts innovation rates towards the industry with lower innovation rate. Because the investment sector has higher innovation rate in the social planner allocation of the calibrated model, introduction of the inter-temporal spillover effect into the social planner problem distorts allocation of innovative resources toward the consumption sector.

Table 5: Relative Industry Innovation Rates

	Consumption / Investment, τ_c/τ_x
Social planner	0.83
Social planner + monopoly power	0.94
Social planner + inter-temporal spillover	0.9
Social planner + net profit	0.7

Notes: Social planner industry innovation rate ratios when externalities added one by one.

6 Innovation Subsidies

There is under-investment in innovation in both sectors as established in the previous section. Building on this result, I analyze the role of R&D subsidies in bringing the innovation rates to socially optimal levels and increasing welfare. I show that long-run welfare of the society can be increased substantially by providing R&D subsidies to incumbent and entering firms in each sector. Considering only time-invariant subsidies, welfare of the society can be increased by as much as 21 percent over the long-run.

In the previous section, various distortions of the economy are explained. My focus in this section is how the government can increase welfare by subsidizing innovation in both sectors, and at what rates the innovative activities in each sector should be subsidized. To answer these questions, I compare the welfare gains of various subsidy systems, which consist of entry subsidy rates in each sector, and the incumbent firm R&D subsidy rates in each sector. The subsidy system is financed by lump-sum taxation of households. Also, in all of the subsidy systems, the entry subsidy rate is adjusted relative to the incumbent firm R&D subsidy rate to make innovative resource allocation within sectors across entering and incumbent firms to be optimal. Therefore, welfare comparisons of subsidy systems reflect welfare changes resulting from total innovation in that sector.

Starting from the balanced growth path of the benchmark economy (described in the calibration section), I alter the subsidy rates (unexpected to agents in the economy) for all the subsequent times and keep them constant. Then I calculate the transition to new balanced growth path under the new subsidy system. Afterwards, I calculate the welfare gain/loss of the subsidized economy relative to the benchmark economy. The algorithm I used to calculate the welfare impacts of the subsidy systems is described as follows.

1. Discount the variables that grow at the balanced growth path with the technology indices that leads to growth.
2. Solve for the steady states of the benchmark economy and subsidized economy.
3. Using the reverse shooting algorithm described by Judd (1998), solve the transition of the economy from the steady state of the benchmark economy to the steady state of the subsidized economy.
4. Starting from the steady state of the benchmark economy and by normalizing the technology indices at this steady state equal to one, simulate the economy forward and generate the consumption sequence (non-discounted). Attain two consumption sequences that will be used to compute welfare gain: 1) the consumption sequence of the benchmark economy, 2) the consumption sequence of the subsidized economy.
5. Calculate the sum of discounted utility of these two consumption sequences. Equation (25) is the closed form solution of the sum of the discounted utility of the benchmark economy which is at the balanced growth path, where C_0 is the consumption amount at the time of subsidy change and g_C is the consumption growth rate. The sum of discounted utility of the subsidized system is calculated using numerical integration over the utility values of consumption sequence,

$$W(C_0, g_C) = \frac{1}{\rho}\left(\ln C_0 + \frac{g_C}{\rho}\right). \tag{25}$$

6. Calculate the consumption equivalent welfare change described in Equation (26). The welfare gain/loss is equal to ξ: the rate of increase in consumption in the benchmark economy that will make the representative household indifferent with moving to the subsidized economy,

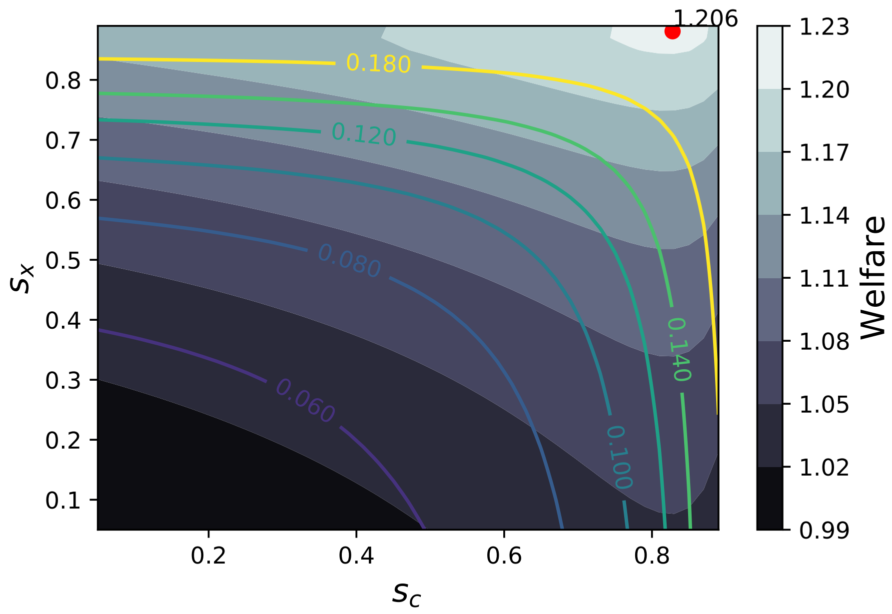
$$W(\xi C_0, g_C) = \int_0^\infty \exp(-\rho t)\ln(C_t^s)dt, \tag{26}$$

where C_t^s is consumption at time t in the subsidized economy.

I calculate the welfare gain of the subsidy systems in this set: $\{(0, s_c, s_x, 1-(1-s_c)\gamma, 1-(1-s_x)\gamma) : s_c = 0.05, .0.07, ..., .89, \quad s_x = 0.05, .07, ..., .89\}$. By considering subsidies from 5 percent to 89 percent, I cover all the relevant subsidy rates. The welfare gains of the subsidies in this set are depicted in Figure 3 as a contour map. The total amount of R&D expenditures these subsidies induce are shown in Appendix C.

There are several results worth highlighting. First, holding subsidy of a sector constant as the subsidy of the other sector increases, so does the welfare gain until a certain point. Afterwards more subsidy results in a reduction in welfare gain. The same result holds when subsidies to both sectors increase simultaneously. Remembering the distortions identified in Section 3.2, the competitive equilibrium innovation rate can be above or below the social planner innovation rate. In this economy, it

Figure 3: Welfare Gain



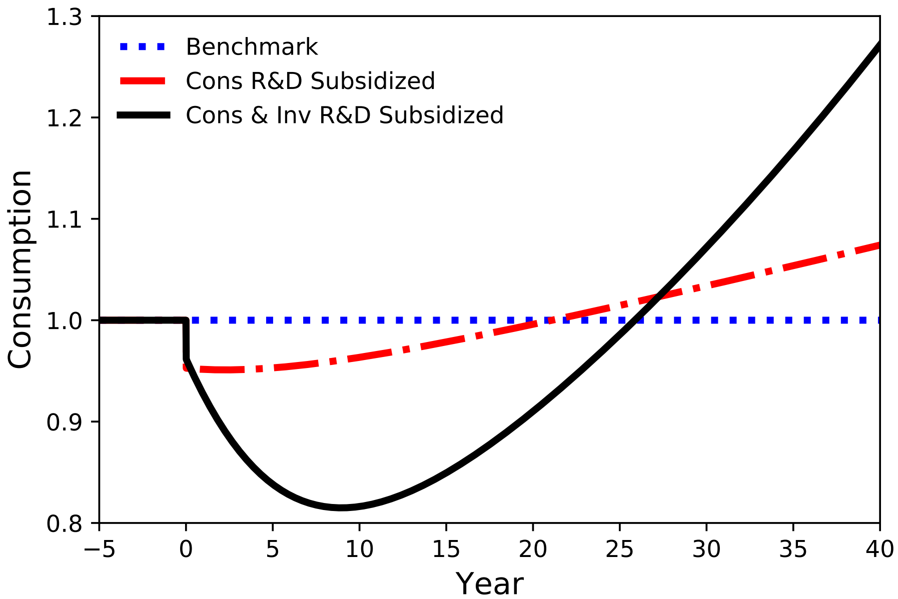
Notes: Contour map of welfare gains of R&D subsidies. The curves on top of contour shades show total R&D labor in the economy at the balanced growth path.

is below. Therefore, raising innovation rate to socially optimal levels leads to higher welfare. When the innovation rates surpass the optimal levels, welfare gains start decreasing. Maximum welfare gain is attained by subsidizing consumption sector R&D by 83 percent and investment sector by 88 percent, which result in about 21 percent welfare gain. However, this result suggests that the level of under-investment in innovation is quite high for both sectors. Second, iso-welfare curves are tilted towards investment sector R&D subsidy. A given rate of subsidy generates higher welfare gain when it is applied to only investment sector than when it is applied only to the consumption sector.

There are two main reasons that correcting for the distortions requires more than 80 percent R&D subsidy to each sector. First, as explained in the Section 5, there are large distortions in the economy. The amount of resources allocated to R&D under the social planner is more than the amount of innovative resources in the market economy. This high level of increase in resource allocation to innovation under the social planner is common in the models based on Klette and Kortum (2004). Lentz and Mortensen (2015) show that social planner increases innovative resources threefold. Similarly, Segerstrom (2007) find that innovation should be heavily subsidized. Second, subsidizing R&D also promotes a higher entry rate by increasing the value of firms. The higher entry rate corresponds to a higher probability for an incumbent firm to shrink by one good. In other words, inter-temporal spillover effect increases which decreases incumbent firms incentive to innovate. To compensate for the higher inter-temporal spillover, firm R&D needs to be subsidized even more.

How does this economy achieve the maximum welfare gain? Analyzing the trajectory of consumption helps us to answer this question. Figure 4 shows the trajectories of consumption in the benchmark economy, when only the consumption sector R&D is subsidized by 83 percent, and a 88 percent investment sector R&D subsidy is added on top of all the other subsidies.[6] For better comparison of consumption paths after the subsidy to the benchmark economy, I discounted each consumption path in the figure with the benchmark economy consumption. Allocating more research labor to innovation results in reduction in consumption goods production in earlier periods but a higher long-run consumption growth rate. Consumption in the consumption sector subsidized economy rebounds more quickly. However, consumption in the investment sector subsidized economy surpasses the consumption subsidized economy in later years. The reason why consumption grows more slowly in earlier periods with the investment sector subsidy lies in the response of capital to subsidies. Subsidizing investment sector R&D leads to higher innovation rates in this sector. This leads to a lower growth rate of the price of investment goods (higher in absolute terms) and higher user cost of capital. Therefore, capital accumulates slowly. Hence, in earlier years consumption grows at a lower rate when investment sector is subsidized. Later on, after the economy reaches the balanced growth path, the higher innovative step of investment sector generates a higher consumption growth rate. Therefore, consumption in this economy catches and surpasses the benchmark and consumption sector subsidized economy.

Figure 4: Sequence of Consumption with Different Subsidies



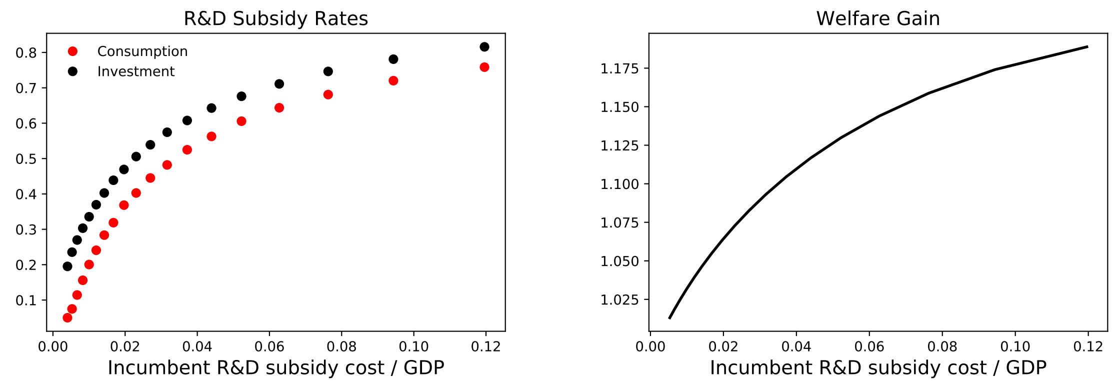
Notes: Consumption paths are relative to benchmark economy consumption level.
Cons R&D Subsidized: Consumption R&D subsidy of 83%,
Cons & Inv R&D subsidized: The subsidy system that maximizes the welfare gain.

[6]Remember that entrant innovation is subsidized at a lower rate to correct for the congestion externality in the entry process.

6.1 Welfare Gains with Limited Transfer Budget

The amount of tax collection required to subsidize incumbent R&D in order to reach peak welfare gains is more than 21 percent of GDP. This amount is unreasonable for two issues not modeled in this paper: the distortionary effects of taxation, and the political economy of taxation. Therefore, a related question concerns the fiscal authority's allocation of subsidies across sectors when its transfer budget is limited by some unmodeled factors. On the one hand, a given rate of investment sector subsidy leads to higher welfare gains than an equal rate of consumption sector subsidy. On the other hand, a given rate of investment sector subsidy costs more than an equal rate of consumption sector subsidy. This trade–off determines the optimal allocation of a limited transfer budget. In this economy, the welfare gain advantage of investment sector dominates.

Figure 5: Cost Constrained Optimal R&D Subsidy

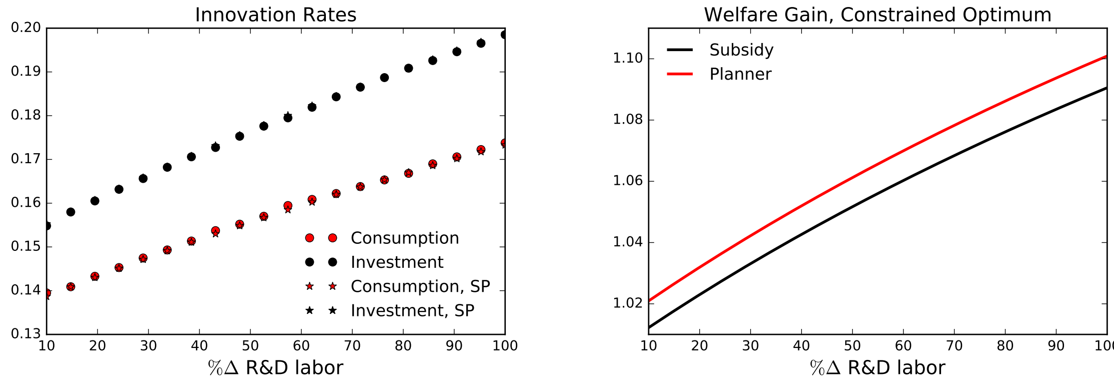


Notes: Optimal R&D subsidies to sectors under limited transfer budget and associated welfare gain.

The left panel of Figure 5 shows optimal government R&D subsidy rate to sectors for different ratios of incumbent firm R&D subsidy cost to GDP at the balanced growth path. It is always optimal to subsidize the investment sector at a higher rate than the consumption sector. For example, if the tax authority increases its incumbent R&D subsidy budget as a share of GDP by 10 percent, it can achieve 0.17 percent welfare gain by subsidizing the consumption sector by 5 percent and the investment sector by 14.1 percent. The right panel of Figure 5 shows the welfare gains associated with optimal R&D subsidies under a limited transfer budget. As the total amount of government budget allocated to R&D subsidies to incumbents increases, the welfare gains in consumption equivalent terms increase at a decreasing rate.

A similar exercise, depicted in Figure 6, analyzes the optimal government subsidy and social planner allocations when the total amount of labor allocated to R&D is constrained. First, the social planner with an R&D labor constraint equal to the total R&D labor in the BGP of the competitive equilibrium can still generate a 0.9 percent welfare gain by increasing the innovation rate in the investment sector from 14.4 percent to 14.9 percent and reducing the innovation rate in the consumption sector from 14.3 percent to 13.4 percent.[7] Second, the constrained social planner allocation that increases the total R&D labor by 10 percent leads to a 2.1 percent welfare gain. In this new allocation, the innovation rate in the investment sector is equal to 15.5 percent and that in the consumption sector is equal to 13.9 percent.

Figure 6: Labor Constrained Optimal R&D Subsidy



Notes: Sectoral innovation rates under an R&D labor constraint, and the associated welfare gains. %Δ R&D labor represents the increase in the total R&D labor relative to the BGP of the benchmark economy, in percentages.

The left panel of Figure 6 depicts innovation rates resulting from the constrained optimum government subsidy and social planner allocations, whereas the right panel shows the welfare gains of such government policies and social planner allocations. Both the constrained optimum government subsidy and the social planner achieve the same innovation rates. Furthermore, in both the social planner allocation and the competitive equilibrium, the innovation rate in the investment sector is higher than the innovation rate in the consumption sector. The social planner achieves higher welfare gains than the government subsidy. This partially stems from the social planner's ability to correct the distortions in the Euler equation, set time varying innovation rates on the transition, and the social planner's greater flexibility when dealing with multiple distortions in the innovation process.[8]

In this section I argue that the government should subsidize investment sector innovation at a higher rate than the consumption sector. Similarly, the social planner sets a higher innovation rate in the investment sector. However, this result may seem contrary to the results of Section 3.4, in which I argue that the social planner sets a higher innovation rate in the consumption sector. In the theoretical analysis of Section 3.4, I assume the industries have identical innovation functions, and capital is not used in investment good production. When industries have identical innovation functions, and industries differ mainly with respect to their location in the supply chain, it is optimal to set higher innovation rates in the consumption sector as it is closer to the final consumption and hence has higher *influence*. However, in this section, the

[7] The social planner also corrects the distortion in the Euler equation as a result of investment goods producers' monopoly power, and this correction contributes to the welfare gain.

[8] Recall that I consider only constant subsidy rates over time.

investment sector has a larger innovative step, $\lambda_x > \lambda_c$. A higher quality improvement in the investment sector following successful innovation makes it socially optimal to subsidize that sector at a higher rate.

7 Comparison to Atkeson and Burstein (2018)

A key contribution of my model is incorporating an investment good sector, distinct from the consumption sector, into Klette and Kortum (2004) (KK) model. To highlight the importance of this heterogeneity in optimal R&D policy design, I compare my results with those obtained using the Atkeson and Burstein (2018) (AB) methodology.

The AB model nests many endogenous and semi-endogenous growth models including the KK model. It has a rich structure in many aspects. For example, it incorporates both expanding variety and quality ladder type innovations as well as own good innovations among incumbent firms. AB develop a methodology to linearly approximate output and productivity trajectories. Using this methodology, AB analyze changes in the output and productivity trajectories following a policy-induced change in the economy's innovation intensity, and how these responses vary by the degree of inter-temporal knowledge spillovers.

My model is not nested in the AB model, which assumes that the allocation of innovative resources is conditionally efficient. In my model, as described in Section 3, externalities in the innovation process and differences in markups and innovation functions across sectors lead to the misallocation of innovative resources across sectors. The social planner can therefore increase welfare by reallocating innovative resources across sectors without altering the total amount of innovative resources. In fact, the constrained optimum social planner allocation increases welfare by 0.9 percent by increasing the innovation rate in the investment sector and reducing the innovation rate in the consumption sector. Furthermore, I analyze sector-dependent optimal government R&D subsidy policy, which is outside the scope of AB. AB does not analyze the impact of non-proportional changes in innovation subsidies when the allocation of innovative resources across firms at the initial BGP is conditionally inefficient.

To compare my results with AB methodology, I analyze the impact of a policy-induced 10% increase in R&D labor on the economy in four cases: i) a one-sector version of my model where consumption and investment goods are produced by the same sector and the model is solved by AB approximation, ii) a one-sector version of my model where consumption and investment goods are produced by the same sector and the model is solved non-linearly iii) a two-sector version of the model with sector-dependent R&D subsidies, iv) a two-sector version of the model with a uniform subsidy across sectors. Since the one-sector version of my model is nested in the AB model, the AB methodology is well-suited to solving for the transition path of the economy. Finally, where applicable, I appeal to AB's results on endogenous growth models.[9]

[9]Specifically, I rely on Proposition 2 of Atkeson and Burstein (2018). To a first-order approximation, the new

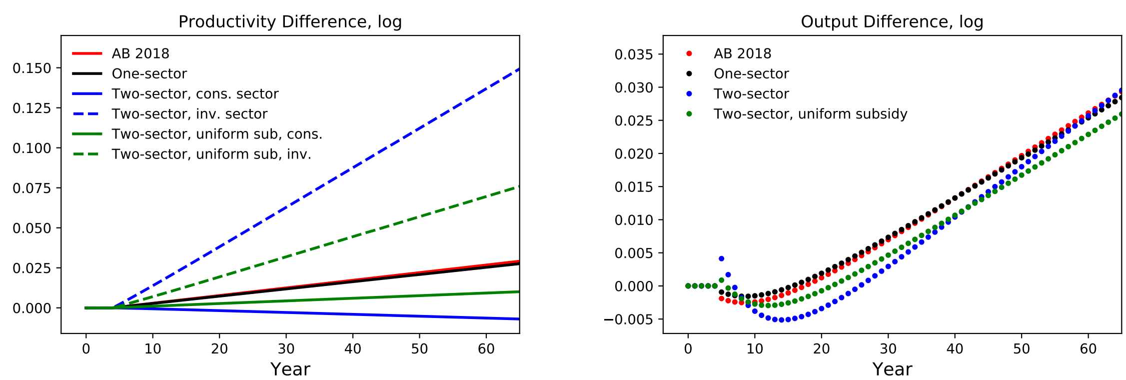
In each case, the chosen subsidy rates maximize consumption-equivalent welfare gains by increasing R&D labor in the new BGP less than 10% of the initial BGP. Table 6 lists the subsidy rates, the resulting BGP growth rates, and consumption-equivalent welfare gains in each case. Figure 7 plots log difference of sectoral productivity relative to the initial BGP productivity, and the log difference of output relative to the initial BGP output in each case.

Table 6: Policy Exercises

Case	Model	Sector	Subsidy	BGP Growth Rate	Welfare Gain
i)	One-sector (AB methodology)	–	17.9%	–	–
ii)	One-sector (non-linear)	–	17.9%	1.63%	1.35%
iii)	Two-sector	Consumption Investment	5% 23.8%	1.65%	1.22%
iv)	Two-sector	Consumption Investment	17.6% 17.6%	1.63%	1.14%

As seen in the left panel of Figure 7, the linear approximation of productivity using the AB methodology closely tracks the non-linear solution of the productivity trajectory in the one-sector version of my model. A permanent increase in R&D subsidy from 10% to 17.9% leads to a reduction in output relative to the initial BGP in earlier periods, and an increase in output in later periods. Output using the AB approximation remains within 0.1% of the non-linear solution in each of the 60 years after the policy change. While hardly surprising since this case is nested by Atkeson and Burstein (2018), it is nonetheless reassuring to establish the closeness of the two solutions.

Figure 7: Productivity and Output Dynamics



Notes: Trajectories of productivity and output after a policy induced 10% increase in R&D labor.

path of productivity, $\{Z'_t\}_{t=1}^{\infty}$, is equal to $\log Z'_{t+1} - \log \bar{Z}_{t+1} \approx \sum_{j=0}^{t} \Theta_e(\log l'_{rt-j} - \log \bar{l}_r)$, where Θ_e is the impact elasticity of the economy with respect to entrant innovation, and $\bar{l}_r$ is the R&D labor (variables with bar represent initial BGP values). In my model, $\Theta_e = \frac{z^{1-\gamma}}{\gamma\psi\chi} \ln \lambda(\psi\chi z^{\gamma} + \chi b^{\gamma})$.

In general, optimal subsidies differ from those implied by the AB methodology whenever the allocation of innovative resources across sectors is conditionally inefficient. Case (iii) corresponds to such an environment, where a 23.8 percent R&D subsidy to investment sector incumbents, and a 5 percent R&D subsidy to consumption sector result in a 10 percent increase in R&D labor. In turn, this leads to an approximately 13% increase in investment sector productivity over 60 years, and a reduction in consumption sector productivity relative to the initial BGP. Immediately after the policy change, consumption increases and investment decreases as a result of the increased rate of decline in the price of investment goods. Even though total production labor falls, some production labor is shifted from investment good production to consumption good production. Following this reallocation, output falls short of initial BGP output as a result of lower production labor and lower investment. Eventually, increased productivity in the investment sector drives output above the initial BGP levels, the economy converges to a 1.65% growth rate, and experiences a 1.22% consumption-equivalent welfare gain. Comparing cases (iii) and (i) shows that the trajectories of the two economies differ mostly in early periods. One year after the change in innovation subsidy policy, output in case iii) is 0.6% higher than output in the AB approximation. However, about 6 years after the policy change, the AB approximation predicts a higher output than the sector-dependent R&D subsidies. Fifteen years after the policy change, predictions of output with the AB approximation are about 0.46% higher than those in the model with sectoral heterogeneity. Long-run trajectories are closer, with the BGP output growth rate at 1.65% in case (iii), and 1.63% in case (ii).

In case (iv) I consider a uniform subsidy – 17.6% R&D subsidy to incumbent R&D in each sector – that increases R&D labor by 10%. Trajectories of sectoral productivity differ from that of case (iii). Output trajectories also differ, with higher BGP growth rate in the sector-dependent subsidy. In the calibration of the model, λ_c is low, and subsidizing consumption sector at a higher rate does not lead to substantial changes in the economy.

Overall, I confirm that the AB methodology provides a powerful tool to approximate output and productivity trajectories following a policy-induced changed in the amount of research labor in a one-sector economy when innovative resource allocations are conditionally efficient. However, these techniques do not apply to the baseline model, which features heterogenous sectors and a conditionally inefficient allocation of innovative resources across sectors.

8 Conclusion

I analyze heterogeneity of innovative activity across sectors in a quantitative environment where firm level innovation is the main driver of the long–run macroeconomic growth. I ask how a government that wants to increase welfare of the society through R&D subsidies should target different sectors on the economy. To answer this and related questions, I develop a quality ladder type of model based on the framework of Klette and Kortum (2004) that features two sectors: consumption goods producers and investment goods producers. These sectors differ mainly in their output's use, R&D cost functions, and quality ladder steps sizes. An industry is classified as a consumption goods industry if household consumption of the industry's output is bigger than investment and inventory allocation made from its output. It is classified as an investment goods industry if vice versa. I calibrate my model using its firm dynamics implications and US data on job creation and destruction. An interesting result of calibration is investment sector firms are more innovative, have a higher quality ladder step, but have a higher cost of innovation.

A sector's contribution to macroeconomic growth and welfare of the society depends on the sector's position in the supply chain of the economy, its innovation rate, and the quality increase (or cost reduction) of the goods after a successful innovation in the sector. Consumption sector innovation affects consumption growth directly, whereas investment sector innovation affects consumption growth indirectly through its effect on the capital stock of the economy. Also, the consumption sector generates about the same amount of innovation as the investment sector. In this sense, the consumption sector contributes more to growth. However, the investment sector is more innovative. Once it innovates, it increases the quality of existing goods more than the consumption sector. The number of innovations, say on central processing units (CPUs), are lower than the number of innovations, say on restaurants. However, once a better CPU is developed, its quality increase is higher than the quality increase of better restaurant food.

The Schumpeterian innovation process described in the model leads to various distortions in the economy and innovation rates in both sectors are lower than socially desirable levels. Therefore, government can increase the welfare of the society in the long run by subsidizing R&D. This welfare gain, in consumption equivalent terms, can reach up to 21 percent. A given rate of R&D subsidy to the investment sector generates more welfare gain than an equal amount of R&D subsidy to the consumption sector.

A more realistic policy question is how the government should allocate a limited transfer budget. Though the investment sector has higher innovativeness, innovation is costly in this sector. In optimality, a subsidy system tilted toward the investment sector generates more welfare gain than a uniform subsidy system with the same cost.

Many of the results rely on quality ladder steps, which are identified by four statistics: the consumption growth rate, the growth rate of the relative price of investment goods, and the innovation rates in each industry. Any mismeasurement of these statistics would lead to biased results. For example, if the growth rate of relative price of investment goods was affected by factors other than the quality increase, the results would not be accurate. Similarly, in the model, the only source of quality improvement is dedicated R&D activity. In this sense, innovation in my model is regarded in the broadest sense: any activity that leads to quality improvement is innovation.

Therefore, this strong assumption also accounts for the large effects found in my paper.

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A Calibration of Sector Dependent Parameters

Sector level variables that I target in my calibration are: innovation rates of entrants, innovation rates of incumbents. In contrast to the stylized model, it is impossible to neatly classify sectors in the data as consumption, investment, or intermediate inputs; output from a given sector serves all three roles. Therefore, to construct sector-level targets in my model, I employ the following steps:

1. Using Business Dynamics Statistics (BDS), compute innovation rates by entrants and incumbents in each sector classified according to SIC system.
2. If a sector level data is not in I-O classification system, using the crosswalk described below, convert the industry level data with SIC classification system into industry level data with I-O classification system.
3. Using the weights constructed from Input-Output tables, compute final good industry targets as the weighted average of I-O industry values.

A.1 Industry Level Targets

In my model, since each product line in a sector employs the same amount of labor, there is a one to one relation between job creation rate and innovation rate in an industry, i.e. number of jobs created by entering firms over total employment in a sector is equal to innovation rates by entrants. Similarly, number of jobs created by expanding firms over total employment in a sector is equal to innovation rate by incumbents in that industry. Therefore, at the industry level, I target job creation rates by entering establishments, and job creation rate by expanding establishments obtained from Business Dynamics Statistics (BDS). Here, I make an identifying assumption that an establishment in the data corresponds to a firm in my model.

A.2 Crosswalk into I-O Industry Classification

While BDS follows the SIC industry classification, the I-O data are classified according to I-O system, a variant of NAICS. I use the following crosswalk to convert SIC industry data to I-O industry data. Let $M_{i,t,SIC}$ be the value of any variable, M, in SIC industry i in year t. I construct the corresponding value in NAICS super-sector s, denoted $M_{s,t,NAICS-SUPER}$, as the employment–weighted share of M across SIC industries:

$$M_{s,t,NAICS-SUPER} \equiv \sum_i e_{s,i}\, M_{i,t,SIC},$$

where $e_{s,i}$ is percentage of employment in NAICS industry s coming from SIC industry i in first quarter of 2001 (Bureau of Labor Statistic (BLS) [10]) Even though some I-O industries correspond to NAICS super sectors, some I-O industries are disaggregates of them. I disaggregate such NAICS super-sectors into 2-digit I-O industries, $M_{j,2-digit}$,

[10]See: https://www.bls.gov/ces/cesnaics02.htm

using Current Employment Statistics data on employment shares of 2-digit industries in NAICS super-sectors in first quarter of 2001 from BLS:

$$M_{j,t,2-digit} = e_{j,s} M_{s,t,NAICS-SUPER},$$

where $e_{j,s}$ is the employment share of 2-digit I-O industry j in NAICS super-sector s in the first quarter of 2001.

A.3 Construction of ω_i^c and ω_i^x

To construct the targets for final goods producing industries in my model, I take weighted average of industry level values where weights are industry-labor requirements to produce one unit of final good. Let M_c and M_x be calibration targets for final consumption and final investment industry. Then, $M_c = \sum_j \omega_j^c M_j$, and $M_x = \sum_j \omega_j^x M_j$, where M_j is the value of the particular target M observed in data for I-O industry j.[11] ω_j^c and ω_j^x are amounts of labor needed from industry j to produce one unit of final good, consumption or investment, as a fraction of total amount of labor needed to produce a unit of final good, consumption or investment.

ω_j^c and ω_j^x are constructed as follows. First, calculate the industry-labor requirements to produce one unit of final output in each industry, L^c and L^x:

$$L^c = l(I-B)^{-1}C, \quad L^x = l(I-B)^{-1}X,$$

where l is a diagonal matrix consisting of $l_{jj} = \frac{N_j}{Y_j}$, employment in industry j (N_j) over gross output in industry j (Y_j), I is the identity matrix, B is input-output matrix with elements $B_{ij} = \frac{y_{ij}}{Y_j}$ representing intermediate input usage of industry j from industry i (y_{ij}) over the gross output of industry j (Y_j), C is a vector of consumption shares across industries, $C_j = \frac{\text{Household consumption of output of industry } j}{\sum_j \text{Household consumption of output of industry } j}$, and X be a vector of investment shares across industries, $X_j = \frac{\text{Output of industry } j \text{ used as investment}}{\sum_j \text{Output of industry } i \text{ used as investment}}$. Second, using industry-labor requirements, I calculate final good weight of an industry as $\omega_i^c = L_j^c / \sum_j L_j^c$, and $\omega_j^x = L_j^x / \sum_j L_j^x$. The data used to construct the above stated variables are obtained from Bureau of Economic Analysis Input-Output use tables after redefinitions.

I then employ the algorithm described above to find sector level aggregates such as number of jobs created by entrants and incumbents, employment, compensation of employees, value added, etc. Notice that, financial services industry (FIRE) is included in the I-O table to construct the labor content of the final goods, however, FIRE is dropped from the analysis while constructing weights, and hence targets for the final goods.

[11]These industries are classified according to I-O system, a variant of NAICS, and corresponds to mining and logging, utilities, construction, manufacturing, wholesale trade, retail trade, transportation and warehousing, information, education and health services, leisure and hospitality, other services, financial activities, professional and business services.

B Details of the Model

B.1 Consumption Goods Producers

I define the problem of a differentiated consumption good producer in two steps. First, I define the static problem: how much to produce, and demand for factor inputs. After solving this problem and establishing the profit from production, I turn to the dynamic problem: how much to invest in R&D to maximize the value of the firm.

Each production unit of a firm has a Cobb-Douglas production function with capital elasticity α. Production of each unit is independent of other production units a firm may possess. Hence, each production unit solves the following cost minimization problem:

$$\min_{l_c,k_c} wl_c + rk_c \quad \text{subject to} \quad k_c^\alpha l_c^{1-\alpha} = c,$$

where w and r are the market wage and capital rental rates. The resulting cost function, $C((w,r),c) = \frac{r^\alpha w^{1-\alpha} c}{\tilde{\alpha}}$, with $\tilde{\alpha} \equiv \alpha^\alpha(1-\alpha)^{1-\alpha}$, is common across all the production units in a sector. As a result, Bertrand competition yields a price $p^j = \lambda_c \frac{r^\alpha w^{1-\alpha}}{\tilde{\alpha}}$ for differentiated product j. Using this price and the demand function for differentiated goods yields the profit of a differentiated good producer:

$$\pi = pc - C((w,r,c)) = \left(1 - \frac{1}{\lambda_c}\right) Z. \tag{27}$$

Note that profits do not vary across differentiated goods in a sector.

Turning to the dynamic problem, I use the profit function in (27) to derive the firm's value function, which can be expressed either as a function of the level of research labor or, more conveniently, the level of innovation arrival rate per good. Let $\phi(\beta, m)$ denote the level of l_R implicitly defined by $\varphi(m, l_R) = \beta$. Since $\varphi(\cdot)$ is strictly increasing in l_R, and homogenous of degree one, $\phi(\cdot)$ is well-defined and homogenous of degree one and convex in β. Set $\phi_c(b_c, m) = m\chi_c b_c^\gamma$, where $b_c = \beta/m$, and $\chi_c > 0$ is a scale parameter.

The Bellman equation of the firm on the balanced growth path is

$$\begin{aligned} RV(m,Z) &= \max_{b_c \geq 0} \Bigg\{ \left(1 - \frac{1}{\lambda_c}\right) mZ - (1-s_c^i) w \phi_c(b_c, m) + \frac{\partial V(m,Z)}{\partial Z} \dot{Z} \\ &+ mb_c[V(m+1,Z) - V(m,Z)] + m\tau_c[V(m-1,Z) - V(m,Z)] \Bigg\}, \end{aligned}$$

where s_c^i is the rate of R&D subsidy for the consumption sector incumbents, and τ_c is the equilibrium Poisson innovation arrival rate in the consumption sector. Given that firm profits, and R&D expenditures are linear in the number of goods, I conjecture that $V(m,Z) = \nu_c mZ$ for some $\nu_c > 0$ and verify this claim. Inserting the guess yields

$$R\nu_c mZ = m\left(1 - \frac{1}{\lambda_c}\right) Z - (1-s_c^i) wm\chi_c b_c^\gamma + \nu_c mZ g_Z + mb_c\nu_c Z - m\tau_c\nu_c Z,$$

where b_c is the optimal innovation intensity, and $g_Z \equiv \frac{\dot{Z}}{Z}$ is the growth rate of household consumption expenditure.

B.2 Solution of the Model

The representative household maximization problem is described in Section 2.1. Consumption is a quality adjusted aggregation of differentiated consumption goods described in Equation (1). Since, I solve for a symmetric equilibrium and limit pricing is assumed, highest quality versions of each differentiated consumption product gets the same positive demand, and the lower quality versions have a demand of zero. This demand function is described in Equation (2). Then we simplify equation 1 into

$$C = \exp\left(\int_0^1 \ln\left(q(\omega)c(\omega)\right) d\omega\right), \tag{28}$$

where $q(\omega)$ is the highest quality level of product ω, and $c(\omega)$ is the consumption of product ω with highest quality. Also, using the fact that the production function of differentiated goods in a sector is identical, and symmetric demand, the labor hired and capital rented across differentiated goods is the same, the production for each differentiated unit becomes $c(\omega) = k_c^\alpha l_c^{1-\alpha}$, and k_c and l_c do not depend on the product. Therefore, the aggregate consumption function turns into

$$C = \exp\left(\int_0^1 \ln\left(q(\omega)k_c^\alpha l_c^{1-\alpha}\right) d\omega\right) \tag{29}$$

$$C = k_c^\alpha l_c^{1-\alpha} \exp\left(\int_0^1 \ln\left(q(\omega)\right) d\omega\right) \tag{30}$$

$$C = k_c^\alpha l_c^{1-\alpha} Q_c, \tag{31}$$

where $Q_c = \exp\left(\int_0^1 \ln\left(q(\omega)\right) d\omega\right)$ is the average quality in the consumption sector. Equation (31) will be used to determine the growth rate of consumption on the balanced growth path. Average price of the industry adjusted for the quality, on the other hand, is equal to

$$P_c = \exp\left(\int_0^1 \ln \frac{p(\omega)}{q(\omega)} d\omega\right) \tag{32}$$

$$= \exp\left(\int_0^1 \ln \frac{\frac{r^\alpha w^{1-\alpha}}{\tilde{\alpha}}}{q(\omega)} d\omega\right) \tag{33}$$

$$= \frac{r^\alpha w^{1-\alpha}}{\tilde{\alpha}} \frac{1}{Q_C} \tag{34}$$

Again, this is a result of identical innovative steps and identical production functions. I normalize the price of the consumption good to 1:

$$1 \equiv P_c = \lambda_c \frac{r^\alpha w^{1-\alpha}}{\tilde{\alpha} Q_c} \tag{35}$$

Similarly, investment is a quality adjusted aggregation of differentiated investment goods. Using the same arguments as above, the demand function of differentiated investment goods can be inserted into the investment aggregator and combined with the identical production functions of differentiated investment goods, aggregate investment is written as

$$X = k_x^{\alpha} l_x^{1-\alpha} Q_x, \tag{36}$$

where $Q_x = \exp\left(\int_0^1 \ln\left(q(\omega)\right) d\omega\right)$ is the average quality in the investment sector. Quality adjusted average price of investment good is also equal price of each differentiated good:

$$P_x = \lambda_x \frac{r^{\alpha} w^{1-\alpha}}{\tilde{\alpha} Q_x}. \tag{37}$$

The two other first order conditions of the household problem, consumption Euler equation and no arbitrage condition, and laws of motion of capital and asset holdings close the consumer part of the model:

$$\frac{\dot{C}}{C} + \frac{\dot{P}_c}{P_c} = R - \rho, \tag{38}$$

$$r = (R + \delta - g_{P_x})(1 - s_{in}) P_x, \tag{39}$$

$$\dot{A} = RA + wL + rK - P_c C - (1 - s_{in}) P_x X, \tag{40}$$

$$\dot{K} = X - \delta K. \tag{41}$$

Turning to the firm side, cost minimization problems of consumption and investment firms lead to

$$r k_c = w l_c \left(\frac{\alpha}{1-\alpha}\right), \tag{42}$$

$$r k_x = w l_x \left(\frac{\alpha}{1-\alpha}\right). \tag{43}$$

And innovation decisions of firms in both sectors generates the following conditions

$$\chi_j \psi_j z_j^{\gamma/(1-\gamma)} = \frac{1}{1-\gamma} \chi_j b_j^{\gamma/(1-\gamma)}, \quad j = c, x, \tag{44}$$

$$(R + \tau_c - b_c) w \chi_c \psi_c z_c^{\gamma/(1-\gamma)} = \pi_c - w \chi_c b_c^{1/(1-\gamma)} + \frac{\partial V(1, Z)}{\partial Z} \dot{Z}, \tag{45}$$

$$(R + \tau_x - b_x) w \chi_x \psi_x z_x^{\gamma/(1-\gamma)} = \pi_x - w \chi_x b_x^{1/(1-\gamma)} + \frac{\partial V(1, I)}{\partial I} \dot{I}, \tag{46}$$

Adding the market clearing condition for labor closes the model:

$$L = l_c + l_x + \sum_{j=c,x} \chi_j \psi_j z_j^{1/(1-\gamma)} + \sum_{j=c,x} \chi_j b_j^{1/(1-\gamma)}. \tag{47}$$

B.3 Balanced Growth Path

To find the growth rates of the variables on the balanced growth path, suppose there exists such a path and then verify it. Let $g_a \equiv \frac{\dot{a}}{a}$ denote the growth rate of any variable a on the balanced growth path. Let Y denote the GDP of the economy, $Y = C + P_x X$. Then the growth rate of consumption is equal to growth rate of investment expenditures, $g_C = g_I = g_{P_x} + g_X$. Using the income approach to GDP, $Y = rK + wL + RA - \dot{A}$, the growth rate of consumption is equal to growth rate of the wage rate, $g_C = g_w = g_r + g_K$. Since the price of consumption is normalized to 1, then Equation (35) implies that $g_{Q_c} = \alpha g_r + (1-\alpha) g_w$. Using the investment price formula, Equation (37), $g_{P_x} + g_{Q_x} = \alpha g_r + (1-\alpha) g_w$. But, in order for the no arbitrage condition to hold, Equation (39), the growth rate of rental rate of capital should be equal to the growth rate of the relative price of investment goods, $g_r = g_{P_x}$. Then, putting the growth rate of consumption and investment price equations together,

$$g_{Q_c} = \alpha g_r + (1-\alpha) g_w,$$
$$g_{Q_x} = (\alpha - 1) g_r + (1-\alpha) g_w,$$

the growth rate of the wage and rental rates of capital can be solved as $g_w = g_{Q_c} + \frac{\alpha}{1-\alpha} g_{Q_x}$, and $g_r = g_{Q_c} - \frac{1-\alpha}{1-\alpha} g_{Q_x}$.

Now, by using the other equilibrium conditions, I can verify that the above growth rates are indeed the balanced growth path rates. First, the growth rate of consumption should be equal to $g_C = \alpha g_K + g_{Q_c}$ by Equation (31):

$$\alpha g_K + g_{Q_c} = \alpha (g_w - g_r) + g_{Q_c} = \frac{\alpha}{1-\alpha} g_{Q_x} + g_{Q_c},$$

where the right hand side of the equation is equal to the growth rate of wage which is equal to growth rate of the consumption. It is straightforward to verify that the other equilibrium conditions are satisfied as well.

B.4 Growth Rates of Average Quality Levels

In this economy innovations occur with a Poisson rate of τ. Hence, in a time interval of t, the probability of exactly m innovations occur is equal to $f(m,t) = \frac{(\tau t)^m \exp(-\tau t)}{m!}$. Assuming the law of large numbers holds, the probability of having exactly m innovations in a time interval is equal to measure of products that had m innovations in that interval [Grossman and Helpman (1991)]. Plugging this back into the average quality level equation,

$$\begin{aligned} Q_t &= \exp\left(\int_0^1 \ln q(\omega) d\omega\right) \\ &= \exp\left(\sum_{m=0}^{\infty} f(m,t) \ln \lambda^m\right) \\ &= \exp\left(\ln \lambda \sum_{m=0}^{\infty} f(m,t) m\right) \\ &= \exp\left(\ln(\lambda)\tau t\right), \end{aligned}$$

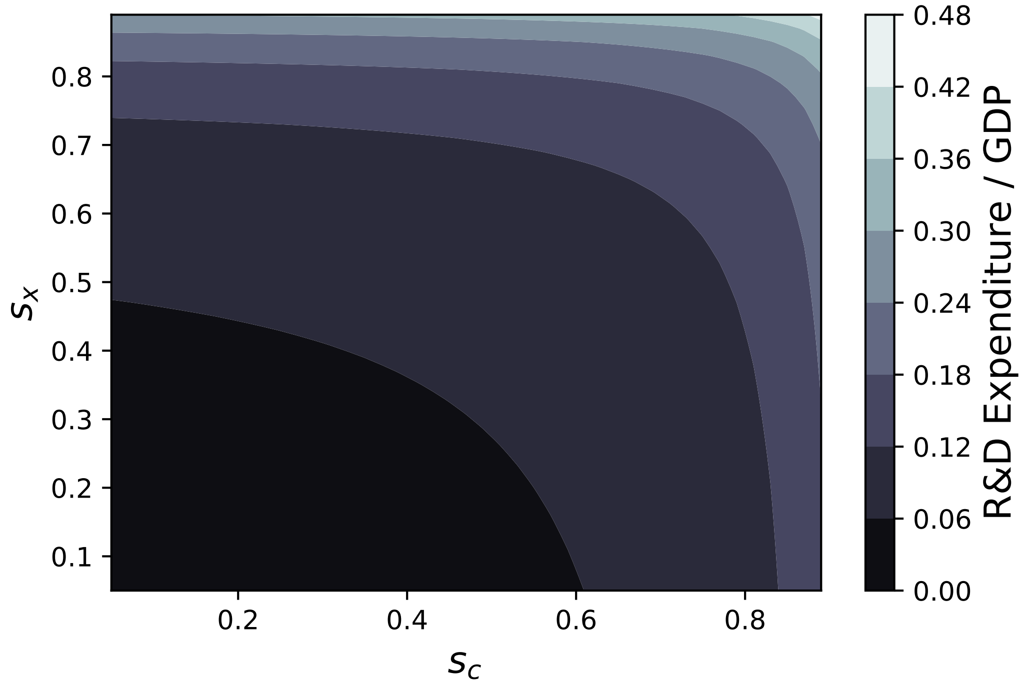
where the latter step is from the expectation of the Poisson distribution. Then the growth rate of average technology in each industry is equal to

$$\frac{\dot{Q}_j}{Q_j} = \tau_j \ln \lambda_j, \quad j = c, x. \tag{48}$$

C R&D Expenditure as a Share of GDP

This section shows the total R&D expenditures as a share of GDP at the balanced growth path.

Figure 8: Total R&D Expenditure as a Share of GDP



Notes: Contour map of R&D expenditure as a share of GDP at the balanced growth path.

D Comparative Statics

As explained before subsidizing firm R&D leads to a trade-off between current consumption and future consumption. An industry's location in the input-output chain impacts both the drop in current consumption and consumption growth rate. In Figures 9 and 10, I plot percent changes in BGP consumption growth rate, production labor, consumption equivalent welfare gain as a result of 10% increase in the R&D subsidy to investment goods producing sector for different α values. Here, α is elasticity of consumption sector output with respect to capital. In these exercise, I keep the elasticity of investment sector output with respect to capital at its benchmark value. To differentiate the α values, I call the latter one, α_x. Here α is related to the influence of investment sector in input-output chain. On the left panel, higher influence of the investment sector leads to higher consumption growth rates. Note that, at the BGP, $g_C = g_{Q_c} + \frac{\alpha}{1-\alpha_x} g_{Q_x}$. As α increases, an increase in rate of technological progress in investment sector, g_{Q_x}, generates higher consumption growth rates. Right panel of the figure 9 shows negative association between influence of the investment sector and change in production labor as a result of R&D subsidy. Industries with low influence leads to smaller reduction in production labor, and hence lower reduction in current period consumption. Left panel suggests subsidizing higher influence industries as they lead to higher growth rates, right panel suggests subsidizing low influence industries as they lead to smaller reduction in current period consumption. The optimal subsidy depends on the which of these opposite factors dominate. Figure 10 plots consumption equivalent welfare gain of an extra 10% subsidy to investment sector against α. As α increases, so does the welfare gain resulting from an extra 10% subsidy to the investment sector. In other words, as the influence of an industry increases, the welfare gain associated with extra subsidy to that industry increases.

Figure 9: Impact of 10% increase in R&D subsidy to investment good producers

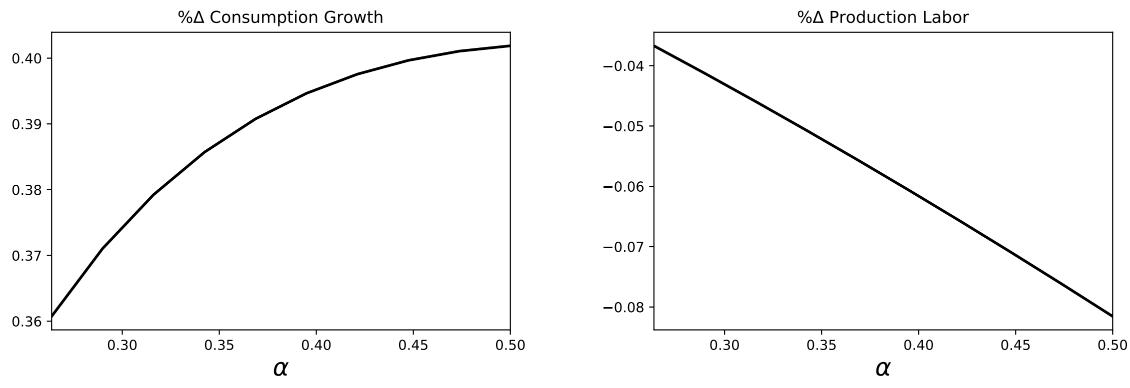
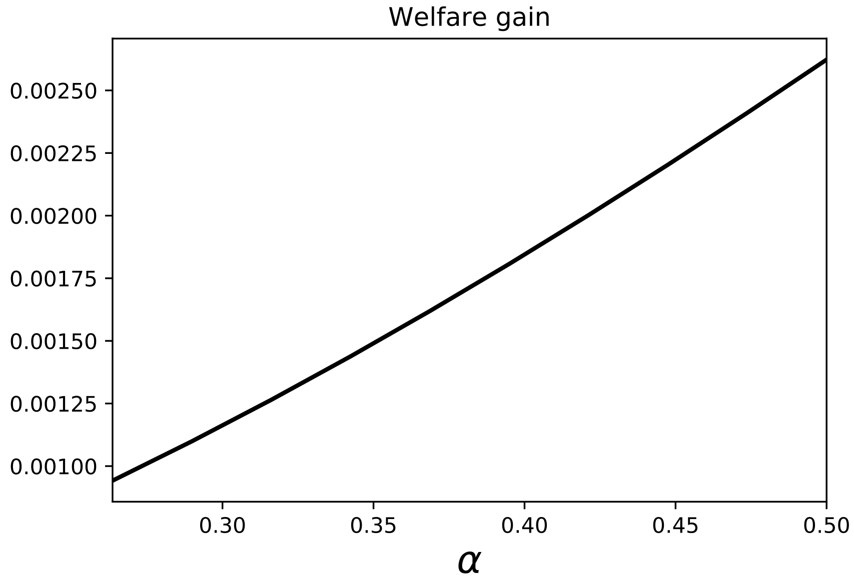


Figure 10: Welfare gain of 10% increase in R&D subsidy to investment good producers



E Robustness

E.1 Alternative Calibration

Entrant innovation not only contributes productivity growth, but also presents a threat to incumbents by stealing production lines of incumbents. Because of this threat, incumbents discount future profit stream of a production line at a higher rate, $\rho + z$, than the social planner, ρ. In the original calibration, $(\rho + z_c)/\rho \approx 2.7$ and $(\rho + z_x)/\rho \approx 2.6$. In both sectors, private discount rate is more than twice of the social planner discount rate. This large difference in discount rates leads to large under investment in innovation in the competitive equilibrium, and derives my results of high optimal subsidy. To check the robustness of my results, I redo the calibration with different targets. In the original calibration, I relied on model's implications on job creation. However, the model also has implications on job destruction rates: number of jobs created by entering firms is equal to number of jobs destructed by exiting firms, number of jobs created by expanding firms is equal to number of jobs destructed by shrinking firms. Table 7 shows target moments calculated from the BDS.

Job destruction rate by death in the data is equal to number jobs destructed by exiting establishments in an industry over the employment in that industry. Job destruction rate by continuers in data is equal to number of jobs destructed in establishments continuing their operations in an industry over total employment in that industry. Here, I retain my identifying assumption that a firm in my model corresponds to an establishment in the data. After job destruction rates are calculated, I construct job destruction rates in consumption and investment final goods sector using the methodology described in detail in Appendix A.

Entrant innovation rate targets go down with Calibration 2 from Calibration 1

Table 7: Alternative Targets

	Variable	Data	Model
Job destruction rate by death, consumption	z_c	0.043	0.043
Job destruction rate by death, investment	z_x	0.043	0.043
Job destruction rate of continuers, consumption	b_c	0.084	0.084
Job destruction rate of continuers, investment	b_x	0.091	0.091
GDP per capita growth rate	g_Y	.016	.016
Growth rate of investment good prices relative to consumption good prices	g_{P_x}	-.027	-.027
Labor's share of income		.714	.714

levels. The entrant innovation rate in the consumption sector goes down to 4.3% from 5%, and the entrant innovation rate in the investment sector goes down to 4.3% from 4.7%. Hence, the threat of entry on incumbents is lower both in the consumption and in the investment sector with Calibration 2. However, the private discount rates in both industries are about 2.4 times the social planner discount rate, still a substantially difference between private and social planner discount rates.

Table 8: Internally calibrated parameters, alternative calibration

	Parameter	Value
Quality ladder step size, investment	λ_x	1.27
Quality ladder step size, consumption	λ_c	1.04
R&D cost function parameter, investment	χ_x	7.84
R&D cost function parameter, consumption	χ_c	6.73
Entry cost function parameter, investment	ψ_x	3.03
Entry cost function parameter, consumption	ψ_c	2.73
Elasticity of output w.r.t capital	α	0.26

Parameter values resulting from this calibration are reported in table 8. The competitive economy still substantially underinvest in innovation. Social planner sets the consumption sector innovation rate to 22% and the investment sector innovation rate to 29%, and GDP growth rate to 3.1% in the long run. Transitioning from competitive equilibrium balanced growth path to social planner allocation results in 20% welfare gain in consumption equivalent terms. Government, on the other hand, can approximate the social planner allocation with an 80% subsidy to the consumption incumbents and an 87% subsidy to the investment incumbents, while adjusting entrant innovation subsidy to correct for the congestion externality in the entry process. This subsidy system generates a welfare gain of 19%. Remember that I don't let the government to correct for the distortions in the Euler equation resulting from monopoly pricing power of the investment good producers.

E.2 Further Robustness Checks on the Entry Rate

To continue analyzing the importance of entry rate, I recalibrate the cost of entry parameters (ψ_c and ψ_x) to match the entry rate in each industry with the estimates from other papers on the contribution of entry to the productivity growth. I keep the other parameter values at the benchmark calibration. Note that estimates of contribution of entry to productivity growth from the papers cited below are at the aggregate level, and I match entrant share in each industry to these estimates. For each calibration, I calculate the social planner innovation rates at the balanced growth path and corresponding welfare gain of moving to social planner allocation from the competitive equilibrium. Table 9 shows results of these exercises.

Table 9: Contribution of entry to sector productivity growth

Study		Social planner		
	Entrant share	τ_c	τ_x	welfare gain
Akcigit and Kerr (2018)	26.0%	0.23	0.29	18%
Garcia-Macia et al. (2016) (1976–1986)	19.1%	0.22	0.27	15%
Garcia-Macia et al. (2016) (2003 – 2013)	12.8 %	0.20	0.25	13%
–	5.0%	0.19	0.24	12%

Akcigit and Kerr (2018) estimate contribution of entry to productivity growth to be about 26 percent. Atkeson and Burstein (2018) use this estimate in their calibration. A lower entry rate in each industry means lower private discount rate of future profit streams from a successful innovation, and hence lower magnitude of distortion. Reduction in the inter-temporal spillover effect as a result of lower entry rate reduces the social planner innovation rates compared to the benchmark calibration. The social planner generates 18 percent welfare gain. Garcia-Macia et al. (2016) estimates contribution of entry to the productivity growth for two time periods, 1976-1986 and 2003-2013. Calibration of my model to their lower estimate, 12.8 percent, reduces the social planner innovation rates and the welfare gain even further. Lastly, if entry shares are targeted to be even lower, 5 percent, welfare gain from the social planner goes down to 12%. This exercise shows the importance of proper target of entry rate, as it is linked to the magnitude of inter-temporal spillover effect. The lower the entry rate, the lower the distortion, and hence the lower the welfare gain from the social planner. However, existence of other externalities leads under investment in innovation and the welfare gain of correcting the other externalities are still high.

E.3 Curvature of R&D Cost Function

I conduct a comparative static exercise with higher curvature R&D cost functions. In the model, the cost of innovation for incumbents in industry i is $\chi_i b_i^{\gamma}$, where b_i is the innovation rate. In the benchmark calibration, the curvature of the R&D cost function is equal to 2.5 ($\gamma = 2.5$). In the proposed exercise, I increase γ to 3.5[12], and calculate the optimal R&D subsidy and the associated welfare gain. The optimal R&D subsidy rates are 77% to the consumption sector and 83% to the investment sector, and the welfare gain of this policy is 8.2%. Even though the optimal subsidies are still high, the associated welfare gain is substantially lower than that in the benchmark model. With higher curvature R&D cost functions, achieving high productivity growth rates is more costly to the society as it require larger reductions in production labor. As a result, the associated welfare gain of innovation policy is smaller. This policy results in larger balanced growth path innovation rates in the consumption and investment sectors (from 14% and 14% respectively, to 20% and 22%). A higher R&D curvature makes it more costly to increase innovation rates, narrowing the gap between the competitive equilibrium and socially optimal innovation rates.

F Multi-sector Model

Consider an extension of my model with n-sectors and intermediate inputs. Gross output production function is Cobb-Douglas in capital, labor and intermediate inputs:

$$y_{fi} = \left(k_{fi}^{\alpha} l_{fi}^{1-\alpha}\right)^{\sigma} \left(\prod_j m_{fij}^{\omega_{ij}}\right)^{1-\sigma},$$

where $y_{f,i}$ is gross output of product f in industry i, k_{fi} and l_{fi} are capital and labor used in production of f, and m_{fij} is the amount of intermediate inputs from industry j used in product f in industry i. A competitive sector purchases firm output and produces industry output with the following production function, $y_i = \exp\left(\int_0^1 \ln\left(q_{fi} y_{fi}\right) df\right)$, where q_{fi} is the quality of product f in industry i. Competitive sectors use output of each industry to produce final consumption and investment goods with the following production functions:

$$C = \prod_j c_j^{\zeta_j}, \text{ and } I = \prod_j i_j^{\phi_j}.$$

The remaining parts of the model are identical to my two-sector model. Representative consumer maximizes discounted sum of logarithmic utility stemming from consumption by choosing consumption and investment, and inelastically supply one unit of labor. Firms, combination of different product lines, make production and invest in R&D. In the model, a firm owns productions lines from only one sector. As result of Bertrand competition and unit elastic demand for each product within a sector, in equilibrium output of sector i is equal to $y_i = \left(k_i^{\alpha} l_i^{1-\alpha}\right)^{\sigma} \left(\prod_j m_{ij}^{\omega_{ij}}\right)^{1-\sigma} Q_i$, where $Q_i \equiv \exp\left(\int_0^1 \ln q_{fi} df\right)$.

Let M be the collection of intermediate inputs supplied to industry i from industry

[12] I also adjust χ_i to keep the total R&D labor required to achieve the benchmark calibration innovation rates constant.

j:

$$\underset{n\times n}{M} = \begin{bmatrix} m_{11} & m_{12} & m_{13} & \dots & m_{1n} \\ m_{21} & m_{22} & m_{23} & \dots & m_{2n} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ m_{n1} & m_{n2} & m_{n3} & \dots & m_{nn} \end{bmatrix}.$$

Collect model parameters in Ω, Z, Φ as follows:

$$\underset{n\times n}{\Omega} = \begin{bmatrix} \omega_{11} & \omega_{12} & \omega_{13} & \dots & \omega_{1n} \\ \omega_{21} & \omega_{22} & \omega_{23} & \dots & \omega_{2n} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ \omega_{n1} & \omega_{n2} & \omega_{n3} & \dots & \omega_{nn} \end{bmatrix}, \quad \underset{n\times 1}{Z} = \begin{bmatrix} \zeta_1 \\ \vdots \\ \zeta_n \end{bmatrix}, \quad \underset{n\times 1}{\Phi} = \begin{bmatrix} \phi_1 \\ \vdots \\ \phi_n \end{bmatrix}.$$

Then, the growth rate of industry level output, consumption, and investment can be written as follows:

$$g_y = \mathbf{B}\begin{bmatrix} g_{Q_1} \\ \vdots \\ g_{Q_n} \end{bmatrix}, \quad g_C = Z \times g_y^T, \quad g_I = \Phi \times g_y^T,$$

where

$$\mathbf{B} \equiv \left(I_n - \alpha\sigma \begin{bmatrix} \phi_1 \cdots \phi_n \\ \vdots \dots \vdots \\ \phi_1 \cdots \phi_n \end{bmatrix} - (1-\sigma)\Omega \right)^{-1}.$$

F.1 Constrained Optimum

Consider an exercise such that the social planner allocates (only) innovative resources across sectors while holding the total innovative input on the level of competitive equilibrium. Since the social planner does not alter the production decisions, optimal allocation of innovative resources is the one that maximizes the growth rate of the economy while holding R&D labor constant:

$$\max_{\{\tau_i\}_{i=0,..,n}} D\mathbf{g_Q}$$

subject to $\sum_{i=0}^{n} C(\tau_i) = L_{R\&D}$, where $D = \mathbf{Z}^T\mathbf{B}$, and $C(\tau_i)$ is the labor cost of having τ_i innovation rate in industry while retaining the competitive equilibrium allocation of innovation in an industry into entrant and incumbent innovation. $C(\tau_i) = A_i\tau_i^{\gamma}$, where

$$A_i \equiv \chi_i \left(\frac{1}{\left(\frac{(1-s_i)\gamma}{(1-s_e)\psi_i}\right)^{1/(\gamma-1)} + 1} \right)^{\gamma} \left(\psi_i \left(\frac{(1-s_i)\gamma}{(1-s_e)\psi_i} \right)^{\gamma/(\gamma-1)} + 1 \right).$$

Solution of this maximization problem is

$$\tau_i = \left(\frac{D_i \ln \lambda_i}{A_i \gamma}\right)^{1/(\gamma-1)} \left(\frac{L_{R\&D}}{\sum A_i \left(\frac{D_i \ln \lambda_i}{A_i \gamma_i}\right)^{\gamma/(\gamma-1)}}\right)^{1/\gamma}.$$

In an environment where sectors have identical innovation functions (both innovative steps and cost of innovation), innovation ratios of sectors i and j boils down to

$$\frac{\tau_i}{\tau_j} = \left(\frac{D_i}{D_j}\right)^{1/\gamma-1}$$

F.1.1 Vertical Economy

Consider an example vertical economy. There are 10 sectors, and each sector i uses labor and intermediate input from sector $i+1$ in production. Industry 10 uses only labor in production. There is no capital in production, $\alpha = 0$. In this specific example, $D_i/D_j = (1-\sigma)^{i-j}$, therefore

$$\frac{\tau_i}{\tau_j} = \left((1-\sigma)^{i-j}\right)^{1/(\gamma-1)}.$$

Contrast constrained optimal allocation to competitive equilibrium allocation:

$$\frac{C'(\tau_i)(\rho + \tau_i - b_i) + c(b_i)}{C'(\tau_j)(\rho + \tau_j - b_j) + c(b_j)} = \frac{\pi_i}{\pi_j} = \left(\frac{1-\sigma}{\lambda}\right)^{i-j}.$$

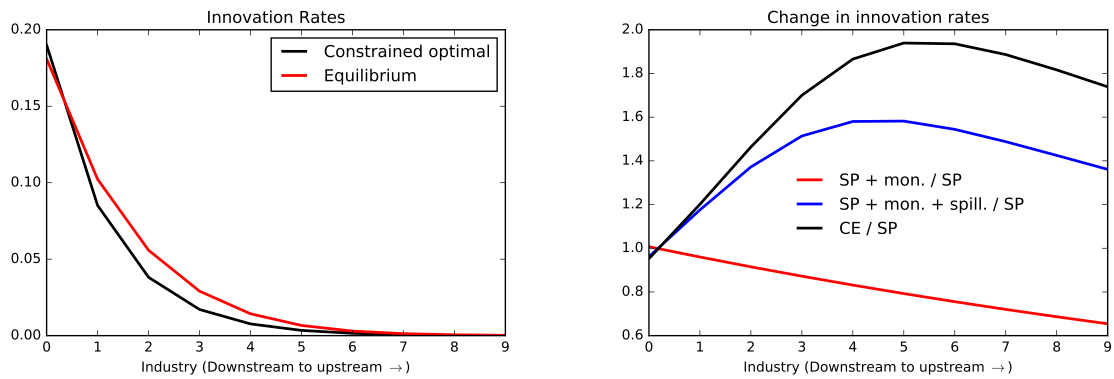
Various distortions in the competitive equilibrium leads sector innovation rates diverge from constrained optimum. First, existence of market power pushes sectoral profit ratios toward downstream industries, beyond their influence on the final consumption: $((1-\sigma)/\lambda)^{i-j} > (1-\sigma)^{i-j}$ when $i < j$, i.e. when i is more downstream. Second, existence of inter-temporal spillovers, the term $\tau_i - b_i$, distorts competitive equilibrium sectoral innovation rates towards upstream industries. Remember that firms discount future stream of profits at a higher rate, $\rho + \tau_i - b_i$, than social planner, ρ, because of the possibility that future innovators may steal incumbent firms' products. To understand how this externality pushes competitive equilibrium innovation towards upstream industries, suppose that ρ is small, and firms optimize using only future gross profit streams, π, not net profit stream, $\pi - c(b)$. Then, $\frac{C'(\tau_i)(\tau_i - b_i)}{C'(\tau_j)(\tau_j - b_j)} \approx \frac{\pi_i}{\pi_j}$. In equilibrium: $\tau_i - b_i = z_i = N\tau_i$[13] Hence, $\frac{\tau_i}{\tau_j} \approx \left(\frac{\pi_i}{\pi_j}\right)^{1/\gamma}$ as opposed to $\frac{\tau_i}{\tau_j} \approx \left(\frac{\pi_i}{\pi_j}\right)^{1/(\gamma-1)}$. If industry i is more downstream than industry j, then $\pi_i/\pi_j > 1$. As $\left(\frac{\pi_i}{\pi_j}\right)^{1/\gamma} < \left(\frac{\pi_i}{\pi_j}\right)^{1/(\gamma-1)}$, existence of intertemporal spillover effect pushes sectoral innovations towards upstream industries. Third, similar to inter-temporal spillover effect, the fact that firms care about net profits, $\pi - c(b)$ instead of gross profits, π,

[13] $N \equiv \frac{\left(\frac{(1-s^i)\gamma}{(1-s^e)\psi}\right)^{1/(\gamma-1)}}{1+\left(\frac{(1-s^i)\gamma}{(1-s^e)\psi}\right)^{1/(\gamma-1)}}.$

pushes equilibrium innovation rates towards upstream industries. To understand it, suppose that there is no inter-temporal spillover effect. Then, $\frac{C'(\tau_i)\rho + c(b_i)}{C'(\tau_j)\rho + c(b_j)} \approx \frac{\pi_i}{\pi_j}$. In equilibrium, $c(b) = M\tau^{\gamma}$[14]. Therefore, $\frac{A\tau_i^{\gamma-1}(\rho + \tau_i M/A)}{A\tau_j^{\gamma-1}(\rho + \tau_j M/A)} = \frac{\pi_i}{\pi_j}$. For this equality to hold, τ_i/τ_j should go down compared to a case where firms consider only gross profit in their R&D decisions. Overall, inter-temporal spillover and net profit considerations of firms increases upstream sector R&D over downstream sector R&D. Monopoly distortions, on the other hand, leads to lower upstream sector R&D over downstream R&D.

A numerical example[15] of constrained social planner allocation is depicted in Figure 11. Social planner allocates more innovative resources to the most downstream industry, while reducing resources to other sectors (left panel). Notice that social planner reduces innovation rates of industries located on the middle of the supply chain at a higher rate than that of more downstream industries. However, the more upstream industries see a slightly lower reduction in innovation rates than that of midstream industries. Overall, this reallocation generates 0.47% welfare gain by increasing the long-run growth rate of the economy from 1.56% to 1.58%.

Figure 11: Constrained Optimum Innovation Rates



Notes: Constrained optimum innovation rates

Right panel of Figure 11 shows impact of various externalities on the allocation of innovation across industries. Introduction of monopoly power increases innovation rate in the most downstream industry but reduces innovation rates in the other industries. The more upstream an industry is, the higher the decline in its innovation rate. This is because monopoly distortions accumulate on upstream industries. When inter-temporal spillovers are added over monopoly distortions, industry innovation rates of upstream industries increase, whereas innovation rate of the most downstream industry decrease. Increase in industry innovation rates is stronger for downstream industries. However joint impact of monopoly distortion and inter-temporal spillover effects leads to non-monotonic increase in innovation rates over social planner allo-

[14] $F \equiv \chi_i \left(\frac{1}{\left(\frac{(1-s_i)\gamma}{(1-s_e)\psi_i}\right)^{1/(\gamma-1)} + 1} \right)^{\gamma}$

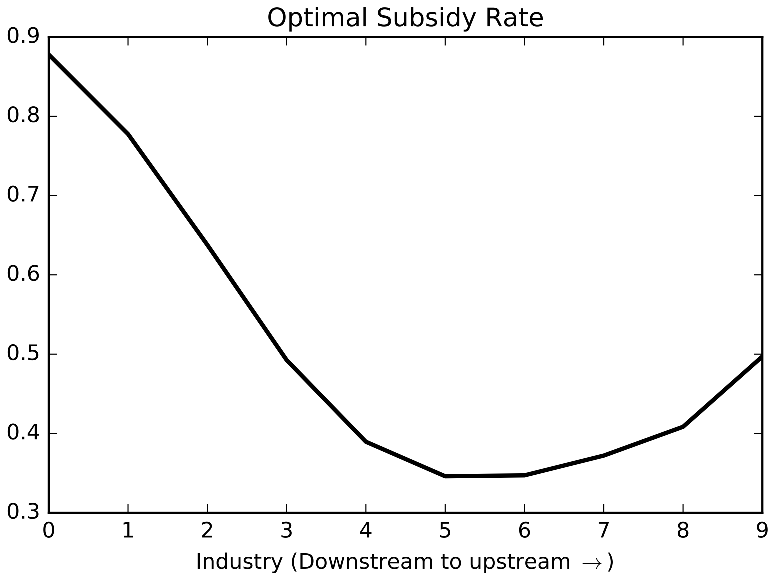
[15] In this parametrization, there are $n = 10$ industries, $\alpha = 0$, $\sigma_i = .7$ for all but the most upstream industry, and $\sigma_n = 1$. Each industry has identical innovation functions, $\lambda_i = 1.074$, $\chi_i = 5.84$, $\psi_i = 2.29$, $\gamma = 2.5$.

cation. The changes in innovation rates increase towards industries in the middle of supply-chain, but the change decreases for more upstream industries while the changes are still positive. Lastly, when net profit consideration of firms added on top of the previous two distortions, the allocation corresponds to the competitive equilibrium. Net profit consideration increases innovation rates in the upstream industries more. As explained above, the parametrization provides an example of above discussed distributional impacts of externalities: monopoly power reduces innovation in the upstream industries (relative to the social planner allocation), where inter-temporal spillover and net profit consideration increase innovation rates in the upstream industries.

F.2 Socially Optimal Innovation Subsidies

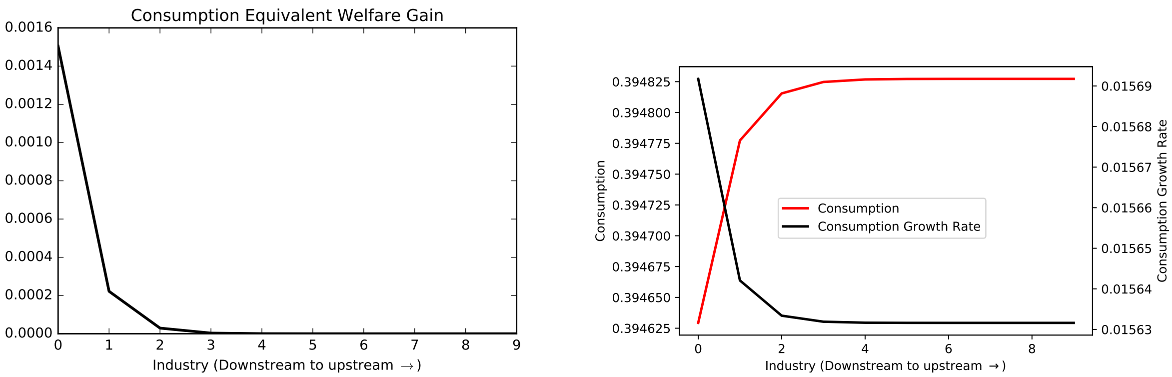
Now, using the above stated example economy, analyze the optimal innovation subsidy to industries. Figure 12 shows the subsidy rates that maximizes the welfare gain. The innovation rate schedule exhibits a U-shape. The most downstream industry is subsidized at the highest rate. The next downstream industry subsidized less, and so on until about the mid-point in the supply chain; thereafter, more upstream industries are subsidized at a higher rate than mid- stream industries.

Figure 12: Optimal Innovation Subsidy



To understand the intuition behind this, I analyze the impact of an extra 10% percent subsidy to one industry at a time. An increase in industry R&D subsidy lowers consumption in the initial period, while increasing the consumption growth rate. The elasticity of consumption growth rate with respect to the innovation subsidy in the industry in question decreases with the distance to final consumption. However, the elasticity (in absolute value) of short-run consumption with respect to industry innovation subsidy also decreases with the distance to final consumption. Subsidizing downstream industries leads to a larger increase in the consumption growth rate, but also a larger decrease in the short run consumption. The former effect dominates, resulting in larger welfare gains. Subsidizing the most upstream industries does not increase the consumption growth rate as much, nor does it significantly decrease the level of short run consumption. The tradeoff between long run growth and short run reductions in initial consumption results in the U-shaped optimal innovation subsidy profile depicted in Figure 13.

Figure 13: 10% increase in industry innovation subsidy



Notes: 10% increase in industry innovation subsidy

G Growth Decomposition

Consumption growth in the long-run is a result of innovative activities in the two sectors. As described in Equation (23), consumption growth rate can be decomposed into contributions of technological progress in consumption goods and technological progress in investment goods. Using the definition of the total innovation rate, which is the sum of entrant innovation and incumbent innovation, consumption growth rate can be further decomposed into contributions of entrants and incumbents:

$$
\begin{aligned}
g_C &= (z_c + b_c)\ln\lambda_c + \frac{\alpha}{1-\alpha}(z_x + b_x)\ln\lambda_x \\
g_C &= \underbrace{z_c \ln\lambda_c}_{\substack{\text{Consumption}\\ \text{Entrants}}} + \underbrace{b_c \ln\lambda_c}_{\substack{\text{Consumption}\\ \text{Incumbents}}} + \underbrace{\frac{\alpha}{1-\alpha} z_x \ln\lambda_x}_{\substack{\text{Investment}\\ \text{Entrants}}} + \underbrace{\frac{\alpha}{1-\alpha} b_x \ln\lambda_x}_{\substack{\text{Investment}\\ \text{Incumbents}}}.
\end{aligned} \tag{49}
$$

Equation (49) decomposes the growth rate by sectors and entrants/incumbents. Table 10 shows each element's contribution as a percentage of the consumption growth rate. The investment sector contributes 72% percent of growth, whereas the consumption sector contributes 28%. The contribution of the investment sector in my estimates is comparable to estimates of Sakellaris and Wilson (2004), who empirically find that embodied technological change in investment goods contributes two thirds of macroeconomic growth. Krusell (1998) develops an endogenous growth model that can account for the decline in the relative price of investment goods. He attributes approximately half of the consumption growth to investment specific technological change.

Table 10: Consumption Growth Decomposition

	Consumption	Investment	Total
Entrant	10%	18%	28%
Incumbent	23%	49%	72%
Total	33%	67%	

Entrants contribute approximately one third to growth. Remember, in the model, the innovativeness of entrants and incumbents are the same. Hence, the difference of entrants and incumbents in terms of contribution to growth stems mainly from different entry and expansion rates. Similar to my results, by using the Census of Manufacturers data from 1977 to 1987, Foster et al. (2001) show that net entry contributes one quarter of multi–factor productivity growth[16]. Overall, investment sector incumbents contribute the most to growth and consumption sector entrants contribute the least. Intuitively, most of the growth comes from companies producing better machines, and less comes from consumption sector entrants like new restaurants.

[16]Notice that I do growth decomposition in this anaylysis, whereas Foster et al. (2001) analyzis contributions to productivity.