

Assignment 11 Papoulis ex 8.30

Gunjit Mittal (AI21BTECH11011)

June 11, 2022

Outline

1 Question

2 Solution

Question

In a production process, the number of defective units per hour is a poisson distributed random variable x with parameter $\lambda = 5$. A new process is introduced, and it is observed that the hourly defectives in a 22-hour period are

$$x_i = 3 \ 0 \ 5 \ 4 \ 2 \ 6 \ 4 \ 1 \ 5 \ 3 \ 7 \ 4 \ 0 \ 8 \ 3 \ 2 \ 4 \ 3 \ 6 \ 5 \ 6 \ 9$$

Test the hypothesis $\lambda = 5$ against $\lambda < 5$ with $\alpha = 0.05$.

Solution

Hypothesis $H_0 \implies \lambda = \lambda_0 = 5$ and $H_1 \implies \lambda < 5$

Given $X = \text{Poisson}(5)$

Using approximation, X can be considered as a Normal $(5, \sqrt{5})$

Since we have to prove the hypothesis against the type $\theta < \theta_0$, We accept H_0 iff $q > z_\alpha$ and $z_\alpha = z_{0.05} = -1.64$

$$q = \frac{\bar{x} - \lambda_0}{s/\sqrt{n}} \quad (2.1)$$

Where $\bar{x} = \frac{90}{22} = 4.09$, s is variance and n is number of observations = 22

$$q = \frac{4.09 - 5}{\sqrt{5}/\sqrt{5}} = -0.91 \quad (2.2)$$

$\implies q > z_\alpha$ so the hypothesis $\lambda = 5$ is accepted.