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# Random Numners

# Gunjit Mittal (AI21BTECH11011)

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#### **CONTENTS**

- 1 Uniform Random Numbers
- **2** Central Limit Theorem
- **3** From Uniform to Other
  - 1 Uniform Random Numbers
- 1.1 **Solution:** Download the following files and execute the C program.

wget https://github.com/gunjitmittal/
Random\_Numbers/blob/main/codes/
coeffs.h
wget https://github.com/gunjitmittal/
Random\_Numbers/blob/main/codes/
exrand.c
gcc exrand.c
./a.out

1.2 **Solution:** The following code plots Fig. 1.1

wget https://github.com/gunjitmittal/ Random\_Numbers/blob/main/codes/ cdf\_plot.py python cdf\_plot.py

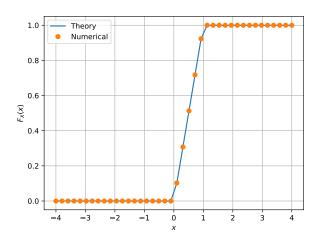


Fig. 1.1. The CDF of U

1.3 **Solution:** Since U is a uniform distribution

$$F_U(x) = x \tag{1.1}$$

1.4 **Solution:** Download the following files and execute the C program.

wget https://github.com/gunjitmittal/ Random\_Numbers/blob/main/codes/ mean.c gcc mean.c ./a.out

> Mean: 0.500004 Variance: 0.083269

1.5 Solution:

$$E[U] = \int_{-\infty}^{\infty} x dF_U(x)$$
 (1.2)

$$= \int_0^1 x dx \tag{1.3}$$

$$= \left[\frac{x^2}{2}\right]_0^1 \tag{1.4}$$

$$=\frac{1}{2}\tag{1.5}$$

$$E[U - E[U]]^2 = E[U^2] - E[U]^2$$
 (1.6)

$$E\left[U^{2}\right] = \int_{-\infty}^{\infty} x^{2} dF_{U}(x) \tag{1.7}$$

$$= \int_0^1 x^2 dx$$
 (1.8)

$$= \left[\frac{x^3}{3}\right]_0^1 \tag{1.9}$$

$$=\frac{1}{3}$$
 (1.10)

$$E[U - E[U]]^2 = \frac{1}{3} - \left(\frac{1}{2}\right)^2$$
 (1.11)

$$=\frac{1}{3} - \frac{1}{4} = \frac{1}{12} \tag{1.12}$$

### 2 CENTRAL LIMIT THEOREM

2.1 **Solution:** Download the following files and execute the C program.

```
wget https://github.com/gunjitmittal/
Random_Numbers/blob/main/codes/
coeffs.h
wget https://github.com/gunjitmittal/
Random_Numbers/blob/main/codes/
exrand2.c
gcc exrand2.c
./a.out
```

2.2 **Solution:** The following code plots Fig. 2.1

```
wget https://github.com/gunjitmittal/
Random_Numbers/blob/main/codes/
cdf_plot2.py
python cdf_plot2.py
```

- CDF reaches 0.5 as it reaches the mean(0)
- It reaches 1 as the graph reaches the end
- Differentiating the graph we obtain the graph of the PDF.

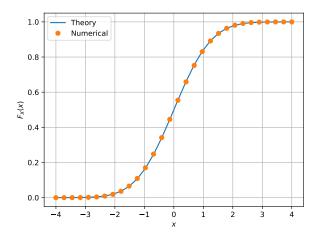


Fig. 2.1. The CDF of X

2.3 **Solution:** The following code plots Fig. 2.2

```
wget https://github.com/gunjitmittal/
Random_Numbers/blob/main/codes/
pdf_plot.py
python pdf_plot.py
```

• PDF is symmetric about x = mean(0)

• Graph is shpaed like a bell

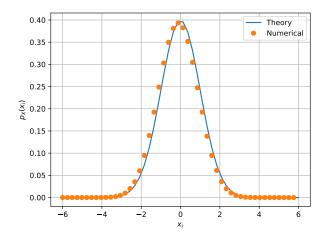


Fig. 2.2. The PDF of X

2.4 **Solution:** Download the following files and execute the C program.

wget https://github.com/gunjitmittal/ Random\_Numbers/blob/main/codes/ mean2.c gcc mean2.c ./a.out

> Mean: 0.000326 Variance: 1.000467

2.5

$$F_X(x) = \int_{-\infty}^x \frac{1}{\sqrt{2\pi}} e^{-\frac{x^2}{2}}$$
 (2.1)

$$= \frac{1}{2}\operatorname{erf}(\frac{x}{\sqrt{2}})\tag{2.2}$$

3.2

$$E[X] = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} x e^{-\frac{x^2}{2}} dx$$
 (2.3)

Taking 
$$\frac{x^2}{2} = t \implies xdx = dt$$
 (2.4)

$$E[X] = \frac{1}{\sqrt{2\pi}} \int_{\infty}^{\infty} e^{-t} dt = 0$$
 (2.5)

$$E[X^2] = \int_{-\infty}^{\infty} \frac{1}{\sqrt{2\pi}} x^2 e^{-\frac{x^2}{2}} dx$$
 (2.6)

$$= \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} x(xe^{-\frac{x^2}{2}}) dx$$
 (2.7)

$$= \frac{1}{\sqrt{2\pi}} \left[ -xe^{-\frac{-x^2}{2}} + \int_{-\infty}^{\infty} e^{-\frac{x^2}{2}} dx \right]_{-\infty}^{\infty}$$
(2.8)

$$=1 (2.9)$$

Variance = 
$$E[X^2] - E[X]^2 = 1 - 0 = 1$$
 (2.10)

#### 3 From Uniform to Other

## 3.1 **Solution:** The following code plots Fig. 3.1

wget https://github.com/gunjitmittal/ Random\_Numbers/blob/main/codes/ cdf\_plot3.py python cdf\_plot3.py

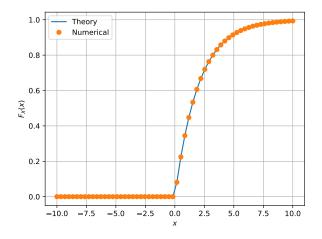


Fig. 3.1. The CDF of V

$$F_V(x) = P(V \le x) \tag{3.1}$$

$$= P(-2\ln(1-U) \le x)$$
 (3.2)

$$= P(U \le 1 - e^{\frac{-x}{2}}) \tag{3.3}$$

$$P(U < x) = \int_0^x dx = x$$
 (3.4)

$$\therefore P(U \le 1 - e^{\frac{-x}{2}}) = 1 - e^{\frac{-x}{2}} \tag{3.5}$$

$$\implies F_V(x) = 1 - e^{\frac{-x}{2}}$$
 (3.6)