Random Numners

Gunjit Mittal (AI21BTECH11011)

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1 UNIFORM RANDOM NUMBERS

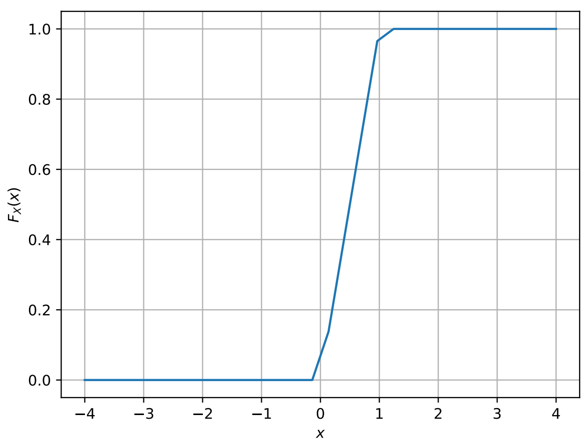

Fig. 1.1. The CDF of U

1.1

1.2

1.3 **Solution:** Since U is a uniform distribution

$$F_U(x) = x \tag{1.1}$$

1.4

1.5 Verify your result theoretically given that

$$E\left[U^k\right] = \int_{-\infty}^{\infty} x^k dF_U(x) \tag{1.2}$$

$$E\left[U\right] = \int_{-\infty}^{\infty} x dF_U(x) \tag{1.3}$$

$$= \int_0^1 x dx \tag{1.4}$$

$$= \left[\frac{x^2}{2}\right]_0^1 \tag{1.5}$$

$$= \frac{1}{2} \tag{1.6}$$

$$E\left[U - E\left[U\right]\right]^2 = E\left[U^2\right] - E\left[U\right]^2 \tag{1.7}$$

$$E\left[U^2\right] = \int_{-\infty}^{\infty} x^2 dF_U(x) \tag{1.8}$$

$$= \int_0^1 x^2 dx \tag{1.9}$$

$$= \left[\frac{x^3}{3}\right]_0^1 \tag{1.10}$$

$$= \frac{1}{3} \tag{1.11}$$

$$E\left[U - E\left[U\right]\right]^2 = \frac{1}{3} - \left(\frac{1}{2}\right)^2 \tag{1.12}$$

$$= \frac{1}{3} - \frac{1}{4} = \frac{1}{12} \tag{1.13}$$

2 CENTRAL LIMIT THEOREM

2.1

2.2

2.3 **Solution:**

- PDF is symmetric about x=0
- Graph is bell shaped
- Mean of graph is situated at the apex point of the bell.

2.4

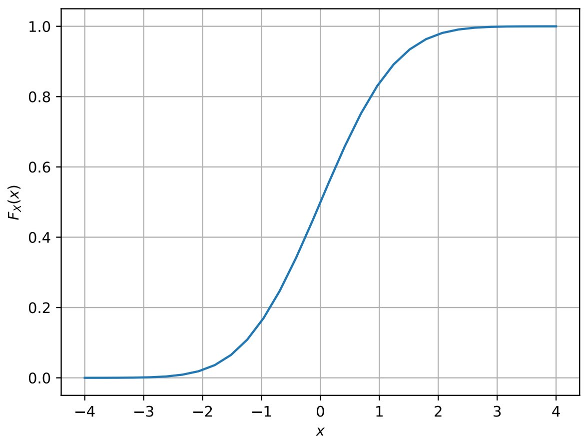

Fig. 2.1. The CDF of X

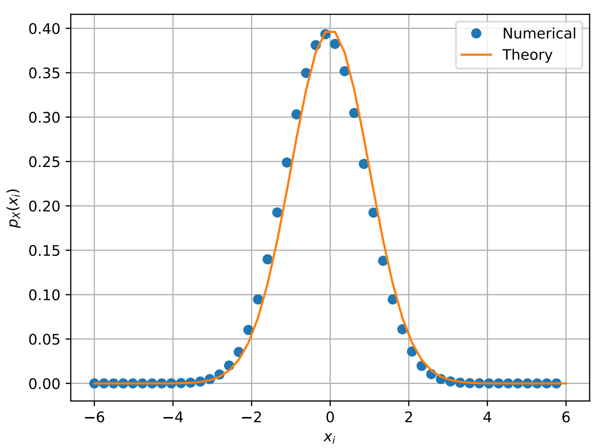

Fig. 2.2. The PDF of X

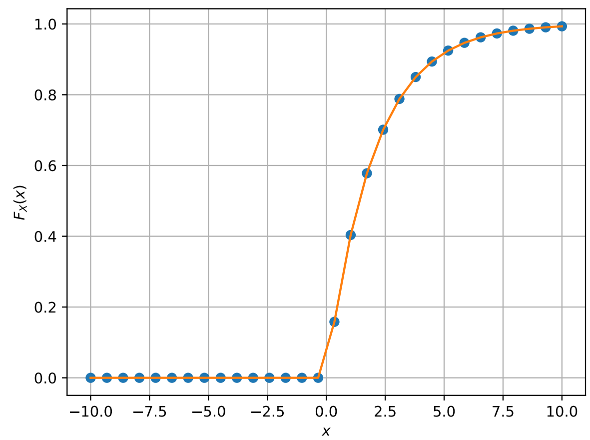

Fig. 3.1. The CDF of V

2.5

$$F_X(x) = \int_{-\infty}^{x} \frac{1}{\sqrt{2\pi}} e^{-\frac{x^2}{2}} \tag{2.1}$$

$$= \frac{1}{2} erf(\frac{x}{\sqrt{2}}) \tag{2.2}$$

$$E[X] = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} x e^{-\frac{x^2}{2}} dx \tag{2.3}$$

$$\text{Taking } \frac{x^2}{2} = t \implies x dx = dt \tag{2.4}$$

$$E[X] = \frac{1}{\sqrt{2\pi}} \int_{\infty}^{\infty} e^{-t} dt = 0 \tag{2.5}$$

$$E[X^2] = \int_{-\infty}^{\infty} \frac{1}{\sqrt{2\pi}} x^2 e^{-\frac{x^2}{2}} dx \tag{2.6}$$

$$= \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} x(x e^{-\frac{x^2}{2}}) dx \tag{2.7}$$

$$= \frac{1}{\sqrt{2\pi}} \left[-x e^{-\frac{-x^2}{2}} + \int_{-\infty}^{\infty} e^{-\frac{x^2}{2}} dx \right]_{-\infty}^{\infty} \tag{2.8}$$

$$= 1 \tag{2.9}$$

$$\text{Variance} = E[X^2] - E[X]^2 = 1 - 0 = 1 \tag{2.10}$$

3 From Uniform to Other

3.1

3.2

$$F_V(x) = P(V \leq x) \tag{3.1}$$
$$= P(-2\ln(1-U) \leq x) \tag{3.2}$$
$$= P(U \leq 1 - e^{\frac{-x}{2}}) \tag{3.3}$$
$$P(U < x) = \int_0^x dx = x \tag{3.4}$$
$$\therefore P(U \leq 1 - e^{\frac{-x}{2}}) = 1 - e^{\frac{-x}{2}} \tag{3.5}$$
$$\implies F_V(x) = 1 - e^{\frac{-x}{2}} \tag{3.6}$$