

CYBER 759 – Assignment 3

Gunnar Yonker

1.

Outcome: X

HHH:3

HHT:2

HTH:2

HTT:1

TTH:2

THT:1

TTH:1

TTT:0

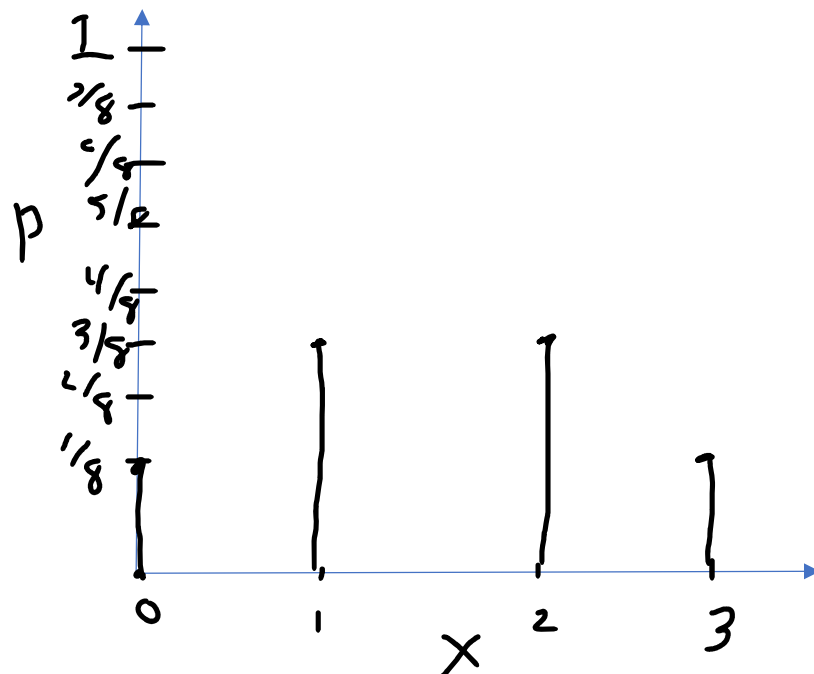
$P(X = 0)$ = Probability of getting 0 heads = Probability of TTT = $1/8$

$P(X = 1)$ = Probability of getting 1 head = Probability of HTT, THT, or TTH = $3/8$

$P(X = 2)$ = Probability of getting 2 heads = Probability of HHT, HTH, or THH = $3/8$

$P(X = 3)$ = Probability of getting 3 heads = Probability of HHH = $1/8$

X	0	1	2	3
P(X)	$1/8$	$3/8$	$3/8$	$1/8$



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2.

$P(H) = 0.70$ (Probability of getting a head)

$P(T) = 0.30$ (Probability of getting a tail)

$P(X = 0) = P(TTT) = 0.3 * 0.3 * 0.3 = 0.0027$

$P(X = 1)$ = Probability of getting 1 head: HTT, THT, TTH

$P(HTT) = .7 * .3 * .3 = 0.063$

$P(THT) = .3 * .7 * .3 = 0.063$

$P(TTH) = .3 * .3 * .7 = 0.063$

$P(X = 1) = 0.063 + 0.063 + 0.063 = 0.189$

$P(X = 2)$ = Probability of getting 2 heads: HHT, HTH, THH:

$P(HHT) = 0.7 * 0.7 * 0.3 = 0.147$

$P(HTH) = 0.7 * 0.3 * 0.7 = 0.147$

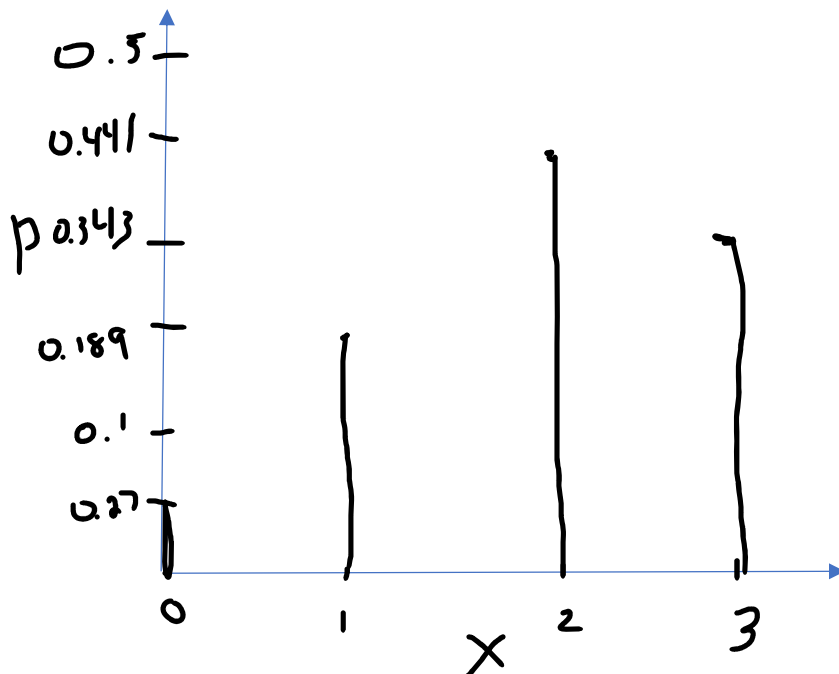
$P(THH) = 0.3 * 0.7 * 0.7 = 0.147$

$P(X = 2) = 0.147 + 0.147 + 0.147 = 0.441$

$P(X = 3)$ = Probability of getting 3 heads: HHH

$P(HHH) = 0.7 * 0.7 * 0.7 = 0.343$

X	0	1	2	3
P(X)	0.027	0.189	0.441	0.343



3. I got a little confused on this question because we cannot directly calculate the probability of an employee having exactly the salary of \$78,000 since the chance of that exact number is approximately zero, using the gaussian distribution formula we can calculate the height of the distribution at that salary though like this:

$$f(78,000) = (1/(10,000\sqrt{2\pi})) * e^{-(78,000-75,000)^2/2(10,000)^2}$$

$$f(78,000) = (1/(10,000\sqrt{2\pi})) * e^{-0.045}$$

$$f(78,000) = (0.00003989422)e^{-0.45}$$

$$f(78,000) = 0.00003813877$$

* I'm not sure if that is how you intended us to solve this question, by plugging in the standard deviation and mean into the gaussian distribution. Otherwise, my other thought was if it was to solve how many employees have a salary larger than \$78,000 using the gaussian distribution which would require the use of a z-score. If neither of these were how you were hoping that we would solve this problem, just let me know and I will make sure that I practice solving it the correct way. Thank you!

$$Z = (X - \mu) / \sigma$$

$$Z = (78,000 - 75,000) / 10,000$$

$$Z = 0.3 \text{ which is a Z-table value of approximately } 0.6179.$$

$$\text{Probability of } P(X > 78,000) = 1 - 0.6179$$

$$P(X > 78,000) = 0.3821 \text{ or } 38.21\%$$

So, the probability that an employee has a salary larger than \$78,000 is approximately 38.21%.

4.

$$P(X = k) = (\lambda^k e^{-\lambda}) / (k!)$$

$P(X = k)$ is the probability of hiring k employees, λ is the mean number of hires per week (5), e is the base of the natural log, and $k!$ is the factorial of k .

$$P(X > 4) = 1 - (P(X = 0) + P(X = 1) + P(X = 2) + P(X = 3) + P(X = 4))$$

$$P(X = 0) = (5^0 e^{-5}) / (0!) = (1e^{-5}) / 1$$

$$P(X = 1) = (5^1 e^{-5}) / (1!) = (5e^{-5}) / 1$$

$$P(X = 2) = (5^2 e^{-5}) / (2!) = (25e^{-5}) / 2$$

$$P(X = 3) = (5^3 e^{-5}) / (3!) = (125e^{-5}) / 6$$

$$P(X = 4) = (5^4 e^{-5}) / (4!) = (625e^{-5}) / 24$$

Summing them up:

$$P(X \leq 4) = e^{-5}(1 + 5 + 12.5 + 20.83 + 26.04)$$

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$$P(X > 4) = 1 - P(X \leq 4)$$

$$P(X > 4) = 1 - e^{-5}(65.38) = 1 - 0.4406$$

$$P(X > 4) \approx 0.5594$$

The probability that the company hires more than 4 employees in a given week is approximately 0.5594 or 55.94%.

5.

Calculate the probability of each employee group making more than \$75,000

a) Male employee making more than \$75,000:

$$P(\text{Salary} > \$75,000 \mid \text{Male}) = 0.30$$

$$P(\text{Male and Salary} > \$75,000) = 0.60 * 0.30 = 0.18$$

b) Female employee making more than \$75,000:

$$P(\text{Salary} > \$75,000 \mid \text{Female}) = 0.60$$

$$P(\text{Female and Salary} > \$75,000) = 0.30 * 0.60 = 0.18$$

c) Non-binary employee making more than \$75,000:

$$P(\text{Salary} > \$75,000 \mid \text{Non-binary}) = 0.10$$

$$P(\text{Non-binary and Salary} > \$75,000) = 0.10 * 0.10 = 0.01$$

Calculate overall probability of an employee making more than \$75,000

Male:

$$P(\text{Salary} > \$75,000) = 0.18 + 0.18 + 0.01 = 0.37$$

Bayes Theorem:

$$P(\text{Male} \mid \text{Salary} \leq \$75,000) = (P(\text{Salary} \leq \$75,000 \mid \text{Male}) * P(\text{Male})) / P(\text{Salary} \leq \$75,000)$$

Given:

$$P(\text{Salary} \leq \$75,000 \mid \text{Male}) = 1 - P(\text{Salary} > \$75,000 \mid \text{Male}) = 0.70$$

$$P(\text{Salary} \leq \$75,000) = 1 - P(\text{Salary} > \$75,000) = 0.63$$

$$P(\text{Male} \mid \text{Salary} \leq \$75,000) = (0.70 * 0.60) / 0.63 = 0.6667 \text{ or } 66.67\%$$

Female:

Bayes Theorem:

$$P(\text{Female} \mid \text{Salary} \leq \$75,000) = (P(\text{Salary} \leq \$75,000 \mid \text{Female}) * P(\text{Female})) / P(\text{Salary} \leq \$75,000)$$

$$\text{Given: } P(\text{Salary} \leq \$75,000 \mid \text{Female}) = 1 - P(\text{Salary} > \$75,000 \mid \text{Female}) = 0.40$$

$$P(\text{Female}) = 0.30 \text{ (given)}$$

Result:

$$P(\text{Female} \mid \text{Salary} \leq \$75,000) = (0.40 * 0.30) / 0.63 = 0.1905 \text{ or } 19.05\%$$

Non-Binary:

Bayes Theorem:

$$P(\text{Non-Binary} \mid \text{Salary} \leq \$75,000) = (P(\text{Salary} \leq \$75,000 \mid \text{Non-Binary}) * P(\text{Non-Binary})) / P(\text{Salary} \leq \$75,000)$$

Given:

$$P(\text{Salary} \leq \$75,000 \mid \text{Non-Binary}) = 1 - P(\text{Salary} > \$75,000 \mid \text{Non-Binary}) = 0.90$$

$$P(\text{Non-Binary}) = 0.10 \text{ (given)}$$

Result:

$$P(\text{Non-Binary} \mid \text{Salary} \leq \$75,000) = (0.90 * 0.10) / 0.63 = 0.1429 \text{ or } 14.29\%.$$

From these probabilities I can make an educated guess that the gender of this employee that is making \$30,000 identifies as a Male. The probability that the employee identifies as a male is approximately 66.67%, as a female at approximately 19.05%, and then as a Non-Binary at approximately 14.29%. This makes it most probable that the employee identifies as a Male.