# Rewarding Doubt: A Reinforcement Learning Approach to Calibrated Confidence Expression of Large Language Models

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### **Abstract**

A safe and trustworthy use of Large Language Models (LLMs) requires an accurate expression of confidence in their answers. We propose a novel Reinforcement Learning approach that allows to directly fine-tune LLMs to express calibrated confidence estimates alongside their answers to factual questions. Our method optimizes a reward based on the logarithmic scoring rule, explicitly penalizing both over- and under-confidence. This encourages the model to align its confidence estimates with the actual predictive accuracy. The optimal policy under our reward design would result in perfectly calibrated confidence expressions. Unlike prior approaches that decouple confidence estimation from response generation, our method integrates confidence calibration seamlessly into the generative process of the LLM. Empirically, we demonstrate that models trained with our approach exhibit substantially improved calibration and generalize to unseen tasks without further fine-tuning, suggesting the emergence of general confidence awareness. We provide our training and evaluation code in the supplementary and will make it publicly available upon acceptance.

#### 1 Introduction

In human intelligence and inter-human interaction, the ability to understand our own uncertainty and communicate our doubts to others is fundamental for effective decision-making, collaboration, and learning [7, 37]. Similarly, for Large Language Models (LLMs) to be safely used in real-world applications, especially when humans and AI systems work together, they must not only generate accurate information but also communicate their confidence in that information. While LLMs have demonstrated impressive capabilities in natural language understanding, question answering and text summarization [32, 6, 1], LLMs still face significant limitations, such as their tendency to generate inaccurate information, often referred to as hallucinations [11]. This raises concerns about their reliability, particularly in real-world applications where trustworthiness is essential. Especially in high-stakes environments such as medical diagnosis, where LLMs are starting to become support tools for professionals [23, 25, 33, 4], overconfident predictions including factual errors or hallucinations could have serious consequences for patient health. Also, in customer service or legal consultation [26, 28], LLMs need to express uncertainty and defer complex queries to human representatives when unsure to avoid misinformed decisions. Reliable confidence estimation and expression would enable these systems to flag uncertain outputs for human review, ensuring that crucial decisions are

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Figure 1: Illustration of our reward design: The model is rewarded for a high confidence if it is correct and punished if it is incorrect. To maximize the reward, the model needs to learn when to predict a higher or lower confidence, considering a possible higher punishment. Our reward function is designed so that the best reward is given when the confidence and the epistemic probability of being correct are the same, thus incentivizing the expression of calibrated confidences.

not made based on uncertain LLM outputs. To allow risk estimation while using LLM-generated output, model confidence should be calibrated, meaning that the expressed numerical confidence should be equal to the probability of the model's answer being correct.

Many previous methods for confidence estimation lack in calibration performance as they do not train the model and instead infer the confidence from the internal state in a zero-shot setup [14, 19, 8]. Additionally, this does not give models an inherent awareness of confidence. Other trained methods in this area decouple the uncertainty estimation from the text generation process [3, 18]. This approach optimizes for calibrated confidence estimation but does not enable the uncertainty-awareness and expression in the model itself.

Targeting these limitations, we propose a novel reinforcement-learning (RL) approach for teaching LLMs to express their calibrated confidence, encouraging a granular, accurate estimation of the confidence level in the training objective. For this, we model confidence estimation as a betting game: a high-confidence answer would warrant a larger bet, reflecting a strong belief in its correctness, while a lower confidence score would suggest caution. Central to our method is a reward function based on the logarithmic scoring rule, a strictly proper scoring rule. We are the first to optimize this function through reinforcement-learning-based policy optimization, leveraging its calibration properties for directly and seamlessly training confidence calibration in LLM generations. This reward function captures the fundamental risk-reward tradeoff in probabilistic decision-making, as illustrated in Figure 1. It increases the reward when a correct answer is given with high confidence, simulating the higher potential return of big bets. Conversely, it penalizes incorrect answers more when they are made with high confidence, discouraging overconfidence. This ensures that both uncertainty and confidence are appropriately factored into the reward system. As a proper scoring rule, optimizing the reward function trains the model to align its predicted confidence with the accuracy of its output, encouraging granular and calibrated confidence scoring. A calibrated confidence estimation will provably result in the highest reward during training. This not only improves the trustworthiness of LLMs in collaborative human-AI scenarios but also helps users better assess when AI tools should be trusted, double-checked, or deferred to human expertise.

## 2 Related Works

Confidence estimation and calibration have a long history in machine learning and natural language processing [34]. With the rise of LLMs, a lot of attention has moved to extracting and improving confidence estimation from these new models. There is a wide range of methods that can roughly be divided into black-box and white-box methods [9].

Black-box methods require only the model generations for uncertainty estimation and do not depend on any knowledge of the model's internal state. Within this group, linguistic confidence methods assess the uncertainty by simply prompting the LLM to verbalize its confidence. Xiong et al. [37]

showed that prompting techniques like chain-of-thought (CoT) [36] can, to a limited extend, help the model in the calibration of its confidence expression. Based on the assumption that uncertainty in its generation would prime the model to hallucinate information, consistency-based methods [21, 35] measure the similarity between multiple generations and show that high variance in the generation correlates with high uncertainty. Wang et al. [35] propose using CoT to produce various reasoning pathways towards an answer. By marginalizing out the reasoning pathways, they provide the most consistent answer as the one most probably correct and offer the consistency measure as an estimation of the model's confidence. Black-box methods are valuable for their simplicity, ease-of-use and universality, however generally lack behind white-box methods in their calibration performance.

White-box methods, on the other hand, depend on access to the internal state of the model, either for the evaluation of logits and activations or for fine-tuning of the model for calibration. Logit-based methods calibrate the confidence by assessing the probabilities or the entropy of tokens in the predicted sequence. Huang et al. [14] calculate the mean probability of the generated tokens with the intuition that a model that generates an answer with high probability tokens also has a high confidence in this answer. Kuhn et al. [19] and [8] expand on this idea by looking at the entropy of the generated tokens while also taking the semantic relevance into account. Kadavath et al. [17] showed that LLMs can self-evaluate the correctness of their own generation. They prompt the model to provide an answer followed by a judgement whether its own answer is "true" or "false". They then compare the probability of the "true" or "false" token to calculate a confidence estimation.

While all the aforementioned methods are zero-shot methods, another line of white-box methods, proposes to train external models for confidence estimation. Xiong et al. [37] propose a method that trains a linear probe for assessing the model's confidence based on the activations of the LLM's hidden state. They evaluate the application of the probe to various layers and train it with correctness labels of the task. This approach does not explicitly train on the granularity of confidence estimation and, therefore, is only indirectly able to express a confidence gradient for fine-grained calibration. Building on this approach, Mielke et al. [22] give an LLM access to the confidence extracted by the external probe and fine-tune it to express this confidence. However, the calibration performance is upper-bounded by the ability of the trained probe to assess the model's confidence accurately. Stengel-Eskin et al. [27] propose a DPO-based method, LACIE, simulating an interaction between a generation and a listener model, where the speaker model generates answers, and the listener model evaluates them based on perceived confidence. By training with Direct Preference Optimization (DPO) on data from TriviaQA, LACIE rewards accurate confidence expression, including correct answers accepted by the listener model and incorrect, rejected ones, while penalizing misleading confidences. The confidence expression is calibrated to how the listener model interprets confidence cues, depending on its ability to judge confidence and aligning perceived confidence rather than a factbased numerical confidence score. However, none of these methods provide theoretical guarantees or provable optimality regarding the trained calibration of the expressed confidence.

Simultaneously, fine-tuning LLMs using Reinforcement Learning from human feedback (RLHF) has received increasing attention for aligning the models with human preferences, avoiding toxicity or the generation of harmful content [24, 20]. Outside of RLHF, RL has been used to fine-tune LLMs for agentic interaction in textual environments [40, 5]. Confidence estimation of LLMs using RL is a novel research direction that, to the best of our knowledge, has only been approached by Tao et al. [30] and Xu et al. [38]. Tao et al. [30] build on the Reinforcement Learning with Human Feedback (RLHF) paradigm and design a reward function that aligns the verbalized confidence of the model with the preference rating predicted by the RLHF reward model. However, this approach can not evaluate model confidence in factual statements and depends on human-annotated preference data. Xu et al. [38] introduce a novel Reinforcement Learning from Knowledge Feedback (RLKF) framework, which aims to teach a model to refuse to answer questions outside their knowledge scope, reducing hallucinations, but does not quantify confidence when giving an answer.

In contrast, our method directly optimizes for factual calibration using a theoretically grounded, proper scoring rule as the reward signal, enabling the model to develop intrinsic uncertainty awareness without requiring external preference models, knowledge supervision, or post-hoc calibration techniques, while at the same time seamlessly integrating calibrated confidence expression into the LLMs response generation.

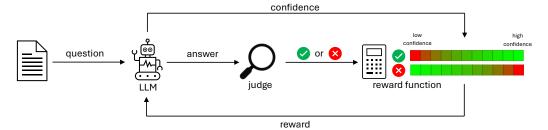


Figure 2: Overview of our reinforcement learning framework: The LLM is prompted to answer a question and provide the confidence in this answer. The answer is checked for correctness by a judge function and the reward is calculated based on the correctness and the confidence. Correct answers with high confidences are rewarded highly, but also penalized heavily when incorrect.

## 3 Rewarding Doubt

We propose a novel reinforcement-learning approach, that improves an LLM's ability to verbalize an accurate numerical confidence in a previously generated answer. The LLM functions as an agent in a simulated environment as shown in Figure 2, that poses challenging question-answering scenarios. It is prompted with task queries such as factual questions and asked to predict both an answer to the query as well as a confidence score. Based on the correctness of the answer, and the expressed confidence, we reward the model, incentivizing it to express a calibrated confidence.

Formally, let the model be provided with a textual question or request q, resulting in an answer-confidence pair  $(a,\hat{p})$  as response, where a is a textual answer with binary correctness value, and  $0 \le \hat{p} \le 1$  is a numerical confidence score representing the subjective probability the model assigns to answer a being correct. We train this subjective probability assessment to align with the true epistemic probability  $p^*$ , which represents the actual likelihood of correctness given the model's internal knowledge state. If  $\hat{p}$  and  $p^*$  are aligned the model is perfectly calibrated, meaning the probability of correctness P(j(a)=1) always equals the expressed confidence:

$$P(j(a) = 1 \mid \hat{p} = x) = x, \quad \forall x \in [0, 1],$$

where  $j(\cdot)$  is a correctness judging function that is 1 if answer a is correct, and 0 otherwise.

The true epistemic probability  $p^*$  is not directly observable, thus supervised learning of calibration is only possible by constructing an artificial ground truth to approximate  $p^*$ . Instead, we model this task as a Markov Decision Process (MDP) defined by the tuple  $(\mathcal{S}, \mathcal{A}, \mathcal{T}, R)$ , where the model learns to generate calibrated confidence scores through reinforcement learning. The MDP is defined by the following components:

- State space (S): A state  $s_t \in \mathcal{S}$  consists of a natural language question q, the model's predicted answer a, and the partial sequence of confidence tokens predicted so far, if any. That is,  $s_t = (q, a, c_{1:t-1})$ , where  $c_{1:t-1}$  represents the previously generated confidence score tokens.
- Action space (A): The action space consists of selecting the next token c<sub>t</sub> in the confidence estimation process from the LLM vocabulary, including numerical tokens (e.g., representing percentages or probability values) and a special end-of-sequence token that finalizes the prediction.
- Transition function  $(\mathcal{T}(s_{t+1} \mid s_t, a_t))$ : The environment transitions deterministically based on the language model's autoregressive token generation process. Given a state  $s_t = (q, a, c_{1:t-1})$  and an action  $c_t$ , the next state is defined as  $s_{t+1} = (q, a, c_{1:t})$ . Once the end-of-sequence token is generated, the episode terminates.
- Reward function (R): The reward R(a,c,j) is computed based on the final confidence score sequence  $c=(c_1,\ldots,c_T)$  and the correctness of the answer j(a).

To promote accurate confidence estimation, the model's reward function must fulfill the requirement of being maximized when  $\hat{p} = p^*$ . The model should receive a higher reward when it correctly predicts an outcome with high confidence. Conversely, incorrect predictions should be penalized

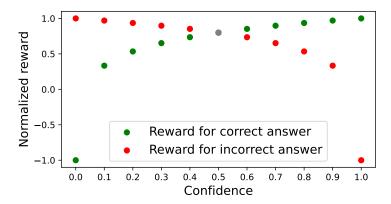


Figure 3: The rewards for each confidence value for correct and incorrect answers. The closer the confidence is to ten or zero, respectively, the higher is the reward. At the same time, the possible punishment increases to a greater extent. The model has to learn when the trade-off between those two possibilities is worthwhile.

more severely when the model exhibits high confidence. In contrast, expressing uncertainty reduces potential penalties, leading to a higher expected reward in uncertain scenarios. This approach incentivizes the model to express high confidence only in cases where certainty is warranted while expressing doubt in ambiguous situations. By penalizing both overconfidence and underconfidence, the model is encouraged to calibrate its confidence accurately, effectively balancing the trade-off between reward maximization and penalty avoidance.

We design our reward as a logarithmic scoring function, fulfilling the described optimality conditions:

$$R(a, \hat{p}, j) = \begin{cases} log(\max(\hat{p}, \epsilon)), & \text{if } j(a) = 1 \text{ (correct)} \\ log(\min(1 - \hat{p}, 1 - \epsilon)), & \text{if } j(a) = 0 \text{ (incorrect)} \end{cases}$$

where  $\epsilon>0$  is a small positive constant introduced for numerical stability to avoid evaluating the logarithm at zero. The normalized reward for correct and incorrect answers for each confidence is visualized in Figure 3.

**Proposition 1** (Optimality implies Calibration). The expected reward  $\mathbb{E}[R(a,\hat{p},j)]$  is maximized for each sample when  $\hat{p} = p^*$  and the optimal policy under the reward design is thus perfectly calibrated.

The proof of Proposition 1 is analogous to the proof that the logarithmic scoring rule is a proper scoring rule. We provide it in full in Appendix B and discuss the influence of the clipping on the optimality of the reward function.

# 4 Experimental Setup

We evaluate our method in both Single-Answer and Multiple-Answer settings. We prompt the model to provide a confidence for each answer as an integer between 0 and 10, which we normalize for the reward calculation. A confidence of zero is defined as the model being certain that the answer is incorrect, while ten is defined as the model being certain the answer is correct. We normalize the reward function to the range of [-1,1].

In the Single-Answer setting we train the model on the TriviaQA dataset [16], which contains question-answer-evidence triplets, from which we only use the questions and answers. For generalization experiments, we evaluate our method on CommonsenseQA [29] and MedQA [15], which are multiple-choice question datasets in the commonsense and medical domain, respectively. For the Multiple-Answer setting, we train on the QAMPARI dataset [2], which contains questions with multiple-answers as well as evidence, again only using the questions and answers.

In the Single-Answer setting we compare our approach on the TriviaQA dataset against the following methods: Chain-of-Thought [37], Top-K [31], Surrogate Token [17], Sequence Probability [14] and Self-Consistency [35] as zero-shot methods, LACIE [27], which uses DPO for optimizing

confidence expression and Trained Probe [3], which employs supervised training of an external probe for estimation model. We also compare to the non-finetuned base model in a zero-shot manner, using the same prompt as our Rewarding Doubt method and refer to this setup as Verbalize. In the Multiple-Answer setting we compare to Trained Probe and Sequence probability, as those methods are the best performing zero-shot and trained baselines in the Single-Answer setting. LACIE does not report results for this dataset, thus we can only compare on TriviaQA.

We report our results using the Expected Calibration Error (ECE) and the Area Under the Receiver Operating Characteristic Curve (AUROC) metric. The ECE measures the difference between a model's predicted confidence and its actual accuracy, assessing how well confidences align with real outcomes. AUROC assesses a model's ability to distinguish between classes, with a score of 0.5 indicating random guessing and 1.0 indicating perfect classification. Additionally, we visualize the calibration with calibration curves, where a well-calibrated model lies close to the 45° line and large deviations show a high miscalibration.

**Response Generation** To calibrate and reward the model only on the confidences and not the answers we separate generation in two steps during training: Answer and confidence generation. Answers are generated first and afterwards treated as fixed inputs alongside the question, while the confidence is generated in a separate generation step and considered as sole target for optimization. Like this, we ensure that answer generation is disentangled from the optimization process, ensuring the answer correctness is not affected by our confidence calibration training.

**Correctness Assessment** For the multiple-choice datasets MedQA and CommonsenseQA, we evaluate correctness using the exact string matching between the model's response and the ground truth answer.

For the TriviaQA and QAMPARI datasets, we use the F1 score of word overlap to measure the similarity between the model's response and the ground truth candidates. The F1 score is calculated for each candidate and the maximum score is considered the final score. We consider an answer as correct if its score exceeds a threshold of 0.5.

## 5 Implementation Details

**Training** As base model for our experiments we use Meta-Llama-3-8B-Instruct [10]. We employ the 4-bit quantized performance-optimized model version by Unsloth AI [12]. Additionally, we apply LoRA fine-tuning [13], reducing the amount of trainable parameters from 8,030,265,345 to 41,947,137. For the Single-Answer setting we train the model for two epochs with a learning rate of 1e-5. For the Multiple-Answer setting, due to the size of the training dataset and the fact that each question yields multiple facts, the model is trained for a limited amount of 24,000 steps with a batchsize of eight and a learning rate of 1e-5 with a reward scale of five. All models are trained on one Nvidia A40 with each training run taking seven days. On average the model generated approximately 3.4 answers per fact. If the model fails to generate an answer in the specified format, it is penalized with an out-of-format reward of -3.

**Baselines** For the Sequence Probability, we compute the average probability for each token in the response. In the Self-Consistency method, we let the model explore ten reasoning pathways, and the similarity of each resulting output is evaluated using the BERTScore metric [39]. For the trained probe [3], the original study introduced a custom dataset comprising short statements classified as either true or false. The model's activations in response to these statements were extracted from specific layers, and a multilayer perceptron (MLP) was subsequently trained on these activations to predict the truthfulness of the statements. To ensure a fair comparison, we adapted this methodology to better align with our data by allowing the model to generate answers to training dataset questions and then extracting its activations from the 24th layer for both the statements and their corresponding answers. The labels for each sample were determined following the same evaluation procedure as described in our evaluation framework. For the architecture of the MLP, we employed the same design as Azaria and Mitchell [3] and train it for four epochs with a learning rate of 1e-4 until convergence. The exact prompts used for each baseline are provided in Appendix A.

#### 6 Results and Discussion

Table 1: Comparison of methods on the TriviaQA dataset in the Single-Answer setting. \*Results are from the original paper [27].

Method	ECE (↓)	AUROC (†)	Accuracy (†)
Verbalize	0.3459	0.5858	0.6310
Chain-of-Thought	0.3065	0.6379	0.6273
Top-K	0.1611	0.6673	0.6023
Surrogate Token	0.3686	0.5923	0.5933
Sequence Probability	0.3156	0.7804	0.5955
Self-Consistency	0.1134	0.8213	0.6224
LACIE*	0.1200	0.7200	n/a
Trained Probe	0.0189	0.8173	0.5925
Rewarding Doubt (ours)	0.0226	0.8592	0.6309

This section presents and discusses the key findings of our experiments for both Single and Multiple-Answer tasks and the generalization to out-of-domain datasets.

**Calibration Performance** To assess how well our approach improves calibration, we compare it against the zero-shot LLM baseline (Verbalize) and several established methods in both Single-Answer and Multiple-Answer question-answering tasks. Results for the Single-Answer setting on TriviaQA are presented in Table 1, and those for the Multiple-Answer setting on QAMPARI appear in Table 2. Across both tasks, Rewarding Doubt substantially improves the model's confidence calibration over zero-shot verbalization.

In the Single-Answer setting on TriviaQA, Rewarding Doubt achieves an ECE of 0.0226 and an AUROC of 0.8592, clearly outperforming all zero-shot baselines as well as LACIE, which is based on DPO-based optimization. The second fine-tuned method, Trained Probe, which relies on supervised fine-tuning, reports a slightly lower ECE (0.0189), both methods achieve near-perfect results. Further the AUROC of Rewarding Doubt is notably higher, suggesting that although both methods offer strong calibration, Rewarding Doubt better discriminates between correct and incorrect answers. In the Multiple-Answer setting on QAMPARI, Rewarding Doubt also outperforms baselines, achieving an ECE of 0.0816 and an AUROC of 0.6947. In comparison, Verbalize, Sequence Probability, and Trained Probe perform notably worse. Our findings support the claim by Azaria and Mitchell [3] that a model's internal state encodes information about the truthfulness of statements, which can serve as an indicator of uncertainty. However, without fine-tuning, the model struggles to utilize this internal information effectively. Our approach enables the model to make use of this correlation and translate it into an accurate expression of the probability that a given answer is correct.

The calibration curves in Figure 4 further illustrate these improvements. For both TriviaQA and QAMPARI, the fine-tuned model's confidence much more closely aligns with the ideal 45° line than the zero-shot Verbalize baseline. Additionally, we observe a shift in the confidence distribution after fine-tuning. As shown in Figure 5, in a zero-shot setting the LLM (Verbalize) predominantly assigns high confidence scores (8 or above), reflecting overconfidence, a pattern also noted by Xiong et al. [37], who attribute it to supervised pretraining that favors confident expressions. After fine-tuning with Rewarding Doubt, the model's confidence scores (shown in Figure 5b) span a wider range, including lower values, indicating a more nuanced expression of uncertainty. This shift suggests that fine-tuning mitigates overconfidence and better aligns the model's confidence with its actual performance.

**Stability of Answer Correctness** Training confidence calibration with our method only targets the uncertainty estimation abilities and does not aim to alter the responses of the model. This is achieved by only rewarding the model on its expressed confidence, while the answer is generated beforehand independently from the model update step. Our results show a stable accuracy for all experiments without notable differences in accuracy between the base model (Verbalize) and the model adapted with Rewarding Doubt, showing that confidence calibration training with Rewarding Doubt does not affect task performance.

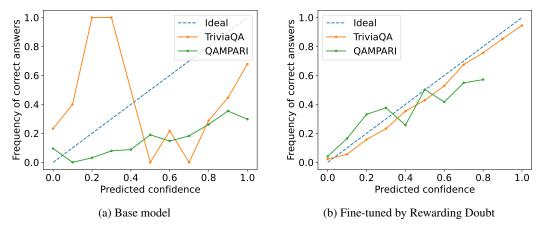


Figure 4: Calibration curves of the zero-shot base model (Verbalize) and the model fine-tuned by Rewarding Doubt.

Table 2: Comparison of methods on the QAMPARI dataset in the Multiple-Answer setting.

Method	ECE $(\downarrow)$	AUROC (†)	Accuracy (†)
Verbalize	0.5319	0.6047	0.2549
Sequence probability	0.5324	0.5942	0.1927
Trained probe	0.1117	0.6481	0.2233
Rewarding doubt (ours)	0.0816	0.6947	0.2480

**Generalization Capabilities** To assess the generalization abilities of Rewarding Doubt, we evaluated the model trained on TriviaQA in out-of-domain settings using the CommonsenseQA [29] and MedQA [15] datasets. Results are shown in Section 6 and Table 4, respectively.

On MedQA, Rewarding Doubt significantly outperforms Verbalize in both metrics, while on CommonsenseQA, it achieves a comparable ECE, however paired with a much higher AUROC. This discrepancy highlights a limitation of relying solely on ECE for evaluating calibration. ECE does not reflect how well a model discriminates between correct and incorrect predictions across different confidence levels. A model consistently assigning moderate confidence values could appear well-calibrated under ECE, yet fail to offer meaningful distinctions between uncertain and certain cases. The AUROC, by contrast, directly measures this discriminative ability. Thus, the substantial improvements in AUROC underscore that Rewarding Doubt produces more useful and actionable confidence

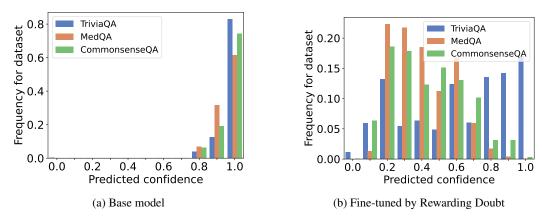


Figure 5: Histograms of predicted confidences of the zero-shot base model (Verbalize) and the model fine-tuned on the TriviaQA dataset.

Table 3: Comparison of generalization results on the CommonsenseQA dataset trained on the TriviaQA dataset.

Method	ECE $(\downarrow)$	AUROC (†)	Accuracy $(\uparrow)$
Verbalize	0.2820	0.5425	0.6860
Trained Probe	0.4819	0.5374	0.7108
Rewarding doubt (ours)	0.2930	0.6385	0.7163

Table 4: Comparison of generalization results on the MedQA dataset trained on the TriviaQA dataset.

Method	ECE $(\downarrow)$	AUROC (†)	Accuracy $(\uparrow)$
Verbalize	0.4480	0.5075	0.5066
Trained Probe	0.2099	0.5513	0.5051
Rewarding doubt (ours)	0.1145	0.6649	0.5161

estimates. Compared to the Trained Probe baseline, Rewarding Doubt consistently outperforms with significantly improved metrics, showing a stronger ability to generalize to new datasets.

We also explore generalization across experimental settings by applying a model trained in a Single-Answer setting to a Multiple-Answer task. Although it does not reach the performance of a model trained specifically for that task, it still outperforms baseline methods considerably, demonstrating transferability of the learning confidence estimation patterns. This suggests promising applications for improving confidence estimation in more complex or less structured scenarios, such as fact verification and calibration in free-text generation, even when specialized training data is unavailable.

Overall, our experiments show that Rewarding Doubt provides a robust and efficient way to enhance calibration, while generalizing across tasks, and maintaining stable task performance, making it an effective approach for accurate confidence calibration and expression in language models. Beyond improvements in calibration quality, our method also offers practical advantages. While fine-tuning requires an initial training investment, inference remains highly efficient, as only a small, constant number of tokens need to be generated to express confidence. In contrast, zero-shot methods like Chain-of-Thought and Self-Consistency have substantial computational overhead during inference by requiring lengthy reasoning chains or multiple generations. Rewarding Doubt introduces no such overhead, does not rely on an additional model, and directly provides actionable confidence estimates through simple verbalization directly by the LLM, making it highly suitable for real-world deployment.

**Limitations** During our experiments, we observed that the policy occasionally converges to always predicting a fixed confidence value, likely caused by training instabilities. While this did not occur consistently, it suggests a potential area for further refinement, such as adjusting the reward structure or introducing additional constraints to encourage a more stable learning of a diverse confidence distribution. Additionally, our approach is currently limited to settings, where a binary correctness measure can be established. In the future this can be extended to also include continuous correctness measures such as usually applied for free text generation.

Table 5: Comparison of the base and fine-tuned model on the Qampari dataset in different settings.

Training	Evaluation	$\mathbf{ECE}\left(\downarrow\right)$	AUROC $(\uparrow)$
Base model	Single fact	0.5875	0.5787
Single fact	Single fact	<b>0.1536</b>	<b>0.7240</b>
Base model	Multi fact	0.5319	0.6047
Single fact	Multi fact	0.1777	0.6617
Multi fact	Multi fact	<b>0.1061</b>	<b>0.7268</b>

### 7 Conclusion

In this work, we propose Rewarding Doubt, a novel approach that enables LLMs to express confidence in their answers more accurately using natural language. We leverage reinforcement learning with a reward function based on the logarithmic scoring rule that incentivizes well-calibrated confidence expressions. Fine-tuning with our method significantly improves the model's ability to estimate a calibrated confidence, effectively reducing the overconfidence patterns commonly observed in LLMs. This not only enhances the trustworthiness in AI-generated responses but also lays the groundwork for more reliable human-AI collaboration, where models can transparently communicate uncertainty, an essential step toward safer and more accountable AI systems.

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#### References

- [1] Josh Achiam, Steven Adler, Sandhini Agarwal, Lama Ahmad, Ilge Akkaya, Florencia Leoni Aleman, Diogo Almeida, Janko Altenschmidt, Sam Altman, Shyamal Anadkat, et al. Gpt-4 technical report. *arXiv*, 2023.
- [2] Samuel Joseph Amouyal, Tomer Wolfson, Ohad Rubin, Ori Yoran, Jonathan Herzig, and Jonathan Berant. Qampari: An open-domain question answering benchmark for questions with many answers from multiple paragraphs, 2023. URL https://arxiv.org/abs/2205. 12665.
- [3] Amos Azaria and Tom Mitchell. The internal state of an llm knows when it's lying, 2023. URL https://arxiv.org/abs/2304.13734.
- [4] David Bani-Harouni, Nassir Navab, and Matthias Keicher. Magda: Multi-agent guideline-driven diagnostic assistance. In *International workshop on foundation models for general medical AI*, pages 163–172. Springer, 2024.
- [5] Thomas Carta, Clément Romac, Thomas Wolf, Sylvain Lamprier, Olivier Sigaud, and Pierre-Yves Oudeyer. Grounding large language models in interactive environments with online reinforcement learning. In *International Conference on Machine Learning*, pages 3676–3713. PMLR, 2023.
- [6] Wei-Lin Chiang, Zhuohan Li, Zi Lin, Ying Sheng, Zhanghao Wu, Hao Zhang, Lianmin Zheng, Siyuan Zhuang, Yonghao Zhuang, Joseph E. Gonzalez, Ion Stoica, and Eric P. Xing. Vicuna: An open-source chatbot impressing gpt-4 with 90%\* chatgpt quality, 2023.
- [7] Leda Cosmides and John Tooby. Are humans good intuitive statisticians after all? rethinking some conclusions from the literature on judgment under uncertainty. *cognition*, 58(1):1–73, 1996.
- [8] Jinhao Duan, Hao Cheng, Shiqi Wang, Alex Zavalny, Chenan Wang, Renjing Xu, Bhavya Kailkhura, and Kaidi Xu. Shifting attention to relevance: Towards the predictive uncertainty quantification of free-form large language models. In *Proceedings of the 62nd Annual Meeting of the Association for Computational Linguistics (Volume 1: Long Papers)*, pages 5050–5063, 2024.
- [9] Jiahui Geng, Fengyu Cai, Yuxia Wang, Heinz Koeppl, Preslav Nakov, and Iryna Gurevych. A survey of confidence estimation and calibration in large language models, 2024. URL https://arxiv.org/abs/2311.08298.
- [10] Aaron Grattafiori, Abhimanyu Dubey, Abhinav Jauhri, Abhinav Pandey, Abhishek Kadian, Ahmad Al-Dahle, et al. The llama 3 herd of models, 2024. URL https://arxiv.org/abs/ 2407.21783.

- [11] Muhammad Usman Hadi, Rizwan Qureshi, Abbas Shah, Muhammad Irfan, Anas Zafar, Muhammad Bilal Shaikh, Naveed Akhtar, Jia Wu, Seyedali Mirjalili, et al. A survey on large language models: Applications, challenges, limitations, and practical usage. *Authorea Preprints*, 2023.
- [12] Daniel Han, Michael Han, et al. Unsloth. 2023. URL http://github.com/unslothai/ unsloth.
- [13] Edward J Hu, yelong shen, Phillip Wallis, Zeyuan Allen-Zhu, Yuanzhi Li, Shean Wang, Lu Wang, and Weizhu Chen. LoRA: Low-rank adaptation of large language models. In *International Conference on Learning Representations*, 2022.
- [14] Yuheng Huang, Jiayang Song, Zhijie Wang, Shengming Zhao, Huaming Chen, Felix Juefei-Xu, and Lei Ma. Look before you leap: An exploratory study of uncertainty measurement for large language models, 2023. URL https://arxiv.org/abs/2307.10236.
- [15] Di Jin, Eileen Pan, Nassim Oufattole, Wei-Hung Weng, Hanyi Fang, and Peter Szolovits. What disease does this patient have? a large-scale open domain question answering dataset from medical exams, 2020. URL https://arxiv.org/abs/2009.13081.
- [16] Mandar Joshi, Eunsol Choi, Daniel S. Weld, and Luke Zettlemoyer. Triviaqa: A large scale distantly supervised challenge dataset for reading comprehension. In *Proceedings of the 55th Annual Meeting of the Association for Computational Linguistics*, Vancouver, Canada, July 2017. Association for Computational Linguistics.
- [17] Saurav Kadavath, Tom Conerly, Amanda Askell, Tom Henighan, Dawn Drain, Ethan Perez, Nicholas Schiefer, Zac Hatfield-Dodds, Nova DasSarma, Eli Tran-Johnson, Scott Johnston, Sheer El-Showk, Andy Jones, Nelson Elhage, Tristan Hume, Anna Chen, Yuntao Bai, Sam Bowman, Stanislav Fort, Deep Ganguli, Danny Hernandez, Josh Jacobson, Jackson Kernion, Shauna Kravec, Liane Lovitt, Kamal Ndousse, Catherine Olsson, Sam Ringer, Dario Amodei, Tom Brown, Jack Clark, Nicholas Joseph, Ben Mann, Sam McCandlish, Chris Olah, and Jared Kaplan. Language models (mostly) know what they know, 2022. URL https://arxiv.org/abs/2207.05221.
- [18] Sanyam Kapoor, Nate Gruver, Manley Roberts, Arka Pal, Samuel Dooley, Micah Goldblum, and Andrew Wilson. Calibration-tuning: Teaching large language models to know what they don't know. In Raúl Vázquez, Hande Celikkanat, Dennis Ulmer, Jörg Tiedemann, Swabha Swayamdipta, Wilker Aziz, Barbara Plank, Joris Baan, and Marie-Catherine de Marneffe, editors, *Proceedings of the 1st Workshop on Uncertainty-Aware NLP (UncertaiNLP 2024)*, pages 1–14, St Julians, Malta, March 2024. Association for Computational Linguistics. URL https://aclanthology.org/2024.uncertainlp-1.1/.
- [19] Lorenz Kuhn, Yarin Gal, and Sebastian Farquhar. Semantic uncertainty: Linguistic invariances for uncertainty estimation in natural language generation. arXiv preprint arXiv:2302.09664, 2023.
- [20] Ximing Lu, Sean Welleck, Jack Hessel, Liwei Jiang, Lianhui Qin, Peter West, Prithviraj Ammanabrolu, and Yejin Choi. Quark: Controllable text generation with reinforced unlearning. *Advances in neural information processing systems*, 35:27591–27609, 2022.
- [21] Potsawee Manakul, Adian Liusie, and Mark JF Gales. Selfcheckgpt: Zero-resource black-box hallucination detection for generative large language models. arXiv preprint arXiv:2303.08896, 2023.
- [22] Sabrina J Mielke, Arthur Szlam, Emily Dinan, and Y-Lan Boureau. Reducing conversational agents' overconfidence through linguistic calibration. *Transactions of the Association for Computational Linguistics*, 10:857–872, 2022.
- [23] Michael Moor, Oishi Banerjee, Zahra Shakeri Hossein Abad, Harlan M Krumholz, Jure Leskovec, Eric J Topol, and Pranav Rajpurkar. Foundation models for generalist medical artificial intelligence. *Nature*, 616(7956):259–265, 2023.

- [24] Long Ouyang, Jeffrey Wu, Xu Jiang, Diogo Almeida, Carroll Wainwright, Pamela Mishkin, Chong Zhang, Sandhini Agarwal, Katarina Slama, Alex Ray, et al. Training language models to follow instructions with human feedback. *Advances in neural information processing systems*, 35:27730–27744, 2022.
- [25] Chantal Pellegrini, Ege Özsoy, Benjamin Busam, Benedikt Wiestler, Nassir Navab, and Matthias Keicher. Radialog: Large vision-language models for x-ray reporting and dialog-driven assistance. In *Medical Imaging with Deep Learning*, 2025.
- [26] Jingzhe Shi, Jialuo Li, Qinwei Ma, Zaiwen Yang, Huan Ma, and Lei Li. Chops: Chat with customer profile systems for customer service with llms, 2024. URL https://arxiv.org/abs/2404.01343.
- [27] Elias Stengel-Eskin, Peter Hase, and Mohit Bansal. Lacie: Listener-aware finetuning for confidence calibration in large language models. *arXiv preprint arXiv:2405.21028*, 2024.
- [28] Jingyun Sun, Chengxiao Dai, Zhongze Luo, Yangbo Chang, and Yang Li. Lawluo: A multiagent collaborative framework for multi-round chinese legal consultation, 2024. URL https://arxiv.org/abs/2407.16252.
- [29] Alon Talmor, Jonathan Herzig, Nicholas Lourie, and Jonathan Berant. CommonsenseQA: A question answering challenge targeting commonsense knowledge. In *Proceedings of the 2019 Conference of the North American Chapter of the Association for Computational Linguistics: Human Language Technologies, Volume 1 (Long and Short Papers)*, pages 4149–4158, Minneapolis, Minnesota, June 2019. Association for Computational Linguistics. doi: 10.18653/v1/N19-1421. URL https://aclanthology.org/N19-1421.
- [30] Shuchang Tao, Liuyi Yao, Hanxing Ding, Yuexiang Xie, Qi Cao, Fei Sun, Jinyang Gao, Huawei Shen, and Bolin Ding. When to trust llms: Aligning confidence with response quality. *arXiv* preprint arXiv:2404.17287, 2024.
- [31] Katherine Tian, Eric Mitchell, Allan Zhou, Archit Sharma, Rafael Rafailov, Huaxiu Yao, Chelsea Finn, and Christopher D Manning. Just ask for calibration: Strategies for eliciting calibrated confidence scores from language models fine-tuned with human feedback. *arXiv* preprint arXiv:2305.14975, 2023.
- [32] Hugo Touvron, Thibaut Lavril, Gautier Izacard, Xavier Martinet, Marie-Anne Lachaux, Timothée Lacroix, Baptiste Rozière, Naman Goyal, Eric Hambro, Faisal Azhar, et al. Llama: Open and efficient foundation language models. *arXiv*, 2023.
- [33] Tao Tu, Shekoofeh Azizi, Danny Driess, Mike Schaekermann, Mohamed Amin, Pi-Chuan Chang, Andrew Carroll, Charles Lau, Ryutaro Tanno, Ira Ktena, et al. Towards generalist biomedical ai. *NEJM AI*, 1(3):AIoa2300138, 2024.
- [34] Cheng Wang. Calibration in deep learning: A survey of the state-of-the-art, 2024. URL https://arxiv.org/abs/2308.01222.
- [35] Xuezhi Wang, Jason Wei, Dale Schuurmans, Quoc Le, Ed Chi, Sharan Narang, Aakanksha Chowdhery, and Denny Zhou. Self-consistency improves chain of thought reasoning in language models. *arXiv preprint arXiv:2203.11171*, 2022.
- [36] Jason Wei, Xuezhi Wang, Dale Schuurmans, Maarten Bosma, Fei Xia, Ed Chi, Quoc V Le, Denny Zhou, et al. Chain-of-thought prompting elicits reasoning in large language models. *Advances in neural information processing systems*, 35:24824–24837, 2022.
- [37] Miao Xiong, Zhiyuan Hu, Xinyang Lu, Yifei Li, Jie Fu, Junxian He, and Bryan Hooi. Can llms express their uncertainty? an empirical evaluation of confidence elicitation in llms, 2024. URL https://arxiv.org/abs/2306.13063.
- [38] Hongshen Xu, Zichen Zhu, Situo Zhang, Da Ma, Shuai Fan, Lu Chen, and Kai Yu. Rejection improves reliability: Training llms to refuse unknown questions using rl from knowledge feedback, 2024. URL https://arxiv.org/abs/2403.18349.

- [39] Tianyi Zhang, Varsha Kishore, Felix Wu, Kilian Q Weinberger, and Yoav Artzi. Bertscore: Evaluating text generation with bert. *arXiv preprint arXiv:1904.09675*, 2019.
- [40] Wei Zhou, Xiangyu Peng, and Mark Riedl. Dialogue shaping: Empowering agents through npc interaction. *arXiv preprint arXiv:2307.15833*, 2023.

# A Prompts

For all the question-answering settings, the model is directly prompted to answer a question without a preceding example or context. For our method the model was prompted to answer the question and additionally provide a verbalized confidence. For the other baselines that do not need a verbalized confidence but infer it indirectly, the model is prompted to only give the correct answer. The specifics for multiple-choice are slightly changed but hold mostly the same meaning. The exact prompts for each method can be seen in Table 6 for open questions and Table 7 for multiple-choice questions. The prompts for each Multi-Answer method can be seen in Table 8. We decided not to give the model a role like "expert" in the system prompt but keep it neutral, as we observed that the role we give the model affects the verbalized confidence.

Table 6: The prompts for each method for the open short form questions.

	o: The prompts for each method for the open short form questions.
Method	Prompt
Verbalize/ Rewarding Doubt	You will get questions. Answer with the correct answer. Additionally provide a confidence between 0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, of how sure you are the answer is correct. A value close to 0 means you think there is a high probability that the answer is wrong. The closer the value is to 10, the higher you think is the probability that the answer is correct. The output should have the format 'Answer: <answer>, Confidence: <confidence>' and nothing else.</confidence></answer>
СоТ	You will get questions. Read the question, analyze step by step, provide your reasoning steps, answer and your confidence in the response. The confidence should be between 0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10. A value close to 0 means you think there is a high probability that you could be wrong. The closer the value is to 5, the lower you think is the chance that you could be wrong. The output should have the format 'Thoughts: <reasoning steps="">, Answer: <just and="" answer="" else="" nothing="" the="">, Confidence: <confidence>'.</confidence></just></reasoning>
Top-K	You will get test questions with possible options. Read the question and enumerate your 5 best guesses and the probability for each that is correct (0% to 100%). Just answer with the options and nothing else. The answer should have the format 1. <answer> <pre> <pre> <pre> <pre></pre></pre></pre></pre></answer>
Sequence Probability/ Trained Probe	You will get questions. Answer with the correct answer only and nothing else.
Surrogate To- ken	(user) You will get questions. Answer with the correct answer only and nothing else. (model) <response> (user) Is the proposed answer: True False \n The proposed answer is:</response>
Self- Consistency	You will get questions. Read the question, analyze step by step, provide your reasoning steps and answer. The output should have the format 'Thoughts: <reasoning steps="">, Answer: <just and="" answer="" else="" nothing="" the="">'</just></reasoning>

Table 7: The prompts for each method for multiple choice questions.

Method	Prompt
Verbalize/ Re-	You will get test questions with possible options. Answer with the correct
warding Doubt	option. Additionally provide a confidence between 0, 1, 2, 3, 4, 5, 6, 7, 8,
	9, 10, of how sure you are the answer is correct. A value close to 0 means
	you think there is a high probability that the answer is wrong. The closer
	the value is to 10, the higher you think is the probability that the answer
	is correct. The output should have the format 'Answer: <answer_index>,</answer_index>
	Confidence: <confidence>' and nothing else.</confidence>
CoT	You will get test questions with possible options. Read the question, analyze
	step by step, provide your reasoningsteps, answer and your confidence in
	the response. The confidence should be between 0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10.
	A value close to 0 means you think there is a high probability that you could
	be wrong. The closer the value is to 5, the lower you think is the chance
	that you could be wrong. The output should have the format 'Thoughts:
	<pre><reasoning steps="">, Answer: <answer_index>, Confidence: <confidence>'</confidence></answer_index></reasoning></pre>
	and nothing else.
Sequence Prob-	You will get test questions with possible options. Answer with the correct
ability/ Trained	option index only and nothing else.
Probe	
Surrogate To-	(user) You will get test questions with possible options. Answer with the
ken	correct option index only and nothing else. (model) <response> (user) Is</response>
	the proposed answer: True False \n The proposed answer is:
Self-	You will get test questions with possible options. Read the question, analyze
Consistency	step by step, provide your reasoningsteps and the correct option index.
	The output should have the format 'Thoughts: <reasoning steps="">, Answer:</reasoning>
	<answer_index>' and nothing else.</answer_index>

Table 8: The prompts for each method for multiple fact questions.

Method	Prompt
	<u> </u>
Verbalize/ Re-	Instructions: 1. You will get a question with multiple possible answers. 2.
warding Doubt	Enumerate all possible answers you know. After each individual answer state
	your confidence in this answer. The format should be 'Answer: <answer>,</answer>
	Confidence: <confidence>\n' for each individual answer. 3. The confidence</confidence>
	should be an integer number between 0 and 10. 0 means you know for
	certain the answer is wrong. 10 means you know for certain the answer is
	correct. 4. Do not say anything else. Do not write multiple answers in one
	answer block. 5. When asked about dates, answer with the specific year.
Sequence Prob-	Instructions: 1. You will get a question with multiple possible answers. 2.
ability/ Trained	Enumerate all possible answers you know. Write each single answer in this
Probe	format "Answer: <answer>\n" . 3. Do not say anything else. Do not write</answer>
	multiple answers in one answer block or any other comments. 4. When
	asked about dates, answer with the specific year. 5. Do not repeat answers.

# **B** Proof

In the following, we prove Proposition 1 with the reward function

$$R(a,\hat{p},j) = \begin{cases} log(\hat{p}), & \text{if } j(a) = 1 \text{ (correct)} \\ log(1-\hat{p}), & \text{if } j(a) = 0 \text{ (incorrect)} \end{cases}$$

*Proof.* The proof is analogous to the proof that the logarithmic scoring function is a proper scoring function.

Let  $f(\hat{p}) = \mathbb{E}[R(a, \hat{p}, j)]$  be the expected reward for all values of  $\hat{p}$  and  $p^*$ :

$$f(\hat{p}) = p^* \log(\hat{p}) + (1 - p^*) \log(1 - \hat{p}).$$

Taking the first derivative w.r.t.  $\hat{p}$ :

$$f'(\hat{p}) = \frac{p^*}{\hat{p}} - \frac{1 - p^*}{1 - \hat{p}}$$

and setting

$$f'(\hat{p}) = 0 \implies p^*(1 - \hat{p}) = \hat{p}(1 - p^*) \implies \hat{p} = p^*$$

showing the only critical point in (0,1) of f' is at  $\hat{p}=p^*$ .

The second derivative:

$$f''(\hat{p}) = -\frac{p^*}{\hat{p}^2} - \frac{1-p^*}{(1-\hat{p})^2}$$

is strictly negative for  $\hat{p} \in (0,1)$ . Hence,  $f(\hat{p})$  is concave and has its global maximum at  $\hat{p} = p^*$ .  $\square$ 

As the logarithm of 0 is undefined, we add a small constant  $\epsilon$  in the reward function we use for training:

$$R(a, \hat{p}, j) = \begin{cases} log(\max(\hat{p}, \epsilon)), & \text{if } j(a) = 1 \text{ (correct)} \\ log(\min(1 - \hat{p}, 1 - \epsilon)), & \text{if } j(a) = 0 \text{ (incorrect)} \end{cases}$$

Through this clipping all confidence predictions between 0 and  $\epsilon$ , and 1 and  $1-\epsilon$ , respectively, are rewarded equally. This leads to the model not being able to differentiate between confidence estimations within these ranges. We argue this effect is minor for a sufficiently small  $\epsilon$  and can be disregarded in practice.

## C Societal Impact

This work introduces a reinforcement learning approach that enables Large Language Models (LLMs) to express calibrated confidence in their factual answers, advancing safe and trustworthy AI deployment. The method improves reliability and uncertainty awareness in LLMs, which is particularly valuable in high-stakes settings such as medicine, law, or customer support, where overconfident errors can have serious consequences. By optimizing a proper scoring rule during training, our method provides a theoretically sound and generalizable mechanism for aligning confidence with factual correctness—supporting human-AI collaboration and informed decision-making. However, expressing numerical confidence may lead users to overly trust AI systems, especially if the model is well-calibrated statistically but still wrong in important individual cases. This risk calls for careful deployment, appropriate user interfaces that contextualize model confidence, and safeguards against overreliance on AI-generated outputs.

#### D Asset Details

The TriviaQA dataset [16] is available at https://nlp.cs.washington.edu/triviaqa/under the Apache License 2.0. The CommonsenseQA dataset [29] (https://www.tau-nlp.sites.tau.ac.il/commonsenseqa) and the MedQA dataset [15] (https://github.com/jind11/MedQA?tab=readme-ov-file) are licensed under the MIT License. The QAMPARI dataset [2] is published at https://samsam3232.github.io/qampari/ and licensed under the CCO 1.0 Universal license. All these licenses allow broad use, distribution and modification. We use the Meta-Llama-3-8B-Instruct [10] language model, provided at https://huggingface.co/unsloth/llama-3-8b-Instruct-bnb-4bit under the Llama 3 Community License Agreement.