Assignment 14 Papoulis example 15.15

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Outline

Question

Consider a type of one-dimensional random walk over $0, 1, 2, \cdots$, where the particle moves from i to i + 1 with probability p or moves back to the origin with probability q.

Solution

This gives

$$p_{ij} = \begin{cases} p & j = i+1 \\ q & j = 0 \\ 0 & \text{otherwise} \end{cases} \tag{2.1}$$

Thus at the nth trial the system is in state e_j only if the previous failure occurred at n - i, and the index i represents the number of uninterrupted successes up to the nth trial. More generally, we can let

$$p_{ij} = \begin{cases} p_i & j = i+1 \\ q_i & j = 0 \\ 0 & \text{otherwise} \end{cases} \tag{2.2}$$

where $p_i + q_i = 1$. In this case the probability that the time between two successive returns to zero equals k is given by the product $p_1 p_2 \cdots p_{k-1} q_k$.