

Random Numners

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1 UNIFORM RANDOM NUMBERS

1.1 **Solution:** Download the following files and execute the C program.

```
wget https://github.com/gunjitmittal/
  Random_Numbers/blob/main/codes/
  coeffs.h
wget https://github.com/gunjitmittal/
  Random_Numbers/blob/main/codes/
  exrand.c
gcc exrand.c
./a.out
```

1.2 **Solution:** The following code plots Fig. 1.1

```
wget https://github.com/gunjitmittal/
  Random_Numbers/blob/main/codes/
  cdf_plot.py
python cdf_plot.py
```

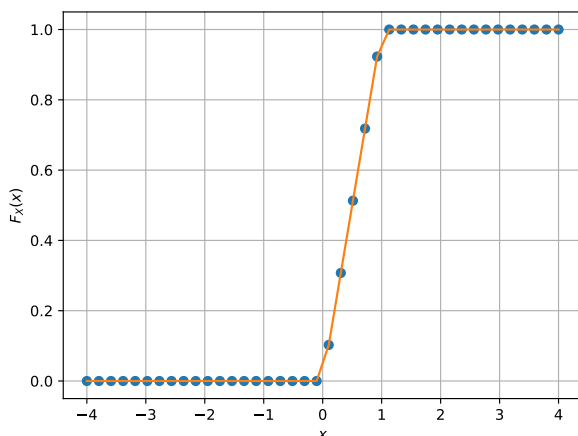


Fig. 1.1. The CDF of U

1.3 **Solution:** Since U is a uniform distribution

$$F_U(x) = x \quad (1.1)$$

1.4 **Solution:** Download the following files and execute the C program.

```
wget https://github.com/gunjitmittal/
  Random_Numbers/blob/main/codes/
  mean.c
gcc mean.c
./a.out
```

Mean: 0.500004
Variance: 0.083269

1.5 **Solution:**

$$E[U] = \int_{-\infty}^{\infty} x dF_U(x) \quad (1.2)$$

$$= \int_0^1 x dx \quad (1.3)$$

$$= \left[\frac{x^2}{2} \right]_0^1 \quad (1.4)$$

$$= \frac{1}{2} \quad (1.5)$$

$$E[U - E[U]]^2 = E[U^2] - E[U]^2 \quad (1.6)$$

$$E[U^2] = \int_{-\infty}^{\infty} x^2 dF_U(x) \quad (1.7)$$

$$= \int_0^1 x^2 dx \quad (1.8)$$

$$= \left[\frac{x^3}{3} \right]_0^1 \quad (1.9)$$

$$= \frac{1}{3} \quad (1.10)$$

$$E[U - E[U]]^2 = \frac{1}{3} - \left(\frac{1}{2} \right)^2 \quad (1.11)$$

$$= \frac{1}{3} - \frac{1}{4} = \frac{1}{12} \quad (1.12)$$

2 CENTRAL LIMIT THEOREM

2.1 Solution: Download the following files and execute the C program.

```
wget https://github.com/gunjitmittal/
  Random_Numbers/blob/main/codes/
  coeffs.h
wget https://github.com/gunjitmittal/
  Random_Numbers/blob/main/codes/
  exrand2.c
gcc exrand2.c
./a.out
```

2.2 Solution: The following code plots Fig. 2.1

```
wget https://github.com/gunjitmittal/
  Random_Numbers/blob/main/codes/
  cdf_plot2.py
python cdf_plot2.py
```

- CDF reaches 0.5 as it reaches the mean(0)
- It reaches 1 as the graph reaches the end
- Differentiating the graph we obtain the graph of the PDF.

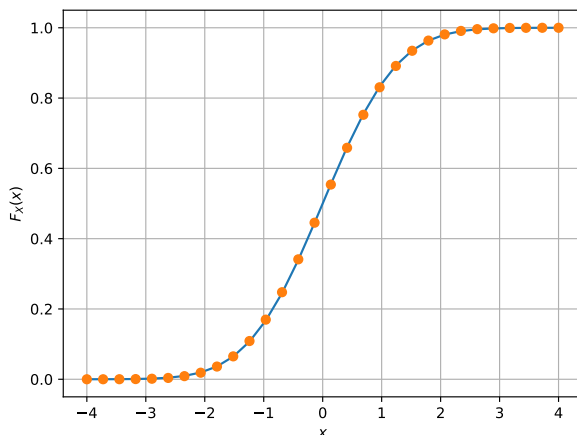


Fig. 2.1. The CDF of X

2.3 Solution: The following code plots Fig. 2.2

```
wget https://github.com/gunjitmittal/
  Random_Numbers/blob/main/codes/
  pdf_plot.py
python pdf_plot.py
```

- PDF is symmetric about $x = \text{mean}(0)$

- Graph is shaped like a bell

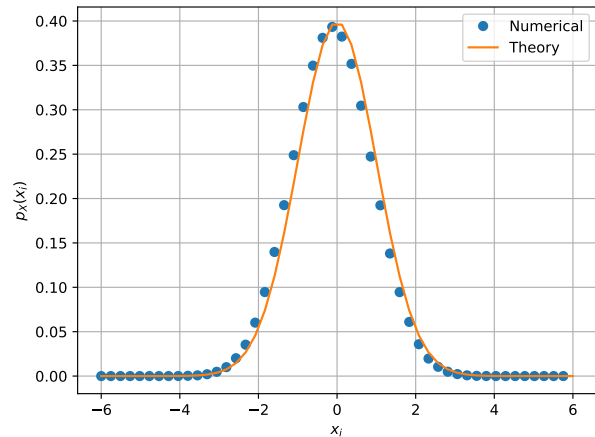


Fig. 2.2. The PDF of X

2.4 Solution: Download the following files and execute the C program.

```
wget https://github.com/gunjitmittal/
  Random_Numbers/blob/main/codes/
  mean2.c
gcc mean2.c
./a.out
```

```
Mean: 0.000326
Variance: 1.000467
```

2.5

$$F_X(x) = \int_{-\infty}^x \frac{1}{\sqrt{2\pi}} e^{-\frac{x^2}{2}} dx \quad (2.1)$$

$$= \frac{1}{2} \operatorname{erf}\left(\frac{x}{\sqrt{2}}\right) \quad (2.2)$$

$$E[X] = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} x e^{-\frac{x^2}{2}} dx \quad (2.3)$$

$$\text{Taking } \frac{x^2}{2} = t \implies x dx = dt \quad (2.4)$$

$$E[X] = \frac{1}{\sqrt{2\pi}} \int_{\infty}^{\infty} e^{-t} dt = 0 \quad (2.5)$$

$$E[X^2] = \int_{-\infty}^{\infty} \frac{1}{\sqrt{2\pi}} x^2 e^{-\frac{x^2}{2}} dx \quad (2.6)$$

$$= \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} x (x e^{-\frac{x^2}{2}}) dx \quad (2.7)$$

$$= \frac{1}{\sqrt{2\pi}} \left[-x e^{-\frac{x^2}{2}} + \int_{-\infty}^{\infty} e^{-\frac{x^2}{2}} dx \right]_{-\infty}^{\infty} \quad (2.8)$$

$$= 1 \quad (2.9)$$

$$\text{Variance} = E[X^2] - E[X]^2 = 1 - 0 = 1 \quad (2.10)$$

3.2

$$F_V(x) = P(V \leq x) \quad (3.1)$$

$$= P(-2 \ln(1 - U) \leq x) \quad (3.2)$$

$$= P(U \leq 1 - e^{-\frac{x}{2}}) \quad (3.3)$$

$$P(U < x) = \int_0^x dx = x \quad (3.4)$$

$$\therefore P(U \leq 1 - e^{-\frac{x}{2}}) = 1 - e^{-\frac{x}{2}} \quad (3.5)$$

$$\implies F_V(x) = 1 - e^{-\frac{x}{2}} \quad (3.6)$$

3 FROM UNIFORM TO OTHER

3.1 **Solution:** The following code plots Fig. 3.1

```
wget https://github.com/gunjitmittal/
Random_Numbers/blob/main/codes/
cdf_plot3.py
python cdf_plot3.py
```

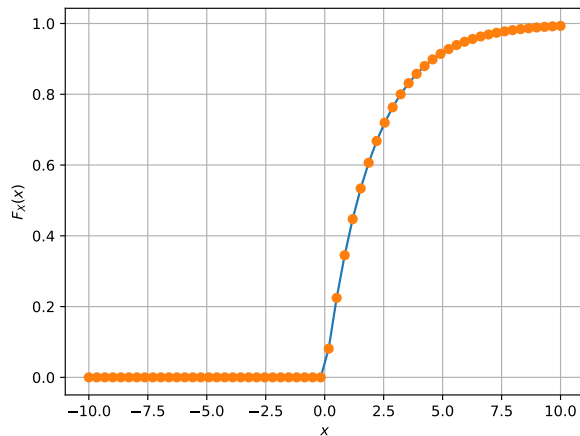


Fig. 3.1. The CDF of V