MIDDLE EAST TECHNICAL UNIVERSITY

ELECTRICAL AND ELECTRONICS ENGINEERING

EE564

DESIGN OF ELECTRICAL MACHINES

-PROJECT 1-

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Question 1

Part A

In this part I made all calculations with MATLAB so I have used MATLAB report generator.

Part A.1

```
% In this part, magnetic flux is assumed homogeneously, and use effective
% length from datasheet.


u0 = 4*pi*10^-7;

core_Al = 5040; % nH/turn^2
core_le = 103e-3; % m  eff length
core_Ae = 138e-6; %m^2 cross sec area
core_height = 18e-3;

core_inner_r = 13.1e-3;
core_outer_r = 20.9e-3;


core_init_ur = 3000;    %initial ur

core_ur = 4500;         % @ 425mT

B_op = 0.425;

N = 20;
Imax = B_op*core_le/(core_ur*u0*N);

I = 0.43;

core_R = core_le/(core_ur*u0*core_Ae); %reluctance

core_L = N^2/core_R     %H
```

```
core_L =


    0.0030
```

Part A.2

```
% L = N*fi/I , fi = NI/R, L = N^2(1/R1+1/R2...)

% in order to modeled this I have calculated 10 different reluctance
% which is effected flux and inductance then calculated inductance

% inner parts of the toroid has small effective length this cause smaller
% reluctance so that magnetic flux density and magnetic flux is high at
% smaller radius and smaller higher radius. Also our assumption of the
% first part is verified.
```

```
core_r_div = linspace(core_inner_r,core_outer_r, 11);

core_r_div_effective = zeros(10,0);
core_R_div_effective = zeros(10,0);
core_le_div_effective    = zeros(10,0);
core_Ae_div          = zeros(10,0);
core_L_div           = zeros(10,0);
core_flux_div        = zeros(10,0);
total_div_flux           = 0;
total_div_inductance     = 0;

for i = 1:10

    core_r_div_effective(i) = (core_r_div(i) + core_r_div(i+1))/2;

    core_le_div_effective(i) = 2*pi*core_r_div_effective(i);

    core_R_div_effective(i) = core_le_div_effective(i)/(core_ur*u0*core_Ae/10);

    core_L_div(i) = N^2 / core_R_div_effective(i);

    core_flux_div(i) = N*I/core_R_div_effective(i);

    total_div_inductance = total_div_inductance + core_L_div(i);
    total_div_flux       = total_div_flux + core_flux_div(i);

end


plot(core_r_div_effective,core_L_div);
title('inductance vs radius part 1');
xlabel('radius');
ylabel('inductance');

figure;

plot(core_r_div_effective,core_flux_div);
title('flux vs radius part 1');
xlabel('radius');
ylabel('flux');

figure;

total_div_flux

total_div_inductance
```

```
total_div_flux =
   6.3957e-05

total_div_inductance =

    0.0030
```

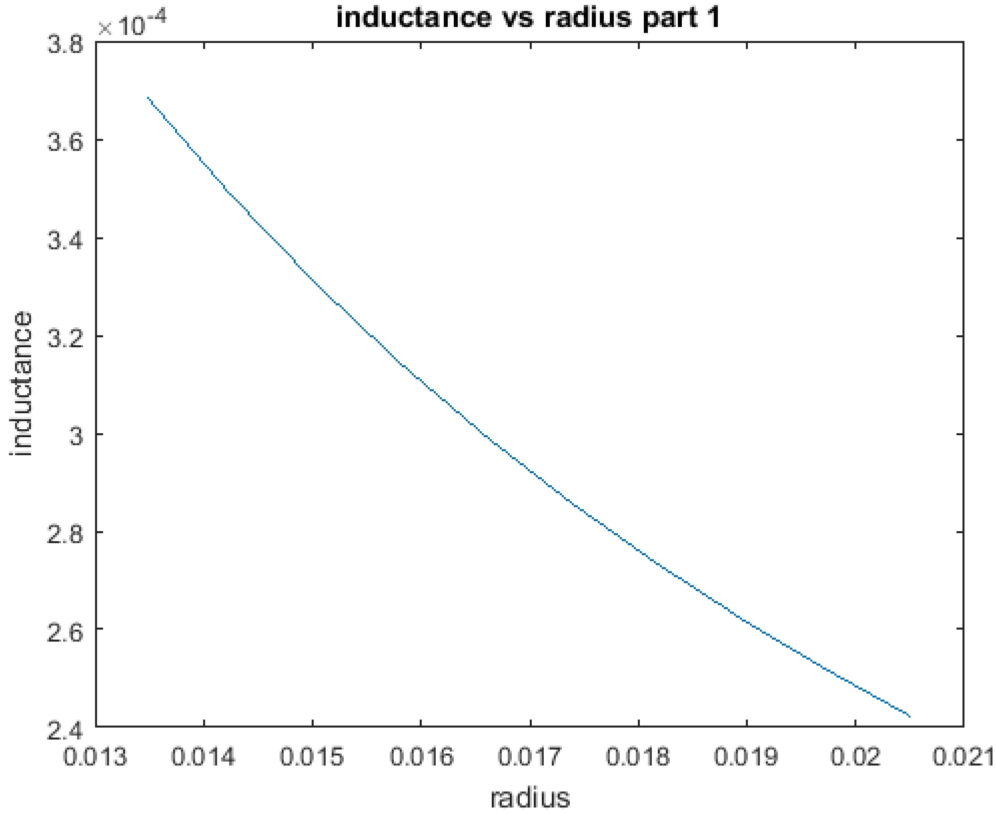


Figure 1. Inductance values for each dividing region

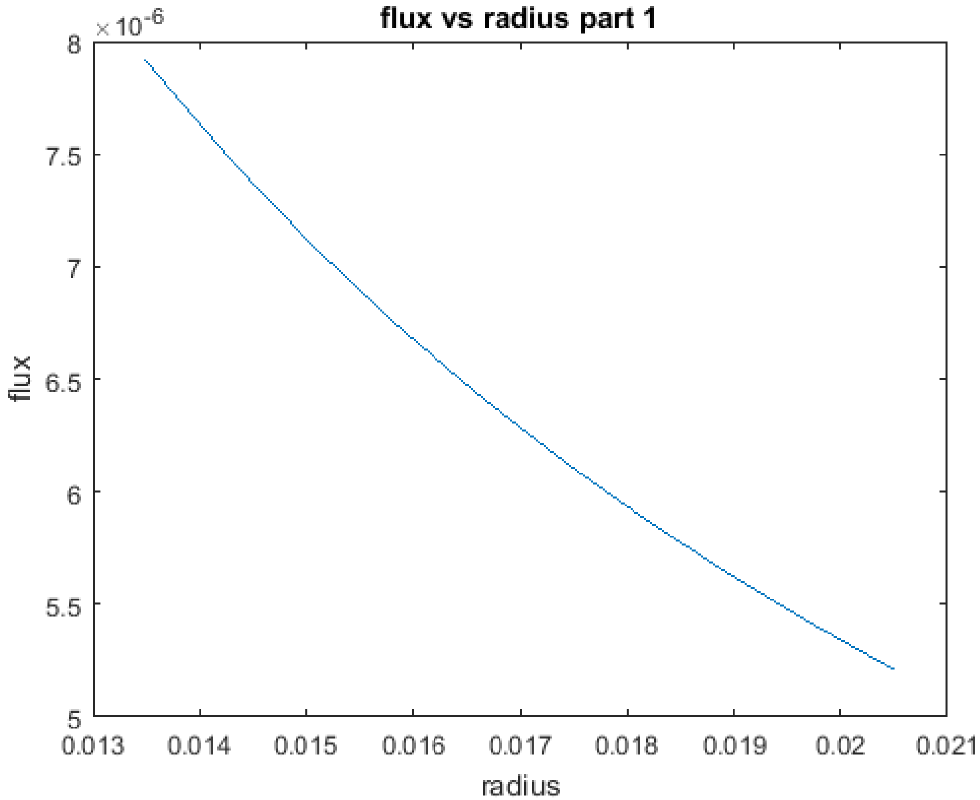


Figure 2. Magnetic flux for each dividing region

Part A.3

```
% In this part magnetic flux density is saturated so that increasing
% current cause to decrease permeability then, inductance of the inductor
% decreases. Moreover, same as part 3, magnetic flux and flux density are higher
% at the smaller radius of the core because of smaller effective distance
% and reluctance.

% In this part magnetic flux density is saturated so that increasing
% current cause to decrease permiability so that inductance of the inductor
% decrease also, same as part 3 magnetic flux and flux density higher
% at the smaller radius of the core because of smaller effective distance
% and reluctance.


% NI = B*A*R
I_part3 = 0.6525;

core_B_sat = 0.53;

core_ur_part3 = 4500*0.53/0.425/1.5;

core_R_part3 = core_le/(core_ur_part3*u0*core_Ae); %reluctance


core_L_part3 = N^2/core_R_part3     %H

core_flux_part3 = N*I_part3/core_R_part3

% part 3 division

core_R_div_effective_part3 = zeros(10,0);
core_L_div_part3  = zeros(10,0);
core_flux_div_part3 =zeros(10,0);
total_inductance_div_part3 = 0;
total_flux_div_part3  = 0;

for i = 1:10

    core_R_div_effective_part3(i) = core_le_div_effective(i)/(core_ur_part3*u0*core_Ae/10);

    core_L_div_part3(i) = N^2 / core_R_div_effective_part3(i);

    core_flux_div_part3(i) = N*I_part3/core_R_div_effective_part3(i);

    total_inductance_div_part3 = total_inductance_div_part3 + core_L_div_part3(i);
    total_flux_div_part3       = total_flux_div_part3 + core_flux_div_part3(i);

end


plot(core_r_div_effective,core_L_div_part3);
title('inductance vs radius part 3');
xlabel('radius');
ylabel('inductance');
```

```
figure;

plot(core_r_div_effective,core_flux_div_part3);
title('flux vs radius part 3');
xlabel('radius');
ylabel('flux');

figure;

total_flux_div_part3

total_inductance_div_part3
```

```
core_L_part3 =

    0.0025


core_flux_part3 =

   8.2200e-05


total_flux_div_part3 =

   8.0685e-05


total_inductance_div_part3 =

    0.0025
```

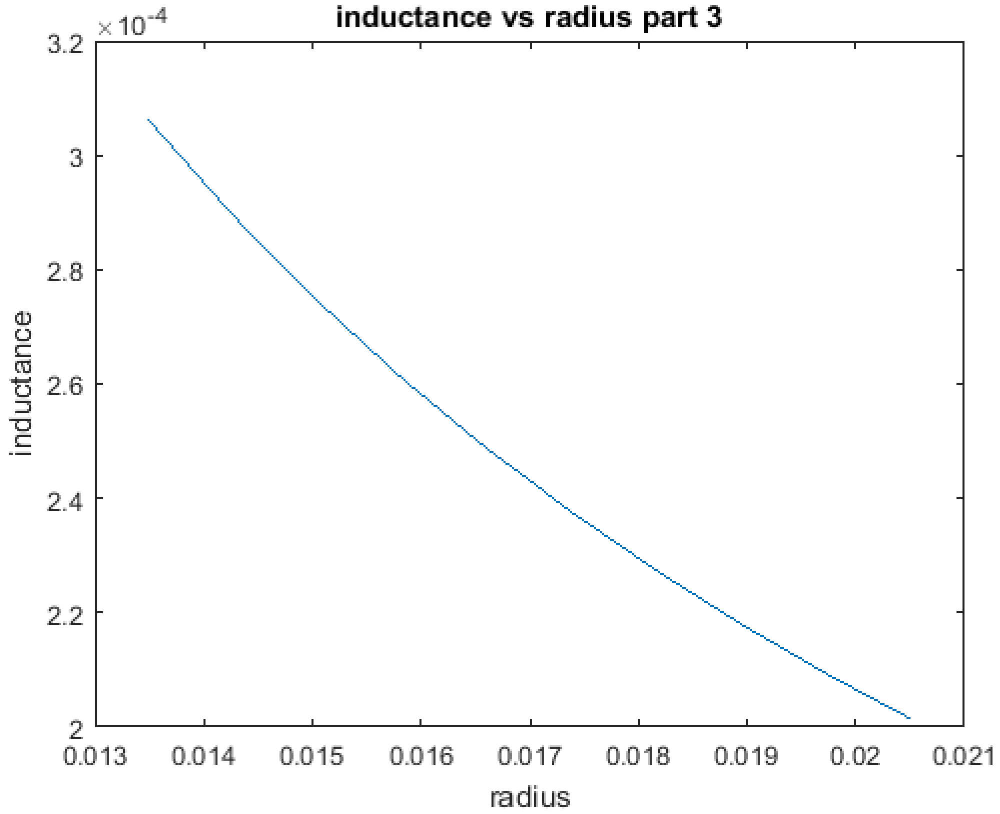


Figure 3. Inductance values for each dividing region for saturated flux density

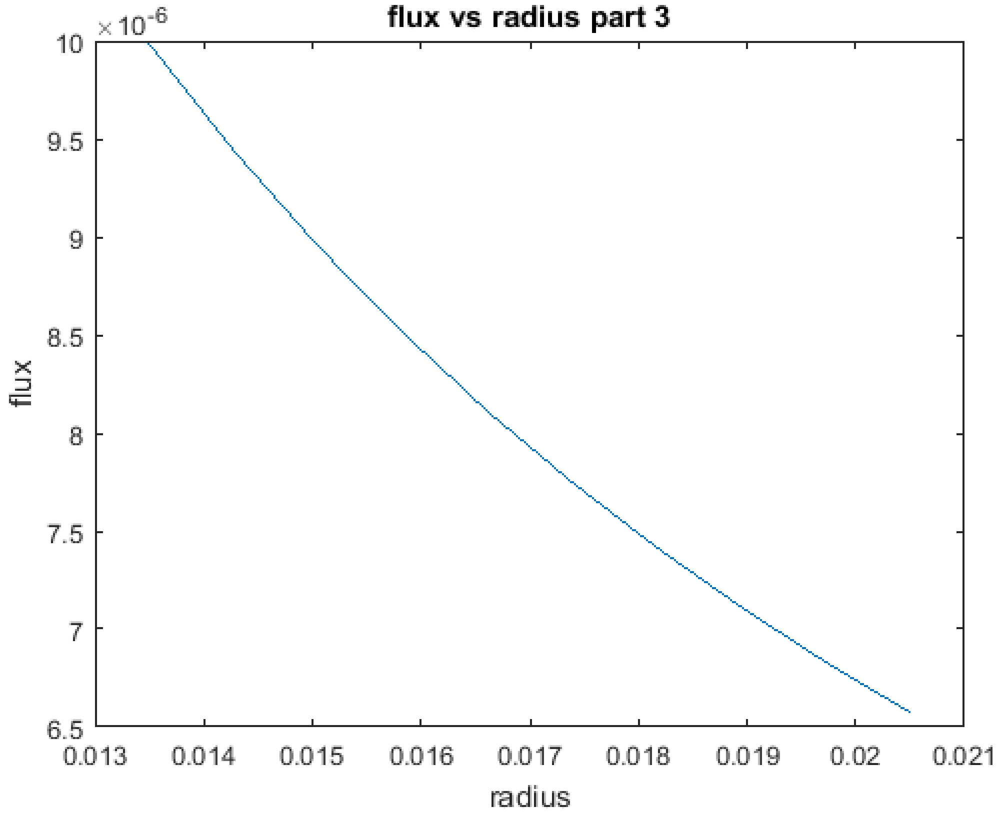


Figure 4. **Magnetic flux for each dividing region for saturated flux density**

Part A.4

```
% By adding gap to core cause to increase the reluctance so the inductance and
% core flux are decreasing. Fringing flux is neglected.

d_gap = 2e-3;

core_le_part4 = core_le - 2e-3;

gap_R = d_gap/(core_Ae*u0);

core_R_part4 = core_le_part4/(core_ur*u0*core_Ae); %reluctance

core_L_part4 = N^2/(core_R_part4+gap_R)     %H
```

```
core_L_part4 =

   3.4298e-05
```

Part A.5

```
% In this part, in order to model fringing flux, I increased the air gap a
% little, in order to decide this distance I found required reluctance to model
% fringing distance. Also, ı have used this reluctance to calculate
% inductance
%
% Formula of fringing flux is flux = 1+(d_gap/sqrt(Acore))*ln(2*window_area/d_gap)
%

area_window= 2*pi*core_inner_r;

fringing_flux = 1+(d_gap/sqrt(core_Ae))* log(2*area_window/d_gap);

d_fringing = N*I*core_Ae*u0/fringing_flux

fringing_R = d_fringing/(core_Ae*u0);

core_L_part5 = N^2/(core_R_part4+gap_R+fringing_R)     %H

% By using this formula, d_fringing is came so small, so that effect of fringing
% could not seen obviously
```

```
d_fringing =
       8.5178e-10


core_L_part5 =
       3.4298e-05
```

Part B

Part B.1

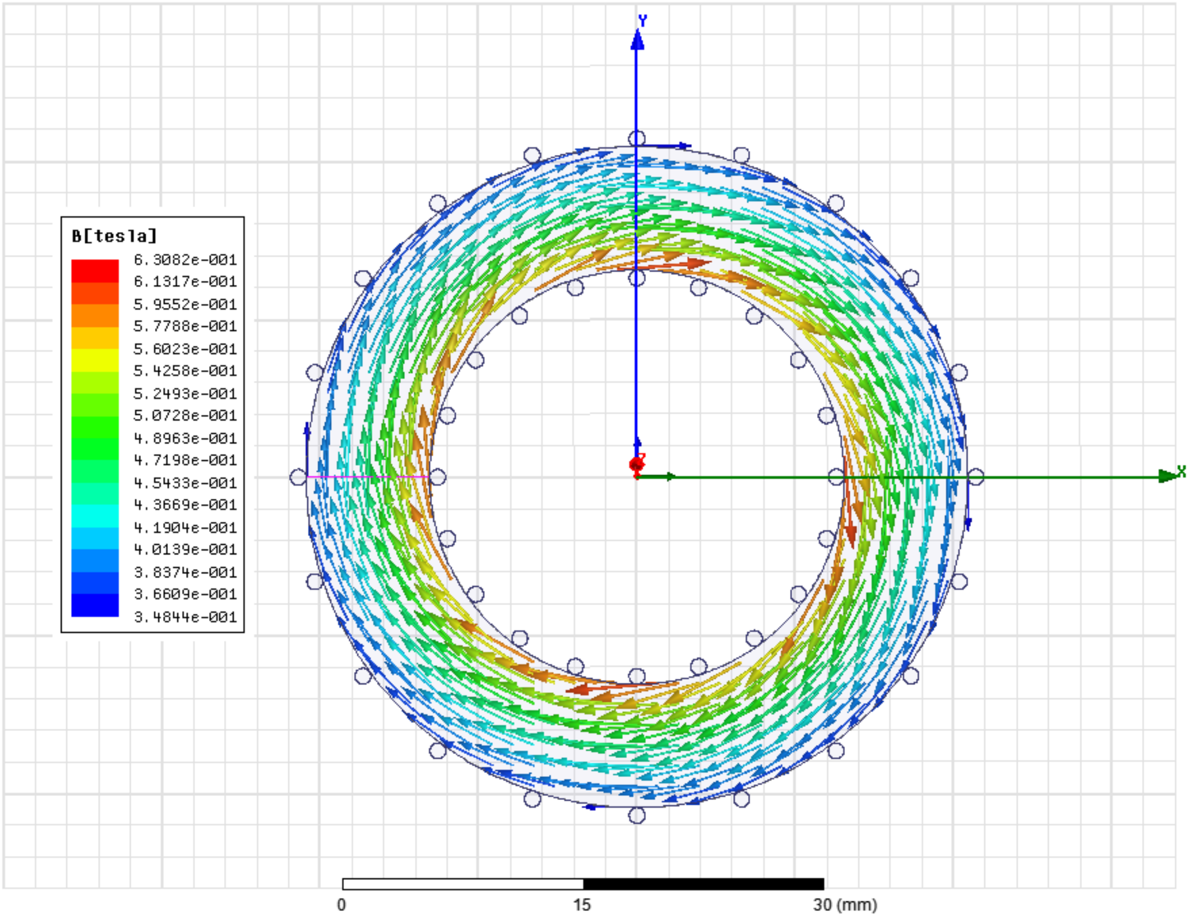


Figure 5. Magnetic flux density for linear core material with symmetrical excitation

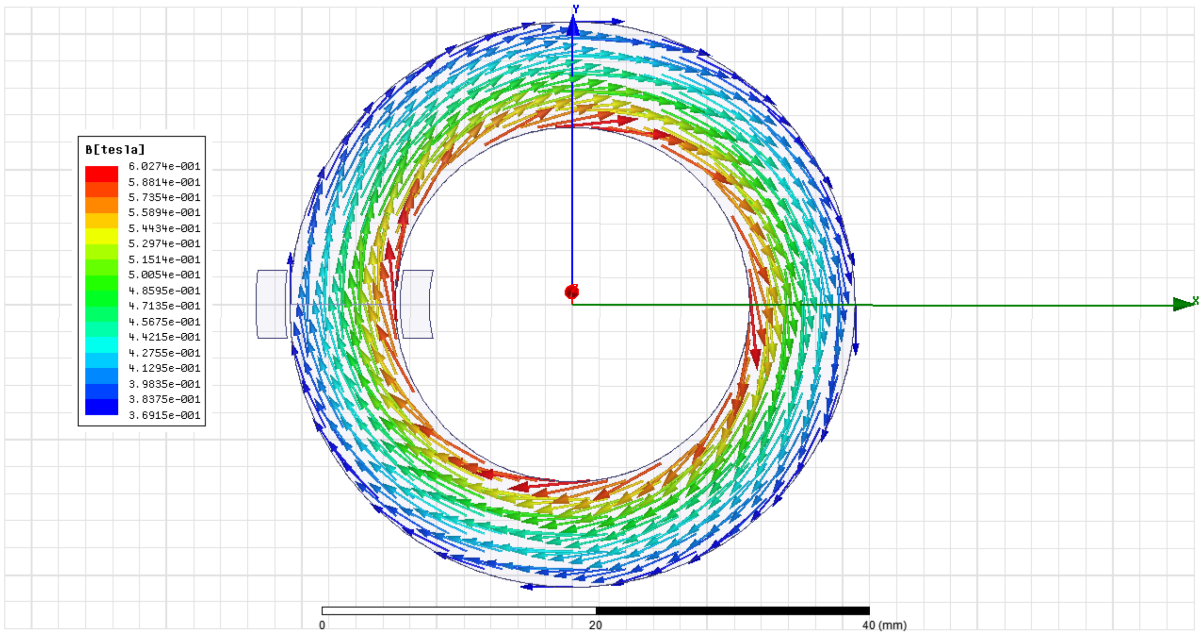


Figure 6. Magnetic flux density for linear core material

Part B.2

Stored energy of the core is calculated from magnetic flux passing on L1 and leakages are modeled with L2 and L3 line fluxes.

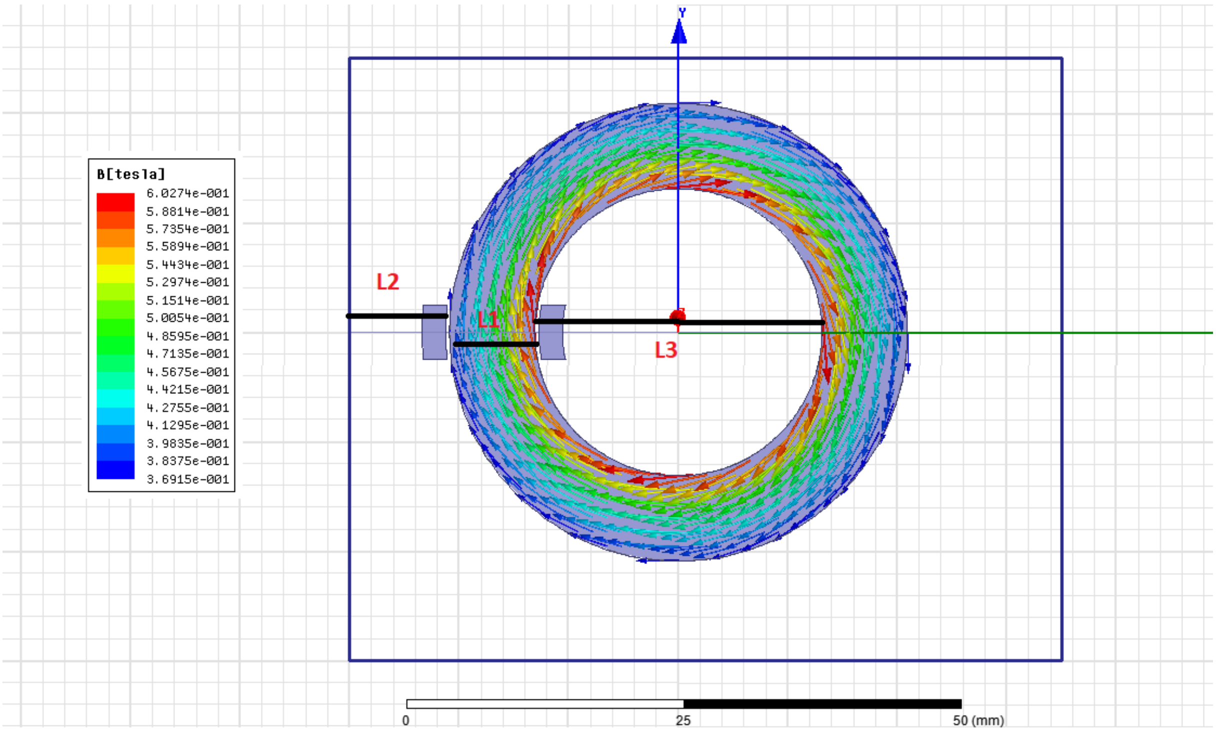


Figure 7: Magnetic flux density for linear core material and lines using to measure passing flux and energy

	Distance [mm]	Mag_B [mTesla] Setup1 : LastAdaptive
1	0.000000	371.728570
2	1.950000	416.034101
3	3.900000	463.220649
4	5.850000	525.611046
5	7.800000	591.554113

Figure 8. Magnitude of magnetic flux density on L1

From figure 8, average magnetic flux density on the core is calculated as 473.2 mT. Then with following formula reluctance is calculated. (N = 20, I = 0.435A, B = 0.473.2T, A = 138mm^2)

$$R = \frac{N * I}{B * A}$$

Reluctance is found as R = 133.22 * 10^3, then with following formula, total inductance is nearly **3mH** as founded analytically.

L = **3mH**

$$L = \frac{N^2}{R}$$

By using previous reluctance and inductance formula, next formula is derived.

$$L = \frac{N * B * A}{I}$$

	Distance [mm]	Mag_B [uTesla] Setup1 : LastAdaptive
1	0.000000	239.507900
2	2.275000	399.312176
3	4.550000	647.760854
4	6.825000	935.165608
5	9.100000	76.019128

	Distance [mm]	Mag_B [uTesla] Setup1 : LastAdaptive
1	0.000000	136.841116
2	6.550000	510.390002
3	13.100000	264.351124
4	19.650000	175.574521
5	26.200000	131.040676

Figure 9. Magnitude of magnetic flux density on L2(left) and L3(right)

Note: I have calculated average magnetic flux density until 9.1 mm outside of core as. Then I used cross section area of this distance.

Average magnetic flux density on L2 is 459uT and on L3 is 243uT, then leakage inductances are (N = 20, I = 0.435, A_1= 18*9.1mm^2, A_2= 18*26.2mm^2)

L_{L1} = 3.45uH, L_{L2} =5.27 uH

$$L_{leakage} = \frac{N(B1 * A1 + B2 * A2)}{I}$$

$L_{leakage}$ = **8.72uH**.

Part B.3

In this part simulations are made for desired current value and %50 higher of it separately.

B	u	H
0	3000	0
50	3200	12,44029
100	3900	20,41483
150	4150	28,77753
200	4350	36,6059
250	5000	39,80892
300	5000	47,7707
350	4950	56,29544
400	4900	64,99415
450	4850	73,87222
510	4800	84,59395
520		90
530		100
530		1050

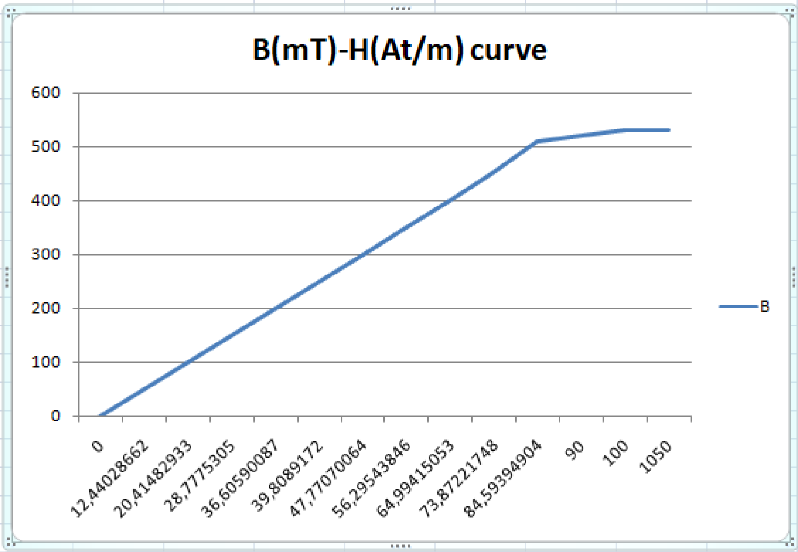


Figure 10. B-H curve of core which is obtained permeability- flux density of the core (at appendix)

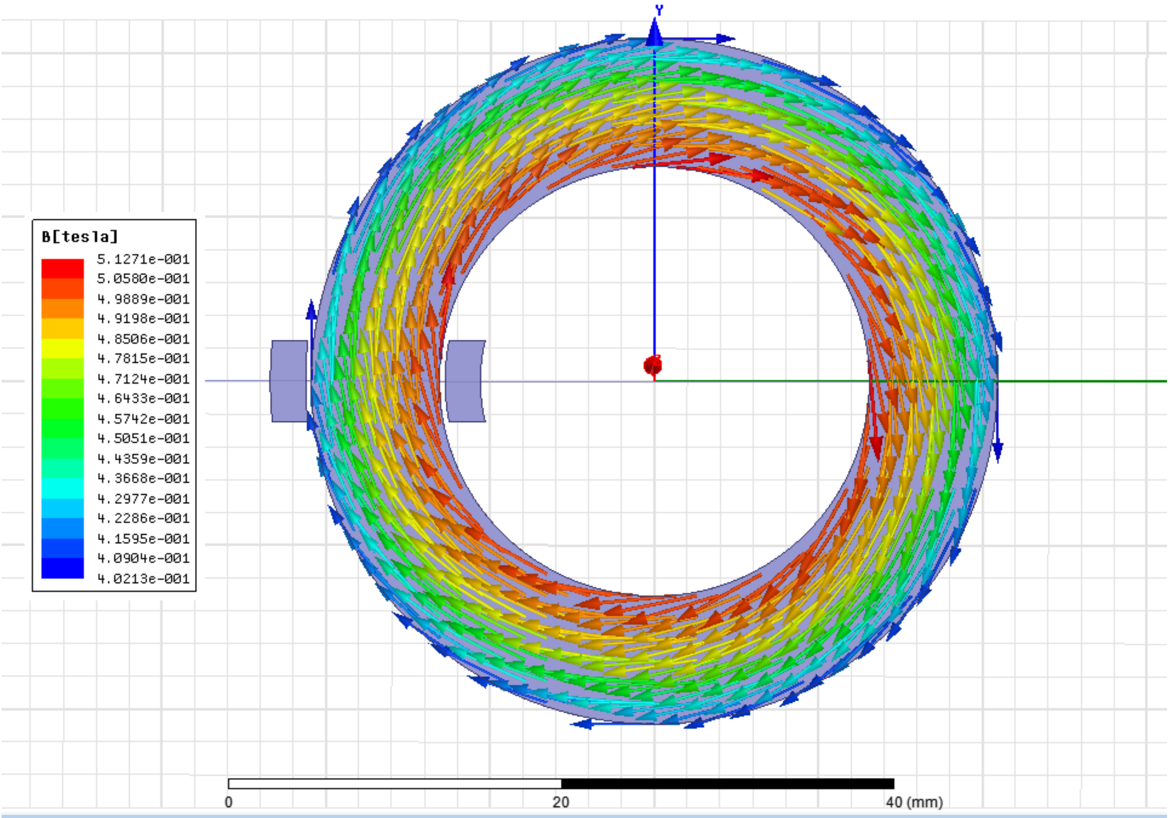


Figure 11. Magnetic flux density for non-linear core material (current = 0.435A)

	Distance [mm]	Mag_B [mTesla] Setup1 : LastAdaptive
1	0.000000	408.693205
2	1.950000	446.736351
3	3.900000	485.824538
4	5.850000	494.516881
5	7.800000	505.334745

Figure 12. Magnitude of magnetic flux density on L1 for non-linear core

	Distance [mm]	Mag_B [uTesla] Setup1 : LastAdaptive
1	0.000000	239.382398
2	2.275000	399.151605
3	4.550000	647.548442
4	6.825000	934.871602
5	9.100000	76.646088

	Distance [mm]	Mag_B [uTesla] Setup1 : LastAdaptive
1	0.000000	135.734123
2	6.550000	509.954403
3	13.100000	264.046887
4	19.650000	175.379173
5	26.200000	131.257045

Figure 13. Magnitude of magnetic flux density on L2 and L3 for non-linear core

Same formulas are used as previous parts. Average magnetic flux on core cross section is Bav = 468.2 mT. (N = 20, I = 0.435, A= 138mm^2)

$L = \frac{NBA}{I} = \mathbf{2.96mH}$

Also, avarege flux densities for leakage inductance are B1 = 459.4uT and B2 = 243uT, then leakage inductances are (N = 20, I = 0.435, A_1= 18*9.1mm^2, A_2= 18*26.2mm^2).

$L_{leakage}$ = **8.71uH**

Inductor is operates linear region of the B-H curve for this current value so that inductance values of Part B.2 and Part B.3 almost same.

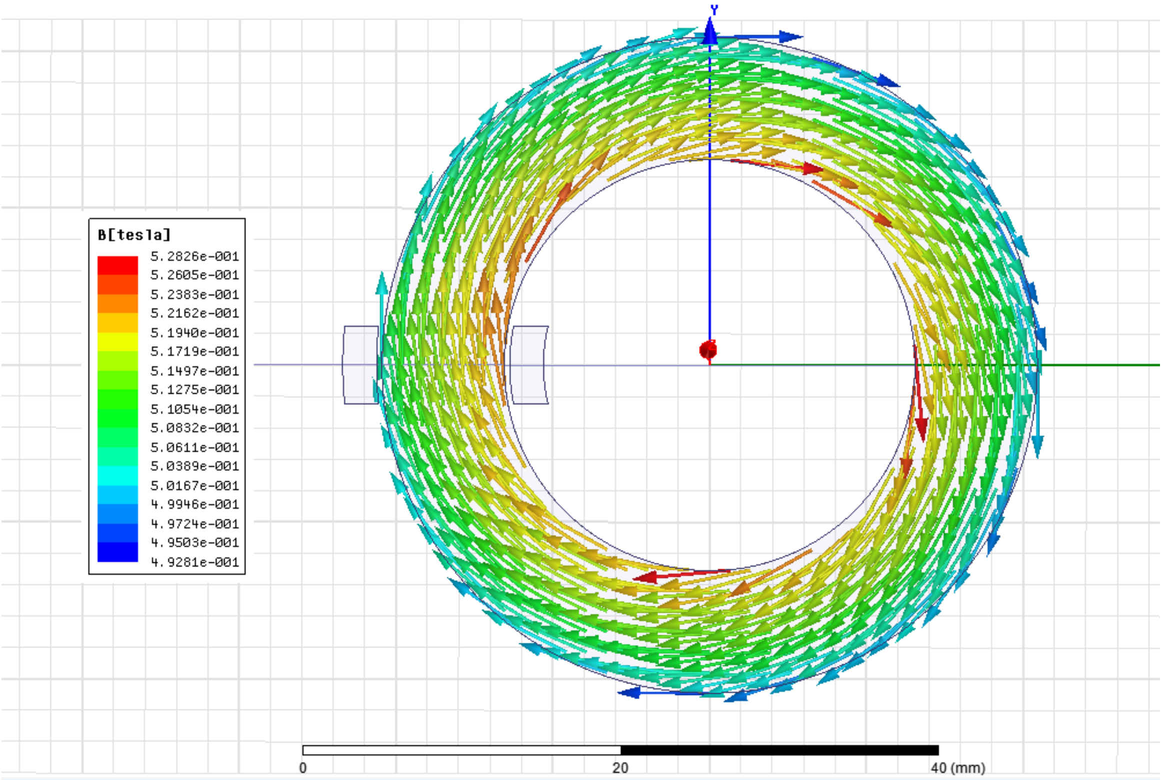


Figure 14. Magnetic flux density for non-linear core material (current = 0.435A*1.5)

	Distance [mm]	Mag_B [mTesla] Setup1 : LastAdaptive
1	0.000000	503.232389
2	1.950000	512.769438
3	3.900000	517.810551
4	5.850000	520.468538
5	7.800000	523.184880

Figure 15. Magnitude of magnetic flux density on L1 for non-linear core(current = 0.435*1.5)

	Distance [mm]	Mag_B [mTesla] Setup1 : LastAdaptive
1	0.000000	0.391158
2	2.275000	0.586189
3	4.550000	0.907778
4	6.825000	1.406396
5	9.100000	0.124275

	Distance [mm]	Mag_B [uTesla] Setup1 : LastAdaptive
1	0.000000	212.260232
2	6.550000	772.893551
3	13.100000	393.699512
4	19.650000	260.777483
5	26.200000	195.601144

Figure 16. Magnitude of magnetic flux density on L2 and L3 for non-linear core (current = 0.435*1.5)

Average magnetic flux on core cross section is Bav = 515.4mT. (N = 20, I = 0.653, A= 138mm^2)

$L = \frac{NBA}{I}$ = **2.17mH**

Also, average flux densities for leakage inductance are B1 = 683uT and B2 = 267uT, then leakage inductances are (N = 20, I = 0.653, A_1= 18*9.1mm^2, A_2= 18*26.2mm^2).

L_{L1} = 2.76uH, L_{L2} =1.08uH

$L_{leakage}$ = **3.84uH**

Inductor is operates non-linear region of the B-H curve so that permeability of core is lower, reluctance of core is higher also flux and flux density is lower so that inductances became smaller for this part. Analytical calculation total inductance values for non-linear core material(partA.4, 2.2mH) is verified this total inductance value.

Part B.4

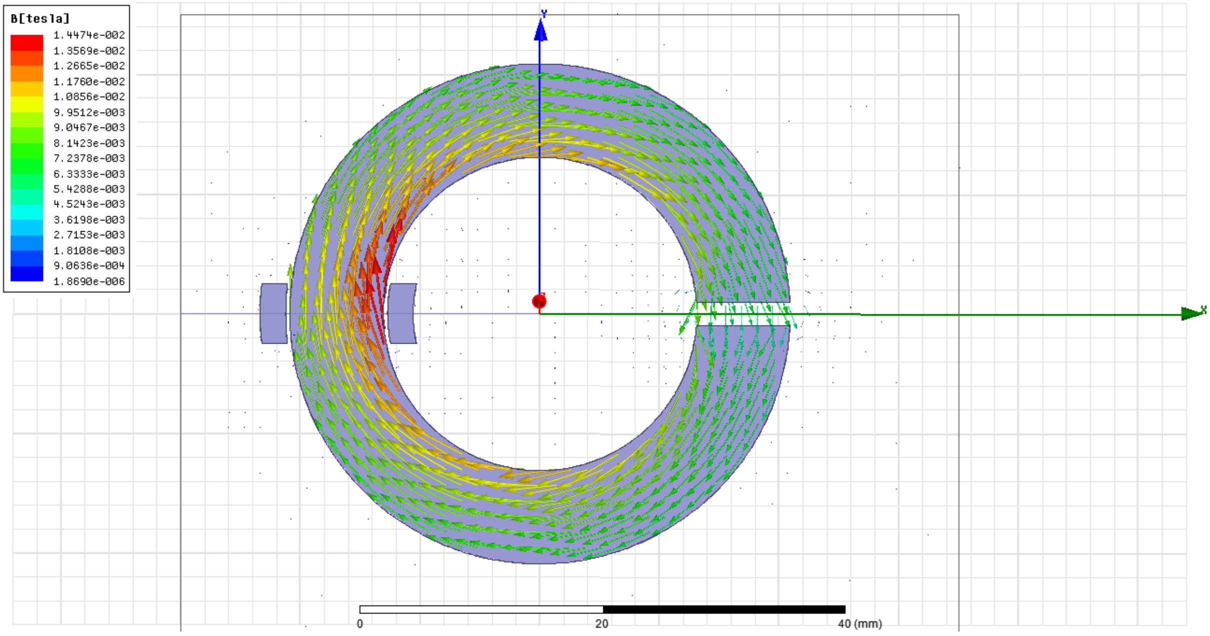


Figure 17. Magnetic flux density for non-linear core material with 2mm gap

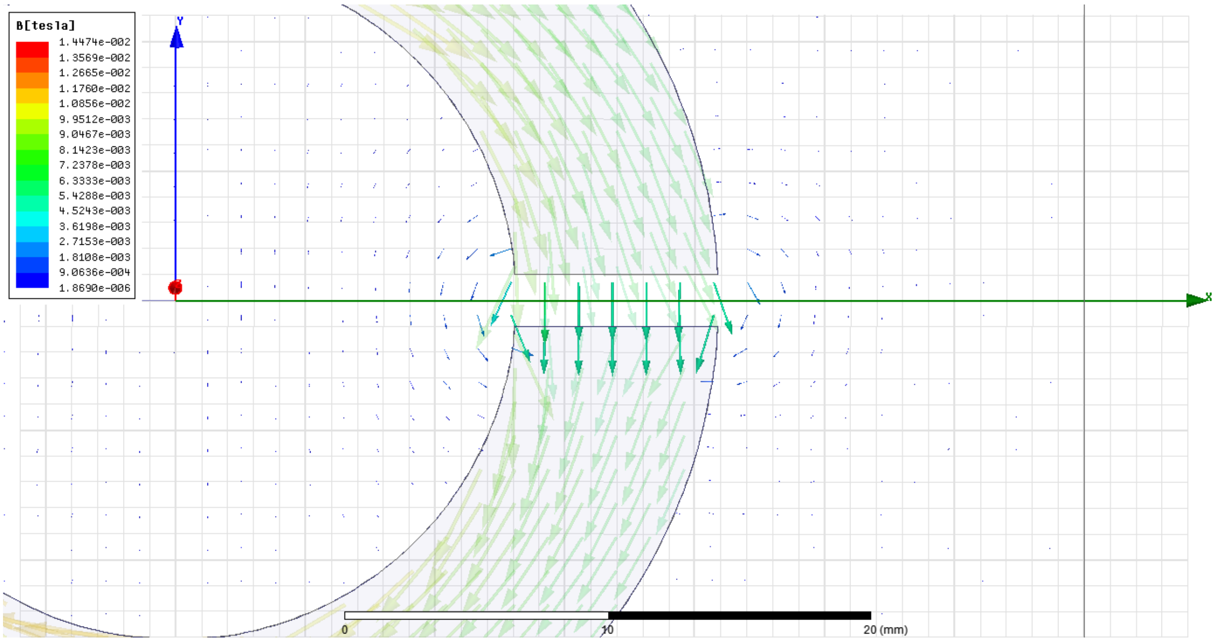


Figure 18. Fringing flux density for non-linear core material with 2mm gap

	Distance [mm]	Mag_B [mTesla] Setup1 : LastAdaptive
1	0.000000	9.666132
2	0.866667	9.958117
3	1.733333	10.259639
4	2.600000	10.570700
5	3.466667	10.891300
6	4.333333	11.400099
7	5.200000	12.116377
8	6.066667	12.867898
9	6.933333	13.654663
10	7.800000	14.476671

Figure 19. Magnitude of magnetic flux density on L1 for non-linear core with 2mm gap

In this part, to calculate fringing inductance with leakage inductance, L4 line is added right of the gap horizontally.

	Distance [mm]	Mag_B [mTesla] Setup1 : LastAdaptive
1	0.000000	0.315490
2	1.011111	0.361304
3	2.022222	0.416040
4	3.033333	0.497973
5	4.044444	0.608047
6	5.055556	0.768216
7	6.066667	1.005566
8	7.077778	0.841355
9	8.088889	0.237044
10	9.100000	0.001869

	Distance [mm]	Mag_B [mTesla] Setup1 : LastAdaptive
1	0.000000	0.005122
2	2.911111	1.217389
3	5.822222	0.757249
4	8.733333	0.595402
5	11.644444	0.535582
6	14.555556	0.527468
7	17.466667	0.583035
8	20.377778	0.746694
9	23.288889	1.293303
10	26.200000	4.551013

	Distance [mm]	Mag_B [mTesla] Setup1 : LastAdaptive
1	0.000000	0.180774
2	1.555556	0.211676
3	3.111111	0.249296
4	4.666667	0.293180
5	6.222222	0.363179
6	7.777778	0.465734
7	9.333333	0.638482
8	10.888889	0.950622
9	12.444444	1.670321
10	14.000000	4.197864

Figure 20. Magnitude of magnetic flux density on L2(top-left), L3(top-right) and L4(bottom) for non-linear core with 2mm gap

In this part, 2mm air gap is added to the core, this cause higher reluctance value of core so that inductance and flux components are decrease.

Average magnetic flux on core cross section is Bav = 11.6 mT. (N = 20, I = 0.435, A= 138mm^2)

$L = \frac{NBA}{I}$ = **73.6uH**

Also, average flux densities for leakage and fringing inductances are B1 = 504uT, B2 = 1.1mT and B3 = 921uT, then leakage inductances are (N = 20, I = 0.653, A_1= 18*9.1mm^2, A_2= 18*26.2mm^2, A_2= 18*14mm^2).

L_{L1} = 3.79uH, L_{L2} =23.8uH, L_{L3} =10.67uH

$L_{leakage+fringing}$ = **38.26uH**

As average flux densities around the air gap, fringing flux is an important effect for kind of applications.

At Part A.5, I have calculated this total inductance 34uH but here total inductance is 73.6uH. In analytical calculation fringing flux is ignored so if we ignore this flux at this part total inductance becomes 35.3uH, which is almost same as analytical value.

Part C

	Analytical		FEA	
Linear core	L_{total} = 3mH		L_{total} = 3mH	$L_{leakage}$ = 8.72uH
Non-linear core (1.5*I)	L_{total} = 2.5mH		L_{total} = 2.17mH	$L_{leakage}$ = 3.83uH
Core with air gap	L_{total} = 34uH	$L_{fringing}$ = ~10nH	L_{total} = 73.6uH	$L_{leakage+fringing}$ = 38.26uH

In analytical calculation, leakage flux could not model because it changes according to specifications of winding. Also, only fringing flux could not be modeled FEA because we could not separate fringing and leakage fluxes. Moreover, calculation of all point on the core by analytically but FEA made it almost all point according to the meshing size.

2D analysis method is appropriate only symmetric geometrical models because we draw only one cross section of model but it can have any salience or indentation such as screw hole. All geometry especially magnetic permeable materials geometry effects magnetic circuit. for these reasons, 3D analysis is more appropriate for non-symmetrical models.

Question 2

There is a lot of trade-offs when designing transformer so that optimization of design really important. All important parameters are explained bottom.

Power, Vin,Vo, Freq, temp, Iin, Iout	These parameters depend on customer needs, main purpose is designer obtaining these needs with maximum efficiency with minimum cost
Temperature	It effects B-H curve directly, so it is important for calculation
Type (square, E, C core)	Calculation of magnetic circuit depends on shape
Lamination type	According to frequency and material, eddy current and core loss decrease with lamination
B,H	Main operating flux density is decided according to material B-H curve
Cable resistivity, radius etc	According to current of transformer, cable is chosen then these parameters obtained
Core cross section	Cross section area effects reluctance, flux, inductance of the transformer, these parameters are so critically for magnetic circuits and also size and core loss of the transformer, it should be optimized
N1, N2	These parameters also effects flux, inductance etc. Choosing this value high leads to high copper loss, choosing low leads to low flux and high core size
Window size	This value is decided according to fill factor, winding height and also cable quantity of the winding cable
Power factor	It is needed for efficiency calculation

Table 1. Important Input Parameters of Transformer Design

Appendix

Toroids - Uncoated (0) | Toroids - Epoxy coated (Z) | Toroids - Parylene coated (Y)

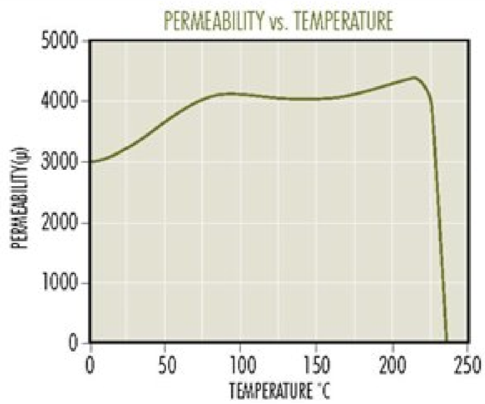
Part #	A_L nH/T² (nominal)									L_e Path Length mm	A_e Cross Section mm²	V_e Volume mm³	WaAc cm⁴	OD mm	ID mm	Height mm
	L 900µ	R 2300µ	P 2500µ	F 3000µ	T 3000µ	J 5000µ	W 10000µ	M 15000µ	C 900µ							
4015		3,867	4,204		5,040	8,408	16,816			103	138	14,205	7.44	41.8	26.2	18

T Material

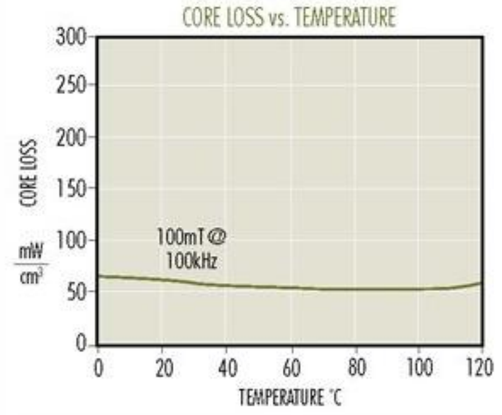
T material is a power material for transformers and inductors operating from 20kHz to 750kHz. T material offers stability in both perm and losses over a wide temperature range.

Initial Perm (10 kHz)	3,000 ± 25%
Saturation Flux Density (5,300 G at 15 Oe, 25°C)	530 mT, 11.9 A•T/cm
Curie Temperature	220°C
Maximum Usable Frequency (50% roll-off)	≤1.5 MHz
Remanence (1,500 G, 25°C)	150 mT
Resistivity	5 Ω-m
Density	4.8 g/cm³

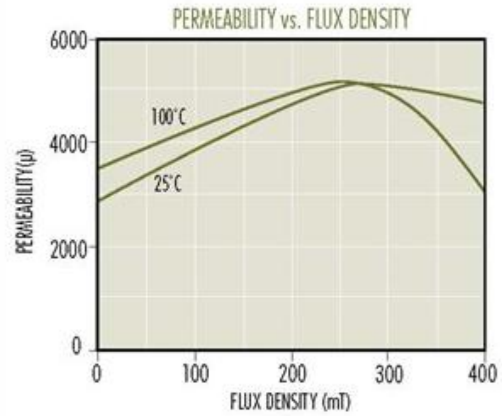
PERMEABILITY vs. TEMPERATURE



CORE LOSS vs. TEMPERATURE



PERMEABILITY vs. FLUX DENSITY



CORE LOSS vs. FLUX DENSITY AT 100°C