Random Numners

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CONTENTS

1 Uniform Random Numbers

1.1 **Solution:** Download the following files and execute the C program.

```
wget https://github.com/gunjitmittal/
    Random_Numbers/blob/main/codes/
    coeffs.h
wget https://github.com/gunjitmittal/
    Random_Numbers/blob/main/codes/
    exrand.c
gcc exrand.c
./a.out
```

1.2 **Solution:** The following code plots Fig. 1.1

```
wget https://github.com/gunjitmittal/
    Random_Numbers/blob/main/codes/
    cdf_plot.py
python cdf_plot.py
```

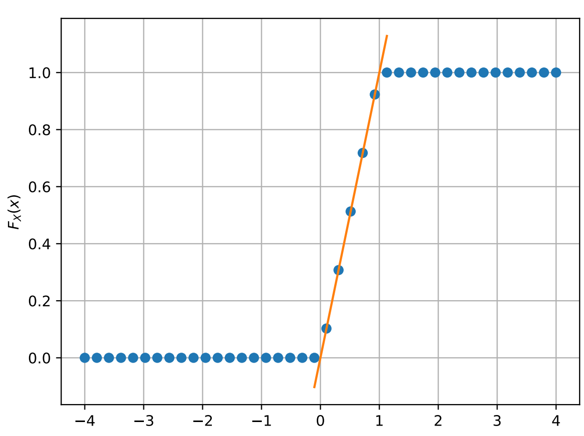

Fig. 1.1. The CDF of U

1.3 **Solution:** Since U is a uniform distribution

$$F_U(x) = x \tag{1.1}$$

1.4 **Solution:** Download the following files and execute the C program.

```
wget https://github.com/gunjitmittal/
    Random_Numbers/blob/main/codes/
    mean.c
gcc mean.c
./a.out
```

```
Mean: 0.500004
Variance: 0.083269
```

1.5 **Solution:**

$$E[U] = \int_{-\infty}^{\infty} x dF_U(x) \tag{1.2}$$
$$= \int_0^1 x dx \tag{1.3}$$
$$= \left[\frac{x^2}{2}\right]_0^1 \tag{1.4}$$
$$= \frac{1}{2} \tag{1.5}$$

$$E[U - E[U]]^2 = E\left[U^2\right] - E[U]^2 \tag{1.6}$$
$$E\left[U^2\right] = \int_{-\infty}^{\infty} x^2 dF_U(x) \tag{1.7}$$
$$= \int_0^1 x^2 dx \tag{1.8}$$
$$= \left[\frac{x^3}{3}\right]_0^1 \tag{1.9}$$
$$= \frac{1}{3} \tag{1.10}$$
$$E[U - E[U]]^2 = \frac{1}{3} - \left(\frac{1}{2}\right)^2 \tag{1.11}$$
$$= \frac{1}{3} - \frac{1}{4} = \frac{1}{12} \tag{1.12}$$

2 Central Limit Theorem

2.1 **Solution:** Download the following files and execute the C program.

```
wget https://github.com/gunjitmittal/
    Random_Numbers/blob/main/codes/
    coeffs.h
wget https://github.com/gunjitmittal/
    Random_Numbers/blob/main/codes/
    exrand2.c
gcc exrand2.c
./a.out
```

2.2 **Solution:** The following code plots Fig. 2.1

```
wget https://github.com/gunjitmittal/
    Random_Numbers/blob/main/codes/
    cdf_plot2.py
python cdf_plot2.py
```

- CDF reaches 0.5 as it reaches the mean(0)
- It reaches 1 as the graph reaches the end
- Differentiating the graph we obtain the graph of the PDF.

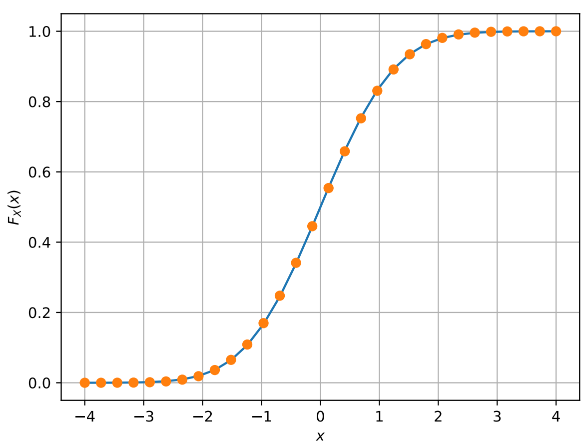

Fig. 2.1. The CDF of X

2.3 **Solution:** The following code plots Fig. 2.2

```
wget https://github.com/gunjitmittal/
    Random_Numbers/blob/main/codes/
    pdf_plot.py
python pdf_plot.py
```

- PDF is symmetric about x = mean(0)
- Graph is shpaed like a bell

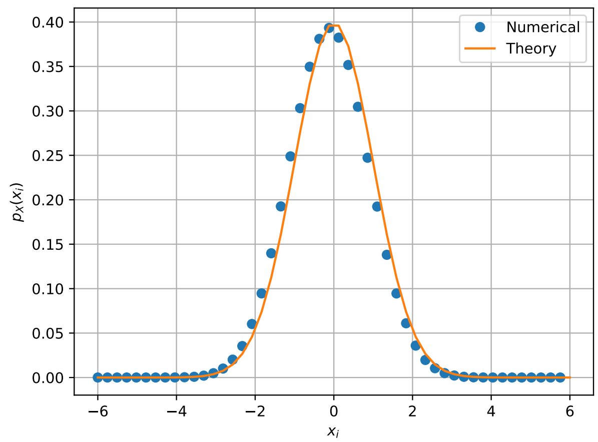

Fig. 2.2. The PDF of X

2.4 **Solution:** Download the following files and execute the C program.

```
wget https://github.com/gunjitmittal/
    Random_Numbers/blob/main/codes/
    mean2.c
gcc mean2.c
./a.out
```

```
Mean: 0.000326
Variance: 1.000467
```

2.5

$$F_X(x) = \int_{-\infty}^{x} \frac{1}{\sqrt{2\pi}} e^{-\frac{x^2}{2}} \tag{2.1}$$

$$= \frac{1}{2}\operatorname{erf}\left(\frac{x}{\sqrt{2}}\right) \tag{2.2}$$

$$E[X] = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} x e^{-\frac{x^2}{2}} dx \tag{2.3}$$

$$\text{Taking } \frac{x^2}{2} = t \implies x dx = dt \tag{2.4}$$

$$E[X] = \frac{1}{\sqrt{2\pi}} \int_{\infty}^{\infty} e^{-t} dt = 0 \tag{2.5}$$

$$E[X^2] = \int_{-\infty}^{\infty} \frac{1}{\sqrt{2\pi}} x^2 e^{-\frac{x^2}{2}} dx \tag{2.6}$$

$$= \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} x (x e^{-\frac{x^2}{2}}) dx \tag{2.7}$$

$$= \frac{1}{\sqrt{2\pi}} \left[-x e^{-\frac{-x^2}{2}} + \int_{-\infty}^{\infty} e^{-\frac{x^2}{2}} dx \right]_{-\infty}^{\infty} \tag{2.8}$$

$$= 1 \tag{2.9}$$

$$\text{Variance} = E[X^2] - E[X]^2 = 1 - 0 = 1 \tag{2.10}$$

3 From Uniform to Other

3.1 **Solution:** The following code plots Fig. 3.1

```
wget https://github.com/gunjitmittal/
    Random_Numbers/blob/main/codes/
    cdf_plot3.py
python cdf_plot3.py
```

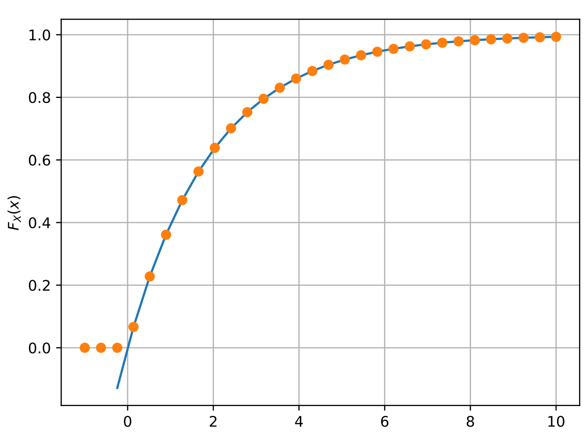

Fig. 3.1. The CDF of V

3.2

$$F_V(x) = P(V \le x) \tag{3.1}$$

$$= P(-2\ln(1-U) \le x) \tag{3.2}$$

$$= P(U \le 1 - e^{\frac{-x}{2}}) \tag{3.3}$$

$$P(U < x) = \int_0^x dx = x \tag{3.4}$$

$$\therefore P(U \le 1 - e^{\frac{-x}{2}}) = 1 - e^{\frac{-x}{2}} \tag{3.5}$$

$$\implies F_V(x) = 1 - e^{\frac{-x}{2}} \tag{3.6}$$