PS2

April 20, 2017

0.0.1 Question 1

To be able to hit the circular plate uniformly we must calculate $P(\theta)d\theta$. In polar coordinates area alement is given by $dA = rdrd\theta$. From geometry, it is obvious that $r = d\tan\theta$, hence $dr = d(1+\tan^2\theta)d\theta$. Since the points will be distributed uniformly, we can write down

$$P(\theta,\phi)d\theta d\phi = \frac{d^2\tan\theta(1+\tan^2\theta)d\theta d\phi}{\pi R^2}$$

where R is the radius of the circular plate.

$$P(\theta,\phi)d\theta d\phi = \frac{\tan\theta(1+\tan^2\theta)d\theta d\phi}{\pi\tan^2\theta_{max}}$$

Since we are only interested in distribution of θ we should integrate this over ϕ.

$$P(\theta)d\theta = \int_{\phi=0}^{\phi=2\pi}\frac{\tan\theta(1+\tan^2\theta)d\theta d\phi}{\pi\tan^2\theta_{max}}$$

Finally we obtain,

$$P(\theta)d\theta = \frac{2\tan\theta(1+\tan^2\theta)d\theta}{\tan^2\theta_{max}}$$

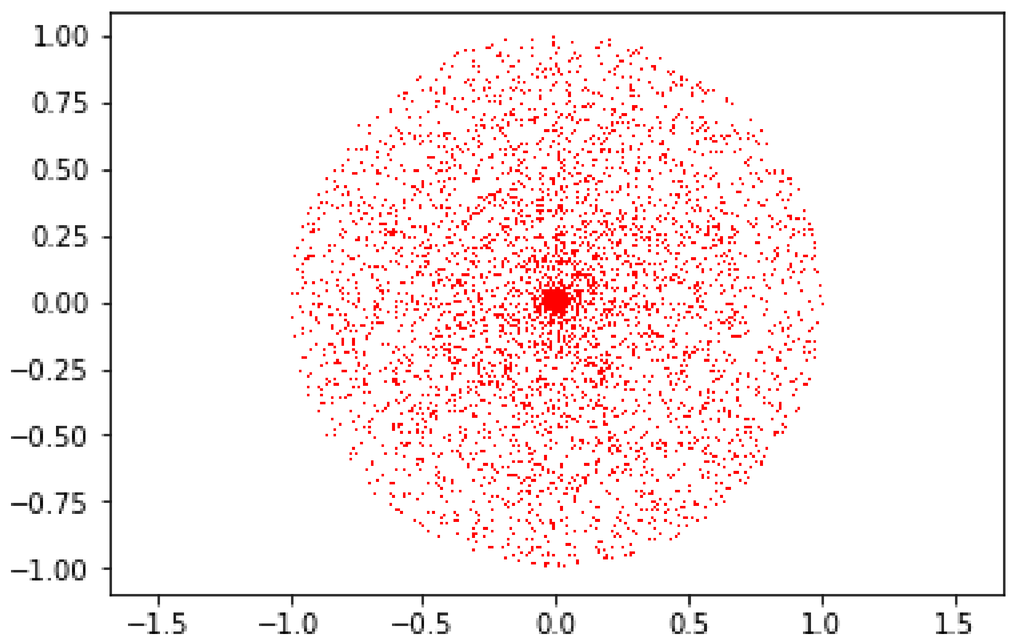
We can check our result by using Metropolis algorithm.
As seen from above figure, points are approximately distributed uniformly.

In [13]:
```
%matplotlib inline
import numpy as np
import matplotlib.pyplot as plt
import math

initialX=1;

currentX=initialX

testX=math.pi/2

delta=0.02

n=5000;
```

```
thetas=[]
angles=2*math.pi*np.random.rand(n)
accepted=0
for i in range(0, n):
    testX=delta*(2*np.random.rand(1)-1)+currentX;
    while testX<0 or testX>1.56079666011:
         testX=delta*(2*np.random.rand(1)-1)+currentX
    val1=0.02*np.tan(currentX)*(1+np.tan(currentX)**2)
    val2=0.02*np.tan(testX)*(1+np.tan(testX)**2)
    ratio=val2/val1
    if ratio >= np.random.rand(1):
        currentX=testX;
        accepted+=1
    thetas.append(currentX[0])
print(accepted)
x=0.01*np.tan(thetas)*np.cos(angles)
y=0.01*np.tan(thetas)*np.sin(angles)
plt.axis('equal')
plt.plot(x,y,',r')
```

```
3104
```

```
Out[13]: [<matplotlib.lines.Line2D at 0x201fd103940>]
```



To find suitable $f(x)$ to generate desired distribution, first we need cumulative probability function.

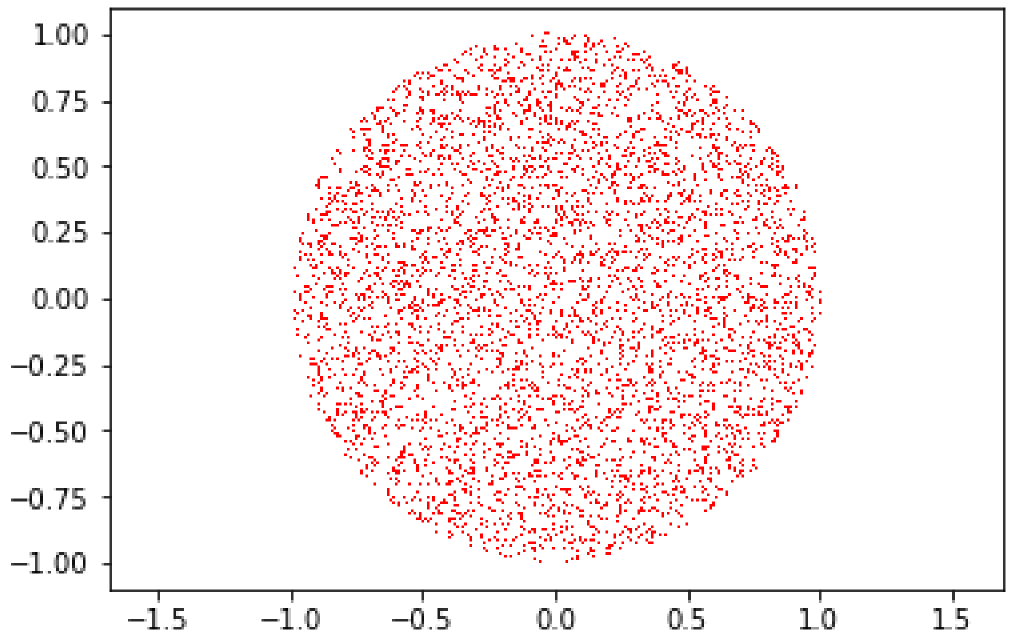
$$F(\theta < \theta_0) = \int_0^{\theta_0} \frac{2\tan\theta(1+\tan^2\theta)d\theta}{\tan^2\theta_{max}}$$
$$= \frac{\tan^2\theta_0}{\tan^2\theta_{max}}$$

To find $f(x)$ we should invert this cumulative distribution function. We obtain $f(x) = \arctan(\tan\theta_{max}\sqrt{x})$. Then we can pick uniform x values and calculate θ values.

```
In [2]: %matplotlib inline
        import numpy as np
        import matplotlib.pyplot as plt
        import math

        N=5000
        phi=2*math.pi*np.random.rand(N)
        r=np.random.rand(N)
        theta=np.arctan(100*np.sqrt(r))

        x=0.01*np.tan(theta)*np.cos(phi)
        y=0.01*np.tan(theta)*np.sin(phi)

        print(np.arctan(100))
        print(np.tan(2.1))
        plt.axis('equal')
        plt.plot(x,y,',r')
```

```
1.56079666011
-1.7098465429
```

```
Out[2]: [<matplotlib.lines.Line2D at 0x15818c5d518>]
```



As seen from above figure, the points are approximately distributed uniformly.