# PS2

April 20, 2017

#### 0.0.1 Question 1

To be able to hit the circular plate uniformly we must calculate $P(\theta)d\theta$. In polar coordinates area alement is given by $dA = rdrd\phi$. From geometry, it is obvious that $r = d\tan\theta$, hence $dr = d(1+\tan^2\theta)d\theta$. Since the points will be distributed uniformly, we can write down

$$P(\theta,\phi)d\theta d\phi = \frac{d^2\tan\theta(1+\tan^2\theta)d\theta d\phi}{\pi R^2}$$

where R is the radius of the circular plate.

$$P(\theta,\phi)d\theta d\phi = \frac{\tan\theta(1+\tan^2\theta)d\theta d\phi}{\pi\tan^2\theta_{max}}$$

Since we are only interested in distribution of $\theta$ we should integrate this over $\phi$.

$$P(\theta)d\theta = \int_{\phi=0}^{\phi=2\pi}\frac{\tan\theta(1+\tan^2\theta)d\theta d\phi}{\pi\tan^2\theta_{max}}$$

Finally we obtain,

$$P(\theta)d\theta = \frac{2\tan\theta(1+\tan^2\theta)d\theta}{\tan^2\theta_{max}}$$

To find suitable $f(x)$ to generate desired distribution, first we need cumulative probability function.

$$F(\theta<\theta_0) = \int_0^{\theta_0}\frac{2\tan\theta(1+\tan^2\theta)d\theta}{\tan^2\theta_{max}}$$
$$= \frac{\tan^2\theta_0}{\tan^2\theta_{max}}$$

To find $f(x)$ we should invert this cumulative distribution function.

We obtain $f(x) = \arctan(\tan\theta_{max}\sqrt{x})$. Then we can pick uniform $x$ values and calculate $\theta$ values.

```
In [4]: %matplotlib inline
        import numpy as np
        import matplotlib.pyplot as plt
        import math

        N=5000
```

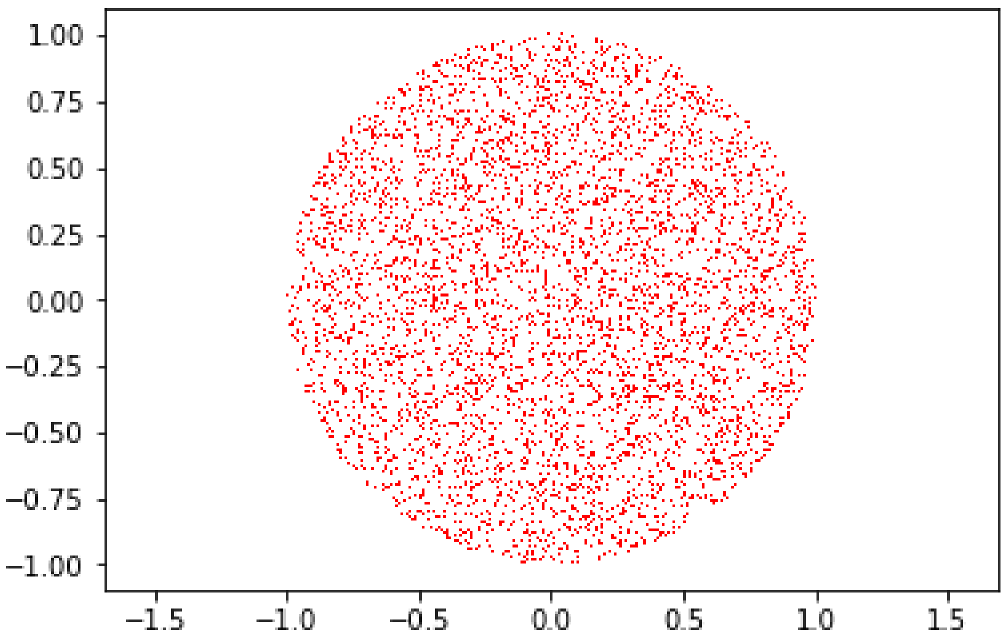
```
phi=2*math.pi*np.random.rand(N)
r=np.random.rand(N)
theta=np.arctan(100*np.sqrt(r))

x=0.01*np.tan(theta)*np.cos(phi)
y=0.01*np.tan(theta)*np.sin(phi)

print(np.arctan(100))
print(np.tan(2.1))
plt.axis('equal')
plt.plot(x,y,',r')
```

```
1.56079666011
-1.7098465429
```

```
Out[4]: [<matplotlib.lines.Line2D at 0x1da4fa549e8>]
```




As seen from above figure, the points are approximately distributed uniformly.