Assignment 10 Papoulis ex 8.23

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Outline

Question

The random variable x has a Poisson distribution with mean θ. Show that the ML estimate of θ equals $\bar{x}$.

Solution

The samples of x are the integers x_i and the joint density of the random variable's x_i equals

$$f(X,\theta) = e^{-n\theta}\prod\frac{\theta^{x_i}}{x_i!} = e^{-n\theta}\frac{\theta^{n\bar{x}}}{\prod x_i!} \tag{2.1}$$

$$\frac{\partial f(X,\theta)}{\partial\theta} = -ne^{-n\theta}\frac{\theta^{n\bar{x}}}{\prod x_i!} + n\bar{x}e^{-n\theta}\frac{\theta^{n\bar{x}-1}}{\prod x_i!} \tag{2.2}$$

$$\frac{\partial f(X,\theta)}{\partial\theta} = e^{-n\theta}\frac{\theta^{n\bar{x}}}{\prod x_i!}\left(-n + \frac{n\bar{x}}{\theta}\right) \tag{2.3}$$

In order to maximise $f(X,\theta)$, $\frac{\partial f(X,\theta)}{\partial\theta}$ must be 0

$$-n + \frac{n\bar{x}}{\theta} = 0 \tag{2.4}$$

$$\implies \hat{\theta} = \bar{x} \tag{2.5}$$